

Private health expenditures and environmental quality

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Abstract

This paper presents a simple two-period overlapping generations model that contains environmental and health issues. It investigates an intergenerational conflict between old and young generations as regards two defensive expenditures offsetting the influence of a worsening environment represented here by health care and environmental investment. Workers support environmental maintenance while retirees prefer investing in health care. The authors have shown that an increasing support for private health expenditure leads to a higher level of capital accumulation and leads also to a higher level of environmental quality if the maintenance efforts are higher than consumption externalities.

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1 Introduction

Age-based matters have a direct impact on population preferences. The old individuals don't profit from long-term expenditure choices. They have a preference for investments which are more useful in the short term, while the young prefer investments giving better effects over a longer period. This is the case of the investments in health status, such as health-care expenditure and environmental maintenance. Both of them improve health situations: the valuable effect of health expenditure is evident and the positive role of environmental quality on health is known as well (EEA 2007).

The workers support however environmental expenditure while the retirees prefer health-care investment. Actually, old individuals appreciate private health-care expenditure instead of the environment investments due to the fact that they take usually more time to be totally effective although, they can last for a longer time. They do not enjoy future environmental improvements. The young generation prefers however environmental expenditure as they yield to results over a longer horizon owing to a longer remaining lifespan in which to enjoy enhanced environmental quality (see Balestra and Davide 2012). It is worth mentioning that we are not claiming that old people are not interested at all in environmental maintenance, but that they are less concerned than the young people.

Health is affected by environmental quality and health care expenditure. However, the weight of these two parameters might be age-based. However, the weight of these two parameters might be age-based. With the knowledge of this intergenerational conflict, we explore the question of how and whether a population of two groups: young and old; support two defensive expenditures represented here by health care and environmental maintenance and affecting capital accumulation and environmental quality.

We set up a simple overlapping generations model OLG built on John and Pecchenino (1994)'s influential work such that the individual chooses his own level of health investment and not publicly through voting. We have shown the effects of the competitive mechanism in terms of health expenditure, environmental quality, capital accumulation and consumption possibilities.

Our contribution is probably closest to the paper of Gutierrez (2008) who studies the impact of health investment on physical capital. She proclaims that the stock of pollution leads to a health cost only when individuals are elderly. This

cost influences saving and thus capital accumulation and. We differ from her paper since we are interested about environmental degradation that comes from consumption instead of pollution stock that comes from production. Also, we include the possibility of maintenance investment via using part of the young wealth to spend in improving the quality of the environment. Hence, we study the repercussion of the intergenerational conflict that may occur between old and young generations as regard two defensive expenditures mitigating the influence of a worsening environment by analyzing the effect of an increase of private health expenditure support on the capital accumulation and the environment.

This study goes along with diverse streams of the existing literature. In this respect, Chakraborty and Das (2005) postulate a positive relationship between the mortality risk and the private health investment and show that in the absence of annuities markets, health stocks can have persistent effects on income distribution. Chakraborty (2004) points out how development traps and persistent inequality may surge when survival probability is endogenous and depend negatively on health expenditure. Tubb (2011) supposes that agents are taxed and that taxation revenue can be spent on either environmental maintenance or on transfers to the old population. Aging enhances the proportion of elderly individuals and consequently enhances political pressure for the public planner to tilt the composition of public spending in favour of a transfer payment to the elderly. The author shows that since young population anticipates that higher longevity implies an increased return from such investment, ageing may simultaneously increase the young generation's demand for environmental investments.

This paper proceeds as follows. In Section 2 we introduce the theoretical model. Section 3 sketches the optimization problem. Section 4 characterizes the equilibrium. In Section 5 we describe the steady state. Section 6 concludes.

2 Theoretical Model

To formalize this model, consider a general equilibrium OLG closed and competitive economy populated by identical individuals. Each generation is alive for two periods.

In the working period, individuals earn wage w_t by supplying inelastically one unit of labor. Individuals divide the income among consumption c_t^1 , savings s_t for the retirement and payments for environmental maintenance m_t .¹ In the retirement period, individuals get the returns $(1+r_{t+1})$ on the savings and can be spent in consumption c_{t+1}^2 or in health expenditure z_{t+1} since environmental degradation affects the health of the individuals, who are forced to expend on health care when they are elderly. These expenditures include expenses for clinical services, annual health screening, necessary medication or the cost of buying nutrients, organic food and so on. Following Gutierrez (2008), we model these health costs z_{t+1} as withdrawals from savings, which means a decrease in consumption in retirement period.²

Individuals face a tension between maintenance investment and health care. The individuals' constraints over the two periods can therefore be summarized as follows:

$$w_t = c_t^1 + s_t + m_t \quad (1)$$

$$c_{t+1}^2 = [(1+r_{t+1})s_t - z_{t+1}] \quad (2)$$

with $c_t^1, c_{t+1}^2, s_t, m_t \geq 0$.

The capital investment enables to form a consumable savings in the second period and also leads to an increase in the technology of future generations through external increasing returns (due to externalities).

The investment in the public through environmental protection expenditure or environmental maintenance improves the environmental quality while agents' consumption damages it for future generations.

Following John and Pecchenino (1994), the motion of the environmental quality is as follows:

$$E_{t+1} = (1-b)E_t - \beta(c_t^1 + c_t^2) + \delta m_t \quad (3)$$

¹ The superscript '1' refers to young individual. The subscript 't' represents period t.

² The superscript '2' refers to an older individual. The subscript 't + 1' represents period t + 1.

where E_t is the environmental quality in period t , E_{t+1} is the environmental quality in period $t + 1$, $b \in (0,1)$ represents the natural rate of deterioration of the environment, $\beta > 0$ stands for the degradation of the environment and $\delta > 0$ is the environmental improvement due to the actions of the young at t . The environment is supposed to be a public good which is affected by two economic actions: consumption and maintenance expenditure. On the one hand, the environmental quality is negatively affected by the consumption activities. On the other hand the environment is positively affected by the payment of environmental maintenance m_t .

The individual's utility U is derived from consumption and environmental quality in first and second periods, where $U' > 0$ and $U'' < 0$. For simplification, preferences of each individual are defined by the log-linear lifetime utility U :

$$U = \ln c_t^1 + p \ln c_{t+1}^2 + \ln E_t + p \ln E_{t+1} \quad (4)$$

Where p defines the psychological discount factor, and it is the same for all individuals. Higher values of p signify a larger preference for current compared to future consumption.

At each period t , the firms produce homogenous good in competitive markets using K the capital, L the labor according to a homogeneous of degree one production function. The production is described by an aggregate production function

$$Y_t = F(K_t, L_t) \quad (5)$$

Supposing that equation (5) fulfill constant returns to scale, the production function in intensive form becomes

$$y_t = f(k_t) \quad (6)$$

Where Y_t is the output in period t , K_t the capital stock, L_t the labor supply, $k_t = K_t / L_t$ the capital-labor ratio, and y_t the output-labor ratio.

3 Optimization problem

The individual takes as given the wage w_t , the return on the savings r_{t+1} , the stock of environment at the beginning of first period E_t and the environmental parameters b, β, δ . Therefore, the competitive life-cycle choice problem of the individual is to choose $c_t^1, c_{t+1}^2, m_t, z_{t+1}$, and s_t according to the maximization program. Hence, the individual maximizes the utility function subject to the evolution of environmental quality (3) and the constraint (1)–(2). This means the individuals' optimization problem gives the following first order conditions (FOCs):

$$\frac{1}{c_t^1} = p(\beta + \delta) \frac{1}{E_{t+1}} \quad (7)$$

$$p(1 + r_{t+1}) \frac{1}{c_{t+1}^2} = \delta \frac{1}{E_{t+1}} \quad (8)$$

Proof: See Appendix A

Equation (7) points out that young individuals choose consumption to equate the marginal rate of substitution between consumption when young and environmental quality in retirement period to the marginal rate of transformation $\beta + \delta$. At the intragenerationally efficient allocation, a decline in utility caused by a decrease in consumption by the young individuals is equal to a raise in utility thanks to the sum of the additional utility from declining consumption externalities β and from raising the environmental maintenance δ .

Equation (8) indicates that individuals choose savings to equate the marginal rate of substitution between consumption in retirement period and environmental quality in retirement period to the marginal rate of transformation $\frac{p\delta}{(1+r_{t+1})}$. At the maximum of the utility, a lower utility due to falling consumption retirement period $(1+r_{t+1})$ is equal to a higher utility due to a rise of environmental maintenance effort $p\delta$.

The firm produces at time t profits:

$$\pi_t = F(K_t, L_t) - w_t L_t - (r_t + \sigma) K_t \quad (9)$$

where L_t indicates the labour input paid at a wage w_t , K_t aggregate physical capital and r_t denotes the return factor on savings from time $t-1$ to time t .

Supposing perfect competition in the factor markets, the profit maximization problem yields the following factor prices which are equal to their marginal productivities.

$$w_t = f(k_t) - k_t f'(k_t) \quad (10)$$

$$r_t = f'(k_t) - \sigma \quad (11)$$

where $\sigma \in (0,1)$ is the depreciation rate of capital and $f'(k_t) > \sigma$.

The first order conditions (FOCs) of the firm's maximization problem are (10) and (11).

4 Characterization of the Equilibrium

A competitive equilibrium for the economy under analysis is a sequence, $\{c_t^1, c_{t+1}^2, m_t, w_t, r_t, s_t, z_{t+1}, k_t, E_t\}_{t=0}^{\infty}$ such that, given the initial conditions of the state parameters k_0 and E_0 : firms maximize profits; old consumers maximize their utility function; and markets clear.

The first-order conditions of the utility maximization are (7)–(8) and the first-order conditions of profit maximization are (10) and (11). A market clearing condition for capital is $K_{t+1} = L_t s_t$ which point out that the total savings by young individuals in population $L_t s_t$ must equal their own addition to the future stock of capita K_{t+1} . Since there is no population growth, this condition is rewritten as

$$k_{t+1} = s_t \quad (12)$$

By plugging equations (7)–(10) and (11) into (1), it gives

$$m_t = f(k_t) - k_t f'(k_t) - \frac{1}{p(\beta + \delta)} E_{t+1} - k_{t+1} \quad (13)$$

Plugging equations (11)–(12) into (2) gives

$$c_{t+1}^2 = [(1 + f'(k_{t+1}) - \sigma)k_{t+1} - z_{t+1}] \quad (14)$$

Proof: See Appendix A

For the sake of simplicity, we standardize the population of generation t as one. Therefore, by plugging equations (7)–(13) and (14) lagged once into (3) yields

$$E_{t+1} = (1-b)E_t - \beta \left[\frac{1}{p(\beta + \delta)} E_t + [(1 + f'(k_t) - \sigma)k_t - z_t] \right] + \delta \left[f(k_t) - k_t f'(k_t) - \frac{1}{p(\beta + \delta)} E_t - k_{t+1} \right] \quad (15)$$

Plugging as well equations (11) and (14) into (8) gives

$$E_{t+1} = \frac{1}{p} \left[\delta k_{t+1} - \frac{\delta z_{t+1}}{(1 + f'(k_{t+1}) - \sigma)} \right] \quad (16)$$

Equations (15) and (16) represent the law of motion for the environment.

Rewrite equations (15) as

$$[(1+p)/p]E_{t+1} - (1-b)E_t + \beta p[(1 + f'(k_t) - \sigma)k_t - z_t] - \delta[f(k_t) - k_t f'(k_t) - k_{t+1}] = 0 \quad (17)$$

Equation (16) is defining E_{t+1} as a function of k_{t+1} . Therefore, rewrite it as

$$E_{t+1} \equiv \phi(k_{t+1}) \quad (18)$$

5 The steady state

Since all parameters are constant in the steady state, time subscripts are eliminated.

Let \bar{k} and \bar{E} indicate steady state values.

In steady state, equation (17) becomes

$$\bar{E} = \frac{p}{1+bp} \left\{ -\beta p[(1 + f'(\bar{k}) - \sigma)\bar{k} - z] + \delta[f(\bar{k}) - \bar{k}f'(\bar{k}) - \bar{k}] \right\} \equiv \psi(\bar{k}) \quad (19)$$

In steady state, equation (18) becomes

$$\bar{E} = \frac{1}{p} \left[\delta \bar{k} - \frac{\delta z}{(1 + f'(\bar{k}) - \sigma)} \right] \equiv \phi(\bar{k}) \quad (20)$$

The stable condition is given by the following equation,

$$k_{t+1} - \bar{k} = \left[\frac{(1-b)\phi' - (\delta + \beta p)k f'' - \beta p(1 + f' - \sigma)}{\delta + [(1+p)/p]\phi'} \right] (k_t - \bar{k}) \quad (21)$$

Proof: See Appendix B

The coefficient on the right-hand side of this equation is less than one if and only if $\phi'(\bar{k}) > \psi'(\bar{k})$ where

$$\phi'(\bar{k}) = \frac{\delta \left[(1 + f'(\bar{k}) - \sigma)^2 + z f''(\bar{k}) \right]}{(1 + f'(\bar{k}) - \sigma)^2}$$

The condition $0 < z < \frac{(1 + f' - \sigma)^2}{-f''}$ is sufficient for $\phi' > 0$. A greater capital stock is associated with greater environmental quality.

$$\psi'(\bar{k}) = \frac{p}{1 + \beta p} \left[-\beta p(1 + f'(\bar{k}) + \bar{k} f''(\bar{k}) - \sigma) - \delta(\bar{k} f''(\bar{k}) + 1) \right]$$

Equations (15) and (16) can be rewritten as

$$\frac{1 + \beta p}{p} \bar{E} = -\beta p \left[(1 + f'(\bar{k}) - \sigma)k - z \right] + \delta \left[f(\bar{k}) - \bar{k} f'(\bar{k}) - \bar{k} \right] \quad (22)$$

$$\bar{E} = \frac{1}{p} \left[\delta \bar{k} - \frac{\delta z}{(1 + f'(\bar{k}) - \sigma)} \right] \quad (23)$$

The following analysis describes the comparative static behaviour of the steady state of this model.

The differentiation of (22) and (23) taking b, β, δ and σ as given yields

$$\begin{bmatrix} \frac{1+bp}{p} & \beta p(1+f'(\bar{k})-\sigma)+(\beta p+\delta)\bar{k}f''(\bar{k})+\delta \\ p(1+f'(\bar{k})-\sigma) & f''(\bar{k})(p\bar{E}-\delta\bar{k})-\delta(1+f'(\bar{k})-\sigma) \end{bmatrix} \begin{bmatrix} \partial\bar{E} \\ \partial\bar{k} \end{bmatrix} = \begin{bmatrix} \beta p \\ -\delta \end{bmatrix} \partial z$$

The determinant of the left-hand-side matrix is

$$|D| = \begin{vmatrix} \frac{1+bp}{p} & \beta p(1+f'(\bar{k})-\sigma)+(\beta p+\delta)\bar{k}f''(\bar{k})+\delta \\ p(1+f'(\bar{k})-\sigma) & f''(\bar{k})(p\bar{E}-\delta\bar{k})-\delta(1+f'(\bar{k})-\sigma) \end{vmatrix}$$

We set $X = \beta p(1+f'(\bar{k})-\sigma)+(\beta p+\delta)\bar{k}f''(\bar{k})+\delta$

and $Y = f''(\bar{k})\left(-\frac{\delta z}{(1+f'(\bar{k})-\sigma)}\right) - \delta(1+f'(\bar{k})-\sigma)$

with; $p\bar{E}-\delta\bar{k} = -\frac{\delta z}{(1+f'(\bar{k})-\sigma)}$

The determinant is $|D| = \frac{1+bp}{p}Y - p(1+f'(\bar{k})-\sigma)X$, where $0 < z < \frac{(1+f'(\bar{k})-\sigma)^2}{-f''}$

is sufficient for $Y < 0$. $X \geq 0$, this condition is not very restrictive. Thus, $|D|$ is negative.

We explore next the repercussion of spending on health care on the steady state equilibrium level of the capital and the environmental quality. Then, we discuss the implication of the result on the economy and we compare it with the literature findings.

Note

$$D = \begin{bmatrix} \frac{1+bp}{p} & \beta p(1+f'(\bar{k})-\sigma)+(\beta p+\delta)\bar{k}f''(\bar{k})+\delta \\ p(1+f'(\bar{k})-\sigma) & f''(\bar{k})(p\bar{E}-\delta\bar{k})-\delta(1+f'(\bar{k})-\sigma) \end{bmatrix}$$

$$\text{and } H = \begin{bmatrix} \beta p \partial z \\ -\delta \partial z \end{bmatrix}$$

$$\text{This gives } D_1 = \begin{bmatrix} \beta p \partial z & \beta p(1+f'(\bar{k})-\sigma)+(\beta p+\delta)\bar{k}f''(\bar{k})+\delta \\ -\delta \partial z & f''(\bar{k})(p\bar{E}-\delta\bar{k})-\delta(1+f'(\bar{k})-\sigma) \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \frac{1+bp}{p} & \beta p \partial z \\ p(1+f'(\bar{k})-\sigma) & -\delta \partial z \end{bmatrix}$$

By Cramer's rule,

$$\begin{aligned} \frac{\partial \bar{k}}{\partial z} &= \frac{|D_2|}{|D|} = \frac{-\frac{1+bp}{p} \delta \partial z - p(1+f'(\bar{k})-\sigma) \beta p \partial z}{|D|} \\ \frac{\partial \bar{k}}{\partial z} &= \frac{1}{|D|} \left\{ -\frac{1+bp}{p} \delta - \beta p^2 (1+f'(\bar{k})-\sigma) \right\} > 0 \end{aligned}$$

Since $|D| < 0$ then $\partial \bar{k} / \partial z > 0$

Our characterization of the steady state implies that greater health costs have positive outcomes on capital accumulation. The intuition is that consumption externalities make individuals acquire health costs when old. Hence, greater environmental degradation implies larger health costs that make individuals save more for old age and accumulate more capital.

This result goes in the same direction of that obtained by Raffin and Seegmuller (2014) where, at the steady state, the indicator of health and the stock of physical capital are positively dependant. This means that the richer the economy, the better the health status, regardless of the pollution's level. So, the rise in health care always offset the harmful impact of environmental degradation. This justifies that welfare increases with capital accumulation. Our result also goes on the same path of that achieved by Gutierrez (2008) but differs from that obtained by John and Pecchenino (1995) who find that economies in which consumption causes greater environmental degradation accumulate less capital. This is so since in his OLG model individuals pay taxes to sustain environmental quality when they are young and consequently an increase in degradation reduces their savings for the futures. Nevertheless, in our model, greater environmental degradation induces higher health costs, which are paid in the old age, so individuals have to raise savings and consequently the capital intensifies.

As for the repercussion of health care depending on environmental quality, we have

$$\frac{\partial \bar{E}}{\partial z} = \frac{|D_1|}{|D|} = \frac{\beta p \partial z [f''(\bar{k})(p\bar{E} - \delta\bar{k}) - \delta(1 + f'(\bar{k}) - \sigma)] + \delta \partial z [\beta p(1 + f'(\bar{k}) - \sigma) + (\beta p + \delta)\bar{k}f''(\bar{k}) + \delta]}{|D|}$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{\delta}{|D|} \left\{ \delta - f''(\bar{k}) \left[\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - \bar{k} \right) - \delta \bar{k} \right] \right\}$$

Since $|D| < 0$ then $\frac{\partial \bar{E}}{\partial z} > 0 \quad \forall \beta p < \delta$

Proof: See Appendix C

As to the environmental quality, our results show that supporting health care is harmful to the environment once the maintenance efforts are less than the consumption externalities $\delta < \beta p$. However, if consumption externalities are lower than the maintenance efforts $\delta > \beta p$, health care support is beneficial to the environment.

Gutierrez (2008) finds that a greater future stock of pollution for any given stock of capital leads to a greater health cost faced by old individuals. This larger future costs makes individuals save more for retirement period, which means that economy accumulate more capital and degrades the environment more. The results of our model are clearer than those obtained by Gutierrez (2008) since she excludes the possibility of maintenance (via using part of the young wealth to invest in improving the quality of the environment). Thus by introducing into our model the maintenance effort gives another channel for decreasing consumption externalities other than to accumulate less capital by reducing savings. In our model we also find the same results as this author regarding the effects of increasing health expenditure on capital. However, their effects on the environment don't always degrade the environment as shown by this same author. They depend on the domination of one of two opposing effects: strength of consumption externalities and maintenance efforts.

Proposition: *Under the stable condition, a rise in private health expenditure (higher z) leads always to a higher level of capital accumulation but leads to a higher level of environmental quality only if $\delta > \beta p$.*

We explain now the intergenerational conflict between old and young populations that has led to the results obtained.

Actually, at an international level, there is an intergenerational conflict between young and old over two types of defensive expenditure due to their contradictory interests. Crucially, the young individuals support environmental care while retirees prefer investing in health care. Old individuals appreciate private health-care expenditure instead of the environmental investments due to the fact that they take usually more time to be totally effective although, they can last for a longer time. They do not enjoy future environmental improvements. The young generation prefers environmental expenditure as they yield to results over a longer horizon. The benefit that the young generation receives when being old from the investment in environmental quality when young generates a longer remaining lifespan in which to enjoy enhanced environmental quality. It is worth mentioning that we are not claiming that old people are not interested at all in environmental maintenance, but that they are less concerned than the young people are.

Elderly cannot enjoy improvements in the quality of future environment. They prefer spending in private health-care expenditure in the detriment of environmental investment in order to live longer and to raise their utility. They continue to invest in this curative option until a critic value which is the total return of capital $z < (1 + f'(\bar{k}) - \sigma)\bar{k}$. By choosing the curative option when young, they also chose to invest more of their wage for the next period (negative consumption effect in the period t). Therefore, they have more precautionary savings which lead to capital accumulation and to worsening the environmental quality by increasing their consumption possibilities (positive consumption effect in the period $t+1$).

For young generation now, environmental expenditure is supported over health-care. They have a good motive to spend in maintaining the environment healthy – as they are going to live longer to benefit from it. So, a higher environmental maintenance at young age forces them to lower their savings and consumption in first period t . Thus, environmental investment has a negative effect on capital accumulation and a positive effect on environment. On the other hand, the young generation consumption possibilities in second period $t+1$ are reduced since their precautionary saving is low due to maintenance effort in t . Then, this is another positive effect on the environmental quality.

It is however important to note that if the young individuals may have a stronger incentive to save and accumulate capital for the next period, in order to increase their consumption and to face the health costs when old, than to invest in maintenance expenditure seeing as the environment is not much of a problem for the time being. As a result, they reach the second period with a quite high capital stock, thus a high production as well, which enhances their second period consumption (positive consumption effect in $t+1$ and worsens the environment. However, young people will need then to invest much more in health care in their second period of life since the environment has been severely damaged due to the lack of expenditure in maintenance in period t . Thus, a disincentive by young generation towards environmental expenditure in aging economy has a negative effect on environment and a positive effect on capital accumulation.

The intergenerational conflict that arises from different attitude of young and old towards environment and health spending leads to contradictory effects on capital accumulation and on environmental quality. In order to recognize whether the positive effects overcome the negative effects or vice versa, we studied the impact of a higher health care support by aging population on capital stock and the environment. We find that it is possible that health and the environment can flourish simultaneously. We have shown that an increase of the support to private health expenditure leads to a higher level of capital accumulation and leads to a higher level of environmental quality if the maintenance efforts are bigger than the consumption externalities. It is worth mentioning that this result holds to many other specifications beyond health care expenditure and still robust to other interpretations of the conflict's source.³

6 Conclusion

We have developed a two-period overlapping generations model where agents are affected by environmental quality. To offset this inconvenience, they can invest in defensive expenditure, either in maintenance or health care (the preventive versus the curative option). Individuals face tension between those two options. We explore the question of how and whether the intergenerational conflict among two

³We thank an anonymous referee for pointing this out.

groups – young and old – affects capital accumulation and environmental quality by analyzing the effects that consumption externalities can have when affecting the population's health. We have shown that an increase of the support to private health expenditure leads to a higher level of capital accumulation and leads to a higher level of environmental quality if the maintenance efforts are higher than the consumption externalities.

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Appendices

Appendix A

Proof of equations (7)–(8)

To type the objective function of the individual, we substitute the constraints (1)–(2) and (3) into the utility function (4)

$$U = \ln c_t^1 + p \ln \left[(1 + r_{t+1})(w_t - c_t^1 - m_t) - z_{t+1} \right] + \ln E_t + p \ln \left[(1 - b)E_t - \beta(c_t^1 + c_t^2) + \delta m_t \right]$$

Deriving U with respect to m_t gives

$$\left[-(1 + r_{t+1}) \frac{1}{c_{t+1}^2} + \delta \frac{1}{E_{t+1}} \right] = 0$$

After rearrangement, we get

$$p(1 + r_{t+1}) \frac{1}{c_{t+1}^2} = \delta \frac{1}{E_{t+1}} \quad (8)$$

Deriving U with respect to c_t^1 gives

$$\frac{1}{c_t^1} + p \left[-(1 + r_{t+1}) \frac{1}{c_{t+1}^2} - \beta \frac{1}{E_{t+1}} \right] = 0$$

Substituting (8) into it gives

$$\frac{1}{c_t^1} = p(\beta + \delta) \frac{1}{E_{t+1}} \quad (7)$$

Proof of equation (16)

Rewrite equation (8) as

$$E_{t+1} = \frac{\delta c_{t+1}^2}{p(1 + r_{t+1})}$$

Plugging equations (2) and (11) into it gives

$$E_{t+1} = \frac{\delta [(1 + f'(k_{t+1}) - \sigma)k_{t+1} - z_{t+1}]}{p(1 + f'(k_{t+1}) - \sigma)}$$

We obtain

$$E_{t+1} = \frac{1}{p} \left[\delta k_{t+1} - \frac{\delta z_{t+1}}{(1 + f'(k_{t+1}) - \sigma)} \right] \quad (16)$$

Appendix B

Proof of equation (21)

To find the stable condition, plug (18) and (18) lagged once into (17) to have the following first-order nonlinear difference equation in k :

$$\begin{aligned} & [(1+p)/p]\phi(k_{t+1}) - (1-b)\phi(k_t) + \beta p[(1+f'(k_t) - \sigma)k_t - z_t] - \delta[f(k_t) - k_t f'(k_t) - k_{t+1}] = 0 \\ \Rightarrow & [(1+p)/p]\phi(k_{t+1}) - (1-b)\phi(k_t) + \beta p(1+f'(k_t) - \sigma)k_t - \beta p z_t - \delta f(k_t) + \delta k_t f'(k_t) + \delta k_{t+1} = 0 \\ \Rightarrow & -[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) - \beta p(1+f'(k_t) - \sigma)k_t + \beta p z_t + \delta f(k_t) - \delta k_t f'(k_t) = \delta k_{t+1} \\ \Rightarrow & \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) - \beta p(1+f'(k_t) - \sigma)k_t + \beta p z_t + \delta f(k_t) - \delta k_t f'(k_t)}{\delta} = k_{t+1} \\ \Rightarrow & \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) + \delta f(k_t) - \delta k_t f'(k_t) + \beta p z_t}{\delta} = k_{t+1} + \frac{\beta p(1+f'(k_t) - \sigma)k_t}{\delta} \\ \Rightarrow & \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) + \delta f(k_t) - \delta k_t f'(k_t) + \beta p z_t}{\delta} = k_{t+1} + \left(1 - 1 + \frac{\beta p(1+f'(k_t) - \sigma)}{\delta}\right) k_t \\ \Rightarrow & \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) + \delta f(k_t) - \delta k_t f'(k_t) + [-\delta - \beta p(1+f'(k_t) - \sigma)]k_t + \beta p z_t}{\delta} = k_{t+1} - k_t \end{aligned}$$

This equation can be linearizing thanks to Taylor's rule that said:

$$f(x) = f(\bar{x}) + \frac{\partial f}{\partial x} (x - \bar{x})$$

$$\begin{aligned}
 &\Rightarrow \frac{-[(1+p)/p]\phi(k_{t+1}) + (1-b)\phi(k_t) + \delta f(k_t) - \delta k_t f'(k_t) + [-\delta - \beta p(1 + f'(k_t) - \sigma)]k_t + \beta p z_t}{\delta} \\
 &= \frac{-[(1+p)/p]\phi + (1-b)\phi + \delta f - \delta k f' + [-\delta - \beta p(1 + f' - \sigma)]k + \beta p z}{\delta} \\
 &+ \frac{-[(1+p)/p]\phi' + (1-b)\phi' - \delta k f'' - \delta - \beta p(1 + f' - \sigma) - \beta p k f''}{\delta} k_{t+1} - k_t \\
 &\Rightarrow k_{t+1} - k_t = 0 + \left[\frac{-[(1+p)/p]\phi' + (1-b)\phi' - \delta k f'' - \delta - \beta p(1 + f' - \sigma) - \beta p k f''}{\delta} \right] (k_t - \bar{k}) \\
 &\Rightarrow k_{t+1} - \bar{k} = \left[\frac{-[(1+p)/p]\phi' + (1-b)\phi' - \delta k f'' - \delta - \beta p(1 + f' - \sigma) - \beta p k f''}{\delta} \right] (k_t - \bar{k}) \\
 &\Rightarrow k_{t+1} - k_t = -k_t + \bar{k} + \left[\frac{\delta - [(1+p)/p]\phi' + (1-b)\phi' - \delta k f'' - \delta - \beta p(1 + f' - \sigma) - \beta p k f''}{\delta} \right] (k_t - \bar{k}) \\
 &\Rightarrow k_{t+1} - \bar{k} + \frac{[(1+p)/p]\phi'}{\delta} = \left[\frac{(1-b)\phi' - (\delta + \beta p)k f'' - \beta p(1 + f' - \sigma)}{\delta} \right] (k_t - \bar{k}) \\
 &\Rightarrow (k_{t+1} - \bar{k}) \left[1 + \frac{[(1+p)/p]\phi'}{\delta} \right] = \left[\frac{(1-b)\phi' - (\delta + \beta p)k f'' - \beta p(1 + f' - \sigma)}{\delta} \right] (k_t - \bar{k}) \\
 &\Rightarrow (k_{t+1} - \bar{k}) \left[\frac{\delta + [(1+p)/p]\phi'}{\delta} \right] = \left[\frac{(1-b)\phi' - (\delta + \beta p)k f'' - \beta p(1 + f' - \sigma)}{\delta} \right] (k_t - \bar{k}) \\
 &\Rightarrow k_{t+1} - \bar{k} = \left[\frac{(1-b)\phi' - (\delta + \beta p)k f'' - \beta p(1 + f' - \sigma)}{\delta} \right] \frac{\delta}{\delta + [(1+p)/p]\phi'} (k_t - \bar{k})
 \end{aligned}$$

We obtain

$$k_{t+1} - \bar{k} = \left[\frac{(1-b)\phi' - (\delta + \beta p)k f'' - \beta p(1 + f' - \sigma)}{\delta + [(1+p)/p]\phi'} \right] (k_t - \bar{k}) \quad (21)$$

Appendix C:

$$\frac{\partial \bar{E}}{\partial z} = \frac{|D_1|}{|D|} = \frac{\beta p \partial z [f''(\bar{k})(p\bar{E} - \delta\bar{k}) - \delta(1 + f'(\bar{k}) - \sigma)] + \delta \partial z [\beta p(1 + f'(\bar{k}) - \sigma) + (\beta p + \delta)\bar{k}f''(\bar{k}) + \delta]}{|D|}$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{1}{|D|} \left\{ \beta p [f''(\bar{k})(p\bar{E} - \delta\bar{k}) - \delta(1 + f'(\bar{k}) - \sigma)] + \delta [\beta p(1 + f'(\bar{k}) - \sigma) + (\beta p + \delta)\bar{k}f''(\bar{k}) + \delta] \right\}$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{1}{|D|} \left[-\frac{\beta p \delta z}{(1 + f'(\bar{k}) - \sigma)} f''(k) + \delta(\beta p + \delta)kf''(k) + \delta^2 \right]$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{1}{|D|} \left[-\frac{\beta p \delta z}{(1 + f'(\bar{k}) - \sigma)} f''(k) + \delta\beta pkf''(k) + \delta^2 kf''(k) + \delta^2 \right]$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{1}{|D|} \left[\delta^2 - f''(k) \left(\frac{\beta p \delta z}{(1 + f'(\bar{k}) - \sigma)} - \delta\beta pk - \delta^2 k \right) \right]$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{1}{|D|} \left\{ \delta^2 - f''(k) \left[\beta p \delta \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k \right) - \delta^2 k \right] \right\}$$

$$\frac{\partial \bar{E}}{\partial z} = \frac{\delta}{|D|} \left\{ \delta - f''(\bar{k}) \left[\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - \bar{k} \right) - \delta\bar{k} \right] \right\}$$

Determination of the sign of: $\delta - f''(k) \left[\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k \right) - \delta k \right]$

• when : $\delta - f''(k) \left[\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k \right) - \delta k \right] > 0$

$$\Rightarrow \delta > f''(k) \left[\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k \right) - \delta k \right]$$

$$\Rightarrow \frac{\delta}{f''(k)} < - \left[\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k \right) - \delta k \right]$$

$$\Rightarrow \frac{\delta}{f''(k)} < -\beta p \left(\frac{z}{(1 + f'(\bar{k}) - \sigma)} - k \right) + \delta k$$

$$\begin{aligned}
 &\Rightarrow \frac{\delta}{f''(k)} - \delta k < -\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) \\
 &\Rightarrow -\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) > \frac{\delta}{f''(k)} - \delta k \\
 &\Rightarrow \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) < -\frac{\delta}{\beta p} \left(\frac{1}{f''(k)} - k \right) < 0 \Leftrightarrow \bar{E} > 0 \\
 &\Rightarrow k - \frac{z}{(1+f'(\bar{k})-\sigma)} > \frac{\delta}{\beta p} \left(\frac{1}{f''(k)} - k \right) \\
 &\Rightarrow \delta \left(k - \frac{z}{(1+f'(\bar{k})-\sigma)} \right) > \frac{\delta^2}{\beta p} \left(\frac{1}{f''(k)} - k \right) \\
 &\Rightarrow E > \frac{\delta^2}{\beta p} \left(\frac{1}{f''(k)} - k \right) \\
 &\Rightarrow \delta \left(-\frac{z}{(1+f'(\bar{k})-\sigma)} + k \right) > \frac{\delta^2}{\beta p} \left(\frac{1-kf''(k)}{f''(k)} \right) \\
 &\Rightarrow \left(-\frac{z}{(1+f'(\bar{k})-\sigma)} + k \right) > \frac{\delta}{\beta p} \left(\frac{1-kf''(k)}{f''(k)} \right) \\
 &\Rightarrow -\frac{z}{(1+f'(\bar{k})-\sigma)} > \frac{\delta}{\beta p} \left(\frac{1-kf''(k)}{f''(k)} \right) - k \\
 &\Rightarrow -z > \frac{\delta(1+f'(\bar{k})-\sigma)}{\beta p} \left(\frac{1-kf''(k)}{f''(k)} \right) - k(1+f'(\bar{k})-\sigma) \\
 &\Rightarrow -z > \frac{\delta(1+f'(\bar{k})-\sigma) - \delta k(1+f'(\bar{k})-\sigma)f''(k) - \beta p k(1+f'(\bar{k})-\sigma)f''(k)}{\beta p f''(k)} \\
 &\Rightarrow z < \frac{-\delta(1+f'(\bar{k})-\sigma) + \delta k(1+f'(\bar{k})-\sigma)f''(k) + \beta p k(1+f'(\bar{k})-\sigma)f''(k)}{\beta p f''(k)}
 \end{aligned}$$

$$\begin{cases} z < \frac{(1 + f'(\bar{k}) - \sigma)[\bar{k}f''(k)(\delta + \beta p) - \delta]}{\beta p f''(k)} \\ z < \frac{(1 + f' - \sigma)^2}{-f''} \end{cases}$$

To resolve this equations system, we proceed by subtraction

$$\begin{aligned} &\Rightarrow \frac{(1 + f' - \sigma)[\bar{k}f''(\delta + \beta p) - \delta]}{\beta p f''} + \frac{(1 + f' - \sigma)^2}{f''} > 0 \\ &\Rightarrow \frac{(1 + f' - \sigma)f''[\bar{k}f''(\delta + \beta p) - \delta] + \beta p(1 + f' - \sigma)^2 f''}{\beta p f''^2} > 0 \\ &\Rightarrow \frac{(1 + f' - \sigma)f''[\bar{k}f''(\delta + \beta p) - \delta + \beta p(1 + f' - \sigma)]}{\beta p f''^2} > 0 \\ &\Rightarrow (1 + f' - \sigma)f''[\bar{k}f''(\delta + \beta p) - \delta + \beta p(1 + f' - \sigma)] > 0 \\ &\Rightarrow \bar{k}f''(\delta + \beta p) - \delta + \beta p(1 + f' - \sigma) < 0 \\ &\Rightarrow \beta p(1 - \sigma + f' + \bar{k}f'') + \delta(\bar{k}f'' - 1) < 0 \\ &\Rightarrow \beta p(1 - \sigma + f' + \bar{k}f'') < -\delta(\bar{k}f'' - 1) \\ &\Rightarrow p < \frac{\delta(1 - \bar{k}f'')}{\beta(1 - \sigma + f' + \bar{k}f'')} \end{aligned}$$

Under our hypothesis;

$$\begin{cases} 1 - \bar{k}f'' > 1 \\ f' - \sigma + 1 + \bar{k}f'' > 1 + \bar{k}f'' \end{cases} \Leftrightarrow \begin{cases} \frac{1 - \bar{k}f''}{f' - \sigma + 1 + \bar{k}f''} > \frac{1}{1 + \bar{k}f''} \\ \frac{1 - \bar{k}f''}{f' - \sigma + 1 + \bar{k}f''} > 0 \end{cases}$$

$$\Leftrightarrow \frac{1}{1 + \bar{k}f''} > 0 \Leftrightarrow 0 < -\bar{k}f'' < 1$$

$$\begin{aligned} \Rightarrow \frac{1}{1+\bar{k}f''} > 1 &\Rightarrow \begin{cases} \frac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} > 1 \\ \frac{\delta}{\beta} \frac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} > p \end{cases} \\ \Rightarrow \frac{1-\bar{k}f''}{f'-\sigma+1+\bar{k}f''} \frac{\beta f'-\sigma+1+\bar{k}f''}{\delta} > \frac{1}{p} &\Rightarrow \frac{\beta}{\delta} > \frac{1}{p} \\ &\Rightarrow \frac{\beta p}{\delta} > 1 \quad \Rightarrow \beta p > \delta \end{aligned}$$

$$\Rightarrow \delta - f''(k) \left[\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) - \delta k \right] > 0 \quad \forall \beta p > \delta$$

Since $|D| < 0$, $\frac{\partial \bar{E}}{\partial z} < 0 \quad \forall \beta p > \delta$

$$\bullet \text{ if } \delta - f''(k) \left[\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) - \delta k \right] < 0$$

$$\Rightarrow \delta < f''(k) \left[\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) - \delta k \right]$$

$$\Rightarrow \frac{\delta}{f''(k)} > - \left[\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) - \delta k \right]$$

$$\Rightarrow \frac{\delta}{f''(k)} > -\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) + \delta k$$

$$\Rightarrow \frac{\delta}{f''(k)} - \delta k > -\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right)$$

$$\Rightarrow -\beta p \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) < \frac{\delta}{f''(k)} - \delta k$$

$$\Rightarrow \left(\frac{z}{(1+f'(\bar{k})-\sigma)} - k \right) > -\frac{\delta}{\beta p} \left(\frac{1}{f''(k)} - k \right) > 0$$

$$\Rightarrow k - \frac{z}{(1+f'(\bar{k})-\sigma)} < \frac{\delta}{\beta p} \left(\frac{1}{f''(k)} - k \right)$$

$$\Rightarrow \delta \left(k - \frac{z}{(1 + f'(\bar{k}) - \sigma)} \right) < \frac{\delta^2}{\beta p} \left(\frac{1}{f''(k)} - k \right)$$

$$\Rightarrow \bar{E} < \frac{\delta^2}{\beta p} \left(\frac{1}{f''(k)} - k \right) < 0$$

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