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Tail Dependence of Financial Stocks and CDS Markets – Evidence Using Copula Methods and Simulation-Based Inference

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Abstract
Using copula methods and simulation-based inference, the authors investigate the association between the performance of a stock index formed by European financial institutions and a basket of CDS contracts of the same sector. Their analysis focuses on (i) assessing the dependence structure of the markets when extreme events occur, and (ii) checking the validity of the conclusion by Merton (*On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, 1974) and other similar structural models that there is an intensification of the relationship between stock prices and credit spreads after large negative shocks in the value of firms' assets. The authors show that there is a large tail dependence between the two portfolios. However, the dependence structure seems to be similar with respect to positive and negative innovations in the indexes. Their findings suggest that credit models' implications do not apply to financial firms, likely because the implicit subsidies from governments to financial institutions are distorting the dependency structure.

JEL G13 G14 G15
Keywords CDS markets; credit risk; Merton's model; copulas; simulation-based inference; banking

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1 Introduction

The market for credit derivatives, and in particular the market for credit default swaps (henceforth CDS), has experienced remarkable development over the last two decades. These markets are often seen as very opaque due to the lack of formally established clearing and settlement mechanisms providing reliable information on prices or volumes. Further, they are still barely subject to any regulation. The turnover of CDS markets has surged over the years, mostly through transactions executed over-the-counter. The transparency of these operations is a concern for financial supervisors, who fear that the concentration of massive risk-taking by a small group of financial intermediaries might jeopardize financial stability. Indeed, the role of these markets in the recent financial crisis has been widely scrutinized by the policy makers and has had extensive media coverage, particularly after the AIG bail-out.

CDS spreads reflect the default risk of the underlying debt instrument. The final payoff of these over-the-counter contracts depends on a credit event, and the spreads indicate the creditworthiness of the reference entity. These derivatives may be used to hedge risk or for speculation; they may also allow investors to separately transact the credit risk of the reference entity and to split funding from default risk.[1] Financial institutions are one of the major participants in the CDS markets, since they allow those institutions to hedge and to diversify their exposure to illiquid bonds and/or loans/receivables. Indeed, some arguments in favour of these instruments are that they provide additional liquidity to the bond market, promote risk sharing between market participants, and allow the creation of synthetic portfolios of bonds.

The rapid growth of this market, along with a severe financial crisis experienced in Europe, induced a relevant discussion in the literature on the impact of credit risk derivatives on financial stability. In fact, this discussion had started in the years prior to the aforementioned crisis, with some authors defending the

[1] CDS is a bilateral financial contract in which one counterparty (the protection buyer or buyer) pays a periodic fee, typically expressed in basis points per annum on the notional amount, in return for a contingent payment by the other counterparty (the protection seller or seller) after a credit event of the reference entity. The contingent payment is designed to reflect the loss incurred by creditors of the reference entity in the event of a default. The settlement mechanism depends on the liquidity and availability of reference obligations.

position that CDS can stimulate financial stability through their ability to improve credit risk allocation, as a consequence of a more liquid and diversified market for credit risk transfers. For instance, Alan Greenspan[2] argued that these new financial instruments allowed the sophisticated financial institutions to reduce their credit risk, transferring it to less leveraged market participants. In contrast, others suggest that the CDS market has been used by large financial institutions to leverage their positions and to perform regulatory arbitrage.

One of the interesting features of these financial instruments is that they provide us a way to assess the interaction between stock performance and credit risk. The linkage between credit spreads and stock prices is sustained by credit risk structural models, such as the Merton (1974) model. The author values equity and debt as contingent claims over the firm's assets. According to Merton, the default probability of a company is a non-linear function of the value of the assets, the asset price volatility, and the debt-equity ratio. Consequently, the returns of debt claims and stocks should be correlated, particularly when default risk surges. This is because the value of debt becomes more sensitive to changes in the asset's value when a firm enters into financial distress. When the credit risk is low, debt claimers hardly benefit from increases in firm assets' value because their upside potential is limited, in contrast with stockholders who own residual claims (with unlimited upside potential).

Duffie (1999) shows that, subject to some assumptions, a long position in a par priced floating rate note and the purchase of a CDS contract with the same face value of protection results in a position with no credit risk. Hence, the CDS spread should be equal to the credit spread of the par priced floating rate note. In that sense, one should expect a similar association between bond credit spreads and stock prices and between CDS spreads and stock prices, because bond credit spreads and CDS spreads are close substitutes. In theory, when the equity and debt rewards are not proper, arbitrage based on the firm capital structure is possible. Thus, if a company CDS spread is higher (lower) than it should be (given the stock price as well), an arbitrageur may obtain riskless profit from selling (buying) CDS

[2] From Greenspan's speech "Economic Flexibility" before Her Majesty's Treasury Enterprise Conference (London, 26 January 2004).
http://www.federalreserve.gov/boarddocs/speeches/2004/20040126/default.htm

contracts and buying (selling) shares. This way, arbitrage forces the equilibrium between the two markets.

Our research addresses the interaction between the credit risk and the performance of financial stocks. This paper pursues two research questions. First, we examine the dependence structure of the markets when extreme events occur. For that reason, the conclusions of this paper may be relevant for risk managers. Second, we aim to check the validity of the conclusion of Merton (1974), and other similar structural models, concerning the upsurge of the association between stock prices and credit spreads when extreme innovations occur. In that sense, we assess the "too-big-to-fail" effect on the association between the performance of financial stocks and credit risk.

This study extends the thriving academic literature on the interaction of credit markets and stock markets. In doing so, we use the theory of copulas. Copula-based models provide a great deal of flexibility in modelling multivariate distributions, permitting the researcher to specify the models for the marginal distributions separately from the dependence structure (copula) that defines the joint distribution. In addition to flexibility, this method also facilitates the estimation of the model in phases, reducing the computational burden. We add to that analysis simulation-based inference with the aim of selecting the type of dependence structure that best fits the empirical data and of ascertaining the robustness of the results.

The contribution of this paper is relevant for several reasons. First, banks played an essential part in triggering the recent financial crisis, as well as being among the worst-hit players. Moreover, they still perform an important role in the economy, namely providing liquidity transformation and monitoring services. After the 2007 financial crisis, the importance of credit risk in the banking sector has increased and CDS spreads are seen as an indicator of a bank's weakness. CDS spreads are used to extract market perceptions about the financial soundness of banking institutions, and of systemically important banks in particular. Thus, an understanding of the relationships between CDS spreads of the financial sector and stock markets could be of interest to evaluate financial stability, and more precisely, it is of crucial importance in terms of market discipline.

Moreover, it is also important to evaluate the "too-big-to-fail" effect on the association between the performance of financial stocks and credit risk, in particular for systemically large banks. In this respect, it is of interest to gauge whether the incentives provided to the banking system are reflected in the association between spreads and stock returns. In that sense, we evaluate whether Merton (1974) and other similar structural model's assertions can be applied to the banking sector. Finally, CDS markets may threaten financial stability due to spillovers to other markets, namely the equity market and the bond market, and thus the results of this paper may help in understanding contagion.

This paper is structured as follows: Section two contains a literature review on this subject; Section three describes the sample; Section four presents the methodology and the empirical results; and finally, Section five displays the conclusions and presents a brief discussion of the implication of the results.

2 Literature Review

The empirical literature about the relationship between stock and debt market's performance is quite extensive. In the 90s, some empirical studies showed an empirical relation between stock returns and bond yield changes. For instance, Blume et al. (1991), Cornell and Green (1991), and Fama and French (1993) report a contemporary and slightly positive but statistically significant association between stocks and bond returns. Kwan (1996) concludes that changes of bond yields are positively influenced by changes of Treasury bond yields and negatively affected by contemporaneous and lagged stock returns. More recently, Alexander and Ferri (2000) show a positive association between the raw daily returns of stocks and bonds of financially distressed firms in the period 1994-1997. However, when stock abnormal returns are used instead of raw returns, the statistical association between the variables becomes non-statistically significant. Hotchkiss and Ronen (2002) do not find evidence that stock markets led bond markets, although they report a modest and positive contemporaneous association between them.

Longstaff et al. (2003) examine Granger causality between (weekly) changes of CDS spreads, changes of bond credit spreads, and stock returns. Their analysis focuses on US markets, and the results indicate that stock markets and CDS markets led corporate bond markets. Campbell and Taksler (2002) document an empirical relationship between the volatility of stock returns and bond yields. Norden and Weber (2009) study the relationships between stock markets, bond markets and CDS markets during the period 2000-2002 for a set of 58 firms [USA (35), Europe (20) and Asia (3)]. They find that (i) CDS markets react to stock market movements, and that the magnitude of that reaction is affected by the credit quality of the firm and by the liquidity of the bond market; and (ii) stock returns lead credit spreads and CDS spreads.

Bystrom (2005) analyses the association between the performance of a CDS iTraxx index and stock market returns during the period 2004–2005, and concludes that stock market returns ‘Granger’ cause CDS spread changes, but the reverse does not occur. Fung et al. (2008) report a negative correlation between CDS and stock indices performance. That correlation is higher amid financially distressed firms and, overall, the correlation surged after July 2007. This outcome is consistent with the Merton (1974) model: the decline of stock prices results in an increase of leverage, contributing to a rise of default risk and CDS spreads. The results also suggest that stock markets lead CDS markets, regardless of the firm’s financial situation. However, the volatility spillovers from the CDS markets to the stock markets are higher than the reverse.

Avramov et al. (2009) show that the effects of rating downgrades on stock prices and CDS spreads are greater amid financially distressed firms. Forte and Peña (2009) show that stock markets lead CDS and bond markets in price discovery. Forte and Lovreta (2009) show that the price discovery process changes with the financial situation of firms. The contribution of stock markets to price discovery is positively influenced by the turnover ratio of the stock market, the credit quality of the firm, and the reduced presence of negative adverse shocks. Stock markets appear to lead CDS markets, but that leadership has been decreasing over time.

The correlation between the two markets also appears to be asymmetric. For instance, Dupuis et al. (2009) conduct an empirical analysis on the influence of credit risk on the performance of stocks from the automobile industry using the theory of copulas. They show that stock returns and CDS spread changes are negatively correlated, being that correlation higher in the tails of the probability density functions (henceforth, p.d.f.). Gatfaoui (2007) also presents evidence of an asymmetric relation between the CDS market and the stock market.

Heyde and Neyer (2010) show that macroeconomic surroundings influence the impact of CDS markets on the stability of the banking sector. During recessions, CDS markets affect the stability of the banking sector regardless of the shock type (idiosyncratic or systematic), increasing the risk of a systemic crisis. However, in periods of moderate economic growth and during booms, idiosyncratic shocks will increase the systemic risk only if there are other channels of contagion as well.

This paper adds to the financial literature by focusing exclusively in the banking sector, which is by its nature and opaqueness very different from other economic sectors. Moreover, instead of analysing the lead-lag association between stock returns and CDS spread changes as other authors have, we concentrate our efforts in investigating whether the association between the markets at extreme conditions – in the tails – is consistent with what is conjectured by financial theory (Merton, 1974 and other structural models) and by conventional wisdom. The next section describes the data used in the remainder of the paper.

3 Data Description

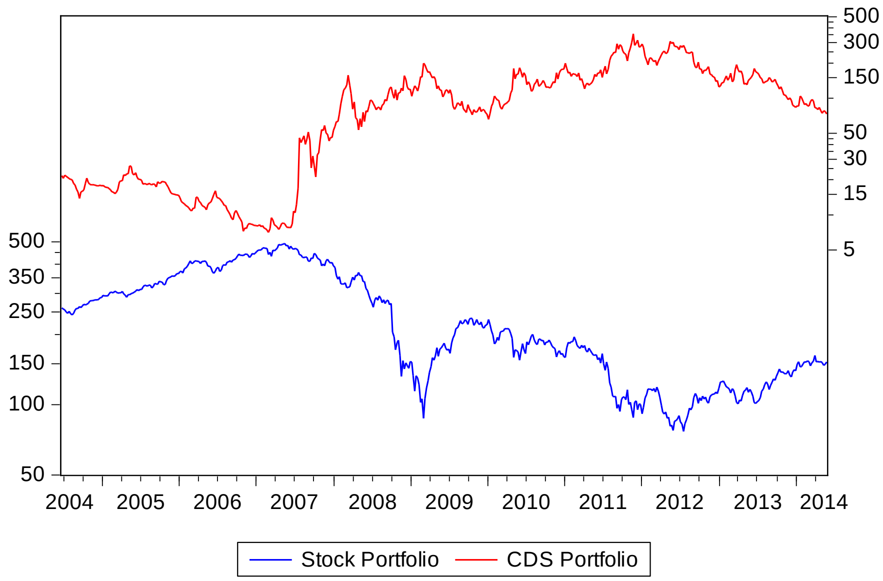
We perform our analysis using weekly data for the period between 03 December 2007 and 28 May 2014. We study the interaction between two well-known European indices of the financial sector: the DJ EuroStoxx Banks 600 (Bloomberg ticker: SX7E) and the iTraxx Europe Senior Financials 5Y TR from Markit (Bloomberg ticker: SNRFIN CDSI GENERIC 5Y Corp). The prices and spreads from these two indices are extracted from Bloomberg.

Although daily information is available for the two indices, we conduct the analysis using weekly data, to make our results immune to the microstructure noise that stems from the bid-ask bounce and non-synchronicity between the two indices. In addition, information is assimilated at different paces by stock prices and spreads within each index, causing autocorrelation of index returns and affecting spuriously their conditional means and variances.

The DJ EuroStoxx Banks 600 index is a capitalization-weighted basket and includes stocks from the banking sector (mostly large and systemically important banks) traded in countries that integrate the European Monetary Union. iTraxx indices are often used as proxies for default risk. These baskets cover firms and sovereign entities from different sectors and regions of the world, and usually display high liquidity and low bid-ask spreads. The iTraxx Europe Senior Financials 5Y TR is a basket of CDS contracts having European financial institutions as references. It is an equally weighted index of twenty-five European financial institutions reference entities (also large and systemically important).

Figure 1 displays the performance of the iTraxx Europe Senior Financials 5Y TR and the DJ EuroStoxx Banks 600. In the period before 2008, CDS spreads were small, denoting the reduced probability of default of the major European financial institutions. As of 2007, the default risk of financial institutions has surged sharply, in particular after the Bear Stearns failure, with investors perceiving a higher probability of default of financial companies. As for stock prices, they experienced pronounced declines between 2007 and mid-2009, and after 2010. Indeed, the figure suggests a negative co-movement between CDS spreads and stock prices. It is clear that news about financial firms has opposite effects on stock prices and CDS spreads. CDS spreads should increase after negative news, in particular when the likelihood of a credit event is greater.

Figure 1: DJ Eurostoxx Banks 600 (Left Axis) and iTraxx European Financial SNR Prices (Right Axis)



In this assessment we exclude the span prior to 2007. Before late-2007, the bid-ask spread associated to the CDS index was very high compared with the rest of the sample. Between mid-2004 and 03 December 2007 the average bid-ask spread was near 12.0%, whereas in the remainder of the span it was below 1.75% (non-tabulated results). In addition, the percentage of trading days with no CDS spread changes was also very high during the first span, which is consistent with the presence of large transaction costs. Short-term frictions in the CDS markets were very high prior to 2008, and that can bias the results. Moreover, as Figure 1 suggests, there is a clear structural break in the time series conditional mean and variance of the series of CDS index returns. For all the above-mentioned reasons, we exclude the span prior to 03 December 2007 from the subsequent analysis. The restricted sample comprises 363 weekly observations. The next section presents the methodology used in the assessment of the interaction between CDS spreads and stock returns.

4 Methodology and Empirical Results

The classical theory of portfolio management and risk management is based on the assumption that returns follow multivariate normal i.i.d. distributions. This assumption is very convenient because it allows practitioners to use correlations as a measure of dependence. However, that might not be a very realistic assumption about the behaviour of returns on financial markets. For instance, equity returns take joint negative extreme values more often than joint positive extremes, leading to the conventional wisdom that "stocks tend to crash together but not boom together". The opposite tends to take place in the CDS market, where the correlation is larger when higher positive extreme values occur.

Another way to assess the correlation structure of the series lies in the concept of copulas. Copula-based multivariate models permit modelling the marginal distributions separately from the dependence structure (copula) that links these distributions to form the joint distribution. This method increases the degree of flexibility in specifying the model, in comparison to other methods.

In some cases, such as in portfolio management, the concordance between extreme (tail) values of random variables is of interest. Very often the marginal distributions are asymmetric and/or the tail dependence is non-linear. This means that the correlation makes no sense as a dependence metric, given that it requires an elliptical multivariate distribution. In our analysis, we address the interaction of equity markets and CDS markets and, in particular, we assess the tail dependence between the two markets. Tail dependence captures the behaviour of the random variables during extreme events. We are interested in the co-movement of CDS spreads and stock prices not only in normal conditions, but especially in extreme distress situations. That requires a dependence measure for the upper and the lower tails of the multivariate distribution of the series. Such a dependence measure is related to the conditional probability that one series exceeds some high value, given that the other series exceeded the same value.

The copula of two variables is simply the function that maps the univariate marginal distributions to a joint distribution. The estimation by the copula method is performed in several stages. First, the marginal distributions are estimated separately from the dependence structure, simplifying the study of high-dimension multivariate problems. Before modelling the dependence structure of the series, one must first model their conditional marginal distributions.

$$Y_{i,t} = \mu_i(Z_{t-1}) + \sigma_i(Z_{t-1}) \times \varepsilon_{i,t}$$

$$\text{for } i=1,2$$

$$Z_{t-1} \in \mathcal{F}_{t-1} \sim F_i(0,1)$$

where $Y_{i,t}$ are the returns, and μ_i and σ_i denote the conditional mean and variance of the returns, respectively. Within this setup, it is assumed that each series will have potential time-varying conditional mean and variance, and that the standardized residual $\varepsilon_{i,t}$ is a white noise; i.e., it has a constant conditional distribution (with zero mean and a variance of one).

Thus, in a first pass, we model the conditional means and variances of the returns of the two indices. In order to capture the conditional mean, we use standard econometric approaches. We begin by calculating and plotting the ACF and PACF of the time series, along with the computation of the Ljung-Box-Pierce test and the Breusch-Godfrey LM test (results not reported). To model the conditional mean, we use ARMA models: we fit an AR (1) for the returns of the iTraxx Europe Financials SNR. In the case of the stock index, we do not detect the presence of autocorrelation. The autocorrelation of the original series is removed after applying the ARMA filters. To model the volatility of the returns, we employ GARCH(1,1) models.

After that, the standardized residuals are calculated as:

$$\hat{\varepsilon}_{i,t} = \frac{Y_{i,t} - \mu_i(Z_{t-1}; \hat{\alpha})}{\sigma_i(Z_{t-1}; \hat{\alpha})}$$

where $\hat{\alpha}$ is the vector of estimated parameters of the ARMA/GARCH model.

To further inspect whether the standardized residuals are i.i.d., we perform two alternative and complementary statistical tests: the runs test and the BDS test. According to the null hypothesis of the runs test, the first-stage noise variables are random. The runs test is a non-parametric statistical test that gauges the randomness of a two-valued data sequence. Specifically, it is used to check whether a sequence of values are mutually independent. The BDS test aims to capture non-linear serial dependence in time series. The results of these tests do not reject the null hypothesis of i.i.d. innovations in either of the series (non-tabulated results).

Estimating the dependence structure between the series entails the transformation of the standardized residuals into a uniform distribution using the marginal distribution function F_i. The estimation of F_i may be performed assuming parametric or empirical margins. Many choices are possible for the parametric model of F_i, including the Normal and the standardized Student's t, among others. We use the former two parametric marginal distributions along with the empirical distribution function (EDF) to ascertain the robustness of the results. As we will see latter on, the results do not seem to be affected by the choice of the marginal distribution. The EDF is calculated according to the following expression (Patton, 2012):

$$\hat{F}_i(\varepsilon) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1 * \{\varepsilon_{i,t} < \varepsilon\}$$

Combining the use of the empirical distribution function (EDF) of the standardized residuals with parametric models for estimating the conditional means and variances turns our model semi-parametric. Inference on the estimated dependence statistics can be performed either using the asymptotic distribution of the parameters of the model or using a bootstrap approach (assuming that the true conditional copula is constant through time). As in Rémillard (2010), we assume that the estimated parameters of the ARMA/GARCH model do not affect the asymptotic distribution of the dependence statistics and thereby the conditional mean and variance may be estimated independently of the copula.

We estimate eight different time-invariant copulas, as follows:

- Normal Copula – the normal copula is flexible as it allows for equal degrees of positive and negative dependence, and includes both Fréchet bounds in its permissible range. Normal copula has zero tail dependence, meaning that in the extreme tails of the distribution of the variables are independent.
- Clayton's Copula – the Clayton copula cannot account for negative dependence. It has been used to model correlated risks characterized by strong lower tail dependence and zero upper tail dependence.
- Rotated Clayton Copula – Copula rotation permits the transformation of copulas such that they may be used to model negative dependence also. When a copula has an upper tail dependence then the associated survival copula has a lower tail dependence. The Rotated Clayton copula imposes zero lower tail dependence and allows only for upper tail dependence.
- Frank Copula – the Frank copula has zero tail dependence. The dependence should be larger in the centre of the marginal distributions than in the case of the Normal copula.
- Gumbel Copula – Gumbel copula has zero lower tail dependence and cannot account for negative dependence. If the outcomes are known to be strongly correlated at high values but less correlated at low values, then the Gumbel copula is the right choice for modelling the concordance of the series.
- Rotated Gumbel Copula – it is similar to Gumbel Copula, but it can only account for negative dependence. Rotated Gumbel copula has zero upper tail dependence.
- Student's t Copula – it provides higher tail dependence than the Normal Copula. Student's t copula entails symmetric tail dependence.

– Symmetrised Joe-Clayton Copula – SJC copula parameters are the tail dependence coefficients, but in reverse order.

Along with time-invariant copulas, we also estimate four dynamic copulas: time-varying Normal copula, time-varying Student's t copula, time-varying Rotated Gumbel copula and time-varying SJC copula.

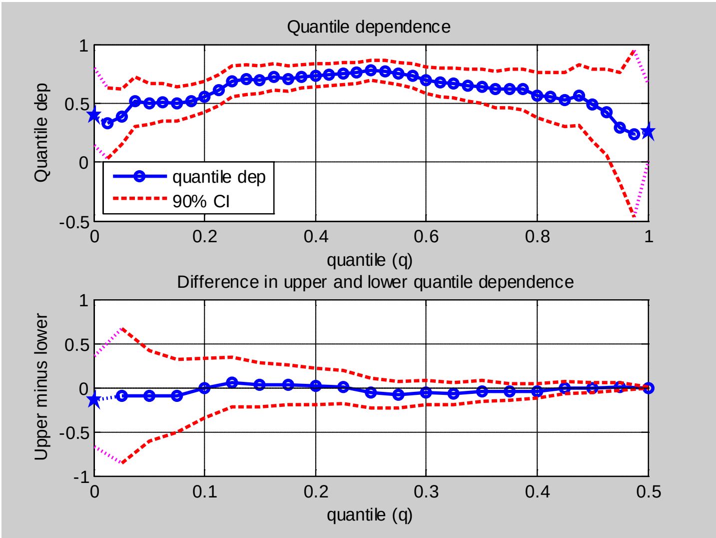
With the aim of making the results easier to interpret, we analyse the dependency structure of a long portfolio of stocks and a short portfolio of CDS contracts. Intuitively, the correlation between the two portfolios is expected to be positive. As a first step, we estimate the quantile dependence of the two time series innovations. The quantile dependence assesses the strength of the dependence between two variables in the joint lower or joint upper tails of their support. Quantile dependence is the probability that both variables lie above or below a given quantile q of their marginal distributions. It provides a good description of the dependence structure of two series.
The empirical quantile dependence of the series is calculated as follows:

$$\hat{\lambda}^q = \begin{cases} \dfrac{1}{T \times q} \times \sum_{t=1}^{T} 1\{U_{Stocks,t} \leq q, U_{CDS,t} \leq q\}, & 0 < q \leq 1/2 \\ \dfrac{1}{T \times (1-q)} \times \sum_{t=1}^{T} 1\{U_{Stocks,t} > q, U_{CDS,t} > q\}, & 1/2 < q \leq 1 \end{cases}$$

where $U_{Stocks,t}$ ($U_{CDS,t}$) corresponds to the implied probability of the filtered stock (CDS) returns at t under the EDF, T is the number of observations for each series and q respects to the quantile under analysis.

Figure 2 shows the (estimated) quantile dependence plot along with a 90% confidence interval based on a bootstrap simulation. The dependency between the two series is concentrated in the median of the margins. Further, that dependence is lower in the tails than in the median of the distribution. In spite of that, there is still a strong tail dependency between the series. We also compute the standard deviation of the quantile dependence through a bootstrap simulation and the corresponding confidence intervals. In effect, the latters are narrower near the median of the distribution than in the tails.

Figure 2: Quantile Dependence for the Eurostoxx Banks 600 Innovations and the iTraxx Financial Europe SNR Innovations



In addition, Figure 2 also presents the difference between the upper and lower tails of the series, along with a pointwise confidence interval for the differential. It suggests that there is no difference between the upper and lower tail quantile dependence frequencies. This aspect will be analysed in more detail later.

By estimating the strength of the dependence between the two variables as we move from the centre of the distribution to the tails, and by comparing the left tail with the right tail, we are able to capture more exhaustive information about the dependence structure than is provided by a scalar indicator such as the linear correlation or the rank correlation. In effect, some copulas, such as the Normal, the Frank and the Student's t copula, assume a symmetric dependence between the variables, and as a consequence this information is useful in choosing the right copula. We use two tests to measure symmetric dependence and tail dependency equality. Under the null hypothesis we have:

$$\lambda^{q} = \lambda^{1-q} \ \forall \in [0,1]$$

where λ is the dependence measure. If the null hypothesis is true, that means that the dependence structure of CDS and stock innovations is symmetric. Figure 2 provides some preliminary insights regarding this issue. Indeed, as highlighted earlier, a closer look to the aforementioned chart suggests that the quantile dependence is symmetric. To further explore that issue, we perform a statistical test proposed by Rémillard (2010). The author proposes a Chi-square test to gauge jointly asymmetric dependence for a set of different q's, instead of testing each q separately. Following Rémillard (2010), we run a co-joint significance test over the dependence measure at different quantiles:

$$H_0: R\lambda = 0$$

where $\lambda = [\lambda^{q1}, \lambda^{q2}, \lambda^{q3} \dots \lambda^{qk}]$ and $q \in \{0.025;\ 0.05;\ 0.10;\ 0.975;\ 0.95;\ 0.90\}$. Rémillard (2010) proposes a bootstrap estimate to implement the Chi-square test, which we also adopt in our analysis (see further details about this test on Rémillard [2010] or Patton [2012]). The test fails to reject the null hypothesis of a symmetric dependence between the variables (Table 1 – Panel A). This signifies that the dependence structure is similar in face of positive and negative innovations of equal absolute magnitude.

The second test addresses tail dependency equality in the tails, namely whether the tail dependence coefficients (i.e., the limits of the quantile dependence functions) are equal. More precisely we test whether:

$$\lambda^U = \lambda^L$$

In other words, it investigates whether right-tail dependence is similar to left-tail dependence. Tail dependence traces out the limiting proportion that one margin exceeds a certain threshold conditional on that the other margin has already exceeded that threshold. It is, thus, a measure of the dependence conditioned to the existence of extreme events. Herein, we test whether the dependence conditional to positive extreme innovations is equal to negative extreme innovations. The test is implemented using bootstrap inference methods (see again Patton [2012] for more details). The t-stat associated with this test is –0.562, which is not statistically significant (Table 1 – Panel B).

In view of these results we may conclude that the dependence structure of the variables appears to be equal in the presence of positive and negative extreme innovations. Taken together, these results reveal that the correlation between

Table 1: Testing for Asymmetric Dependence and Tail Dependence Equality

Panel A - Testing for asymmetric dependence

	Chi-stat	p-value
Testing for asymmetric dependence	0.114	0.990

Panel B - Testing for tail dependence equality

	t-stat	p-value
Testing for tail dependence equality	-0.562	0.574

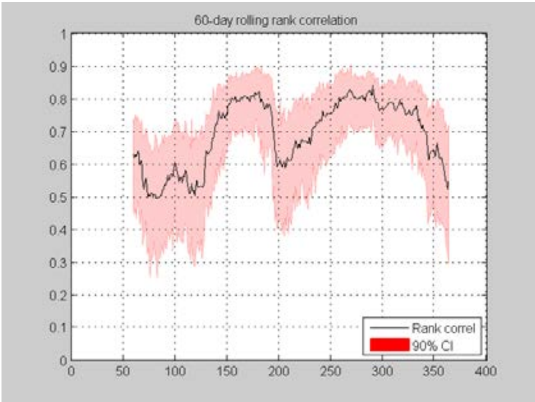
financial stocks and CDS is not affected by the sign of the innovations, contradicting Merton (1974), in that the correlation between the stock returns and spread changes should increase in the presence of large negative movements in the value of the firm's assets.

Figure 3 plots the 60 days rolling rank correlation for the innovations of stock and CDS returns and a bootstrap confidence interval for that correlation. The rank correlation between the standardized residuals ranges between 0.5 and 0.85 in the time frame covered by the analysis. Notice that the correlation is higher during the peak of the 2008 financial crisis and in the period marked by the sovereign debt crisis in Europe. The variability of the rank correlation through time suggests the presence of time-varying dependence. In effect, testing the presence of time-varying dependence could be informative, for example, before specifying a functional form or choosing between a dynamic and a static copula specification. We implement two different types of tests to evaluate structural breaks and time-varying dependence.

The first evaluates a break in the rank correlation at some specified point in the sample. Under the null hypothesis, the dependence measure before and after the breakpoint is equal to:

$$H_0: \rho^1 = \rho^2$$

Figure 3: 60 Day Rolling Rank Correlation between Financial Stocks and CDS Returns



where ρ^1 and ρ^2 denote the rank correlation before and after the breakpoint. The critical value for this test derives from an i.i.d bootstrap simulation. By generating the bootstrap samples, we obtain draws that impose the null hypothesis. Though simple to implement, this entails a prior knowledge by the researcher about the dependence structure of the variables. The critical value for the difference between the rank correlations of both sides of the sample (before and after some specified point in the sample) is obtained using i.i.d bootstrap. The p-values are obtained through 1,000 bootstrap simulations. We account for three different break points (25%, 50% and 75% points of the sample). As one can see in Table 2, we do not detect structural breaks in the first half of the sample; nevertheless, the structural break test hints at a possible break in the rank correlation in the middle of the second half of the sample.

The second test for time-varying dependence checks the break in the rank correlation coefficient at some unknown date. We follow Andrews (1993) in the implementation of the test. A critical value for this test is obtained again by using an i.i.d bootstrap. The null hypothesis of no structural break is again rejected, but this time at a 10% significance level (see Table 2).

The final test concerning time-varying dependence is based on the “ARCH LM” test for conditional variance proposed by Engle (1982). Instead of testing for one discrete one-time break in the dependence structure, it addresses the autocorrelation of a measure of dependence (rank correlation) using an

Table 2: Testing for Time-Varying Dependence and Structural Breaks

		p-value
Break	0.25	0.238
	0.50	0.174
	0.75	0.021
	Anywhere	0.065
AR (p)	1	0.834
	5	0.94
	10	0.42

autoregressive-type test. The null hypothesis of no autocorrelation of the dependence structure of the variables is not rejected. The table below outlines the results of the tests for time-varying correlation between the innovations.

Next, we estimate several copulas in order to find the one that better fits and depicts the data. Copulas are defined in terms of random variables *U1* and *U2* with standard uniform marginal distributions. Herein, we use the empirical distribution function to obtain uniform margins. So, along with the estimation of F_i as described earlier, a Kolmogorov-Smirnov test is performed for each of the standard uniform variables. The test does not reject the null hypothesis that the transformed standardized residuals are uniformly distributed (non-tabulated results).

As copulas separate the marginal distributions from the dependence structures, the appropriate copula for a particular application is the one that best captures the dependence features of the innovations. A first step to choose the right copula (the one that best fits the data) consists in evaluating AIC and BIC measures. Table 3 shows the log likelihood, AIC and BIC measures, and the lower and the upper tail derived from the estimated parameters of the copulas. The results of the estimation suggest that the copula that better fits the data is the Student's t copula, since it is the one that exhibits lower AIC and BIC values. Student's t copulas display strong and symmetric tail dependence. The coefficients of tail dependency translate the probability of two random variables both taking extreme values. In the present case, the tail dependence coefficient equals 0.73 if one attends to the results of the Student's t copula. Thus, one way to interpret the results is that 73% of extreme innovation episodes in one of the series are followed by an extreme innovation with the same sign in the other series.

Table 3: Summary Results from the (Static) Copula Estimation

	-LL	AIC	BIC	Lower Tail	Upper Tail	Par1	Par2
Normal	–119.08	–238.17	–238.18	0	0	0.694	NaN
Clayton	–03.98	–207.96	–207.97	0.6175	0	1.438	NaN
Rot Clayton	–4.69	–169.38	–169.40	0	0.5643	NaN	1.211
Frank	–121.92	–243.84	–243.86	0	0	6.062	NaN
Gumbel	–108.49	–216.99	–217.00	0	0.5552	1.884	NaN
Rot Gumbel	–121.45	–242.90	–242.91	0.5737	0	NaN	1.952
Student's t	–124.47	–248.95	–248.97	0.7317	0.7317	0.565	0.421
SJC	–117.52	–235.05	–235.07	0.5649	0.4211	NaN	0.712

One may also conclude for the existence of a symmetric tail dependence between CDS returns and stock returns of the financial sector. Student’s t copula outperforms others as Clayton, Rotated Clayton, Gumbel and Rotated Gumbel copulas that posit asymmetric tail dependence. Recall that Merton’s model postulates that the correlation between bonds and stocks should be greater when large negative movements in the value of the firms occurs, or put in another way, the conditional probability of having positive large movements in prices of a basket of stocks and in a short position on a basket of CDS contracts should be lower than having large negative movements in those instruments. Our findings suggest that this relationship is not observed for financial firms.

Table 4 displays the standard errors of the copula parameters estimates. We present three standard error types: naïve, bootstrapped standard errors, and corrected standard errors. Naïve standard errors are obtained from the matrix of the second derivatives of the likelihood function (Hessian). Bootstrapped standard errors are advocated by Chen et al. (2006) and Rémillard (2010), and are retrieved from a bootstrapped simulation in tandem of the (uniform) empirical distributions. Under the assumption that the copula is constant over time, we perform an i.i.d. bootstrap to calculate standard errors: (i) we randomly draw with replacement in tandem from the matrix of standardized residuals, (ii) and estimate the dependence measures from the bootstrapped sample; (iii) the before mentioned procedure is repeated 1000 times t; (iv) then, we calculate the standard errors of the parameters. Lastly, corrected standard errors are obtained from a correction of the standard errors à la White as in Chen et al. (2006). A closer look at the results for the Student's t copula reveals that the standard errors are very similar for the alternative approaches.

Table 4: Standard-Errors of the Copula Parameters Estimates

	Naive s.e.		Boot s.e.		Corrected s.e.	
	Par. 1	Par. 2	Par. 1	Par. 2	Par. 1	Par. 2
Normal	0.023		0.032		0.032	
Clayton	0.123		0.157		0.155	
Rot Clayton	0.113		0.144		0.132	
Frank	0.417		0.445		0.623	
Gumbel	0.081		0.091		0.096	
Rot Gumbel	0.084		0.093		0.104	
Student's t	0.025	0.055	0.025	0.073	0.025	0.065
SJC	0.033	0.081	0.073	0.049	0.041	0.119

Several authors have shown that AIC and BIC measures may be inappropriate to compare non-nested models. Thus, as an alternative to the AIC and BIC criterions, we also present the goodness-of-fit test of Chen et al. (2006), the PLR test, a pseudo-likelihood test that compares the ability of a copula to fit the data against another copula candidate using in-sample data. The results from this test are displayed in Table 5. Negative values of the test signify that copulas listed in columns outperform copulas presented in rows. We test Student's t copula against the remaining alternatives. The table below displays the test results and confirms Student's t copula as the one that better fits the data.

All in all, copulas displaying tail dependency equality and dependence symmetry dominate their peers. In the next subsection, we ascertain whether the results hold when using parametric margins instead of the EDF.

Table 5: In-sample PLR Tests of Student's t Copula against the Remaining Specifications

	Student's t
Normal	–0.95
Clayton	–2.19
Rot Clayton	–4.90
Frank	–0.47
Gumbel	–3.37
Rot Gumbel	–0.55
SJC	–3.70

Robustness Tests – Results for Alternative Parametric Marginal DistributionFunctions and Time-Varying Dependence

Up to now, we showed the results of copulas estimation using a semi-parametric approach. We estimate the conditional mean and variance using parametric models and use the empirical distribution function of the standardized residuals to conduct copula estimations. Indeed, as the true distribution function of the residuals is unknown, one feasible approach is to use their empirical distribution function.

One alternative method resides in using parametric marginal distribution functions instead. Two alternative parametric functions commonly used to fit the returns of financial assets are the Gaussian and Student's t marginal distributions (Horta et al, 2010). We estimate the copula functions using these parametric marginal distributions. The assumption that returns are Gaussian is used in theoretical literature, such as the mainstream option pricing theory. Empirically, it has been shown that returns are skewed and display positive excess kurtosis. Student's t marginal distribution, which accommodates a higher kurtosis, is an alternative to the Gaussian distribution.

We convert the standardized residuals of the returns into a uniform distribution assuming alternatively that they follow Gaussian and Student's t marginal distribution functions. If one attends to the AIC and BIC criterions, Student's t copula still outperforms the remaining alternatives when it is assumed that the marginal distribution function is Gaussian or Student's t (see Table 6).

We also compare the goodness-of-fit of Student's t copula with several specifications of dynamic copulas when using parametric margins. Student's t copula appears to outperform its peers that display a dynamic specification, since it is the one that presents lower AIC/BIC measures. This result is in accordance with the one obtained when testing for time-varying dependence (see Table 2), wherein the time-varying dependence hypothesis is rejected by the data.

Indeed, the earlier results are also corroborated when using Chen et al. (2006) PLR test. According to this test, none of the alternatives performs better than Student's t copula function (see Table 7).

Table 6: Summary Results from Copula Estimation – Parametric Marginal Distribution Functions

	Gaussian marginal distribution function			Student's t marginal distribution function		
	LL	AIC	BIC	LL	AIC	BIC
Normal	–100.19	–200.39	–200.40	–115.80	–231.61	–231.62
Clayton	–73.67	–147.35	–147.36	–97.96	–195.93	–195.94
Rot Clayton	–89.74	–179.49	–179.50	–92.96	–185.93	–185.94
Frank	–123.35	–246.71	–246.72	–120.38	–240.76	–240.77
Gumbel	–115.89	–231.79	–231.80	–115.72	–231.45	–231.46
Rot Gumbel	–101.13	–202.26	–202.27	–117.66	–235.33	–235.35
Student's t	–127.51	–255.02	–255.04	–128.40	–256.81	–256.83
SJC	–105.86	–211.74	–211.76	–118.23	–236.47	–236.49
TV rotated Gumbel	–105.65	–211.284	–211.252	–124.257	–248.498	–248.466
TV Normal	–108.913	–217.81	–217.778	–120.452	–240.887	–240.855
TV Clayton	–85.811	–171.606	–171.574	–85.0917	–170.167	–170.135
TV SJC	–105.34	–210.648	–210.583	–119.391	–238.75	–238.685
TV Student's t	–112.866	–225.699	–225.634	NaN	NaN	NaN

Table 7: In-sample PLR Tests of the Student's t Copula against the Remaining Specifications

	Student's t copula	
	Gaussian margins	Student's t margins
Normal	–1.14	–0.93
Clayton	–2.53	–2.34
Rot Clayton	–4.61	–4.86
Frank	–0.42	–0.47
Gumbel	–3.28	–3.32
Rot Gumbel	–0.58	–0.60
SJC	–1.21	–2.58

Robustness Tests – Results for Different Subsamples

In order to ascertain whether copula functions are stable over time, we also divide the sample into two different subsamples. In doing so, we attend to the results of structural break tests exhibited in Table 2. In effect, those tests suggest a possible break in the last tercile of observations in the sample. As such, we form two groups of observations. The first comprises the initial 199 observations, and the second covers the remaining observations. Then, we re-estimate the static copula functions in each subsample. It is important to keep in mind that the partitioning of the sample may result in greater estimation error. Notwithstanding that, the results are very similar to the ones reported earlier, in that Student's t copula is the one that best adjusts to the data.

As we can see in Table 8, Student's t copula outperforms other copulas in both subsamples. These results are confirmed not only by AIC/BIC criterions, but also by Chen et al. (2006) PLR test (see Table 9).

One aspect worth mentioning is that, in the first subsample, the second-best copula is the Frank copula, while in the second sub-period it is the rotated Gumbel. One possible interpretation for this outcome is that tail dependence is higher in the second subsample. In fact, we also observe an increase of the tail dependence coefficient associated to the Student's t copula (from 0.71 to 0.76; non-tabulated results).

Table 8: Summary Results from Copula Estimation – Different Subsamples

	Obs. 1-199			Obs. 200-363		
	LL	AIC	BIC	LL	AIC	BIC
Normal	–51.79	–103.59	–103.61	–69.26	–138.54	–138.56
Clayton	–46.65	–93.32	–93.33	–58.52	–117.06	–117.08
Rot Clayton	–36.13	–72.28	–72.29	–51.46	–102.93	–102.95
Frank	–54.19	–108.39	–108.41	–68.52	–137.06	–137.08
Gumbel	–45.82	–91.66	–91.68	–64.62	–129.25	–129.27
Rot	–53.70	–107.42	–107.43	–68.84	–137.69	–137.71
Student's t	–54.20	–108.41	–108.44	–71.19	–142.41	–142.45
SJC	–51.22	–102.46	–102.50	–67.98	–135.99	–136.02

Table 9: In-sample PLR Tests of Student's t Copula against the Remaining Specifications – Different Subsamples

	Obs. 1–199	Obs. 200–363
Normal	–0.719	–0.913
Clayton	–1.312	–1.790
Rot Clayton	–3.250	–3.373
Frank	–0.002	–0.686
Gumbel	–2.674	–1.867
Rot Gumbel	–0.138	–0.617
SJC	–3.074	–2.098

5 Conclusions

Merton (1974) provides the setup for the analysis of the relationship between CDS market's and stock market's performance. According to the model, a high debt-equity ratio would imply a greater correlation between stock and bond returns than a low debt-equity ratio. Intuition suggests that debt has a limited upside potential, and when the firm is performing well the bondholders do not profit from that situation as stockholders do. On the contrary, when the firm is in distress both stockholders' and bondholders' wealth are highly influenced by the market value of the firm's assets. Concurrently, large negative jumps in the firm's asset value should have a greater effect on the value of debt than positive jumps, *ceteris paribus*. In that sense, the relationship between stock prices and CDS spreads should increase with financial distress. This implies a non-linear association, where the co-movement intensifies when large negative movements in the value of the assets of the firms occur.

Understanding the relationships between CDS spreads of the financial sector and stock markets is important to evaluate financial stability, and more precisely is of crucial importance in terms of supervision, regulation and market discipline. Moreover, it allows us to evaluate the "too-big-to-fail" effect on the association between financial stock performance and credit risk, in particular for systemically large banks.

Using a copula-based approach we address the association between stocks of European financial institutions and CDS markets. We aim to accomplish two purposes: (i) analysing the dependence structure of the markets when extreme events occur, taking into account that sometimes banks are too big to fail; and (ii) checking the validity of the conclusion of Merton (1974) and other similar structural models regarding the intensification of the relationship between stock prices and CDS spreads during financial distress periods. We focus our analysis in the banking sector. We show that the conclusions of Merton (1974) do not apply to financial firms (banks). Our major findings are that (i) the structure of dependence between the two markets appears to be symmetric, and (ii) there is symmetric tail dependency between financial stock returns and CDS spread changes.

These findings contrast with Merton's (1974) assertion that large positive movements in firms' asset values should imply a lower dependence between stock prices and credit claims spreads than large negative movements. One possible reason for the inexistence of a higher negative tail dependence between the innovations of the series may reside in the too-big-to-fail effect; i.e., credit holders receive a subsidy from governments protecting them from bankruptcy costs, in contrast with equity holders whose capital is wiped out if the bank fails. In such cases, the poor financial situation of a bank is likely to affect severely stockholders' wealth, whereas bond holders are bailed out. As a consequence, spreads and stock returns do not co-move as they would if that subsidy did not exist, which in turn makes negative and positive tail dependence statistically similar.

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