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Relative Profit Maximization and Bertrand Equilibrium with Convex Cost Functions

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Abstract
This study derives pure strategy Bertrand equilibria in a duopoly in which two firms produce a homogeneous good with convex cost functions and seek to maximize the weighted sum of their absolute and relative profits. The study shows that there exists a range of equilibrium prices in duopolistic equilibria. This range of equilibrium prices is narrower and lower than the range of equilibrium prices in duopolistic equilibria under pure absolute profit maximization. Moreover, the larger the weight on the relative profit, the narrower and lower the range of equilibrium prices. In this sense, relative profit maximization is more aggressive than absolute profit maximization.

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Keywords Bertrand equilibrium; convex cost function; relative profit maximization

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1 Introduction

Using a model developed by Dastidar (1995), we study pure strategy Bertrand equilibria in a duopoly in which two firms produce a homogeneous good with convex cost functions and seek to maximize the weighted sum of their absolute and relative profits instead of their absolute profits. The relative profit of a firm is the difference between its absolute profit and the absolute profit of a rival firm.

For analyses of relative profit maximization, please see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013), Miller and Pazgal (2001), Schaffer (1989), and Vega-Redondo (1997).[1]

We believe that seeking relative profit or utility is based on human nature. Even if a person earns a large salary and if his brother/sister or close friend earns more, then he is not sufficiently happy and may be disappointed. In contrast, even if a person is very poor and if his neighbor has even less, then he may be consoled by that fact. In addition, firms not only seek to improve their own performance but also to outperform rival firms in the industry. The TV audience-ratings race and market share competition by breweries, automobile manufacturers, convenience store chains, and mobile-phone carriers (especially in Japan) are examples of such firm behavior.

We show that there exists a range of equilibrium prices in duopolistic equilibria. This range of equilibrium prices is narrower and lower than the range of equilibrium prices in duopolistic equilibria under pure absolute profit maximization.[2] Moreover, the larger the weight on the relative profit, the narrower and lower the range. In this sense, relative profit maximization is more aggressive than absolute profit maximization.

In Satoh and Tanaka (2013), we see a similar result in the case of linear demand functions and quadratic cost functions. In this study, we extend that result to a case of general demand functions and convex cost functions.

[1] In Vega-Redondo (1997) it was shown that the equilibrium in a Cournot oligopoly with a homogeneous good under relative profit maximization is equivalent to the competitive equilibrium. However, as shown in this paper the equilibrium in a Bertrand duopoly with a homogeneous good under relative profit maximization may not be equivalent to the competitive equilibrium.

[2] Dastidar (1995) proved that there exists a range of equilibrium prices in duopolistic equilibria under absolute profit maximization.

In the last section, we compare our results with the results obtained by other authors, such as Dastidar (1995) and Dastidar and Sinha (2011).[3]

2 The Model

There are two firms, A and B. They produce a homogeneous good. The price of the good of Firm A is p_A and the price of the good of Firm B is p_B. The outputs of Firms A and B are denoted by x_A and x_B, respectively. The firms set the prices of their goods, and consumers buy the good from the firm whose price is lower. Let $p = \min\{p_A, p_B\}$. Consumers' demand is represented by the demand function $D(p)$. The cost functions of Firms A and B are $c_A(x_A)$ and $c_B(x_B)$, respectively.

Similar to the model in Dastidar (1995) we make the following assumptions.

1. $D(p)$ is continuous and twice continuously differentiable.
2. There exists finite positive numbers p^{max} and x^{max} such that $D(p^{max}) = 0$, $D(0) = x^{max}$, and $D'(p) < 0$ for $0 \le p \le p^{max}$.
3. $c_A(x_A)$ and $c_B(x_B)$ are continuous, twice continuously differentiable and strictly convex.

Further, we assume that there is no fixed cost and the two firms have the same cost function. Thus, $c_i(0) = 0$ for $i \in \{A, B\}$.

If $p_A = p_B$, then each firm acquires a half of the demand, and the two firms constitute a duopoly. Thus, if $p_A = p_B$, then we have $x_A = x_B = \frac{1}{2}D(p)$. In contrast, if $p_A < p_B$ (or $p_B < p_A$), then Firm A (or Firm B) acquires total demand, and it becomes a monopolist.

If $p_A < p_B$, then the absolute profit of Firm A is as follows:

$$\pi_A^M(p) = pD(p) - c_A(D(p)).$$

[3] Ogawa and Kato (2006) and Dastidar and Sinha (2011) consider pure strategy Bertrand equilibria in a mixed duopoly with one private firm whose objective is maximization of its absolute profit and one public firm whose objective is maximization of the weighted sum of its absolute profit and social welfare. They show that the range of equilibrium prices in the mixed duopoly is the same as Dastidar (1995). If both firms in a duopoly are public, however, then the range of equilibrium prices is larger than in the case of a mixed duopoly (Dastidar and Sinha 2011).

M indicates *monopoly*. Of course the profit of Firm B is zero. Similarly, if $p_B < p_A$, then we have the following:

$$\pi_B^M(p) = pD(p) - c_B(D(p)).$$

The profit of Firm A is zero.

In contrast, if $p_A = p_B$, then the absolute profits of Firms A and B are as follows:

$$\pi_A^D(p) = \frac{1}{2}pD(p) - c_A(x_A),\ x_A = \frac{1}{2}D(p),$$

and

$$\pi_B^D(p) = \frac{1}{2}pD(p) - c_B(x_B),\ x_B = \frac{1}{2}D(p).$$

D indicates *duopoly*. In this case, $p = p_A = p_B$.

The objective of Firm A is the weighted sum of its absolute and relative profits. In a duopoly, it is expressed as follows:

$$\Pi_A^D = (1-\alpha)\pi_A^D + \alpha(\pi_A^D - \pi_B^D) = \pi_A^D - \alpha\pi_B^D,$$

and the objective of Firm B is

$$\Pi_B^D = (1-\alpha)\pi_B^D + \alpha(\pi_B^D - \pi_A^D) = \pi_B^D - \alpha\pi_A^D,$$

where

$$0 < \alpha < 1.$$

We call a firm in a duopoly a duopolist.[4] Since, at a duopolistic equilibrium $\pi_A^D = \pi_B^D$, we have

$$\Pi_A^D = \Pi_B^D = (1-\alpha)\pi_A^D.$$

[4] These objective functions are the same as that in Matsumura, Matsushima and Cato (2013). They consider a case where each firm in a duopoly maximizes its relative profit defined as $\pi_i^D - \alpha\pi_j^D$ where $j \neq i$ and $-1 < \alpha < 1$. The difference between us and them is the value of α. We think this should be positive, because the firm who cares about its relative profit wants to obtain its absolute profit, which is more than that of another firm.

We assume

$$\max_p \pi_i^M(p) > 0, \text{ and } \max_p \pi_i^D(p) > 0 \text{ for } i \in \{A,B\}.$$

In a monopoly, the absolute profit of a firm other than the monopolist is zero. Thus, the absolute profit and the relative profit of the monopolist are equal, and the objective of the monopolist is its absolute profit. If Firm A is a monopolist, then

$$\Pi_A^M = \pi_A^M,$$

and if Firm B is a monopolist, then

$$\Pi_B^M = \pi_B^M.$$

Without loss of generality, we assume $p_A \leq p_B$.

3 Preliminary Results

According to Dastidar (1995) for $i \in \{A,B\}$ we define the following:

$$\hat{p}_i \text{ such that } \pi_i^D(\hat{p}_i) = 0,$$

$$\tilde{p}_i \text{ such that } \pi_i^M(\tilde{p}_i) = 0,$$

$$\bar{p}_i \text{ such that } \pi_i^D(\bar{p}_i) = \pi_i^M(\bar{p}_i).$$

By symmetry of the model $\hat{p}_A = \hat{p}_B$, $\tilde{p}_A = \tilde{p}_B$, and $\bar{p}_A = \bar{p}_B$. So, we denote them by $\hat{p}$, $\tilde{p}$ and $\bar{p}$, respectively. In Dastidar (1995) the following results have been proved:

1. There exists a unique $\hat{p}$ in $[0, p^{max})$. (Lemma 1 in Dastidar 1995).
2. There exists a unique $\tilde{p}$ in $[0, p^{max})$. (Lemma 4 in Dastidar 1995).
3. There exists a unique $\bar{p}$ in $[0, p^{max})$. (Lemma 5 in Dastidar 1995).
4. $\hat{p} < \tilde{p} < \bar{p}$. (Lemma 6 in Dastidar 1995).

Now we define another critical price p_i^* by

$$\Pi_i^D(p_i^*) = \Pi_i^M(p_i^*).$$

In addition, by symmetry we have $p_A^* = p_B^*$, and so denote them by p^*. We show the following lemmas.

Lemma 1. *There exists a unique* p^* *in* $[0, p^{max})$, *and* $\tilde{p} < p^* < \bar{p}$ *for* $0 < \alpha < 1$.

Proof. See Appendix A. □

Lemma 2. p^* *is decreasing with respect to* α.

Proof. See Appendix B. □

4 Pure Strategy Bertrand Equilibrium

We verify the following result:

Lemma 3. *For* $p > \tilde{p}$ *we have*

$$\Pi_i^M(p) = \pi_i^M(p) > 0,$$

and for $p < \tilde{p}$

$$\Pi_i^M(p) = \pi_i^M(p) < 0.$$

Proof. See Appendix C. □

First we show the non-existence of a monopolistic equilibrium.

Theorem 1. *There is no monopolistic equilibrium.*

Proof. A monopolistic equilibrium is an equilibrium where Firm A is the monopolist. Suppose that $p_A < p_B$ and $p_A > \tilde{p}$, then Firm B can set p_B slightly lower than p_A and earn the positive absolute profit. If $p_A < p_B$ and $p_A = \tilde{p}$, then Firm A can set p_A slightly higher than $\tilde{p}$ but lower than p_B and earn the positive absolute profit, or Firm B can set $p_B = p_A$ and earn the positive absolute profit in a duopoly ($\hat{p} < \tilde{p}$). Of course, $p_A < \tilde{p}$ is not profitable for Firm A. □

Next, we show

Theorem 2. *There exists a range of equilibrium prices* $[\hat{p}, p^*]$ *in a duopoly.*

Proof.

1. Suppose $p_A = p_B = p$ and $p^* < p < \bar{p}$. The relative profits of the firms are zero. Firm B (or A) can set p_B (or p_A) slightly lower than p, and earn the positive absolute profit as a monopolist. Although that absolute profit is smaller than its absolute profit in a duopolistic equilibrium (because $p < \bar{p}$), its relative profit is positive and it is equal to its absolute profit because the profit of the rival firm is zero; thus we have

$$\Pi_A^M > \Pi_A^D.$$

2. Suppose $p_A = p_B = p$ and $\hat{p} \le p \le p^*$. If Firm B (or A) sets p_B (or p_A) lower than p, then it becomes a monopolist; however, in this case we have

$$\Pi_A^D \ge \Pi_A^M.$$

   Thus, there is no incentive to deviate from the equilibrium.

3. Of course, if $p_A = p_B = p$ and $p < \hat{p}$, the absolute profits of the firms are negative and their relative profits are zero; so each firm can set its price higher than the price of the rival firm, making its absolute profit zero and its relative profit positive since the absolute profit of the rival firm is negative because $\hat{p} < \tilde{p}$.

□

The range of equilibrium prices in a duopoly under absolute profit maximization is $[\hat{p}, \bar{p}]$ (Proposition 1 in Dastidar 1995), and $p^* < \bar{p}$. Therefore, the range of equilibrium prices in a duopoly under relative profit maximization is lower and narrower than that under absolute profit maximization.

Lemma 2 means that the larger the weight on the relative profit, the narrower and lower the range of equilibrium prices.

5 Comparison of the Ranges of Equilibrium Prices

In this section we compare the ranges of equilibrium prices among several cases.

Some of the critical prices are repeated below:
$\hat{p}$: the price at which the profit of each firm in a duopoly is zero.
$\tilde{p}$: the price at which the profit of a monopolist is zero.
$\bar{p}$: the price at which the profit of each firm in a duopoly is equal to the profit of a monopolist.

In Dastidar (1995) it was shown that if the cost functions are convex, $\hat{p} < \tilde{p} < \bar{p}$.

In both absolute and relative profit maximization, the range of equilibrium prices is represented as $[\hat{p}, p^*]$. p^* is the price at which the value of the objective function of a firm in a duopoly is equal to the profit of a monopolist. Note that, in a monopoly, the absolute and relative profits are equal.

1. Pure absolute profit maximization (Dastidar 1995)

   Since $p^* = \bar{p}$, the range of equilibrium prices is $[\hat{p}, \bar{p}]$.

2. Pure relative profit maximization

   If the firms have the same cost function in a duopoly (i.e., the model of the duopoly is symmetric), then the relative profit of each firm in the duopoly at equilibrium is zero. Then, p^* is equal to $\tilde{p}$, and the range of equilibrium prices is $[\hat{p}, \tilde{p}]$. Since $\tilde{p} < \bar{p}$, the range of equilibrium prices under pure relative profit maximization is lower and narrower than that under pure absolute profit maximization.

3. Maximization of weighted average of absolute and relative profits

   In this case, p^* satisfies the relation $\tilde{p} < p^* < \bar{p}$. Therefore, the range of equilibrium prices in this case is lower and narrower than that under pure absolute profit maximization, but higher and wider than that under pure relative profit maximization. The larger the weight on the relative profit, the lower and narrower the range of equilibrium prices.

4. Public firms (Dastidar and Sinha 2011)

   If both firms in a duopoly are public firms, at equilibrium

   > the value of the objective function of a firm in a duopoly
   > $= (1-\beta)\times$profit of a duopolist$+\beta\times$(joint profits+consumers' surplus)
   > $= (1+\beta)\times$profit of a duopolist$+\beta\times$consumers' surplus,

   where $0 < \beta < 1$, and

   > the value of the objective function of a monopolist
   > $= (1-\beta)\times$profit of a monopolist$+\beta\times$(profit of a monopolist+consumers' surplus)
   > =profit of a monopolist$+\beta\times$consumer surplus.

   The condition for them to be equal is equivalent to the following condition.

   > $(1+\beta)\times$profit of a duopolist=profit of a monopolist.

   This condition corresponds to a case where the weight on the relative profit is negative ($-\beta$) in the case of maximization of weighted average, or a case of $\alpha = -\beta$ in the model of this study although we assume $0 < \alpha < 1$ in the previous sections. If the weight on the relative profit is zero, then $p^* = \bar{p}$. Since p^* is decreasing in α, $p^* > \bar{p}$ in the case of public firms; thus the upper bound of the range of equilibrium prices is higher than that under absolute profit maximization.

   The lower bound of the range in the cases of pure absolute profit maximization, pure relative profit maximization, and weighted average maximization is the price at which the absolute profit of a firm in a duopoly is zero. However, in the case of public firms the lower bound is the price at which "$(1-\beta)\times$profit of a duopolist$+\beta\times$(joint profits+consumers' surplus)" is zero. This lower bound seems to be lower than $\hat{p}$; thus the range of equilibrium prices in the case of public firms expands in both directions.

We depict a comparison of the ranges of equilibrium prices in Figure 1. π^M is the profit of the monopolist, π^D is the profit of a firm in a duopoly, and CS denotes consumer' surplus.

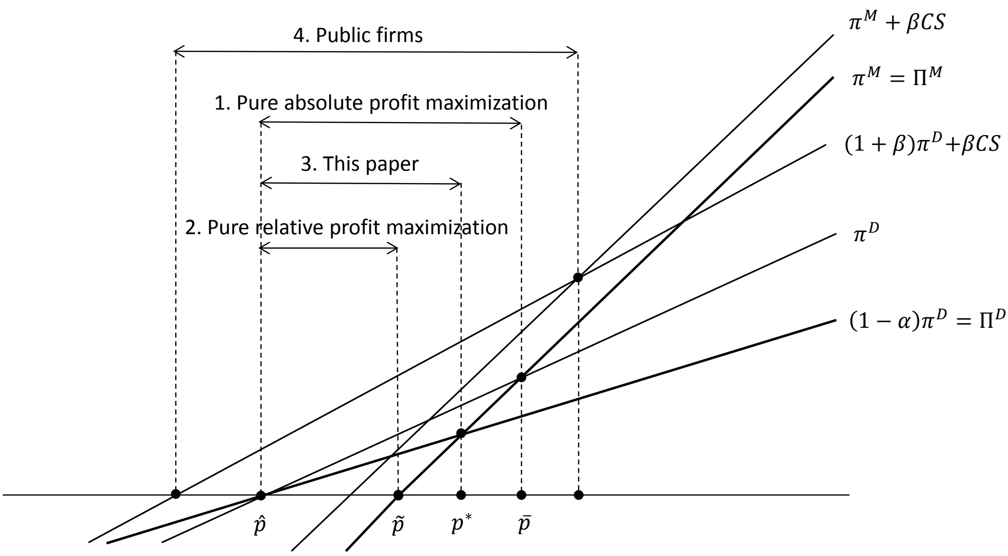

Figure 1: Comparison of the ranges of equilibrium prices

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Appendix

A Proof of Lemma 1

Note that

$$\Pi_i^D(p) = (1-\alpha)\pi_i^D(p),$$

and

$$\Pi_i^M(p) - \Pi_i^D(p) = \pi_i^M(p) - (1-\alpha)\pi_i^D(p).$$

If $p = \bar{p}$, $\pi_i^M(p) - \pi_i^D(p) = 0$, and so

$$\Pi_i^M(\bar{p}) - \Pi_i^D(\bar{p}) = \alpha\pi_i^D(\bar{p}).$$

If $p = \tilde{p}$, $\pi_i^M(p) = 0$, and so

$$\Pi_i^M(\tilde{p}) - \Pi_i^D(\tilde{p}) = -(1-\alpha)\pi_i^D(\tilde{p}).$$

Now

$$\frac{\partial \pi_i^D(p)}{\partial p} = \frac{1}{2}\left\{D(p) + D'(p)\left[p - c_i'\left(\frac{1}{2}D(p)\right)\right]\right\}. \tag{1}$$

When $\pi_i^D(p) \leq 0$, we have $-c_i\left(\frac{1}{2}D(p)\right) \leq -\frac{1}{2}pD(p)$.

Since $c_i(\cdot)$ is strictly convex,

$$c_i\left(\frac{1}{2}D(p)\right) - c_i(0) = c_i\left(\frac{1}{2}D(p)\right) < \frac{1}{2}D(p)c_i'\left(\frac{1}{2}D(p)\right),$$

or

$$-c_i\left(\frac{1}{2}D(p)\right) > -\frac{1}{2}D(p)c_i'\left(\frac{1}{2}D(p)\right).$$

This means

$$-\frac{1}{2}D(p)c_i'\left(\frac{1}{2}D(p)\right) < -\frac{1}{2}pD(p).$$

Therefore,

$$p < c_i'\left(\frac{1}{2}D(p)\right).$$

Since $D'(p) < 0$, from (1) we find that when $\pi_i^D(p) \leq 0$, $\frac{\partial \pi_i^D(p)}{\partial p} > 0$ holds. Thus, the continuity of $\pi_i^D(p)$ and the uniqueness of $\hat{p}$ means that $\pi_i^D(\bar{p}) > 0$, and so $\Pi_i^M(\bar{p}) - \Pi_i^D(\bar{p}) > 0$ for $\alpha > 0$ because $\hat{p} < \bar{p}$.

Similarly $\Pi_i^M(\tilde{p}) - \Pi_i^D(\tilde{p}) = -(1-\alpha)\pi^D(\tilde{p}) < 0$ for $0 < \alpha < 1$ because $\hat{p} < \tilde{p}$.

Therefore, by the continuity of $\pi_i^M(p)$ and $\pi_i^D(p)$ there exists p^* such that $\Pi_i^M(p^*) - \Pi_i^D(p^*) = 0$ between $\tilde{p}$ and $\bar{p}$, that is, $\tilde{p} < p^* < \bar{p}$.

We show uniqueness of p^*. Note that

$$\begin{aligned}\Pi_i^M(p) - \Pi_i^D(p) =& pD(p) - c_i(D(p)) - (1-\alpha)\left[\frac{1}{2}pD(p) - c_i\left(\frac{1}{2}D(p)\right)\right] \\ =& \frac{1+\alpha}{2}pD(p) - c_i(D(p)) + (1-\alpha)c_i\left(\frac{1}{2}D(p)\right).\end{aligned}$$

Now

$$\begin{aligned}\frac{\partial}{\partial p}\left[\Pi_i^M(p) - \Pi_i^D(p)\right] =& \frac{1+\alpha}{2}D(p) + D'(p)\left\{\frac{1+\alpha}{2}\left[p - c_i'(D(p))\right]\right. \\ &\left. + \frac{1-\alpha}{2}\left[c_i'\left(\frac{1}{2}D(p)\right) - c_i'(D(p))\right]\right\}\end{aligned} \tag{2}$$

When $\Pi_i^M(p) - \Pi_i^D(p) \leq 0$, we have $-c_i(D(p)) + (1-\alpha)c_i\left(\frac{1}{2}D(p)\right) \leq -\frac{1+\alpha}{2}pD(p)$.

Since $c_i(\cdot)$ is strictly convex,

$$c_i(D(p)) - c_i(0) = c_i(D(p)) < D(p)c_i'(D(p)),$$

$$c_i(D(p)) - c_i\left(\frac{1}{2}D(p)\right) < \frac{1}{2}D(p)c_i'(D(p)),$$

or

$$-D(p)c_i'(D(p)) < -c_i(D(p)),$$

$$-\frac{1}{2}D(p)c_i'(D(p)) < -c_i(D(p)) + c_i\left(\frac{1}{2}D(p)\right),$$

From them

$$-\frac{1+\alpha}{2}D(p)c_i'(D(p)) < -c_i(D(p)) + (1-\alpha)c_i\left(\frac{1}{2}D(p)\right).$$

It means

$$p < c_i'(D(p)).$$

Also we have

$$c_i'\left(\frac{1}{2}D(p)\right) < c_i'(D(p)).$$

Since $D'(p) < 0$, from (2) we find that when $\Pi_i^M(p) - \Pi_i^D(p) \leq 0$, $\frac{\partial}{\partial p}\left[\Pi_i^M(p) - \Pi_i^D(p)\right] > 0$ holds. Since $\Pi_i^M(p)$ and $\Pi_i^D(p)$ are continuously differentiable, this fact implies that p^* is unique.

B Proof of Lemma 2

p^* satisfies

$$\Pi_i^M(p^*) - \Pi_i^D(p^*) = p^*D(p^*) - c_i(D(p^*)) - (1-\alpha)\left[\frac{1}{2}p^*D(p^*) - c_i\left(\frac{1}{2}D(p^*)\right)\right] = 0.$$

Differentiating this with respect to α,

$$\left(\frac{\partial}{\partial p}\left[\Pi_i^M(p^*) - \Pi_i^D(p^*)\right]\right)\Bigg|_{p=p^*}\frac{dp^*}{d\alpha} = -\left[\frac{1}{2}p^*D(p^*) - c_i\left(\frac{1}{2}D(p^*)\right)\right].$$

This is negative because $\pi_i^D(p^*) = \frac{1}{2}p^*D(p^*) - c_i\left(\frac{1}{2}D(p^*)\right) > 0$ for $\hat{p} < p^*$.

C Proof of Lemma 3

Now

$$\frac{\partial \pi_i^M(p)}{\partial p} = \left\{D(p) + D'(p)\left[p - c_i'(D(p))\right]\right\}. \tag{3}$$

When $\pi_i^M(p) \leq 0$, we have $-c_i(D(p)) \leq -pD(p)$.

Since $c_i(\cdot)$ is strictly convex,

$$c_i(D(p)) < D(p)c_i'(D(p)),$$

or

$$-c_i(D(p)) > -D(p)c_i'(D(p)).$$

This means

$$-D(p)c_i'(D(p)) < -pD(p).$$

Therefore,

$$p < c_i'(D(p)).$$

Since $D'(p) < 0$, from (3) we find that when $\pi_i^M(p) \leq 0$, $\frac{\partial \pi_i^M(p)}{\partial p} > 0$ holds. Thus, the continuity of $\pi_i^M(p)$ and the uniqueness of $\tilde{p}$ means that $\pi_i^M(p) > 0$ for $p > \tilde{p}$ and $\pi_i^M(p) < 0$ for $p < \tilde{p}$.

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