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Pricing as a Risky Choice: Uncertainty and Survival in a Monopoly Market

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Abstract
Roy (*Safety First and the Holding of Assets*, 1952) argues that decisions under uncertainty motivate firms to avoid bankruptcy. In this paper, the authors ask about the behaviour of a monopolist who pre-commits to price when she has only probabilistic knowledge about demand. They argue that pricing in order to maximise the likelihood of survival explains anomalies such as inelastic pricing, why the firm takes on more risk as gains become less likely, and asymmetric responses to demand and cost changes. When demand is a linear demand, the monopolist's response to an increase in the marginal cost is similar to the response when mark-up pricing is used. That is, there is a one-to-one relationship between an increase of the marginal cost and an increase in price.

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1 Introduction

Following Sandmo (1971), one of the shortcomings of assuming that firms maximise expected profits when they make decisions under uncertainty, is that they ignore the consequences of losses, including the possibility of bankruptcy. For example, consider a firm that has two possible strategies. Suppose the first strategy offers marginally higher expected profit than the second strategy. If pure profit maximisation guides the firm's decision, it chooses the first strategy even if this strategy involves a strictly higher likelihood of bankruptcy. As pointed out by Roy (1952), it makes sense that firms maximise the expected profit or expected utility as a function of profit if performances in good states of nature finance bad outcomes. In general, though, investors and firms do not take part in a lottery that is repeated a large number of times. Realistically, a poor performance does not always trigger another chance to make up for a current dread event.

One of the alternatives to profit maximisation, with respect to decision-making under uncertainty, is that the primary objective of most investors and firms is to avoid results that close down a business (Roy, 1952). Under the safety-first principle, firms minimise the probability of bankruptcy or profits falling below some threshold that means the end of further business.[1] It can be argued that the safety-first principle puts too much emphasis on risk relative to income, and Telser's (1955) subsequent extension of Roy's decision principle includes the possibility that firms maximise expected profits subject to some upper limit on the probability of bankruptcy. A second alternative is that firms maximise expected profits less the weighted standard deviation of profit (Turnovsky, 1968). In this paper, we apply the safety-first criterion to a monopolistic firm and ask to what extent the resulting price behaviour is in agreement with some of the observation anomalies of firms' pricing practices.

There are plenty of results on the topic of the non-expected utility-maximising behaviour of individual decision makers and, with respect to firms, this literature suggests that it is relevant to look for motives that supplement profit maximisation

[1] From a practical perspective notice that the most widely used measures in risk management are varieties of the safety-first principle (see Chiu et al., 2012). Also, executive directors have incentive to follow the safety-first principle because, as shown by Hilger et al. (2013), financial distress significantly increases the likelihood of executive dismissal. Schmidt (1997) analyses how the threat of liquidation is used in managerial incentive schemes.

as an explanation for pricing behaviour. With respect to setting prices, Cyert and March (1963) suggest that monopolies apply routine decisions in the form of cost-plus pricing. Empirical studies of firms' pricing behaviour by Koutsoyiannis (1984) and van Dalen and Thurik (1998) dismiss the assumption that they maximise profit. More recently, Fabiani et al. (2007) show that a significant number of firms in Europe apply mark-up pricing. Secondly, empirical findings on the pricing of performance goods (Marburger,1997; Forrest et al., 2002; Krautman and Berri, 2007) suggest that firms engage in inelastic pricing. This is also confirmed for the Swedish Tobacco Monopoly's sale prices (Asplund, 2007). Thirdly, Kahneman and Tversky (1979) suggest that risk becomes more acceptable as gains become less likely. Finally, Kahneman, Knetsch and Thaler (1986) argue that concerns for fairness can explain why price responds more strongly to cost changes than to demand changes. Of course, it is not likely that all firms follow a safety-first strategy, but we find it of some interest that the pricing behaviour of a price-setting monopolist who is motivated by the safety-first principle agrees, to a certain extent, with the abovementioned observations.

The model we apply is that of a price-setting monopolist in a market with stochastic demand under the assumption that the monopolist sets a price that determines her sale.[2] Because the price decision occurs before the actual value of demand becomes known, the monopolist runs the risk of realising profits below a minimum acceptable return. Our focus is (primarily) the case where the minimum acceptable return is zero and, therefore, we examine the pricing behaviour that minimises the likelihood of negative profits. But more generally, the safety-first principle takes into consideration poor performances that correspond to the tail part of a profit's distribution.[3] Thus, applying the safety-first criterion to monopolistic pricing, we think of pricing as a risky choice, essentially looking at the situation where the firm cannot diversify away the risk of ruin by drawing on gains in future time-periods, by engaging in multiproduct activities or in physically separate markets, etc., or through insurance. When the monopolistic firm's

[2] Allen and Hellwig (1986) notice that price setting describes an essential feature of many markets.

[3] When the gain is normally distributed, the safety-first principle implies that the objective is to maximise the difference between expected return and a threshold return relative to the standard deviation. In symbols, the objective function is $(E(r) - \bar{r})/\sqrt{Var(r)}$ where $\bar{r}$ is the minimum gain, $E(r)$ the expected gain, and $\sqrt{Var(r)}$ the standard deviation. When the minimum acceptable gain is zero the firm maximises $E(r)/\sqrt{Var(r)}$.

decision leaves profit's variance unaffected, this means that the firm maximises expected profit.

In comparison to standard results, there are several consequences when the safety-first principle guides the monopolist's pricing decision. First, when a higher price means higher variance of profit, the price under the safety-first criterion falls short of the price that maximises expected profit. This conclusion is general in the sense that the result—that the optimum price under the safety-first criterion is below the profit maximising price—applies when the objective is to maximise a utility function defined over expected profit and the risk of bankruptcy. Also, as long as the profit function's variance is increasing in price, our results apply irrespective of the threshold being negative (when the firm has access to credit) or positive (which seems relevant when the manager is fired as a consequence of profits below a certain threshold).

Second, with respect to comparative statics, when demand increases by rotation, the optimum price under the safety-first principle goes down while, under a linear demand curve, a shift in willingness to pay does not affect the optimum price. In contrast, for a monopolist who maximises expected profit, comparative static results suggest that the price is unchanged and increasing. With respect to cost-changes, a higher fixed cost increases the optimum price under the safety-first criterion while the price is unrelated to a fixed cost when firms maximise expected profit. Also, when the firm applies the safety-first principle and demand is convex, the pass-through for an increase of the unit cost exceeds 1 and, as we show, this can result in profit-increasing taxes.

Previous work on monopolistic firms' behaviour under demand uncertainty applies the assumption that the monopolist maximises the utility of expected profit. Thus, Baron (1971), Leland (1972), and more recently Hau (2004), analyse the monopolist's optimal response to risk under the assumption that the firm survives under any realisation of demand. Kimball (1989), who assumes that marginal costs are convex, shows that a monopolist who commits to price before observing demand will charge a higher price as uncertainty increases. This result is based on the observation that marginal production cost increases in all states for a mean-preserving spread, and, thus, ignores the chance that the firm ends up in a situation of financial distress. Although Sattinger (2013) applies the safety-first principle to savings decisions, and Chiu at al. (2012) use it to discuss portfolio choice, problems relating to economic survival seem to be ignored in recent discussions of firms' behaviour under risk. Our discussion is probably most closely related to Day et al. (1971) and Arzac (1976). Discussing safety margins versus expected profit maximisation for a quantity-setting monopolist, Day et al. (1971) show that output falls short of the quantity that maximises expected profits when avoidance of ruin matters. This conclusion stands when the monopolist follows a pure safety-first criterion and when she uses some other rules for aggregating expected profit and safety margin into one performance measure. Later, Arzac (1976) affirms these results without making any specific assumption about the shock, and, under the pure safety-first criterion, shows some comparative static results: an increase in willingness to pay as well as an increase of a unit tax rate leaves the optimum value of output unchanged. Our findings reverse existing conclusions on the implications of the safety-first principle, as we show that a price-setting monopolist hedges against risk by lowering the price relative to the one that maximises expected profit. Also, as we noted, a change of the constant marginal cost, inclusive of an increase of the unit tax rate, tends to drive up the optimum price.

The paper proceeds as follows. Section 2 models how a monopolist prices following the safety-first principle. Section 3 explores comparative statics and the possibility of profit-increasing excise taxes, and illustrates the asymmetries in adjusting for cost and demand changes. Section 4 discusses the possibility of pricing in the inelastic part of the demand curve. Section 5 concludes.

2 Price decisions and survival

We consider a monopolistic firm that sets a price before the exact value of demand is observed. More precisely, the firm is placed in a market where the relationship between sale, called $x(p,\theta)$, and the price the monopolist charges, called p, is stochastic according to:

$$x(p,\theta) = f(p) + \theta, f'(p) < 0. \tag{1}$$

Here θ is a continuous stochastic variable with an expected value of zero and a variance of ${\sigma_\theta}^2$. Although the monopolist knows the distributional properties of θ, she is ignorant about the exact realisation of the stochastic variable when she sets price. Following, say, Brown and Johnson (1969) the demand function under conditions of uncertainty is in general given by $\tilde{x}(p,\theta)$ where actual demand is thus determined by the announced price and conditions not known to the firm when the price is set (for example the business cycle, change in taste, emergence of competing products). In the literature there is particular interest in the additive and multiplicative varieties of $\tilde{x}(p,\theta)$. We use the additive variety and a change in the additive component of demand is a change in willingness to pay, while a change in the number of consumers corresponds to a demand change by rotation. Under additive uncertainty, for a given price, the monopolist's prediction of sale is equally accurate across the realisations of the stochastic innovation.[4]

For simplicity, we focus on a production technology of the type where there is a fixed cost, called F, and constant marginal cost called c. That is, the firm's production cost is:

$$C(x) = F + cx. \tag{2}$$

Realised profit, called π, is a stochastic variable given by:

$$\pi = p(f(p) + \theta) - F - c(f(p) + \theta). \tag{3}$$

Of course, when the monopolist commits to a price, her beliefs about the expected value of profit and the variance of that profit are both functions of the price. More precisely, the expected value of profit and variance are given by the following Equations (4) and (5), respectively.

$$\bar{\pi}(p) = pf(p) - F - cf(p) \tag{4}$$

$${\sigma_{\pi(p)}}^2 = (p-c)^2{\sigma_\theta}^2. \tag{5}$$

It is easy to see from equations (4) and (5) that actual profits can be negative even when expected profits are positive. For example, suppose that p^π maximises

[4] In the general case of $x = \varphi(p,\theta)$, the monopolist's choice of price affects the variance of sale. In this regard, the monopolist changes the accurateness of her prediction on sales by changing the price she sets.

expected profit, and assume that $\bar{\pi}(p^{\pi})$ is positive. Clearly, actual profits are negative when the shock satisfies $\bar{\pi}(p^{\pi}) + \theta(p^{\pi} - c) < 0$.

Because the price co-determines the distribution of profit, the monopolist's choice of price affects the chance of financial distress. By setting the price, the monopolist decides on the mean of profits, $\bar{\pi}(p)$, and the standard variation of profits, $(p - c)\sigma_{\theta}$. Supposing that profits are normally distributed, we can write actual profit as $\pi(p) = \bar{\pi}(p) + u\sigma_{\theta}(p - c)$, and the probability that profits are negative equals the probability that $u < -\big(\sigma_{\theta}(p - c)\big)^{-1}\bar{\pi}(p)$. Under this assumption about the distribution of profits, the firm minimises the likelihood of negative profits by maximising $\big(\sigma_{\theta}(p - c)\big)^{-1}\bar{\pi}(p)$. In general, when profits follow some other distribution than the normal distribution, Tchebycheff's inequality implies that the firm minimises the value of an upper bound on the likelihood of negative profits by maximising $\big(\sigma_{\theta}(p - c)\big)^{-1}\bar{\pi}(p)$. Thus, when the firm follows the safety-first principle, the relevant optimisation problem is: [5]

$$Max_{p}\, \big((p - c)\sigma_{\theta}\big)^{-1}\big(pf(p) - F - cf(p)\big), \tag{6}$$

under the assumptions that the second-order condition is satisfied and when we assume that expected profits are non-negative for $c < p_l \leq p \leq p_h$, where $\bar{\pi}(p_l) = \bar{\pi}(p_h) = 0$. The restriction that $p_l > c$ follows because the monopolist produces with an average loss equal to the fixed cost, should she use marginal cost pricing. The arguments leading to equation (6) show that the safety-first principle implies that the monopolist maximises expected profits when her decision on price leaves variance unchanged. The optimisation problem is alternatively written as:

$$Max_{p}\, f(p) - (p - c)^{-1}F. \tag{7}$$

Equation (7) implies that the monopolist decides on a price, called p^{*}, which maximises the difference between expected sales, $\bar{x}(p,\theta) = f(p)$, and (expected) zero-profit sale. The latter term, the price that implies a zero-profit sale, is critical in the sense that setting the price higher produces, on average, negative profits.

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[5] We relax this assumption in Propositions 2 and 3 below and analyse the case where the monopolist is active when $(p - c)f(p) - F \geq k$, where k can be either positive or negative.

Therefore, the price that, on average, gives a zero-profit sale is the upper bound on the price the monopolist will set. Maximising the distance between expected sale and the zero-profit sale, the firm maximises the safety margin, i.e., sets a price that minimises the probability of revenue falling short of costs inclusive of the fixed cost. All in all, the optimum price satisfies:

$$f'(p^*)+(p^* - c)^{-2}F = 0. \tag{8}$$

We can compare the bankruptcy-minimising price with the price that maximises expected profit. To do so, rewrite the first-order condition as $(p^* - c)f'(p^*) + f(p^*) = f(p^*) - (p^* - c)^{-1}F$. In comparison, the price that maximises expected profit implies $(p^\pi - c)f'(p^\pi) + f(p^\pi) = 0$. Under the assumption that the monopolist engages in production if average profit is positive, i.e., $f(p^*) - (p^* - c)^{-1}F > 0$, we have $p^* < p^\pi$. That is, pricing that aims at minimising the likelihood of financial default reduces the price in comparison with the price that maximises expected profit. We summarise this as Proposition 1.

Proposition 1. Under the assumptions that expected profits are positive and variance of profits is increasing in price, the monopolist who uses price to minimise the probability of negative profits, sets a price that is less than the price that maximises expected profit.

To explain the intuition of the result, notice that the pricing policy under the safety-first principle reduces expected profits due to the second-order condition which implies $\bar{\pi}(p^*) < \bar{\pi}(p^\pi)$. However, the reduction in expected profits is compensated for by a gain in terms of reduced risk because ${\sigma_{\pi(p^*)}}^2 < {\sigma_{\pi(p^\pi)}}^2$. To see why the firm prefers to reduce risk by pricing aggressively, notice that the price p^* maximises $\Omega = \left((p-c)\sigma_\theta\right)^{-1}\bar{\pi}(p)$. Consider the value of $d\Omega/dp$ evaluated at $p = p^\pi$. Because $d\bar{\pi}/dp = 0$ at $p = p^\pi$, it follows that a marginally lower price around p^π lowers $\bar{\pi}(p)$ but the decrease is of the order of ε^2. In contrast, the increase of $(p-c)\sigma_\theta$ is of the order of ε meaning that $d\Omega/dp$ is positive around $p = p^\pi$. In other words, if the monopolist charges the price that maximises expected profit and considers a deviation from this price, the effect on profit is vanishing because the monopolist deviates from a top. However, by reducing the price, she reduces the variance of profit, and this shows why the optimal price is less than the monopoly price. Incidentally, this way of understanding the result points to a generalisation—that the optimal price is less than the monopoly price whenever the profit's variance is increasing in the price, and is and higher than the monopoly price whenever the profit's variance is a decreasing function of the price.

Proposition 2. Whenever the variance of profit is increasing (decreasing) in price, a monopolist who minimises the probability that profit falls short of k sets a price that is less (greater) than the price that maximises expected profit.

As noted, Day et al. (1971) and Arzac (1976) analyse the behaviour of a quantity-setting monopolist under demand fluctuation. They show that the monopolist minimises the risk of negative profits by reducing output to below the output that maximises expected profit. Following Arzac (1976) the monopolist chooses output under a demand function that is of the form $p(x,\theta) = g(x) + \theta h(x)$, where $E(\theta) = 1$ and $g'(x) + h'(x) < 0$. Setting the quantity, the firm accepts uncertainty with respect to revenue but production costs are the same irrespective of the shock. Thus, the variance of profit is ${\sigma_{\pi(x)}}^2 = \left(xh(x)\right)^2 {\sigma_\theta}^2$. Under the assumption that $h(x) + xh'(x)$ is strictly positive, the monopolist reduces the variance of profit by reducing output. The difference between this result and Propositions 1 and 2 derives from the way decisions affect profit variability. Suppose the monopolistic firm considers changing from setting output to setting a price so that the relationship between price and quantity is given by the average demand curve. Under preset quantity there is uncertainty with respect to revenue and setting price adds uncertainty since there is also cost uncertainty.[6]

Assuming that the aim is to maximise expected profit, Kimball (1989) analyses, as we do, how a monopolist sets price when there is only probabilistic knowledge about demand at the time when the price is chosen. Unlike our approach, Kimball (1989) analyses the situation of multiplicative demand uncertainty. When the monopolist adjusts production to meet demand at the preannounced price, uncertainty drives up the optimum price if production occurs under convex marginal costs. The explanation for this result is that marginal costs increase by a lot, comparatively speaking, when demand increases above its mean relative to the cost decrease that comes with demand decreases below the mean.

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[6] Day et al. (1971) and Arzac (1976) show results for a cost function of the form $c(x) + F$ and their results therefore apply to $cx + F$ as a special case. Our proposition 2 covers the case of $c(x) + F$.

To hedge against the consequences of extraordinary high costs under high demand, the firm finds it attractive to set a high price. This argument, based on the observation that a low price forces the firm to substantial sale and costs in states with high demand, ignores the fact that a high price is more likely to lead to actual profits that are negative. Our result shows that, when the monopolistic firm uses price to hedge against the risk of bankruptcy, the optimum price is less than the price that maximises the expected profit, and that a higher price implies higher variability of profit.

Finally, notice that in Proposition 1, a monopolist who follows the safety-first principle, reduces her price relative to the price-maximising expected profit (positive or negative) and generalises to more general preferences than those examined so far. To see this, suppose that the financial restriction is that profits less than k are unacceptable. In principle, k can take on a negative value if the monopolist can borrow or have other sources of finance. Under the restriction that k is the least acceptable profit, the monopolist is active when $(p-c)f(p) \geq F + k$. With a reasoning that parallels the above, the monopolist thus maximises ${\sigma_\theta}^{-1}h(p)$, where $h(p) = (p-c)^{-1}\big((p-c)f(p) - F - k\big)$, when she decides according to the safety-first principle. Let us assume that the monopolistic firm's objective is to maximise $U\big(\bar{\pi}(p), h(p)\big)$ where utility is increasing in expected profits, $U_1 = \partial U(.)/\partial\bar{\pi}(p)$ is positive and decreasing in the probability of realising profits below the threshold, and that $U_2 = \partial U(.)/\partial h(p)$ is positive. We have:

Proposition 3. When $Max_p\ h(p)$ has a solution, the solution to $Max_p U\big(\bar{\pi}(p), h(p)\big)$ implies that the price is less than the price that solves $\partial\bar{\pi}(p)/\partial p = 0$ under the assumption that $U_1(.,.) > 0$ and $U_2(.,.) > 0$.

3 Comparative statics

3.1 Demand changes

Baldenius and Reichelstein (2000) discuss comparative statics in monopoly and distinguish between uniform changes in consumers' willingness to pay and demand-increases by rotation. Consider uniform changes in willingness to pay. In the standard example of a monopolist in a market with linear demand, the profit-maximising price increases by a rate of half of the size of the shift. When the monopolist who operates under the same linear demand curve follows a safety-first strategy, a uniform change in willingness to pay does not affect the optimum price. The latter conclusion follows from inspection of $(p-c)f'(p)+f(p)=f(p)-(p-c)^{-1}F$, which shows that the optimum price equalises marginal expected profit (the left-hand side) to expected profit per profit margin (the right-hand side). Under a linear demand curve, a shift in willingness to pay affects both measures by the same amount. Looked at this way, it is straightforward that a parallel shift of expected demand leaves the optimum price unchanged when the monopolist aims at maximising the likelihood of survival.[7]

With respect to a change of demand by rotation, suppose that actual sale is $sf(p)+\theta$, where s is the change. Under this shift the effect on average demand depends on the price, but the variance of demand remains the same. Concerning a change of s the first-order condition is:

$$p^{*}sf'(p^{*})+sf(p^{*})-csf'(p^{*})=sf(p^{*})-(p^{*}-c)^{-1}F. \tag{9}$$

It is easy to see that $dp^{*}/ds=-(p^{*}-c)f'(p^{*})/\Lambda_{pp}$ where $\Lambda_{pp}<0$ is the second-order condition corresponding to equation (9). That is, $dp^{*}/ds<0$. The reason that the price goes down when demand increases by rotation is that the variance of profit goes up. This makes the incentive for cautiousness stronger and explains why the price goes down. To the contrary, a monopolist who maximises expected profit chooses the price according to $p^{\pi}f'(p^{\pi})+f(p^{\pi})-cf'(p^{\pi})=0$ and it is straightforward that a demand change by rotation leaves the price unchanged.

3.2 Cost changes

It is easy to see from the first-order condition, equation (9), that an increase of the fixed cost results in a higher optimum price. That is, when the monopolist is exposed to higher risk of financial distress through a higher fixed cost, she

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[7] It is easy to verify that the optimum price responds negatively to an increase in Δ when demand is given by $x=(p-\Delta)^{-\alpha}+\theta$ in situations where the monopolist maximises expected profits or minimises the likely occurrence of a dread event.

responds by increasing the price she charges. Kahneman and Tversky (1979) suggest that potential gains and losses account for decisions involving risk. In particular, agents might take on more risk when gains have moderate probabilities. Actually, this is what is happening when the fixed cost increases. Other things equal, this makes the likely occurrence of success go down, and the response of increasing the price implies more risk in the sense that the variance of profit goes up.

It is well known that under linear demand, an increase of the fixed marginal cost increases the price by fifty percent of the cost increase when the monopolist maximises expected profit. In contrast, when optimal pricing follows the safety-first criterion we have:

$$dp^*/_{dc} = \left((p^* - c)^3 2F - f''(p^*)\right)^{-1}(p^* - c)^3 2F. \tag{10}$$

Equation (10) shows, in the case of linear demand, that the final price changes one-to-one with a change in the constant marginal cost. When demand is convex, it is evident that the pass-through rate of marginal cost increases is more than a hundred percent. To expand on the intuition why the safety-first principle suggests (relatively speaking) strong price-responses to changes in the fixed marginal cost, notice that, under the safety first principle, the monopolist is concerned with the difference between expected sale and expected zero profit sale. Under linear demand and cost, the change in expected sale following a price change is just the slope of the demand curve. Thus, adjusting the price on a one-to-one basis to cost leaves the equality between expected sale and expected zero profit sale intact. In contrast, a monopolist maximising expected profits realises that increasing price on a one-to-one basis to cost reduces marginal revenue by more than it reduces marginal cost showing that the price increase is less strong.

This observation has implications for analysis of tax incidence. Under an excise tax with a tax rate of $t,$ the first-order condition in equation (8) modifies to $(p - c - t)f'(p) + (p - c - t)^{-1}F = 0$ which (we assume) solves for $p^*(t)$ when the corresponding second-order condition is met. In this case, $dp^*(t)/dt > 1$

when demand is convex.[8,9] The fact that the tax pass-through exceeds unity might imply that an increase of the tax rate increases the monopolist's expected profit. In fact, under a linear demand curve we have (which is proved in the Appendix):[10]

Proposition 4. Let $\mu(p) = pf'(p)/f(p)$ and $\xi(p) = pf''(p)/f'(p)$. When $\mu(p^*(t)) > ½\xi(p^*(t))$, the monopolist's expected profit increases with an increase of the tax rate.

Proposition 4 summarises the conditions that give rise to profit-increasing taxes. The existence of profit-increasing taxation owes to the fact that the firm, when trying to avoid a dread event, sets a price that is lower than the price that maximises expected profit. Notice in passing that Proposition 4 reflects a difference between a quantity and a price-setting monopolist. When the monopolist sets price before observing demand, demand uncertainty implies cost uncertainty. Therefore, the unit tax affects pricing because it affects profit's variance. As explained, when the monopolist sets quantity as in Day et al. (1971) and Arzac (1976) the introduction of a tax does not change profit's variance and the firm's decision is unchanged.

Because the tax drives up the price, there is a beneficial profit effect of taxation. When this effect is sufficiently strong, profits go up. Broadly speaking, this happens when the elasticity of $f(p)$ exceeds the elasticity of $f'(p)$ to a sufficient degree, or when the graph of $f(p)$ is sufficiently curvy (evaluated at $p^*(t)$). This means that the price can increase a lot, comparatively speaking, without strong negative effects on sale. However, an increase of the tax rate increases the likelihood of negative profits. This follows since the tax's effect

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8 We have $dp^*(t)/dt = \Lambda_{pp}{}^{-1}(f'(p^*(t)) + (p^*(t) - c - t)^{-2}F)$, where $\Lambda_{pp} = 2f'(p^*(t)) + (p^*(t) - c - t)^{-2}f'^*(p^*(t)) < 0$ is the second-order condition after rewriting by use of the first-order condition. $\Lambda_{pp} < 0$ when the demand curve is not "too curvy."

9 We can compare, under the assumption of normally distributed profits, to the situation where the monopolist maximises the expected utility of profits by maximising $E(U(\pi)) = E(\pi) - \beta VAR(\pi)$, where β reflects the monopolist's attitude to risk. When $\Lambda_{pp} < 0$ is the second order condition we have $dp^*(t)/dt = \Lambda_{pp}{}^{-1}(1 - 2\beta\sigma_\theta)$. Thus, $dp^*(t)/dt > 1$ when $0 > \Lambda_{pp} > (1 - 2\beta\sigma_\theta)$.

10 Some manipulations show that Proposition 4 is consistent with second order conditions when $½\xi(p^*(t)) < \mu(p^*(t)) < ½(p^*(t) - c - t)\xi(p^*(t))$.

through the price cancels due to the first order condition leaving only the direct negative effect on the firm's cost.[11]

In a partial model, the standard measure of welfare is the sum of consumer and producer surplus. The positive profit effect of the tax, which comes about because the price goes up, is equivalent to an increase in producer surplus. The price increase is harmful with respect to consumers and it is straightforward to verify, in spite of the positive effect on the monopolist's profit, that the standard conclusion still applies, that the sum of changes in consumer and producer surplus exceeds the tax burden. Following Weyl and Fabinger (2013), the change of the value of expected consumer surplus is:

$$E\left(\Delta CS_{t_0}^{t_1}\right) = -E\left(\int_{t_0}^{t_1} \rho(t)\big(f\big(p(t)\big) + \theta\big)dt\right), \tag{11}$$

where $\rho(t) = dp^*(t)/dt$ is the pass-through. Because $E(\theta) = 0$, the change in consumer surplus follows:

$$E\left(\Delta CS_{t_0}^{t_1}\right) = -\int_{t_0}^{t_1} \rho(t) f\big(p(t)\big)dt, \tag{12}$$

which implies that $E(dCS/dt) = -\rho(t)f\big(p(t)\big)$. The effect on expected profit is $d\bar{\pi}/dt = \Big((p-c-t)f'\big(p(t)\big) + f\big(p(t)\big)\Big)\rho(t) - f\big(p(t)\big)$. Proposition 4 suggests that $d\bar{\pi}/dt$ is positive for some parameter configurations. In this case, we have $\rho(t) > 1$ and it follows that the total burden net of the tax revenue, which is $-(p-c-t)f'\big(p(t)\big)\rho(t)$, is positive.

4 Inelastic Pricing

When the monopolist behaves cautiously and lowers her price, it is possible that the optimum price is in the inelastic region of the demand schedule. To examine this possibility we can rewrite Equation (8) as:

[11] Differentiate $f(p) - (p-c-t)^{-1}F$ with respect to the tax rate to get

$d\big(f(p) - (p-c-t)^{-1}F\big)/dt = -2F/(p-c-t)^{-2} < 0.$

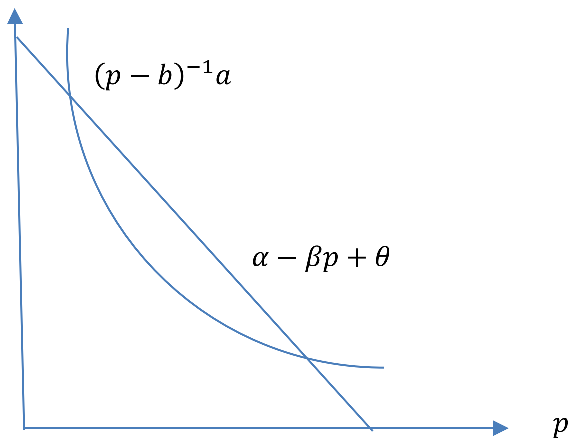
$$f(p^*)(E_d(p^*)+1) = cf'(p^*) + f(p^*) - (p^* - c)^{-1}F, \tag{13}$$

where $E_d(p^*)$ is the price elasticity of demand at the price p^*. If the right-hand side in equation (13) is positive, the monopolist decides on a price lying in the inelastic part of the demand schedule. In fact, the numerical value of the elasticity is $|E_d(p^*)| = (E_d(p^*) - b)^{-2} f(p^*)^{-1} F p^*$. Let us consider the situation where the marginal cost is zero. In this situation, a few manipulations reveal that the firm engages in inelastic pricing when $p^* f(p^*) > F$, saying that the optimal price is in the inelastic range when the monopolist expects positive on-average profits and when the marginal cost is vanishing. We summarise this as:

Proposition 5. When the monopolistic firm follows the safety-first principle, the optimum price under zero marginal cost involves inelastic pricing under the condition that the firm expects positive profits.

To further explain the occurrence of inelastic pricing, suppose that demand follows $x = \alpha - \beta p + \theta$. In this situation the first-order condition for p^* solves for $p^* = c + \sqrt{\beta^{-1}F}$. The price that maximises expected revenue is given by $p^r = ½\beta^{-1}\alpha$. In turn, the monopolistic firm engages in inelastic pricing whenever the revenue-maximising price exceeds the optimal price, or $\alpha \geq 2\left(c\beta + \sqrt{\beta F}\right)$. It is easy to understand this condition by using Figure 1. The slope of the demand function is $-\beta$ and the slope of $(p-c)^{-1}F$ is $-(p-c)^{-2}F$. Now, the optimal price is in the inelastic segment of the demand schedule when, evaluated at $p = p^r$, $-(p-c)^{-2}F \geq -\beta$ which is the same thing as $\alpha \geq 2\left(c\beta + \sqrt{\beta F}\right)$.

Figure 1



It is immediately clear that a high value of α and low values of c, β and F are conducive to inelastic pricing. Because an increase of α drives up the revenue-maximising price while it leaves the optimal price unchanged, a higher α means that it is more likely that p^* falls short of p^r. To state this in a slightly different way, notice that a high value of α, other things equal, produces, on average, profits that are significantly higher than the critical profit value (which we set to zero). In this situation, the firm reduces the price in order to reduce the variance. A low value of α means that the firm is close to the critical value and it is not beneficial to sacrifice profits for reduced variance. In addition, a decrease of F, c or β drives up the expected profit so that it exceeds the critical value, and the firm then sacrifices some of this profit to obtain reduced variance. Formally, notice that lower values of F and c drive down the optimal price without affecting the revenue-maximising price, showing why low values of c and F are associated with inelastic pricing. The effect of a change in β is more complex because the optimal and the revenue-maximising prices change. However, $p^r/p^* = ½\left(c\beta + \sqrt{F\beta}\right)^{-1}\alpha$, showing that the optimal price falls short of the revenue-maximising price as $\beta \to 0$.

We shall mention that Andersen and Nielsen (2013) show that inelastic pricing is consistent with the behaviour of a risk averse monopolist who maximises the expected utility of profit. When comparing the two results it must be noted that the safety-first principle and expected utility maximisation are not consistent. Nevertheless, the two approaches share qualitatively the result on pricing. However, the comparative statics differ since Andersen and Nielsen (2013) report that an increase of the fixed cost drives down the price. Oppositely, we show that the price goes up as the fixed cost goes up—in agreement with Kahneman and Tversky (1979) who suggest that risk becomes more acceptable as gains become less likely.

5 Discussion

In this paper we ask to what extent the pricing behaviour of a price-setting monopolist guided by the safety-first principle mimics actual pricing practices. In the introduction we listed some of the observations that have been made with respect to pricing. Firstly, the case appears that many firms diverge from the pure profit-maximising behaviour because they use mark-up pricing. When a monopolist applies the safety-first principle, a change in the unit cost, inclusive of an increase in a unit tax, is passed through to price on a one-to-one basis for the case of linear demand. This is not the case for a monopolist who maximises expected profit. Secondly, it seems that monopolies in the market for performance goods, such as selling access to football, price in the inelastic range of the demand curve. A similar finding applies to the monopoly market for tobacco in Sweden. For both cases it seems reasonable to say that production is characterised by a fixed cost and negligible variable cost. This is also the behaviour that is predicted when we apply the safety-first principle to a price setting monopolist.[12] Thirdly, Kahneman and Tversky (1979) argue that risk a higher fixed cost (which makes it more likely that the profit is unacceptably low) implies that the optimum price—and, therefore, risk—goes up. Finally, the hypothesis that the monopolist uses the safety-first principle goes some way in explaining the fact that price responds more strongly to cost changes than to demand changes (cf. Kahneman et al., 1986).

Previous examinations of monopolistic pricing under uncertainty, for example Baron (1971), Leland (1972), Kimball (1989) and Hau (2004), analyse the monopolist's optimal response to uncertainty under the assumption that the firm always survives. Our discussion supplements the standard analyses of monopolistic pricing under demand uncertainty because, in contrast, we assume that the monopolistic firm uses price to minimise the risk of financial default. The safety-first principle takes into account poor performances that correspond to the tail part of a profit's distribution. This concern seems relevant unless, for example, the firm's future gains fund current poor performance (Roy, 1952). Day et al. (1971) and Arzac (1976) show that a quantity-setting monopolist sets output lower than the quantity that maximises expected profits when avoidance of ruin matters. Our result extends this because we show that the safety-first criterion implies a lower price when a reduction in price reduces the variance of profit, and a higher price whenever the profit's variance is a decreasing function of the price. Together, these results show that it matters whether a monopolist sets price or quantity when the aim is to minimise the probability of a financially poor performance.

[12] Andersen and Nielsen (2013) show that a monopolist who maximises the expected utility of profits, rather than profits, might engage in inelastic pricing.

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Appendix

Proof of Proposition 2. Let us write the firm's profit as $\pi(p) = \bar{\pi}(p) + u\sigma(p)$. A dread event is $\pi(p) \leq k$. The dread event occurs when $u \leq -\sigma(p)^{-1}(\bar{\pi}(p) - k)$. Thus the firm chooses price to maximise $\sigma(p)^{-1}(\bar{\pi}(p) - k)$ which gives $\sigma(p)^{-2}\big(\bar{\pi}'(p)\sigma(p) - (\bar{\pi}(p) - k)\sigma'(p)\big) = 0$. That is:

$$\bar{\pi}'(p) = \sigma(p)^{-1}(\bar{\pi}(p) - k)\sigma'(p).$$

When expected profits satisfy $\bar{\pi}(p) > k$ we see that $\sigma'(p) > 0$ implies $\bar{\pi}'(p) > 0$ and $\sigma'(p) < 0$ implies $\bar{\pi}'(p) < 0$.

Proof of Proposition 3. When $Max_p\, h(p)$ has a solution—meaning that the second-order condition is satisfied—the solution to $Max_p U\big(\bar{\pi}(p), h(p)\big)$ implies that the price is less than the price that solves $\partial\bar{\pi}(p)/\partial p = 0$.

The solution to $Max_p\, h(p)$ is $h'(\tilde{p}) = 0$ and $h''(\tilde{p}) < 0$ because the second-order condition is satisfied. The first-order condition implies:

$$(\tilde{p} - c)f'(\tilde{p}) + f(\tilde{p}) = f(\tilde{p}) - (\tilde{p} - c)^{-1}(F + k), \tag{A.1}$$

showing that $\tilde{p} < \hat{p}$ where $\hat{p}$ satisfies $(\hat{p} - c)f'(\hat{p}) + f(\hat{p}) = 0$. The solutions are depicted in Figure 1. Consider knowing the solution to $Max_p U\big(\bar{\pi}(p), h(p)\big)$. Denote this by p^*. Now, inspect:

$$Z(p) = U_1\bar{\pi}'(p) + U_2 h'(p). \tag{A.2}$$

The first-order condition is $Z(p^*) = 0$. Clearly, at $p = \hat{p}$ we have $Z(\hat{p}) = U_2 h'(\hat{p}) < 0$ and at $p = \tilde{p}$ we have $Z(\tilde{p}) = U_1\bar{\pi}'(\tilde{p}) > 0$ showing that $\tilde{p} < p^* < \hat{p}$.

Proof of Proposition 4. The expected profit is:

$$\bar{\pi}(p) = (p - c - t)f(p) - F$$

and

$$d\bar{\pi}(p)/dt = (dp/dt - 1)f(p) + (p - c - t)f'(p)\, dp/dt,$$

where and $p = p^*(t)$. Now, when $dp/dt > 1$ it is possible that $d\bar{\pi}(p)/dt$ is positive which happens when:

$$1 - (dp/dt)^{-1} > -(p - c - t)\, f'(p)/f(p).$$

Using the expression for (dp/dt) this reduces to

$$f'(p)/f(p) > f''(p)/(f'(p) - (p - c - t)^{-2}F).$$

Using the first-order condition, this becomes $f'(p)/f(p) > ½\, f''(p)/f'(p)$.