Economics
The Open-Access, Open-Assessment E-Journal

Vol. 8, 2014-28 | August 21, 2014 | http://dx.doi.org/10.5018/economics-ejournal.ja.2014-28
Version 3 - August 23, 2016

The Simple Analytics of Helicopter Money: Why It Works — Always

Willem H. Buiter

Abstract
The paper offers a rigorous analysis of Milton Friedman's parable of the 'helicopter' drop of money – a temporary fiscal stimulus funded through an increase in the stock of fiat base money that is never completely reversed in present discounted value (PDV) terms. A monetized fiscal stimulus is more expansionary than a debt-financed one because a monetized expansion of the Central Bank balance sheet is profitable: it relaxes the intertemporal budget constraint of the State – it creates fiscal space. It is up to the fiscal authority to make appropriate use of this fiscal space. Four conditions must be satisfied for a helicopter money drop to boost aggregate demand. First, it must not be reversed fully in PDV terms. Second, there must be benefits from holding fiat base money other than its pecuniary rate of return: that is, the interest rate on any additional base money issued is below the rate of return on the Central Bank's assets. Third, fiat base money is irredeemable – an asset to the holder but not a liability to the issuer. Fourth, the price of money is positive. Given these conditions, there always exists – even in a permanent liquidity trap – a combination of monetary and fiscal policy actions that boosts private and/or public demand demand) – in principle without limit. Deflation, 'lowflation' and secular stagnation are therefore policy choices. Other conclusions are: (1) the increase in the monetary base need not be permanent for helicopter money to be effective; (2) Treasury debt cancellation by the Central Bank or the purchase by the Central Bank of perpetuities (with zero, negative or positive coupons) rather than finite maturity debt are fundamentally irrelevant policy actions. At most they have signaling value; (3) dropping perishable helicopter money will make it more effective if households are liquidity-constrained.

JEL E2 E4 E5 E6 H6
Keywords Helicopter money; liquidity trap; seigniorage; secular stagnation; central bank; quantitative easing

Authors
Willem H. Buiter, ✉ Citigroup Global Markets Inc., 388 Greenwich Street, New York, NY 10013, USA, willem.buiter@citi.com

Citation Willem H. Buiter (2016). The Simple Analytics of Helicopter Money: Why It Works — Always. *Economics: The Open-Access, Open-Assessment E-Journal*, 8 (2014-28): 1—45 (Version 3). http://dx.doi.org/10.5018/economics-ejournal.ja.2014-28

Received May 21, 2014 **Published as Economics Discussion Paper** June 13, 2014
Revised August 6, 2014 **Accepted** August 17, 2014 **Published** August 21, 2014

1 Introduction

> *"Let us suppose now that one day a helicopter flies over this community and drops an additional $1000 in bills from the sky, ... Let us suppose further that everyone is convinced that this is a unique event which will never be repeated," (Friedman 1969, pp. 4–5).*

This paper aims to provide a rigorous analysis of Milton Friedman's famous parable of the 'helicopter' drop of money. A helicopter drop of money is a temporary fiscal stimulus (tax cut or increase in public spending) financed through an increase in the stock of Central Bank fiat money that is not completely reversed in present discounted value (PDV) terms.[1] Central Bank fiat money is also referred to as the monetary base or the stock of base money. In practice, base money consists of the stock of currency/cash and the stocks of required and excess reserves held by banks and other eligible institutions with the Central Bank. Helicopter money focuses on the *fiscal* dimension of an increase in the monetary base: that any increase in the monetary base that is not completely reversed in PDV terms relaxes the intertemporal budget constraint of the consolidated Central Bank and Treasury, that is, it increases the fiscal space of the State. The up-front fiscal stimulus that is the other half of the combined monetary and fiscal stimulus known as helicopter money, ensures that the increase in the fiscal space is actually utilized.

1.1 Imperfect asset substitutability

In conventional monetary economics, an increase in the stock of fiat base money has an effect (on interest rates, financial and real asset prices, the general price level and/or the level of real economic activity) because base money is assumed to

[1] The formal model in Section 2 shows that monetized central bank balance sheet expansion relaxes the intertemporal budget constraint of the State (increases fiscal space) if the present discounted value of the sequence of additional current and future base money issuance net of interest paid on the monetary base is positive. This is equivalent to the condition that the present discounted value of the profits made by investing the current and future stocks of base money in non-monetary assets *plus* the present discounted value of the terminal stock of base money minus the initial stock of base money is positive.

be an imperfect substitute for all other financial and real assets. Its unique liquidity properties, its unique usefulness in certain kinds of transactions, its legal tender status, the unique creditworthiness of the issuer (the Central Bank) etc. make it unlike any other store of value. It is therefore willingly held by the private sector even though it may be dominated as regards its pecuniary rate of return by other, safe, non-monetary stores of value. In addition, the Central Bank is assumed to have the monopoly of its issuance and to be able to produce it at zero marginal cost.

Because of this, in the imperfect substitutes approach, an increase in the stock of base money changes the equilibrium configuration of prices and quantities (except in special circumstances like a permanent liquidity trap, with risk-free asset yields at the effective lower bound (ELB) for all maturities)[2] even though the stock of base money is an 'inside' financial instrument: for every creditor there is an debtor. Unlike commodity monies (gold, Rai (the stone money of the Isle of Yap) and bitcoin) fiat base money is in zero net supply. But the fact that private entities value it but cannot costlessly produce it or create it themselves, gives the monopoly issuer – the Central Bank, a lever to influence equilibrium prices and/or quantities. There is no Modigliani-Miller theorem for base money issuance – no home-made arbitrage-cum-leverage that allows private actors to undo, costlessly, an increase in the stock of base money engineered by the Central Bank (see Wallace 1981, Sargent and Smith 1987). This is why Central Bank balance sheet expansion, with base money expanding on the liability side of the balance sheet and sovereign debt or other financial or real assets on the asset side, is effective, except in a permanent liquidity trap. QE works in such a world.

Sometimes the assumption of imperfect substitutability is extended to limited substitutability among non-monetary financial and real instruments. In such a Tobin/Brunner–Meltzer world, there can be general equilibrium impacts from

[2] A liquidity trap at time *t* is a situation where the interest rate on non-monetary financial instruments equals the nominal interest rate on base money at time *t*. The economy is at the effective lower bound at time *t*. A permanent liquidity trap is a situation where the risk-free interest rate on non-monetary financial instruments equals the nominal interest rate on base money at all time and at all maturities. The economy is permanently at the ELB. In a world where markets for non-monetary financial instruments are inefficient and 'inside' financial asset supplies matter, this definition of a permanent liquidity trap can be extended to include the requirement that the yields on risk and illiquid assets also be at the ELB at all durations.

changes in the composition of the assets of the Central Bank's balance sheet even if the size of the balance sheet and the stock of base money are unchanged (see Tobin 1969, Brunner and Meltzer 1968, 1972). Qualitative easing or credit easing policies, including Operation Twist are effective in such a world (see e.g. Lenza et. al. 2010).

1.2 The fiscal dimension of monetary base expansion

Helicopter money focuses on a neglected implication of the assumed uniqueness of the monetary base. Because of its unique, valuable and valued liquidity properties, it yields a non-pecuniary utility (or some other productive service or benefit in exchange or production), that makes private agents willing hold it even though it is pecuniary-rate-of-return-dominated by risk-free non-base money stores of value. This is one reason why central banking tends to be profitable or, more precisely, why a monetized increase in the size of the balance sheet of the Central Bank tends to be profitable.

In this paper we too make the assumption that holding base money yields unique non-pecuniary returns that cause base money to be willingly held even though it is rate-of-return-dominated. We also attribute another form of uniqueness to the monetary base: base money is *irredeemable*. X dollars' worth of base money never gives the holder a claim on the issuer (the Central Bank) for anything other than X dollars' worth of base money. Thus, although base money is definitely viewed as an asset by the holders, it is not viewed in any meaningful sense as a liability by the issuer (the Central Bank). This leads to an asymmetric treatment of base money in the intertemporal budget constraints of private agents and of the Central Bank. Even though the monetary base looks like an 'inside' financial asset, the irredeemability of base money turns it, in a sense made precise in the body of the paper, into an 'outside' financial asset, that is, into net wealth. This irredeemability of base money implies that, even in a permanent liquidity trap, when the pecuniary rate of return on the monetary base is equal to the safe nominal interest rate on non-monetary instrument at all maturities, central banking can be profitable.

Because the Treasury is the beneficial owner of the Central Bank, the profits made by the Central Bank sooner or later accrue to the Treasury. This relaxes the intertemporal budget constraint of the Treasury (whose accounts really ought to be

consolidated with those of the Central Bank) and provides the Treasury with additional fiscal space to cut taxes or raise public spending. If the Treasury takes advantage of this relaxation of its intertemporal budget constraint, a fiscal stimulus funded through monetized Central Bank balance sheet expansion that is not completely reversed in PDV terms can be more expansionary than the same fiscal stimulus funded by Treasury borrowing in the markets and never monetized.

There are two classic examples of a monetized Central Bank balance sheet expansion. The first is helicopter money – a temporary fiscal stimulus (say a one-off transfer payment to households, as in Friedman's example, or an increase in infrastructure investment), funded through an increase in the stock of base money that is not completely reversed in PDV terms. The second is an increase in the stock of base money through an open market purchase by the Central Bank of non-monetary sovereign debt held by the public – that is, QE – that is not completely reversed in PDV terms.

1.3 Four conditions for helicopter money effectiveness

There are four conditions that must be satisfied for helicopter money – a monetized temporary fiscal stimulus – to boost aggregate demand. First, the increase in the stock of base money is never completely reversed in PDV terms. Second, there must be benefits from holding additional fiat base money other than its pecuniary rate of return. Only then will (additional) base money be willingly held despite being dominated as a store of value by non-monetary assets with a risk-free nominal rate of return that is typically positive and almost always higher than the interest rate(s) on required or excess reserves held by commercial banks with the Central Bank. Third, fiat base money is irredeemable: it is viewed as an asset by the holder but not as a liability by the issuer. This is necessary for helicopter money to relax the intertemporal budget constraint of the State and of the consolidated private sector and State, even in a pure liquidity trap, with risk-free nominal interest rates on non-monetary instruments equal to the interest rate on base money for all maturities and for all time. The fourth (somewhat pedantic) condition, necessary to rule out barter equilibria with a zero price of fiat base money when nominal prices are flexible, is that the price of money is positive.

The paper shows that, when the State can issue unbacked, irredeemable fiat money/base money which (1) is willingly held in positive amounts by the private

sector although it carries a nominal interest rate that is less than or equal to the short nominal interest rate on risk-free non-monetary financial instruments, (2) can be produced at zero marginal cost, there always exists a combined monetary and fiscal policy action that boosts private demand – in principle without limit. Deflation, inflation below target, 'lowflation', 'subflation' and secular stagnation are therefore unnecessary (see Summers 2013 and Buiter et. al. 2014).[3] They are policy choices.

The feature of irredeemable base money that is key for the 'helicopter money effectiveness' result to hold even in a permanent liquidity trap is that the acceptance of payment in base money by the State to a private agent constitutes a final settlement between that private agent (and any other private agent with whom he exchanges that base money) and the State. It leaves the private agent without any further claim on the State, now or in the future.

Outside the permanent liquidity trap equilibrium, 'helicopter money effectiveness' follows from the assumption that base money is pecuniary-rate-of-return-dominated by the assets of the Central Bank. In this paper these assets are just holdings of one-period (strictly speaking zero duration) safe Treasury debt.

1.4 Helicopter money ineffectiveness when central banking is not profitable

The feature of the model that, under normal conditions, with market interest rates above the own interest rate on base money, central banking is profitable would seem non-controversial. However, it is rejected in a recent blog by Fergus Cumming (2015). Cumming slips in the assumption that the additional assets acquired by the Central Bank as part of a his real-world 'helicopter money drop' (a

[3] The term 'lowflation' is, I believe, due to Moghadam et al. (2014). The term 'subflation' has been around the blogosphere for a while. I use it to refer to an inflation rate below the target level or lower than is optimal. 'Secular stagnation' theories go back to Alvin Hansen (1938). I refer here to the Keynesian variant, which holds that there will be long-term stagnation of employment and economic activity without government demand-side intervention. There also is a long-term supply side variant, associated e.g. with Robert Gordon (2014), which focuses on faltering innovation and productivity growth. Larry Summers (2013) marries the demand-side and supply-side secular stagnation approaches by invoking a number of hysteresis mechanisms. For a formal model see Eggertsson and Mehrotra (2014).

temporary fiscal stimulus funded by the Treasury issuing additional sovereign debt that is bought by the Central Bank), earn a rate of return that is always equal to the interest rate on the additional reserves held by banks with the Central Bank.[4] That, plus the implicit assumption that either there is no (zero interest currency) or that currency cannot be issued on a sufficient scale by the Central Bank to implement a helicopter money drop that would have a material impact on activity, produces a slew of paradoxes and 'helicopter money ineffectiveness results'. When it engages in helicopter money on a large enough scale (or cancels enough sovereign debt it holds) the Central Bank can only remain solvent (in the sense of able to pay all its bills, including interest and principal refinancing, now and in the future), if it is willing to lose control of the size of its balance sheet and/or of the inflation rate.

The reason this paper gets different results is that Cumming drops the two key requirement for helicopter money to be effective, stated in this paper: (1) that (newly issued) base money is pecuniary-rate-of-return-dominated by the assets the

---

[4] Cummings considers the special case of an increase in the stock of base money that is permanent, rather than never completely reversed in PDV terms. This is not material to the difference in the results we obtain. The same would hold if the Central Bank bought additional sovereign debt in an open market purchase, paying for it with additional interest-bearing reserves and either holding that additional sovereign debt and the additional reserves permanently, or cancelling the additional sovereign debt but holding the additional reserves permanently. The additional reserves held by the Central Bank (or more generally, the additional base money issued by the Central Bank as the result of a 'helicopter money drop' or a permanent open market purchase (permanent QE) should be thought of *counterfactually*: we are comparing two parallel universes identical in history and differing only in one respect. One universe has a permanent QE operation today and the Treasury makes whatever changes in future transfers, taxes and/or public spending are necessary to continue to satisfy, now and in the future, the intertemporal budget constraints of the Central Bank, the Treasury and the State. There will in general be (infinitely) many alternative paths of future public spending and revenues that meet this criterion. The other universe does not. With a bit of hand waving, one can re-interpret the parallel universes story as the impact of a completely unexpected helicopter money drop/permanent QE in a single universe. This should also make it clear that the QE or the monetization of a temporary fiscal stimulus does not have to be permanent for it to be effective in stimulating private and/or public demand for goods and services. Even with a temporary QE operation (or with the temporary monetization of a temporary fiscal stimulus), there will, in general, either be a positive PDV of future interest saved (if the additional base money is pecuniary-rate-of-return dominated by the additional assets acquired by the Central Bank), or a positive PDV of the terminal stock of base money, if the stock of nominal base money grows forever at a proportional rate no less than the nominal interest rate on the additional base money issued (see the formal model of Section 2).

Central Bank acquires and (2) that base money is irredeemable. Central Bank balance sheet expansion is not profitable in the Cumming universe.

We are happy to agree with Cumming on the analytical point that helicopter money makes no sense if Central Bank balance sheet expansion is not profitable. We would put it slightly differently, however: helicopter money (with or without cancellation of Treasury debt – the two operations are equivalent except for 'signaling' or 'commitment considerations' – is effective when and to the extent that central banking (strictly speaking Central Bank balance sheet expansion) is profitable. In modern advanced economies it almost always is. Whether you want to engage in a helicopter money drop depends on whether the economy needs a stimulus to aggregate demand, what the possible undesirable side effects of such a demand stimulus are, and what other means of boosting demand are available.

1.5 Is base money net wealth?

The helicopter money drop effectiveness issue is closely related to the question as to whether State-issued fiat money is net wealth for the private sector, despite being technically an 'inside asset', where for every creditor that holds the asset there is a debtor who owes a claim of equal value (see Patinkin 1956/1965, Gurley and Shaw 1960 and Pesek and Saving 1967, Weil 1991). The discussions in Hall 1983, Stockman 1983, King 1983, Fama 1983, Sargent and Wallace 1984, Sargent 1987 and Weil 1991 of outside money, private money and the payment of interest on money ask some of the same questions as this paper, but do not offer the same answer, because they don't address the irredeemability of fiat base money. Sims (2001, 2004), Buiter (2003a, 2003b), Eggertsson 2003 and Eggertsson and Woodford (2003, 2006), Eggertsson and Krugman 2012 and Eggertsson and Mehrotra 2014 all stress that to boost demand in a liquidity trap, base money increases should not be, or expected to be, reversed. None of these papers recognized that even a permanent increase in the stock of base money will not have an expansionary wealth effect in a permanent liquidity trap unless money is irredeemable in the sense developed here; without this, there is no real balance effect in a permanent liquidity trap. Ben Bernanke (who spent years living down the moniker "helicopter Ben" which he acquired following a (non-technical) discussion of helicopter money (Bernanke 2003)) has recently asserted (Bernanke 2016) that the increase in the monetary base that funds the temporary fiscal

stimulus should be permanent. This paper shows that for there to be a positive wealth effect from base money expansion, it is neither necessary nor sufficient that the increase in the monetary base be permanent. The issue has also been revisited by Buiter (2003a, 2003b, 2007), in earlier versions of this paper and, in an informal manner, by Turner (2013, 2015) and by Reichlin et al. (2013).

The paper shows that, because of its non-pecuniary returns and its irredeemability, State-issued fiat money is indeed net wealth to the private sector, in a very precise way: the initial stock of base money plus the present discounted value of all future net base money issuance (net of any interest paid on the outstanding stock of base money) is net wealth, an 'outside' asset to the private sector, even after the intertemporal budget constraint of the State (which includes the Central Bank) has been consolidated with that of the household sector.

The paper also demonstrates that fiat base money issuance is effective in boosting household demand regardless of whether there is Ricardian equivalence (debt neutrality).

2 The model

All important aspects of how helicopter money drops work and what makes helicopter money unique can be established without the need for a complete dynamic general (dis)equilibrium model. All that is needed is a complete specification of the choice process of the household sector in a monetary economy, the period budget identity and solvency constraint of the consolidated general government/Treasury and Central Bank – the State – and the no-arbitrage conditions equating (in principle risk-adjusted) returns on all non-monetary stores of value and constraining the instantaneous nominal interest rate on non-monetary safe assets not to be less than the interest rate on base money.

I shall show that, as long as the price of money is positive, the issuance of fiat base money can boost household consumption demand *by any amount*, given the inherited stocks of financial and real assets, given current and future wages and prices, and given current and future values of public spending on goods and services. Alternatively, given the inherited stocks of financial and real assets, current and future wages and prices, and current and future values of taxes net of transfers, public spending on real goods and services can be boosted by any

amount. Whether such helicopter money drops change asset prices and interest rates, goods prices, wages and/or output and employment depends on the specification of the rest of the model of the economy – including, in more general models, the behavior of the financial sector and of non-financial businesses in driving investment demand, production and labor demand, the rest of the 'supply side' of the economy and the rest of the world, if the economy is open. The point of this paper is to show that, whatever the equilibrium configuration we start from, helicopter money drops will boost household demand and/or public sector demand and must disturb that equilibrium. What 'gives' ultimately, in a fully articulated dynamic general equilibrium model, is not a concern of this paper.

The model of household behavior I use is as stripped-down and simple as I can make it without raising concerns that the key results will not carry over to more general and 'realistic' models. The continuous-time Yaari-Blanchard version of the OLG model is used to characterize household behavior (see Yaari 1965, Blanchard 1985, Buiter 1988 and Weil 1989). This model with its easy aggregation and its closed-form aggregate consumption function includes the conventional (infinite-lived) representative agent model as a special case (when the birth rate is zero). With a positive birth rate, there is no Ricardian equivalence or debt neutrality in the Yaari-Blanchard model. With a zero birth rate there is Ricardian equivalence. This permits me to show that helicopter money drops can boost household demand regardless of whether there is Ricardian equivalence or not. Apart from the uncertain lifetime that characterizes households in the Yaari-Blanchard model (which plays no role either in Ricardian equivalence or the effectiveness of helicopter money drops), the model has no uncertainty. To save on notation I consider a closed economy.

2.1 The household sector

We consider the household and government sectors of a simple closed economy. The holding of intrinsically worthless fiat base money is motivated through a 'money-in-the-direct utility function' approach, but alternative approaches to making money essential (cash-in-advance, legal restrictions, money-in-the transactions-function or money-in-the-production-function, say) would work also. For expository simplicity, there is only private capital. The helicopter money we discuss could, however, be used equally well to fund government investment

programs, including infrastructure investment, as tax cuts or transfer payments that benefit households, or to boost current exhaustive public spending.

2.1.1 Individual household behavior

At each time $t \geq 0$, a household born at time $s \leq t$ maximizes the following utility functional:

$$\max E_t \int_t^{\infty} e^{-\theta(v-t)} \ln\left[\bar{c}(s,v)^{\alpha}\left(\frac{\bar{m}(s,v)}{P(v)}\right)^{1-\alpha}\right] dv$$
$$\{\bar{c}(s,v), \bar{m}(s,v), \bar{b}(s,v), \bar{k}(s,v); s \leq t, v \geq t\} \tag{1}$$

$$\bar{c}(s,v), \bar{m}(s,v) \geq 0, \theta > 0, 0 < \alpha < 1$$

where E_t is the conditional expectation operator at time t, $\theta > 0$ is the pure rate of time preference, $\bar{c}(s,v)$ is consumption at time v by a household born at time s, $\bar{m}(s,v)$, $\bar{b}(s,v)$ and $\bar{k}(s,v)$ are, respectively, the stocks of nominal base money, nominal risk-free constant market value bonds and real capital held at time v by a household born at time s, and $P(v) \geq 0$ is the general price level at time v.[5]

Each household faces a constant (age-independent) instantaneous probability of death, $\lambda \geq 0$. The remaining expected life time λ^{-1} is therefore also age-independent and constant. The randomness of the timing of one's demise is the only source of uncertainty in the model. It follows that the objective functional in (1) can be re-written as:

$$\max \int_t^{\infty} e^{-(\theta+\lambda)(v-t)} \ln\left[\bar{c}(s,v)^{\alpha}\left(\frac{\bar{m}(s,v)}{P(v)}\right)^{1-\alpha}\right] dv \tag{2}$$
$$\{\bar{c}(s,v), \bar{m}(s,v), \bar{b}(s,v), \bar{k}(s,v); v \geq t\}$$

[5] If a unit of real capital is interpreted as an ownership claim to a unit of capital (equity), then $\bar{k}$ can be negative, zero or positive. If it is interpreted as a unit of physical capital itself, $\bar{k}$ has to be non-negative.

Households act competitively in all markets in which their operate, and asset markets are complete and efficient, with free entry. In particular, there exist actuarially fair annuities markets that offer a household an instantaneous rate of return of λ on each unit of non-financial wealth it owns for as long as it lives, in exchange for the annuity-issuing entity claiming the entire stock of financial wealth owned by the household at the time of its death.

The household has three stores of value: fiat base money, which carries a nominal rate of interest i^M which is set by the State and is an irredeemable financial instrument issued by the State (the consolidated Treasury (or general government) and Central Bank), nominal instantaneous bonds with an instantaneous risk-free nominal interest rate i and real capital yielding an instantaneous gross real rate of return ρ. Capital goods and consumption goods consist of the same physical stuff and can be costlessly and instantaneously transformed into each other. Capital depreciates as the constant instantaneous rate $\delta \geq 0$. The real wage earned at time *v* by a household born at time *s* is denoted $\overline{w}(s,v)$ and the lump-sum tax paid (lump-sum transfer payment received if negative) at time *v* by a household born at time *s* is $\overline{\tau}(s,v)$. Labor supply is inelastic and scaled to 1.

Competition ensures that pecuniary rates of return on bonds and capital are equalized. With money yielding positive utility, there can be no equilibrium with a nominal interest rate on risk-free non-monetary assets (bonds) less than the nominal interest rate on base money. Let $r(t)$ be the instantaneous risk-free real interest rate on non-monetary financial instruments and $\pi(t) = \frac{\dot{P}(t)}{P(t)}$ the instantaneous rate of inflation. It follows that condition (3) has to hold in any equilibrium in which both money and bonds are held, and that condition (4) has to hold in any equilibrium in which both bonds and capital are held.

$$i(t) \geq i^M(t) \tag{3}$$

$$\rho(t) - \delta(t) = r(t) = i(t) - \pi(t) \tag{4}$$

For some purposes it would have been interesting to disaggregate base money into its three components. Let *M* be the aggregate stock of nominal base money, *N* the stock of currency, R^r the stock of required reserves and R^e the stock of

excess reserves. The nominal interest rates on the three components of base money are i^N, i^r and i^e respectively. Normally, we would expect that $i^N = 0$. With $M \equiv N + R^r + R^e$ and the average interest rate on base money i^M defined as $i^M \equiv \frac{i^N N + i^r R^r + i^e R^e}{M}$ (a weighed arithmetic average of the interest rates on the three components, with the weight given by each component's share in the aggregate base money stock). The reason this is not pursued here is expositional simplicity. To bring reserves into the model would require modeling a banking sector. The alternative – adding the three components separately into the household's utility function – would not add any new insight and would look messy. Assuming the three components are perfect substitutes in the household utility function makes for notational simplicity but results in corner solutions unless $i^N = i^r = i^e = i^M$. This is effectively the assumption we have to make if our set-up is to accommodate multiple base money components.

The instantaneous budget identity of a household born at time $s \leq t$ that has survived till period t is:

$$\dot{\bar{k}}(s,v) + \frac{\dot{\bar{m}}(s,v) + \dot{\bar{b}}(s,v)}{P(v)} \equiv$$

$$\left(\rho(v) - \delta + \lambda\right)\bar{k}(s,v) + \left(i(t) + \lambda\right)\frac{\bar{b}(s,v)}{P(v)} + \left(\lambda + i^M(t)\right)\frac{\bar{m}(s,v)}{P(v)} + \bar{w}(s,v) - \bar{\tau}(s,v) - \bar{c}(s,v) \tag{5}$$

[6]

The real value of total non-human wealth (or financial wealth) at time v of a household born at time s is

$$\bar{a}(s,v) \equiv \bar{k}(s,v) + \frac{\bar{m}(s,v) + \bar{b}(s,v)}{P(v)} \tag{6}$$

The flow budget identity (5) can, using (4) and (6) be written as:

---

[6] The notational convention is that $\dot{\bar{k}}(s,v) \equiv \frac{\partial \bar{k}(s,v)}{\partial v}$.

$$\dot{\bar{a}}(s,v) \equiv \left(r(v)+\lambda\right)\bar{a}(s,v) - \left(i(v) - i^{M}(v)\right)\frac{\bar{m}(s,v)}{P(v)} + \bar{w}(s,v) - \bar{\tau}(s,v) - \bar{c}(s,v) \qquad (7)$$

The no-Ponzi finance solvency constraint for the household is that the present discounted value of its terminal financial wealth be non-negative in the limit as the time horizon goes to infinity:

$$\lim_{v\to\infty} \bar{a}(s,v) e^{-\int_t^v (r(u)+\lambda)} du \geq 0$$

Because the instantaneous utility function is increasing in both consumption and the stock of real money balances, the solvency constraint will bind:

$$\lim_{v\to\infty} \bar{a}(s,v) e^{-\int_t^v (r(u)+\lambda)} du = 0 \qquad (8)$$

Note that base money is viewed as an asset by the holder (the household). The terminal net financial wealth whose present discounted value (PDV) must be non-negative includes the household's stock of base money.

The optimality conditions of the household's choice problem imply the following decision rules for the household:

$$\bar{c}(s,t) = (1-\alpha)\left(\theta+\lambda\right)\left(\bar{a}(s,t) + \bar{h}(s,t)\right) \qquad (9)$$

$$\bar{h}(s,t) = \int_t^{\infty} \left(\bar{w}(s,t) - \bar{\tau}(s,t)\right) e^{-\int_t^v (r(u)+\lambda) du} dv \qquad (10)$$

$$\frac{\bar{m}(s,t)}{P(t)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{i(t)}\bar{c}(s,t) \qquad (11)$$

The net present discounted value of household after-tax labor income, $\bar{h}(s,t)$, will be referred to as human capital. A shorter life expectancy (higher value of λ) raises the marginal propensity to consume out of comprehensive wealth, $a + h$

2.1.2 Aggregation

We assume that there is a constant and age-independent instantaneous birth rate $\beta \geq 0$. The size of the cohort born at time t is normalized to $\beta e^{(\beta-\lambda)t}$. The size of the surviving cohort at time t which was born at time $s \leq t$ is therefore $\beta e^{(\beta-\lambda)s} e^{-\lambda(t-s)}$. Total population at time t is therefore given, for $\beta > 0$ by $\beta e^{-\lambda t} \int_{-\infty}^{t} e^{\beta s} ds = e^{(\beta-\lambda)t}$. For the case $\beta = 0$ we set the size of the population at $t = 0$ to equal 1, so population size at time t is again $e^{(\beta-\lambda)t} = e^{-\lambda t}$. For any individual household variable $\overline{x}(s,t)$, we define the corresponding population aggregate *X(t)* as follows:

$$X(t) = \beta e^{-\lambda t} \int_{-\infty}^{t} \overline{x}(s,t) e^{\beta s} ds \quad \text{if } \beta > 0$$
$$= \overline{x}(0,t) e^{-\lambda t} \quad \text{if } \beta = 0$$

We assume that each household earns the same wage and pays the same taxes, regardless of age:

$$\overline{w}(s,t) = \overline{w}(t)$$
$$\overline{\tau}(s,t) = \overline{\tau}(t)$$

It follows that each household, regardless of age, has the same human capital:

$$\overline{h}(s,t) = h(t)$$

Finally, there are neither voluntary nor involuntary bequests in this model, so

$$\overline{a}(s,s) = 0 \tag{12}$$

By brute-force aggregation, if follows that aggregate consumption is determined as follows:

$$C(t) = (1-\alpha)(\theta+\lambda)\big(A(t) + H(t)\big) \tag{13}$$

$$\frac{M(t)}{P(t)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{i(t)}C(t) \tag{14}$$

$$\dot{A}(t) \equiv r(t)A(t) - \left(i(t) - i^{M}(t)\right)\frac{M(t)}{P(t)} + W(t) - T(t) - C(t) \tag{15}$$

$$H(t) = \int_{t}^{\infty}\left(W(v) - T(v)\right)e^{-\int_{t}^{v}(r(u)+\beta)du}dv \tag{16}$$

$$A(t) = K(t) + \frac{M(t) + B(t)}{P(t)} \tag{17}$$

For future reference, the solvency constraint of the aggregate household sector is

$$\lim_{v\to\infty} A(v)e^{-\int_{t}^{v} r(u)\mathrm{du}} = 0$$

or (18)

$$\lim_{v\to\infty}\left(K(v) + \frac{M(v) + B(v)}{P(v)}\right)e^{-\int_{t}^{v} r(u)\mathrm{du}} = 0$$

Comparing the aggregate household financial wealth dynamics equation (15), with the individual *surviving* household financial wealth dynamics equation (7) shows that the return on the annuities, λA is missing from the aggregate dynamics. This is as it should be, because $\lambda A(t)$ is both the extra returns over and above the risk-free rate earned by all surviving households at time *t* and the amount of wealth paid to the annuities sellers by the (estates of the) fraction λ of the population that dies at time *t*.

Comparing the aggregate human capital equation (16) – describing the human capital of all generations currently alive but not of those yet to be born – and the individual surviving household's human capital equation (10), we note that if the households alive at time *t* were to discount *all* future after-tax labor income at the individually appropriate, annuity premium-augmented rate of return $r + \lambda$, they would fail to allow for the fact that the labor force to whom that after-tax labor income accrues includes the surviving members of generations born after time *t*. In

the absence of the institution of "inherited slavery", those currently alive cannot claim the labor income of the future surviving members of generations as yet unborn. Population and labor force grow at the proportional rate $\beta - \lambda$, so the appropriate discount rate applied to the future aggregate streams of labor income is $r + \beta$.

2.2 The State

The State whose budget identity and solvency constraint we model is the consolidated general government (the Treasury in what follows) and Central Bank (until Section 3.5 when we consider the Central Bank and the Treasury separately). Let G denote real public spending on goods and services (exhaustive public spending, current and/or capital in principle, in this paper only public consumption spending to save on algebra). The State's budget identity and solvency constraint are given in equation (19) and (20) respectively.

$$\frac{\dot{M}(t)+\dot{B}(t)}{P(t)} \equiv i(t)\frac{B(t)}{P(t)} + i^{M}(t)\frac{M(t)}{P(t)} + G(t) - T(t) \tag{19}$$

Because of the irredeemability of base money, the solvency constraint of the State requires that the present discounted value of its terminal net *non-monetary* liabilities be non-positive, not that the present discounted value of its terminal net financial liabilities be non-positive.

$$\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\,\mathrm{du}} \leq 0 \tag{20}$$

Equation (20) is the natural way to formalize the familiar notion that fiat base money is an asset (wealth) to the holder (the owner – households in this simple model) but does not constitute in any meaningful sense a liability to the issuer (the 'borrower' – the State or the Central Bank as an agent of the State). The owner of a $20 dollar Federal Reserve Note may find comfort in the fact that "This note is legal tender for all debts, public and private", but she has no claim on the Federal Reserve, now or ever, other than for an amount of Federal Reserve Notes adding up to $20 in current value. UK currency notes worth £X carry the proud

inscription *"… promise to pay the bearer the sum of £X"*, but this merely means that the Bank of England will pay out the face value of any genuine Bank of England note no matter how old. The promise to pay stands good for all time but simply means that the Bank will always be willing to exchange one (old, faded) £10 Bank of England note for one (new, crisp) £ 10 Bank of England note (or even for two £ 5 Bank of England notes). Because it promises only money in exchange for money, this 'promise to pay' is, in fact, a statement of the irredeemable nature of Bank of England notes.

I believe that the irredeemability property of fiat currency – that it is an asset to the holder but not a liability of the issuer – extends also to the other component of base money (commercial bank reserves held with the Central Bank), but the simple theoretical model does not depend on this and does distinguish between currency, required reserves and excess reserves.

Equation (20) implies that

$$\frac{M(t)+B(t)}{P(t)} \equiv \int_t^{\infty}\left(T(v)-G(v)+\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)+B(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{21}$$

Because of the irredeemability of base money (reflected in equation (20)) the intertemporal budget constraint of the State is

$$\frac{M(t)+B(t)}{P(t)} \leq \int_t^{\infty}\left(T(v)-G(v)+\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du} \tag{22}$$

Equation (22) holds with equality if equation (20) holds with equality. The inequality in (22) is strict if the inequality in equation (20) is strict.

Substituting the intertemporal budget constraint of the State into the aggregate consumption function (13), using (16) and (17), and rearranging yields:

$$C(t) \le (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^{\infty}\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv \\ -\int_t^{\infty}T(v)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\frac{1}{P(t)}\left(\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right) \end{array}\right] \tag{23}$$

[7]

Equation (23) holds with equality if equation (20) holds with equality. The inequality in (23) is strict if the inequality in equation (20) is strict.

From integration by parts it follows that

$$\begin{aligned} &\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du} \\ &=\int_t^{\infty}\left(\dot{M}(v)-i^M(v)M(v)\right)e^{-\int_t^v i(u)du}dv+M(t) \end{aligned} \tag{24}$$

It follows that (23) can be rewritten as:

[7] Note that $$\begin{aligned} &\int_t^{\infty}\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}dv+\lim_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du} \\ &=\frac{1}{P(t)}\left(\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right) \end{aligned}$$

$$C(t) \le (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^\infty \left(W(v) - G(v)\,\mathrm{e}^{\beta(v-t)}\right) e^{-\int_t^v \left(r(u)+\beta\right) du}\, dv \\ -\int_t^\infty T(v) e^{-\int_t^v \left(r(u)+\beta\right) du}\left[1 - e^{\beta(v-t)}\right] dv \\ +\dfrac{1}{P(t)}\left(M(t) + \int_t^\infty \left(\dot{M}(v) - i^M(v) M(v)\right) e^{-\int_t^v i(u) du}\, dv \right) \end{array}\right] \quad (25)$$

Equation (25) holds with equality if equation (20) holds with equality. The inequality in (25) is strict if the inequality in equation (20) is strict.

2.3 Debt Neutrality

When the birth rate is zero, the consumption function is equivalent to the consumption function of the representative agent model. Pure fiscal stabilization policy – a cut in lump-sum taxes today accompanied by a credible commitment to an increase in future taxes equal in net present value to the up-front tax cut - will not boost household demand. With $\beta > 0$, an up-front tax cut and the credible announcement of a future increase in taxes of equal net present discounted value when discounted at the riskless rate *r* boosts the human capital of those currently alive because some of the deferred taxes will fall on as yet unborn generations. With $\beta = 0$ the wedge between the government's discount rate for future taxes, *r*, and the effective discount rate of the private sector for future taxes, $r + \beta$ disappears, and Ricardian equivalence or debt neutrality prevails. With $\beta = 0$, the aggregate consumption function (25) becomes

$$C(t) \le (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^\infty \left(W(v) - G(v)\right) e^{-\int_t^v r(u) du}\, dv \\ +\dfrac{1}{P(t)}\left(\int_t^\infty \left(i(v) - i^M(v)\right) M(v) e^{-\int_t^v i(u) du}\, dv \right. \\ \left. + \lim_{v \to \infty} M(v) e^{-\int_t^v i(u) du} \right) \end{array}\right] \quad (26)$$

or, equivalently

$$C(t) \leq (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_{t}^{\infty} \left(W(v) - G(v)\right) e^{-\int_{t}^{v} r(u)du} dv \\ \\ + \frac{1}{P(t)} \left(\begin{array}{l} M(t) \\ + \int_{t}^{\infty} \left(\dot{M}(v) - i^{M}(v) M(v)\right) e^{-\int_{t}^{v} i(u)du} dv \end{array} \right) \end{array}\right] \tag{27}$$

Lump-sum taxes disappear from the aggregate consumption function once the intertemporal budget constraint of the State is used to substitute out the initial values of the private sector's holdings of monetary and non-monetary sovereign debt. The first line on the RHS of equations (26) and (27) shows the result, familiar from non-monetary representative agents models that the bite taken out of private comprehensive wealth by the government is measured by the net present discounted value of future exhaustive public spending.

2.4 Helicopter money with debt neutrality

Assume until further notice that equation (3) holds as a strict inequality and base money is pecuniary-rate-of-return-dominated by bonds as a store of value. Even in a representative agent model with debt neutrality/Ricardian equivalence, monetary injections in excess of the interest paid on the outstanding base money stock $\dot{M}(v) > 0$ will boost private consumption demand, holding constant the sequences of current and future spending on real goods and services $\{G(v); v \geq t\}$, prices, wages and interest rates, provided these monetary injections raise the PDV of current and future monetary injections net of interest paid on the outstanding stocks of base money:

$$\int_t^{\infty} \left(\dot{M}(v) - i^M(v)M(v)\right) e^{-\int_t^v i(u)du} dv = \int_t^{\infty} \left(i(v) - i^M(v)\right) M(v) e^{-\int_t^v i(u)du} dv$$

$$+ \lim_{v \to \infty} M(v) e^{-\int_t^v i(u)du} - M(t)$$

The path of lump-sum taxes and of non-monetary debt is irrelevant with $\beta = 0$, as long as the State satisfies its intertemporal budget constraint (22).

Assume that equation (20) (and therefore equations (22), (23), (26) and (27)) holds with equality: the State is not a 'bond creditor' in the long run. It is immediately obvious from equations (26) and (27) – holding with equality – that, holding constant the sequence of current and future real exhaustive public spending constant, monetary injections will always boost consumption demand, as long as the price level $P(t)$ is positive. We can think of monetary injections, holding constant the path of current and future exhaustive public spending, as being introduced either through up-front lump-sum transfer payments, *T*, (helicopter money) or by purchasing non-monetary debt (sovereign bonds) from the private sector (QE or quantitative easing) followed by later tax cuts (since equation (20) is assumed to hold with equality).

If the State, starting at time *t*, increases the stock of base money by buying back non-monetary public debt from the public, say with $\dot{M}(v) = -\dot{B}(v) > 0$ for $t \le v \le t', t' > t$, it is clear from the intertemporal budget constraint of the State, equation (21), that, provided the interest rate on bonds exceeds the interest rate on base money, that is, equation (3) holds as a strict inequality (and holding constant the current and future paths of the price level and of all interest rates), the State will have to raise the PDV of future public spending minus taxes to *strictly* satisfy its intertemporal budget constraint (that is to make use of the fiscal space created by the base money stock expansion). Under these conditions, permanent open market purchases of non-monetary public debt by the Central Bank (irreversible QE) are deferred helicopter money if the State takes advantage of the relaxation of its intertemporal budget constraint it offers: future taxes will be cut and/or future public spending will have be raised if the State is to continue to satisfy its intertemporal budget constraint with strict equality.

2.5 The creditor state and the waste of fiscal space

Remember that equation (20) does not have to hold with strict equality. The same holds for equations (22), (23), (25), (26) and (27). Consider the case

$$\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} < 0,$$

where the State is a net (non-monetary) creditor to the private sector, even in the very long run. The consumption functions (22), (23), (26) and (27) also now hold with strict inequality. In an endowment economy ($\bar{k}(s,t) = K(t) \equiv 0$), the aggregate household solvency constraint (18) implies

$$-\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} = \lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} > 0 \text{ or}$$

$$-\lim_{v\to\infty} B(v)e^{-\int_t^v i(u)\mathrm{du}} = \lim_{v\to\infty} M(v)e^{-\int_t^v i(u)\mathrm{du}} > 0.$$

The state is a permanent creditor to the household sector, something it can do when the long-run growth rate of fiat base money is at least as high as the long-run nominal interest rate, since

$$\lim_{v\to\infty} M(v)e^{-\int_t^v i(u)\mathrm{du}} > 0 \text{ requires}$$

$$\lim_{v\to\infty}\frac{\dot{M}(v)}{M(v)} \geq \lim_{v\to\infty} i(v) > 0.$$

From the government's intertemporal budget constraint (22) it is clear that the fiscal space created by

$$\lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} > 0$$

can be used to cut future taxes or increase future public spending. However, the State could instead ignore, or waste, the additional fiscal space provided by the exploitation of the irredeemability of base money.

Consider the case where, in the long run, the State grows the nominal stock of fiat base money at a proportional rate strictly below the instantaneous risk-free nominal interest rate, that is,

$$\lim_{v \to \infty} M(v)e^{-\int_t^v i(u)du} = 0 \cdot$$

In the representative agent case ($\beta = 0$) the consumption function becomes, when $\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\,\mathrm{du}} < 0$:

$$C(t) < (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^\infty \left(W(v) - G(v)\right)e^{-\int_t^v r(u)du}\,dv \\ + \frac{1}{P(t)}\left(\int_t^\infty \left(i(v) - i^M(v)\right)M(v)e^{-\int_t^v i(u)du}\,dv\right) \end{array}\right]$$

The State can boost household consumption demand by monetary injections, for given sequences of exhaustive public spending, the general price level and interest rates, provided it cuts current and/or future taxes. When the State instead builds up a permanently larger net non-monetary creditor position vis-à-vis the private sector, private consumption will not be boosted because the intertemporal budget constraint of the households is not relaxed. The same conclusion stares one in the face even more clearly when we use the equivalent expression for the seigniorage blessings of monetary issuance, shown in equation (27). The wealth-creating effect of seigniorage is the outstanding stock of base money plus the PDV of future base money issuance:

$$M(t) + \frac{1}{P(t)}\int_t^\infty \left(\frac{\dot{M}(v)}{M(v)} - i^M(v)\right)M(v)e^{-\int_t^v i(u)du}\,dv\,.$$

Again this can be made arbitrarily large for given sequences of *G*, *P* and *i* and i^M by increasing the stock of base money at a proportional rate greater than the own interest rate on base money (which could of course be negative).

In the remainder of the paper we assume that equation (20) holds with equality.

2.6 Helicopter money in a liquidity trap

Consider an economy stuck in the ultimate liquidity trap with the nominal interest rate on bonds equal to the own interest rate on base money forever. With $i(v) = i^M(v), v \geq t$, monetary injections lose none of their potency. Sure, the PDV of the current and future interest saved by issuing base money rather than non-monetary securities (bonds) is zero:

$$\int_t^{\infty} \left(i(v) - i^M(v)\right) M(v) e^{-\int_t^v i(u)du} dv = 0$$

when $i(v) = i^M(v), v \geq t$. But the PDV of the terminal stock of base money can be made anything the State (the monetary authority) wants it to be in a permanent liquidity trap. Assume the initial date is *t = 0*.

$$\lim_{\ell \to \infty} M(\ell) e^{-\int_t^{\ell} i(u)du} = \lim_{\ell \to \infty} M(\ell) e^{-\int_t^{\ell} i^M(u)du} = M(t) \lim_{\ell \to \infty} e^{\int_t^{\ell} \left(\frac{\dot{M}(u)}{M(u)} - i^M(u) \right) du}$$

when $i(v) = i^M(v), v \geq t$. From equation (24), the alternative expression for the wealth represented by the seigniorage monopoly of the State in a permanent liquidity trap is:

$$M(t) + \lim_{\ell \to \infty} \int_t^{\ell} \left(\frac{\dot{M}(v)}{M(v)} - i^M(v) \right) M(v) e^{-\int_t^v i^M(u)du} dv .$$

This again shows that the authorities can use helicopter money to boost consumer demand even in the severest of all conceivable liquidity traps. What they have to do is set a growth rate for the nominal stock of base money, for some finite period of time (forever is possible, but would result in an infinite increase in wealth), that is greater than the own interest rate on base money:

$$\frac{\dot{M}(v)}{M(v)} > i^M(v), t \leq t_1 \leq v \leq t_2,\ t_2 > t_1$$

What this means is that a fiat money economy where the State controls the issuance of fiat money, a liquidity trap is a choice, not a necessity. Most general

equilibrium completions of a model with the consumption function used in this paper will have the property that if, in a perpetual liquidity trap or effective lower bound (ELB) equilibrium, real demand is boosted by a sufficiently large magnitude, the permanent liquidity trap vanishes.

Equations (26) or (27) (or their more general versions without Ricardian equivalence) make it clear that it is also possible for the State to boost public spending on real goods and services, current or capital, and avoid any negative impact of the anticipation of higher future taxes on demand by monetizing the resulting public sector deficits.

2.7 Helicopter money without Ricardian equivalence

The way helicopter money affects household demand is the same in the overlapping generations model (the Yaari–Blanchard model with $\beta > 0$) as in the representative agent model ($\beta = 0$). A comparison of equations (23) and (25) with equations (26) and (27) shows that the nominal value of the comprehensive wealth term in the aggregate consumption function is augmented by base money issuance to the tune of

$$\int_t^\infty \left(i(v) - i^M(v)\right) M(v) e^{-\int_t^v i(u)du} dv + \lim_{v \to \infty} M(v) e^{-\int_t^v i(u)du}$$

or, equivalently,

$$M(t) + \int_t^\infty \left(\dot{M}(v) - i^M(v)\right) e^{-\int_t^v i(u)du} dv$$

It is clear from the model without Ricardian equivalence that monetary base expansions of a given (positive) magnitude in PDV terms will now have different effects when they are implemented through up-front lump-sum transfer payments/tax cuts than through up-front QE (open market purchases of sovereign bonds) followed by deferred transfer payments or tax cuts. Because the deferred tax cuts will in part be enjoyed by generations not yet born today, the 'up-front QE and deferred transfer payment boost' version will be less expansionary, for a given

PDV of base money issuance, than the version with the up-front transfer payment boost – helicopter money.

3 Some further considerations

3.1 Fiat base money is special

In this model unbacked fiat base money is unique for two reasons. First, it performs liquidity or transactions functions that cause it to be willingly held by private agents despite being pecuniary-rate-of-return-dominated. Currency carries a pecuniary rate of return of zero and required or excess reserves carry nominal rates of interest (or own rates of interest) that are almost always below those yielded by other safe nominally denominated assets with the same (short) maturity, like domestic-currency denominated sovereign debt: $i^M < i$. In exceptional circumstances, at the effective lower bound or ELB, nominal interest rates on safe monetary assets can get close to or even equal those on currency and Central Bank reserves: $i \approx i^M$. It is hard to think of circumstances where safe non-monetary debt yields a consistently lower pecuniary rate of return than base money or even than Central Bank reserves: $i^M > i$. Unless Central Banks engage in risky enterprises (acting as a de-facto agricultural development bank, as was the case in Peru under the first Garcia government, purchasing outright debt issued by a sovereign at high risk of default at better than fair prices, or accepting such debt as collateral for loans to banks that would likely default themselves were the sovereign to default, as the ECB has done in the case of Greece) central banking is a profitable business.

I shoe-horned this uniqueness of base money into the model by having base money as an argument in the household's direct utility function. This is not very satisfactory. The only justification is simplicity and the robustness of the results of the paper to using other mechanisms for making fiat base money a superior asset (money in the production function, cash–in-advance or legal restrictions).

What makes something (or some class of objects) desirable because of its unique transactions-facilitating properties differs in the many different approaches that have been adopted for generating a willingness to hold something that is pecuniary-rate-of-return-dominated as a store of value. It is the outcome of a

collective, decentralized social choice. It may help if something is granted legal tender status by the State, but this not a necessary condition. Should fiat base money issued by the State lose this unique advantages it has in facilitating transactions, it will have to pay interest at the same rate as the other safe, liquid financial assets – bonds in this model, or it will not be held voluntarily by private agents. We are in the Wallace (1981, 1990) world of the Modigliani-Miller theorem for open market operations. The net present discounted value of future interest saved is, of course zero in this case. However, if the monetary asset is irredeemable, the PDV of the terminal base money stock would still be net wealth. For this to be positive, the growth rate of the nominal stock of base money would have to be at least equal to the nominal rate of interest (on both base money and other safe assets) in the long run. In the liquidity trap case, with a the safe rate on non-monetary assets equal to the interest rate on base money forever, a helicopter money drop would still be effective in boosting household consumption demand, even though a helicopter bond drop would not be.

3.2 The importance of being precise about the 'irreversibility' of a helicopter drop

The increase in the monetary base need not be permanent or irreversible for it to boost aggregate demand (private and/or public).[8] Permanence or irreversibility of the increase in the stock of base money is neither necessary nor sufficient for a base money-financed fiscal stimulus to be more expansionary than the same size and composition fiscal stimulus financed through the issuance of Treasury debt - with no current and/or future monetization of this additional Treasury debt. Helicopter money will be more expansionary than Treasury debt financing if and only if it is part of a sequence of current and/or future base money issuance which has the property that the present discounted value (PDV) of this sequence of current and future base money increases *minus* any interest paid on the current and future stocks of base money, is positive. I will use the shorthand: "not/never

[8] In earlier versions of this paper I defined a helicopter money drop as a permanent/irreversible increase in the stock of base money. In a number of footnotes I then demonstrated why the permanence/irreversibility of the base money increase was neither necessary nor sufficient for it to be effective. I am in good company here, see e.g. Turner (2015) and Bernanke (2016).

completely reversed in PDV terms" for such an expansionary base money stock issuance sequence.

QE, viewed as the purchase of non-monetary financial assets by the Central Bank funded through an increase in the stock of base money that is not completely reversed in PDV terms, relaxes the intertemporal budget constraint of the State (the consolidated Treasury and Central Bank). Consequently, there will have to be some combination of current and future tax cuts or current and future increases in public spending to ensure that the intertemporal budget constraint of the State remains satisfied with strict equality (no '(bond) creditor state').[9] QE is therefore not a complete description of a policy action by the State; both temporary and permanent QE relax the intertemporal budget constraint of the consolidated Central Bank and Treasury if the nominal rate of return on the assets of the Central Bank exceeds the nominal interest rate on base money, and/or fiat base money is irredeemable. Some other fiscal or financial action(s) will have to be undertaken, now or in the future, to ensure that the intertemporal budget constraint of the State is satisfied with equality. In our simple model, QE is the purchase by the Central Bank of sovereign debt funded through money issuance. The same results would hold, however, if the Central Bank purchased private securities outright instead of sovereign debt, or expanded its balance sheet through collateralized lending, as long as such balance sheet expansion is profitable.

3.3 Fiat base money is net wealth

Fiat base money is net wealth for the consolidated private sector and State sector. Despite fiat money technically being inside money and an inside asset (issued by one economic agent and held by another), fiat base money behaviorally or

9 Assuming that the intertemporal budget constraint held with strict equality in the benchmark equilibrium. Note that even temporary (eventually reversed) QE or a temporary fiscal stimulus funded temporarily by base money issuance but eventually by issuing non-monetary debt will be expansionary, compared to an otherwise identical set of policy actions in which there is never any issuance of additional base money. What matters is effect of these operations on the PDV of the interest saved (profit made) by funding through base money issuance rather than 'bond' issuance, plus the present discounted value of the terminal stock of base money, which is the same as the PDV of the sequences of current and future base money increases, minus any interest paid on the current and future stocks of base money.

effectively is like nature's bounty: an asset and wealth to the owner but not a claim on or liability of the issuer.

Indeed, looking at the Ricardian version of the aggregate consumption function in equation (26) or (27), note that the term

$$\frac{1}{P(t)}\left(M(t) + \int_t^{\infty} \left(\dot{M}(v) - i^M(v) \right) e^{-\int_t^v i(u)du} dv \right)$$

could equally well represent true 'outside assets', like intrinsically worthless pet rocks or Rai, the stone money used on the Isle of Yap. The stock of rare bits of rock deposited on earth by meteorites, say, could be represented by *M(t)* and the net present value of future meteorite deposits could be could be

$$\int_t^{\infty} \left(\dot{M}(v) - i^M(v) \right) e^{-\int_t^v i(u)du} dv .$$

It would be hard to rationalize a non-zero own rate of interest on pet rocks or Rai, but if (intrinsically worthless) rabbits were a socially preferred currency, a positive own rate of interest would be quite plausible. With some slight modifications, almost intrinsically worthless commodities like gold and intrinsically worthless virtual media of exchange like Bitcoin could also fit into our consumption function. Both are, of course, costly to produce or 'mine'. Helicopter drops of Rai, gold or Bitcoin would not share with fiat base money the property that they are issued by the State and can be used to fund the State. They don't roll off the printing presses but are gifts from nature (Rai and gold) and from human ingenuity (in the case of Bitcoin).

3.4 When should a helicopter money drop be preferred to a bond financed fiscal stimulus?

When there is no Ricardian equivalence, aggregate demand can also be stimulated through tax cuts (even through cuts in lump-sum taxes) or higher public spending that) that are financed by the issuance of non-monetary Treasury debt that is never monetized. A temporary balanced-budget increase in public spending on goods and services will boost demand even with there is Ricardian equivalence. When is

helicopter money the best choice? The answer depends on how the economy functions, on the equilibrium it is in and on policy preferences and on political economy considerations.

Almost regardless of the model of the economy one trusts most, it will be the case that aggregate demand will indeed be stimulated, if the fiscal space created by a monetized Central Bank balance sheet expansion that is not reversed completely in PDV terms, is used to provide an up-front fiscal stimulus to boost aggregate demand (at benchmark prices and quantities). Whether this is desirable depends on the nature of the benchmark equilibrium: if there is economic slack (involuntary unemployment and idle capital), inflation is below-target, the neutral real interest rate is worryingly low, the sovereign non-monetary debt burden is sustainable and the external current account deficit is manageable, a boost to aggregate demand makes sense.

If there are no idle resources, inflation is at or above-target, the non-monetary sovereign debt is high and the current account balance is uncomfortably low (or negative), no demand stimulus of any kind, monetized or not, is called for. Government debt issuance may be still desirable for tax-smoothing or other 'micro-efficiency' reasons, of course.

In the current economic conditions faced by the euro area, Japan, China, the UK (since the Brexit vote) and, to a lesser degree, by the US, monetizing a fiscal stimulus would seem to be the obvious first choice. It is correct, as pointed out by Larry Summers (Summers 2015) that, when there is economic slack, inflation is below target and the rate of return on well-chosen infrastructure in the US is much higher than the 2.28% percent fixed nominal interest rate the Federal government can borrow at for 30 years, it makes sense to borrow at that rate.[10] However, by instead monetizing the infrastructure funding, the State formally borrows overnight at 0.50%, if this is done by increasing Excess Reserves held at the Fed. The State may appear to be facing future interest rate risk as it rolls over this very short-term debt. However, the State effectively borrows at an interest rate of –100%, because the monetary debt that is issued never has to be repaid – it is irredeemable – and the interest bill can be paid for by continued base money issuance – growing (at least) at the Rate on Excess Reserves. From a financial

[10] The 30 year Treasury yield was 2.28% on 11 August, 2016. The Rate on Required Reserves and on Excess Reserves on 11 August 2016 was 0.50%.

perspective, the State gets the infrastructure for free. Whether it makes economic sense depends on the inflationary and other consequences of monetary financing in the long run.

An important political economy consideration is whether there is a risk that the political masters of the Treasury become 'addicted' to monetizing public debt and deficits, and would be unwilling or unable to stop once they start the process, even after the economic conditions that justified helicopter money have changed and the common good is no longer served by extending the duration or scope of the helicopter money drops. The fact that hyperinflations have occurred only in countries at the losing end of a war and settled with a heavy reparations bill (Weimar Germany) or in failed states (Zimbabwe, Argentina, Peru) should give some comfort to the proponents of helicopter money.

3.5 The institutional implementation of helicopter money drops

In most contemporary advanced economies, the issuance of fiat base money (often with legal tender status) is performed by an agency of the State, the Central Bank, that has some degree of operational independence (and in a few cases even a measure of target independence) in the design and implementation of monetary policy. Some Central Banks can act as fiscal agents for the State but none that we know of can act openly as fiscal principals. Central Banks typically transfer their profits (over and above what they want to add to reserves or provisions) to their beneficial owner, the central government or federal Treasury.[11] Specifically, Central Banks cannot levy taxes, make transfer payments or pay overt subsidies to other domestic economic entities, nor can they engage in exhaustive public spending other than what is inevitably involved in the running of the Central Bank (payroll, capital expenditure on buildings and equipment, supplies, utilities etc.). The fact that many Central Banks have engaged in large-scale quasi-fiscal interventions, most recently during and after the North-Atlantic financial crisis of

[11] The European Central Bank (ECB) is unique in that its shareholders are the national Central Banks (NCBs) of the 28 (as of May 2014) European Union member States. The profits of the ECB are distributed to the 18 (as of May 2014) NCBs of the EU member States that are also members of the euro area.

2007–2008, does not change the basic legal and institutional reality that a Central Bank cannot implement helicopter money on its own.

Cooperation and coordination between the Central Bank and the Treasury are required for the real-world implementation of helicopter money drops. In practice, to implement the temporary fiscal stimulus financed through the issuance of fiat base money that is closest to the original Friedman helicopter money parable – a lump-sum transfer payment to households funded through base money issuance that is kept constant in PDV terms (allowing for any interest paid on the monetary base) –, the following coordinated fiscal-monetary actions would take place. There would be a one-off cash transfer to all eligible households by the Treasury. The Treasury funds these payments by selling Treasury debt to the Central Bank, which credits the account held by the Treasury with the Central Bank (which is not normally counted as part of the monetary base but constitutes a non-monetary liability of the Central Bank). As the Treasury pays out the cash to the eligible households, the Treasury's account with the Central Bank is drawn down. The monetary base increases because the transfer payment to the households either ends up as increased cash/currency held by households, corporates or banks or as increased bank reserves held with the Central Bank. A virtually identical story can be told if instead of a transfer payment to the household sector, the Treasury were to engage in a program of exhaustive current or capital expenditure.

3.6 The irrelevance of the cancellation of Treasury debt held by the Central Bank.

From a fundamental economic perspective, it makes no difference whether the Central Bank cancels the sovereign bonds it buys (as proposed e.g. by Turner 2013) or holds them indefinitely (rolling them over as they mature). This is because the Treasury is the beneficial owner of the Central Bank. The Treasury therefore receives the Central Bank's profits. Their accounts (including balance sheets and P&L account) therefore can be – or indeed ought to be – consolidated to get a proper perspective on the flow of funds and balance sheet accounts that matter. The only reason to prefer cancellation of sovereign debt held by the Central Bank over the Central Bank holding the sovereign debt permanently is that cancellation may be seen as a more credible commitment device or 'signal' of the irreversibility of the monetary injection. If the Central Bank holds assets other than

Treasury Bills and Bonds, like private securities, repos or other collateralized loans to the private sector, the Treasury is entitled to the profits on these investments also. In the consolidated State accounts, the Treasury debt held as an asset by the Central Bank is netted out against the corresponding Treasury liabilities, and the other assets of the Central Bank become assets of the consolidated State. Consolidating the Central Bank and Treasury accounts highlights that as regards financial and other conventional assets and liabilities, what matters for fiscal sustainability/fiscal space is the value of the net non-monetary assets of the State.

The disaggregated period (instantaneous) budget identity, the intertemporal budget identity and the solvency constraint of the Treasury are given in equations (28), (29) and (30). Those of the Central Bank are given in equations (31), (32) and (33). As before, *B* stands for Treasury debt held outside the Central Bank. B^{cb} denotes Treasury debt held by the Central Bank. *T* is the real value of taxes paid by the private sector, T^{cb} is the real value of payments made by the Central Bank to the Treasury. Our Central Bank is extremely frugal and does not spend on real goods and services. To keep things simple, we consider a closed economy: the Central Bank does not hold any foreign exchange reserves.

$$\frac{\dot{B}(t)+\dot{B}^{cb}(t)}{P(t)} \equiv i(t)\left(\frac{B(t)+B^{cb}(t)}{P(t)}\right)+G(t)-T(t)-T^{cb}(t) \tag{28}$$

$$\begin{aligned}\frac{B(t)+B^{cb}(t)}{P(t)} \equiv & \int_t^{\infty}\left(T(v)+T^{cb}(t)-G(v)\right)e^{-\int_t^v r(u)du}dv \\ & +\lim_{v\to\infty}\left(\frac{B(v)+B^{cb}(t)}{P(v)}\right)e^{-\int_t^v r(u)du}\end{aligned} \tag{29}$$

$$\lim_{v\to\infty}\left(\frac{B(v)+B^{cb}(t)}{P(v)}\right)e^{-\int_t^v r(u)du} \le 0 \tag{30}$$

$$\frac{\dot{M}(t)-\dot{B}^{cb}(t)}{P(t)} \equiv T^{cb}(t)-i(t)\frac{B^{cb}(t)}{P(t)}+i^{M}(t)\frac{M(t)}{P(t)} \tag{31}$$

$$\frac{M(t)-B^{cb}(t)}{P(t)} \equiv \int_t^{\infty}\left(-T^{cb}(v)+\left(i(v)-i^{M}(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)-B^{cb}(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{32}$$

$$\lim_{v\to\infty}\left(\frac{B^{cb}(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \geq 0 \tag{33}$$

The Treasury's intertemporal budget identity and solvency constraint imply the Treasury's intertemporal budget constraint:

$$\frac{B(t)+B^{cb}(t)}{P(t)} \leq \int_t^{\infty}\left(T(v)+T^{cb}(t)-G(v)\right)e^{-\int_t^v r(u)du}dv \tag{34}$$

The Central Bank's intertemporal budget identity and solvency constraint, which recognizes the irredeemability of fiat base money, imply the Central Bank's intertemporal budget constraint, which we can write as the requirement that the Comprehensive Net Worth (Comprehensive Capital or Comprehensive Equity) of the Central Bank, Ω_t be non-negative, that is

$$\Omega_t = \frac{B^{cb}(t)-M(t)}{P(t)} - \int_t^{\infty}\left(T^{cb}(v)-\left(i(v)-i^{M}(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \geq 0 \tag{35}$$

The Treasury, as the beneficial owner of the Central Bank, receives all its profits and absorbs all its losses (not necessarily when they are earned/incurred), subject to the constraint that the Comprehensive Net Worth, Ω_t of the Central Bank be equal to some non-negative value $\overline{\Omega}_t \geq 0$ at all times. The sequence of current and future net payments of the Central Bank to the Treasury therefore satisfies:

$$\int_t^{\infty} T^{cb}(v)e^{-\int_t^v r(u)du}dv =$$

$$-\overline{\Omega}_t + \frac{B^{cb}(t) - M(t)}{P(t)}$$

$$+\int_t^{\infty}\left(\left(i(v) - i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \qquad (36)$$

or equivalently:

$$\int_t^{\infty} T^{cb}(v)e^{-\int_t^v r(u)du}dv = -\overline{\Omega}_t + \frac{B^{cb}(t)}{P(t)} + \int_t^{\infty}\left(\frac{\dot{M}(v) - i^M(v)M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv$$

A very simple rule that satisfies (36) provided the initial ($t = 0$, say) value of the Central Bank's comprehensive net worth equals $\overline{\Omega}_0 \geq 0$, would be for the Central Bank to have a continuously balanced budget:

$$T^{cb}(t) = i(t)\frac{B^{cb}(t)}{P(t)} - i^M(t)\frac{M(t)}{P(t)} \qquad (37)$$

which implies that

$$\dot{M}(t) = \dot{B}^{cb}(t) \qquad (38)$$

Briefly, it does not matter whether the Central Bank today cancels an amount $B^{cb}(t)$ of debt owed to it by the Treasury and as a result does not pay out as profits to the Treasury an infinite future stream of Central Bank profits $\left\{i(v)B^{cb}(t); v \geq t\right\}$ (whose PDV is, of course, $B^{cb}(t)$), or whether it keeps its existing holdings of Treasury debt on its books and pays out as profits to the Treasury an infinite stream of future profits $\left\{i(v)B^{cb}(t), v \geq t\right\}$. This equivalence of the Central Bank cancelling forgiving) a given amount of Treasury debt versus holding it forever (rolling it over when required) is, of course, consistent with the consolidated intertemporal budget constraint of the Central Bank and the Treasury in equations (21) and (22), in which the Central Bank's holdings of Treasury debt are absent. Not that this equivalence holds regardless of the maturity of the

Treasury debt held by the Central Bank – it could be perpetuities – and regardless of the terms on which the Central Bank buys these securities from the Treasury (positive coupon, zero coupon or negative coupon). Treasury debt purchases by the Central Bank could also be replaced by a credit line, overdraft facility or loan from the Central Bank to the Treasury (on any financial terms).

Would Treasury debt cancellation by the Central Bank be viewed as a credible commitment device, or costly signal, not to reverse (in PDV terms) a helicopter money drop? It is hard to see how this would be viewed as anything more than cheap talk. The reduction in the conventional equity/capital/net worth of the central bank cause by the Treasury debt cancellation – indeed if the Treasury debt cancellation were large enough central bank conventional equity could become negative – might cause some temporary market jitters, but it is difficult to see why there would be any lasting cognitive impact on the private sector of such accounting window dressing operations.

3.7 Could buying perpetuities be an effective signal of non-reversibility?

The same considerations that lead one to question the effectiveness as a signal of non-reversibility, of Treasury debt cancellation by the Central Bank also lead one to question the effectiveness of swapping the Central Bank's holdings of finite maturity Treasury debt for perpetual Treasury bonds or perpetuities. After all, the Central Bank could buy the perpetuities today and sell them in the markets again tomorrow, using the proceeds to contract the monetary base. As a signal, buying perpetuities would seem to be rather unconvincing. It also makes no fundamental difference whether the Central Bank buys zero coupon, negative coupon or positive coupon perpetuities from the Treasury, or gives the Treasury a permanent overdraft facility or infinite duration loan at zero, negative, positive, fixed or floating interest rates. All this is fundamentally no different from the Central Bank buying one-month Treasury Bills and rolling them over, as long as the markets believe that this is what the Central Bank will do.

To be credible, signals must be costly to the party sending the signal. It is hard to see how buying perpetuities (like cancelling Treasury debt) would be a costly signal for the Central Bank.

3.8 Should the helicopter drop perishable money?

The argument is sometimes made that helicopter money drops would benefit from a dose of 'Gesell'(see Gesell 1949 and Skidelsky 2016). Gesell proposed paying negative interest on money, to encourage it to be spent more quickly. The same can be achieved by the Treasury sending vouchers with a fixed expiration date to the private sector and funding the operation by borrowing from the Central Bank, which monetizes the Treasury debt it purchases or the loan it extends to the Treasury.

Households receiving a depreciating or perishable Gesell transfer payment would have a strong incentive to spend it early rather than to save it. This could make a significant difference to the consumption behavior of liquidity-or debt-constrained households whose current income from other sources is less than the value of the Gesell transfer payment. A household that is free to borrow and lend could spend the Gesell transfer payment in full but likely would reduce consumption funded out of other income, by borrowing or dissaving and increase its aggregate consumption only by the permanent income equivalent of the transfer payment.

The government can of course make sure that the helicopter money is spent in full on currently produced goods and services by boosting public spending on infrastructure, health, education or defense or any other public purpose.

3.9 Helicopter money drops and the ECB

Matters are slightly more complicated for the ECB, whose equity is held by the national Central Banks (NCBs) of the member States that are part of the euro area. Each NCB has its national Treasury as its beneficial owner. Cancelling an amount $B_i^{cb}(t)$ of sovereign debt of euro area member state *i* (which has an equity stake η_i in the ECB), represents ultimately a wealth transfer of $(1-\eta_i)B_i^{cb}(t)$ to the Treasury of member State *i* from the Treasuries of all other member States. Holding $B_i^{cb}(t)$ indefinitely on the balance sheet of the ECB would result in an infinite stream of profits $\left\{i(v)\eta_i B_i^{cb}(t), v \geq t\right\}$ to the NCB of country *i* and thus ultimately to the Treasury of country *i* and $\left\{i(v)(1-\eta_i)B_i^{cb}(t), v \geq t\right\}$ to the NCBs

of the remaining euro area member States and thus ultimate to their national Treasuries.

This real-world implementation of helicopter money drops is legal and easily implemented everywhere except in the euro area. Article 123.1 of the Treaty on the Functioning of the European Union States:

> *"Overdraft facilities or any other type of credit facility with the European Central Bank or with the Central Banks of the Member States (hereinafter referred to as 'national Central Banks') in favour of Union institutions, bodies, offices or agencies, central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of Member States shall be prohibited, as shall the purchase directly from them by the European Central Bank or national Central Banks of debt instruments."* [12]

This clause has commonly been interpreted as ruling out the financing of government deficits in the euro area through government debt sales to the ECB (or to the NCBs of the Eurosystem) and their monetization by the Eurosystem. Unless this can be fudged by the Eurosystem purchasing the sovereign debt in the secondary markets (as it did under the Securities Markets Programme and proposes to do under the Outright Monetary Transactions programme (should it ever be activated)), Article 123.1 deprives the euro area of the one policy instrument – a temporary fiscal stimulus funded by and monetized by the Central Bank – that is guaranteed to prevent or cure deflation, "lowflation" or secular stagnation. It is time for Article 123 to be scrapped in its entirety if the euro area does not wish to face an unnecessary risk of falling into any of these traps.

4 Conclusion

A helicopter drop of irredeemable fiat base money that is not completely reversed in present discounted value terms boosts demand both when Ricardian equivalence

12 http://www.lisbon-treaty.org/wcm/the-lisbon-treaty/treaty-on-the-functioning-of-the-european-union-and-comments/part-3-union-policies-and-internal-actions/title-viii-economic-and-monetary-policy/chapter-1-economic-policy/391-article-123.html

does not hold and when it holds, and even in a permanent liquidity trap - when nominal yields on all risk-free assets are at the Effective Lower Bound. It makes secular stagnation a policy choice, not something driven by circumstances beyond national policy makers' control.[13]

Acknowledgement: The views and opinions expressed are those of the author alone and should not be attributed to Citigroup or to any other organization the author is associated with. I would like to thank Larry Summers from prodding me to write a short note setting out the essence of the helicopter money argument.

[13] In dynamic general equilibrium with flexible nominal prices, there always exists an equilibrium with a zero price of money in all periods and all States of nature – the barter equilibrium or non-monetary equilibrium. Obviously, helicopter money drops won't boost demand in such an equilibrium.

References

Bernanke, Ben S. (2003). "Some Thoughts on Monetary Policy in Japan", speech, Tokyo, May. http://www.federalreserve.gov/BoardDocs/Speeches/2003/20030531/

Bernanke, Ben S. (2016). What Tools Does the Fed Have Left? Part 3: Helicopter Money. Brookings Blog, Monday, April 11, https://www.brookings.edu/2016/04/11/what-tools-does-the-fed-have-left-part-3-helicopter-money/

Blanchard, Olivier J. (1985). Debt, Deficits and Finite Horizons. *Journal of Political Economy* 93(2): 223–247. http://ideas.repec.org/a/ucp/jpolec/v93y1985i2p223-47.html

Brunner, Karl and Allan H. Meltzer (1968). Liquidity Traps for Money, Bank Credit and Interest Rates. *Journal of Political Economy* 76(1): 1–37. http://www.journals.uchicago.edu/doi/abs/10.1086/259378

Brunner, Karl and Allan H. Meltzer (1972). Money, Debt, and Economic Activity. *Journal of Political Economy* 80(5): 951–977. http://repository.cmu.edu/cgi/viewcontent.cgi?article=1659&context=tepper

Buiter, Willem H. (1988). Death, Birth, Productivity Growth and Debt Neutrality. *Economic Journal* 98(June): 279–293. http://ideas.repec.org/a/ecj/econjl/v98y1988i391p279-93.html

Buiter, Willem H. (2003a). Deflation: Prevention and cure. NBER Working Paper W9623, Apr. 2003. http://willembuiter.com/def.pdf. For a technical appendix, click here: http://willembuiter.com/defap.pdf.

Buiter, Willem H. (2003b). Helicopter money: Irredeemable Fiat Money and the Liquidity Trap. NBER Working Paper No. W10163, Dec. http://www.willembuiter.com/helinber.pdf

For an abstract, click here: http://www.willembuiter.com/abheli.pdf. For an appendix on a cash-in-advance version of the representative agent model, click here: http://www.willembuiter.com/heliap1.pdf. For an appendix on the continuous-time version of the model, click here: http://www.willembuiter.com/heliap2.pdf . A more recent, shorter version, can be found here: http://www.willembuiter.com/helijpe.pdf

Buiter, Willem H. (2007). Seigniorage. *Economics*: *The Open-Access, Open-Assessment E-Journal* 1(2007–10): 1–49. http://dx.doi.org/10.5018/economics-ejournal.ja.2007-10

Buiter, Willem H., Ebrahim Rahbari and Joseph Seydl (2014). Secular Stagnation: Only If We Really Ask for It. *Citi Research, Economics, Global, Global Economics View*, 13 January. https://ir.citi.com/SHP9jQovS7%2BiLAYopQ%2F8XAuZL6iSYJm8QGwYXyK64oJ5eO3Iz86d9Dv2CS%2BlgKn2

Cumming, Fergus (2015). Helicopter Money: Setting the Tale Straight. Bank Underground Blog, 5 August 2015. http://bankunderground.co.uk/2015/08/05/helicopter-money-setting-the-tale-straight/

Eggertsson, Gauti (2003). How to Fight Deflation in a Liquidity Trap: Committing to Being Irresponsible. IMF Working Paper WP/03/63, March. http://www.imf.org/external/pubs/cat/longres.aspx?sk=16331

Eggertsson, Gauti and Paul Krugman (2012). Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach. *The Quarterly Journal of Economics* 127(3): 1469–1513. http://qje.oxfordjournals.org/content/127/3/1469

Eggertsson, Gauti and Neil Mehrotra (2014). A Model of Secular Stagnation. Discussion Paper, July 4. http://www.econ.brown.edu/fac/gauti_eggertsson/papers/Eggertsson_Mehrotra.pdf

Eggertsson, Gauti and Michael Woodford (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity* 34(1): 139–233. http://www.brookings.edu/about/projects/bpea/papers/2003/zero-bound-interest-rates-monetary-policy-eggertsson

Eggertsson, Gauti and Michael Woodford (2006). Optimal Monetary and Fiscal Policy in a Liquidity Trap, in *NBER International Seminar on Macroeconomics*, Richard H. Clarida, Jeffrey Frankel, Francesco Giavazzi and Kenneth D. West (eds). MIT Press. http://www.nber.org/chapters/c0076

Fama, Eugene F. (1983). Financial Intermediation and Price Level Control. *Journal of Monetary Economics*, 12(1): 7–28. http://www.sciencedirect.com/science/article/pii/0304393283900454

Friedman, Milton (1948). A Monetary and Fiscal Framework for Economic Stability. *The American Economic Review* 38 (3): 245–264. http://www.jstor.org/stable/1810624?__redirected

Friedman, Milton (1969). The Optimum Quantity of Money, in Milton Friedman, *The Optimum Quantity of Money and Other Essays*, Chapter 1. Adline Publishing Company, Chicago.

Gordon, Robert J. (2014). The Demise of U. S. Economic Growth: Restatement, Rebuttal, and Reflections. NBER Working Paper 19895, February. http://econpapers.repec.org/paper/nbrnberwo/19895.htm

Gali, Jordi (2014). The Effects of a Money-Financed Fiscal Stimulus. Unpublished paper, September. http://crei.cat/people/gali/gmoney.pdf

Gesell, S. (1949), *Die natürliche Wirtschaftsordnung*, available in English as *The Natural Economic Order*, London: Peter Owen Ltd, 1958.

Gurley, John G. and Edward S. Shaw (1960). *Money in a Theory of Finance*. The Brookings Institution, Washington D.C.

Hall, Robert E. (1983). Optimal Fiduciary Monetary Systems. *Journal of Monetary Economics* 12(1): 33–50. http://web.stanford.edu/~rehall/Optimal_Fiduciary_Monetary_Systems.pdf

Hansen, Alvin H. (1938), *Full Recovery or Stagnation?* WWW. Norton.

King, Robert G. (1983). On the Economics of Private Money. *Journal of Monetary Economics* 12(1): 127–158. http://www.sciencedirect.com/science/article/pii/0304393283900533

Lenza, Michele, Huw Pill and Lucrezia Reichlin (2010). Monetary Policy in Exceptional Times. *Economic Policy* 62, 295–339. http://dx.doi.org/10.1002/9781444390261.ch3

Moghadam, Reza, Ranjit Teja and Pelin Berkmen (2014). Euro Area – "Deflation" Versus "Lowflation". IMFdirect, The international Monetary Fund's global economy forum, March 4. http://blog-imfdirect.imf.org/2014/03/04/euro-area-deflation-versus-lowflation/ .

Patinkin, Don (1956). *Money, Interest and Prices; An Integration of Monetary and Value Theory*. First Edition, Row, Peterson and Company, Evanston, Ill..

Patinkin, Don (1965). *Money, Interest and Prices; An Integration of Monetary and Value Theory*. Second Edition, Harper & Row, Publishers, New York.

Pesek, Boris P., and Thomas R. Saving (1967). *Saving, Money, Wealth, and Economic Theory*. New York: The Macmillan Company,.

Reichlin, Lucrezia, Adair Turner and Michael Woodford (2013). Helicopter Money as a Policy Option. VoxEU, 20 May 2013, http://www.voxeu.org/article/helicopter-money-policy-option .

Sargent, Thomas. J. (1987). *Dynamic Macroeconomic Theory*, Cambridge Massachusetts: Harvard University Press.

Sargent, Thomas J. and Bruce D. Smith (1987). Irrelevance of Open Market Operations in Some Economies with Government Currency Being Dominated in Rate of Return. *The American Economic Review* 77(1): 78–92. http://www.jstor.org/stable/1806730

Sargent, Thomas J. and Neil Wallace (1984). Interest on Reserves. *Journal of Monetary Economics* 15(3): 279–290. http://ideas.repec.org/a/eee/moneco/v15y1985i3p279-290.html

Sims, Christopher A. (2001). Fiscal Aspects of Central Bank Independence. CESifo Working Paper Series No. 547. http://ideas.repec.org/p/ces/ceswps/_547.html

Sims, Christopher A. (2004). Limits to Inflation Targeting, in Ben S. Bernanke and Michael Woodford (eds). *The Inflation-Targeting Debate*, NBER Books, National Bureau of Economic Research, Inc, number bern04-1, July. http://ideas.repec.org/b/nbr/nberbk/bern04-1.html

Skidelsky, Robert (2016). A Tweak to Helicopter Money Will Help the Economy Take Off. *Financial Times*, August 4. http://www.ft.com/cms/s/0/a36c5a26-5997-11e6-9f70-badea1b336d4.html.

Stockman, Alan C. (1983). Comments; "Optimal Fiduciary Monetary Systems" by Robert E. Hall. *Journal of Monetary Economics* 12(1): 51–54. http://www.sciencedirect.com/science/article/pii/030439328390048X

Summers, Lawrence (2013). IMF 14th Annual Research Conference in Honor of Stanley Fisher, International Monetary Fund, November 8. http://larrysummers.com/imf-fourteenth-annual-research-conference-in-honor-of-stanley-fischer/

Summer, Lawrence (2015). "Reflections on Secular Stagnation", speech at the Julius-Rabinowitz Center, Princeton University, February 19. http://larrysummers.com/2015/02/25/reflections-on-secular-stagnation/

Tobin, James (1969). A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking* 1():1: 15–29. http://www.jstor.org/stable/1991374

Turner, A (2013). "Debt, Money and Mephistopheles", speech at Cass Business School, 6 February. http://www.fsa.gov.uk/library/communication/speeches/2013/0206-at .

Turner, A. (2015). The Case for Monetary Finance – An Essentially Political Issue. IMF Jacques Polak Research Conference, 5th November, 2015. https://www.imf.org/external/np/res/seminars/2015/arc/pdf/adair.pdf

Wallace, Neil (1981). A Modigliani-Miller Theorem for Open Market Operations. *American Economic Review* 71(3): 267–274. http://ideas.repec.org/a/aea/aecrev/v71y1981i3p267-74.html

Wallace, Neil (1990). A Suggestion for Oversimplifying the Theory of Money. *Federal Reserve Bank of Minneapolis Quarterly Review* 13(1): 19–26. http://ideas.repec.org/a/fip/fedmqr/y1990iwinp19-26nv.14no.1.html

Weil, Philippe (1989). Overlapping Families of Infinitely-lived Agents. *Journal of Public Economics* 38(2): 183–198. http://ideas.repec.org/a/eee/pubeco/v38y1989i2p183-198.html

Weil, Philippe (1991). Is Money Net Wealth? *International Economic Review* 32(1, February): 37–53. http://ideas.repec.org/a/ier/iecrev/v32y1991i1p37-53.html

Woodford, M (2012). Methods of Policy Accommodation at the Interest-Rate Lower Bound. Proceedings – Economic Policy Symposium – Jackson Hole: 185–288. http://ideas.repec.org/a/fip/fedkpr/y2012p185-288.html

Yaari, M. E. (1965). Uncertain Lifetime, Life Insurance and the Theory of the Consumer. *Review of Economic Studies* 32(2): 137–150. http://restud.oxfordjournals.org/content/32/2/137.extract