Economics
The Open-Access, Open-Assessment E-Journal

Vol. 8, 2014-28 | August 21, 2014 | http://dx.doi.org/10.5018/economics-ejournal.ja.2014-28
Version 2 – September 2, 2015

The Simple Analytics of Helicopter Money: Why It Works — Always

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Abstract

The paper offers a rigorous analysis of Milton Friedman's parable of the 'helicopter' drop of money – a permanent/irreversible increase in the nominal stock of fiat base money which respects the intertemporal budget constraints of the Central Bank, the Treasury and the consolidated State. One example is a temporary fiscal stimulus funded permanently through an increase in the stock of base money. Another one is permanent QE – an irreversible, monetized open market purchase by the Central Bank of Treasury debt or private debt, when it is recognised that this permanent up-front open market purchase will have to be followed sooner or later by public spending increases or tax cuts, to ensure that the intertemporal budget constraints of the Central Bank and the Treasury remain satisfied. Three conditions must be satisfied for a helicopter money drop always to boost aggregate demand. First, there must be benefits from holding fiat base money other than its pecuniary rate of return: that is, the interest rate on any additional base money issued is below the rate of return on the Central Bank's assets: central banking, and specifically Central Bank balance sheet expansion, is profitable. Second, fiat base money is irredeemable – viewed as an asset by the holder but not as a liability by the issuer. Third, the price of money is positive. Given these three conditions, there always exists – even in a permanent liquidity trap – a combined (set of) monetary and fiscal policy action(s) that boosts private demand (or public demand) – in principle without limit. Under these conditions, a helicopter money drop will not cause 'policy insolvency': the Central Bank does not lose control of the size of its balance sheet or of the inflation rate. Deflation, 'lowflation' and secular stagnation are therefore policy choices.

JEL E2 E4 E5 E6 H6
Keywords Helicopter money; liquidity trap; seigniorage; secular stagnation; central bank; quantitative easing

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Citation Willem H. Buiter (2015). The Simple Analytics of Helicopter Money: Why It Works — Always. *Economics: The Open-Access, Open-Assessment E-Journal*, 8 (2014-28): 1—38(Version 2). http://dx.doi.org/10.5018/economics-ejournal.ja.2014-28

Received May 21, 2014 **Published as Economics Discussion Paper** June 13, 2014
Revised August 6, 2014 **Accepted** August 17, 2014 **Published** August 21, 2014

1 Introduction

> *"Let us suppose now that one day a helicopter flies over this community and drops an additional $1000 in bills from the sky, ... Let us suppose further that everyone is convinced that this is a unique event which will never be repeated," (Friedman 1969, pp 4-5).*

This paper aims to provide a rigorous analysis of Milton Friedman's famous parable of the 'helicopter' drop of money. A helicopter drop of money is a permanent/irreversible increase in the nominal stock of fiat base money with a zero nominal interest rate, which respects the intertemporal budget constraint of the consolidated Central Bank and fiscal authority/Treasury – henceforth the State. An example would be a temporary fiscal stimulus (say a one-off transfer payment to households, as in Friedman's example), funded permanently through an increase in the stock of base money. It could also be a permanent increase in the stock of base money through an irreversible open market purchase by the Central Bank of non-monetary sovereign debt held by the public – that is, permanent QE. The reason is that QE, viewed as an irreversible or permanent purchase of non-monetary financial assets by the Central Bank funded through an irreversible or permanent increase in the stock of base money, relaxes the intertemporal budget constraint of the State. Consequently, there will have to be some combination of current and future tax cuts or current and future increases in public spending to ensure that the intertemporal budget constraint of the State remains satisfied.[1] QE is therefore not a complete description of a policy action by the State; both temporary and permanent QE relax the intertemporal budget constraint of the consolidated Central Bank and Treasury if the nominal rate of return on the assets of the Central Bank exceeds the nominal interest rate on base money, and/or fiat base money is irredeemable. Some other fiscal or financial action(s) will have to be undertaken, now or in the future, to ensure the intertemporal budget constraint of the State is satisfied. In our simple model, QE is the purchase by the Central

[1] Even temporary (eventually reversed) QE and a temporary fiscal stimulus funded temporarily by base money issuance but eventually by issuing non-monetary debt will be expansionary. What matters is effect of these operations on the net present discounted value of the interest saved (profit made) by funding through base money issuance rather than 'bond' issuance, plus the present discounted value of the terminal stock of base money.

Bank of sovereign debt funded through money issuance. The same results would hold, however, if the Central Bank purchased private securities outright instead of sovereign debt, or expanded its balance sheet through collateralized lending.

There are three conditions that must be satisfied for helicopter money as defined here to always boost aggregate demand. The first, and by far the most important, condition is that here must be benefits from holding additional fiat base money other than its pecuniary rate of return. Only then will (additional) base money be willingly held despite being dominated as a store of value by non-monetary assets with a risk-free nominal rate of return that is typically positive and almost always higher than the interest rate(s) on required or excess reserves held by commercial banks with the Central Bank. The second condition is that fiat base money is irredeemable: it is viewed as an asset by the holder but not as a liability by the issuer. This is necessary for helicopter money to relax the intertemporal budget constraint of the State and of the consolidated private sector and State, even in a pure liquidity trap, with risk-free nominal interest rates on non-monetary instruments equal to the interest rate on base money for all maturities and for all time. The third condition, necessary to rule out barter equilibria with a zero price of fiat base money when nominal prices are flexible, is that the price of money is positive.

The paper shows that, when the State can issue unbacked, irredeemable fiat money or base money with (1) a nominal interest rate that is less than or equal to the short nominal interest rate on non-monetary financial instruments, (2) can be produced at zero marginal cost and (3) is willingly held in positive amounts by households and other private agents despite the availability of risk-free securities carrying nominal rates of return that are greater than or equal to that on base money, there always exists a combined monetary and fiscal policy action that boosts private demand – in principle without limit. Deflation, inflation below target, 'lowflation', 'subflation' and secular stagnation are therefore unnecessary.[2] They are policy choices.

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[2] The term 'lowflation' is, I believe, due to Moghadam et al. (2014). The term 'subflation' has been around the blogosphere for a while. I use it to refer to an inflation rate below the target level or lower than is optimal. 'Secular stagnation' theories go back to Alvin Hansen (1938). I refer here to the Keynesian variant, which holds that there will be long-term stagnation of employment and economic activity without government demand-side intervention. There also is a long-term supply side variant, associated e.g. with Robert Gordon (2014), which focuses on faltering innovation and productivity

The feature of irredeemable base money that is key for the 'helicopter money effectiveness' result to hold even in a permanent liquidity trap (with the interest rate on non-monetary instruments always equal to the interest rate on base money, is that the acceptance of payment in base money by the State to a private agent constitutes a final settlement between that private agent (and any other private agent with whom he exchanges that base money) and the State. It leaves the private agent without any further claim on the State, now or in the future.

Outside the permanent liquidity trap equilibrium, 'helicopter money effectiveness' follows from the assumption that base money is pecuniary-rate-of-return-dominated by the assets of the Central Bank. In this paper these assets are just holdings of one-period (strictly speaking zero duration) safe Treasury debt.[3] The feature of the model that under normal conditions, with market interest rates above the own interest rate on base money, Central Banking is profitable would seem non-controversial. However, it is rejected in a recent blog by Fergus Cumming (2015), who slips in the assumption that the additional assets acquired by the Central Bank as part of a real-world 'helicopter money drop' (a temporary fiscal stimulus funded by the Treasury issuing additional sovereign debt that is bought by the Central Bank and held permanently by it), earn a rate of return that is always equal to the interest rate on the additional reserves held by banks with the Central Bank - that are the other side of the real-world helicopter money drop.[4]

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growth. Larry Summers (2013) marries the demand-side and supply-side secular stagnation approaches by invoking a number of hysteresis mechanisms. For a formal model see Eggertson and Mehrotra (2014).

[3] A liquidity trap at time t is a situation where the interest rate on non-monetary financial instruments equals the nominal interest rate on base money at time t. The economy is at the effective lower bound (ELB) at time t. A permanent liquidity trap is a situation where the risk-free interest rate on non-monetary financial instruments equals the nominal interest rate on base money at all time and at all maturities. The economy is permanently at the ELB.

[4] The same would hold if the Central Bank bought additional sovereign debt in an open market purchase, paying for it with additional interest-bearing reserves and either holding that additional sovereign debt and the additional reserves permanently, or cancelling the additional sovereign debt but holding the additional reserves permanently. The additional reserves held by the Central Bank (or more generally, the additional base money issued by the Central Bank as the result of a 'helicopter money drop' or a permanent open market purchase (permanent QE) should be thought of *counterfactually*: we are comparing two parallel universes identical in history and differing only in one respect. One universe has a permanent QE operation today and the Treasury makes whatever changes in future transfers, taxes and/or public spending are necessary to continue to satisfy, now and

That, plus the implicit assumption that either there is no currency or that currency cannot be issued on a sufficient scale by the Central Bank to implement a helicopter money drop that would have a material impact on activity, produces a slew of paradoxes and 'helicopter money ineffectiveness results'. When it engages in helicopter money on a large enough scale (or cancels enough sovereign debt it holds) the Central Bank can only remain solvent (in the sense of able to pay all its bills, including interest and principal refinancing, now and in the future), if it is willing to lose control of the size of its balance sheet and/or of the inflation rate.

The reason this paper gets different results is that Cumming drops the key first requirement for helicopter money to be effective, stated in this paper: that (newly issued) base money is pecuniary-rate-of-return-dominated by the assets the Central Bank holds. Central Bank balance sheet expansion is not profitable in the Cumming universe. Clearly, currency (because of its unique liquidity and anonymity properties and because of the unique creditworthiness of its issue, the Central Bank, when it comes to meeting its obligations when these are denominated in domestic currency), is almost always pecuniary-rate-of-return dominated in the real world.[5] Central Bank reserves likewise are uniquely liquid and safe. If there is any degree of substantive Central Bank independence (which means that the Treasury cannot raid the coffers of the Central Bank at will), the creditworthiness of the Central Bank is indeed higher than that of the Treasury. We see examples of this in the Economic and Monetary Union. We are happy to

in the future, the intertemporal budget constraints of the Central Bank, the Treasury and the State. There will in general be (infinitely) many alternative paths of future public spending and revenues that meet this criterion. The other universe does not. With a bit of hand waving, one can re-interpret the parallel universes story as the impact of a completely unexpected helicopter money drop/permanent QE in a single universe. This should also make it clear that the QE or the monetization of a temporary fiscal stimulus does not have to be permanent for it to be effective in stimulating private and/or public demand for goods and services. Even with a temporary QE operation (or with the temporary monetization of a temporary fiscal stimulus), there will, in general, either be a positive present discounted value (NPV) of future interest saved (if the additional base money is pecuniary-rate-of-return dominated by the additional assets acquired by the Central Bank), or a positive NPV of the terminal stock of base money, if the stock of nominal base money grows forever at a proportional rate no less than the nominal interest rate on the additional base money issued.

[5] The irredeemability of currency may be a bit of a drawback, but presumably its legal tender status makes up for that. In any case, most private financial claims are not redeemable in intrinsically valuable goods and services, but in currency.

agree with Cumming on the analytical point that helicopter money makes no sense if Central Bank balance sheet expansion is not profitable. We would put it slightly differently, however: helicopter money (with or without cancellation of Treasury debt – the two operations are equivalent except for 'signaling' or 'commitment considerations – is effective when and to the extent that central banking (strictly speaking Central Bank balance sheet expansion) is profitable. In modern advanced economies it almost always is. Whether you want to engage in a helicopter money drop depends on whether the economy needs a stimulus to aggregate demand and what other means of boosting demand are available.

The helicopter money drop effectiveness issue is closely related to the question as to whether State-issued fiat money is net wealth for the private sector, despite being technically an 'inside asset', where for every creditor that holds the asset there is a debtor who owes a claim of equal value (see Patinkin 1956, 1965, Gurley and Shaw 1960 and Pesek and Saving 1967), Weil (1991). The discussions in Hall (1983), Stockman (1983), King (1983), Fama (1983), Sargent and Wallace (1984), Sargent (1987) and Weil (1991) of outside money, private money and the payment of interest on money ask some of the same questions as this paper, but do not offer the same answer, because they don't address the irredeemability of fiat base money. Sims (2001, 2004), Buiter (2003a, 2003b), Eggertsson (2003) and Eggertsson and Woodford (2003, 2006), Eggertsson and Krugman (2012) and Eggertsson and Mehrotra (2014) all stress that to boost demand in a liquidity trap, base money increases should not be, or expected to be, reversed. None of these papers recognised that even a permanent increase in the stock of base money will not have an expansionary wealth effect in a permanent liquidity trap unless money is irredeemable in the sense developed here; without this, there is no real balance effect in a permanent liquidity trap. Ben Bernanke spent years living down the moniker "helicopter Ben" which he acquired following a (non-technical) discussion of helicopter money (Bernanke 2003). The issue has also been revisited by Buiter (2003a, 2003b, 2007) and, in an informal manner, by Turner (2013) and by Reichlin et al. (2013).

The paper shows that, because of its irredeemability, State-issued fiat money is indeed net wealth to the private sector, in a very precise way: the initial stock of base money plus the present discounted value of all future net base money issuance (net also of any interest paid on the outstanding stock of base money) is net wealth, an 'outside' asset to the private sector, even after the intertemporal budget constraint of the State (which includes the Central Bank) has been consolidated with that of the household sector.

The paper also demonstrates that fiat base money issuance is effective in boosting household demand regardless of whether there is Ricardian equivalence (debt neutrality).

2 The model

All important aspects of how helicopter money drops work and what makes helicopter money unique can be established without the need for a complete dynamic general (dis)equilibrium model. All that is needed is a complete specification of the choice process of the household sector in a monetary economy, the period budget identity and solvency constraint of the consolidated general government/Treasury and Central Bank – the State – and the no-arbitrage conditions equating (in principle risk-adjusted) returns on all non-monetary stores of value and constraining the instantaneous nominal interest rate to be non-negative.

I shall show that, as long as the price of money is positive, the issuance of fiat base money can boost household consumption demand *by any amount*, given the inherited stocks of financial and real assets, given current and future wages and prices, and given current and future values of public spending on goods and services. Whether such helicopter money drops change asset prices and interest rates, goods prices, wages and/or output and employment depends on the specification of the rest of the model of the economy – including, in more general models, the behavior of the financial sector and of non-financial businesses in driving investment demand, production and labor demand, the rest of the 'supply side' of the economy and the rest of the world, if the economy is open. The point of this paper is to show that, whatever the equilibrium configuration we start from, helicopter money drops will boost household demand and must disturb that equilibrium. What 'gives' ultimately, in a fully articulated dynamic general equilibrium model, is not my concern.

The model of household behavior I use is as stripped-down and simple as I can make it without raising concerns that the key results will not carry over to more general and intricate models. The continuous-time Yaari-Blanchard version of the

OLG model is used to characterize household behavior (see Yaari 1965, Blanchard 1985, Buiter 1988 and Weil 1989). This model with its easy aggregation and its closed-form aggregate consumption function includes the conventional (infinite-lived) representative agent model as a special case (when the birth rate is zero). With a positive birth rate, there is no Ricardian equivalence or debt neutrality in the Yaari-Blanchard model. With a zero birth rate there is Ricardian equivalence. This permits me to show that helicopter money drops boost household demand regardless of whether there is Ricardian equivalence or not. Apart from the uncertain lifetime that characterized households in the Yaari-Blanchard model (which plays no role either in Ricardian equivalence or the effectiveness of helicopter money drops), the model has no uncertainty. To save on notation I consider a closed economy.

2.1 The household sector

We consider the household and government sectors of a simple closed economy. The holding of intrinsically worthless fiat base money is motivated through a 'money-in-the-direct utility function' approach, but alternative approaches to making money essential (cash-in-advance, legal restrictions, money-in-the transactions-function or money-in-the production function, say) would work also. For expository simplicity, there is only private capital. The helicopter money we discuss could, however, be used equally well to fund government investment programs as tax cuts or transfer payments that benefit households, or boost to current exhaustive public spending.

2.1.1 Individual household behavior

At each time $t \geq 0$, a household born at time $s \leq t$ maximizes the following utility functional:

$$\max_{\{\bar{c}(s,v),\bar{m}(s,v),\bar{b}(s,v),\bar{k}(s,v);s\le t,v\ge t\}} E_t\int_t^{\infty} e^{-\theta(v-t)}\ln\left[\bar{c}(s,v)^{\alpha}\left(\frac{\bar{m}(s,v)}{P(v)}\right)^{1-\alpha}\right]dv \tag{1}$$

$$\bar{c}(s,v),\bar{m}(s,v)\ge 0,\theta>0,0<\alpha<1$$

where E_t is the conditional expectation operator at time t, $\theta>0$ is the pure rate of time preference, $\bar{c}(s,v)$ is consumption at time v by a household born at time s, $\bar{m}(s,v)$, $\bar{b}(s,v)$ and $\bar{k}(s,v)$ are, respectively, the stocks of nominal base money, nominal risk-free constant market value bonds and real capital held at time v by a household born at time s, and $P(v)\ge 0$ is the general price level at time v.[6]

Each household faces a constant (age-independent) instantaneous probability of death, $\lambda\ge 0$. The remaining expected life time λ^{-1} is therefore also age-independent and constant. The randomness of the timing of one's demise in the only source of uncertainty in the model. It follows that the objective functional in (1) can be re-written as:

$$\max_{\{\bar{c}(s,v),\bar{m}(s,v),\bar{b}(s,v),\bar{k}(s,v);v\ge t\}} \int_t^{\infty} e^{-(\theta+\lambda)(v-t)}\ln\left[\bar{c}(s,v)^{\alpha}\left(\frac{\bar{m}(s,v)}{P(v)}\right)^{1-\alpha}\right]dv \tag{2}$$

Households act competitively in all markets in which the operate, and asset markets are complete and efficient, with free entry. In particular, there exist actuarially fair annuities markets that offer a household an instantaneous rate of return of λ on each unit of non-financial wealth it owns for as long as it lives, in exchange for the annuity-issuing entity claiming the entire stock of financial wealth owned by the household at the time of its death.

The household has three stores of value: fiat base money, which carries a nominal rate of interest i^M which is set by the State and is an irredeemable

[6] If a unit of real capital is interpreted as an ownership claim to a unit of capital (equity), then $\bar{k}$ can be negative, zero or positive. If it is interpreted as a unit of physical capital itself, $\bar{k}$ has to be non-negative.

financial instrument issued by the State (the consolidated general government and Central Bank, in this note), nominal instantaneous bonds with an instantaneous risk-free nominal interest rate i and real capital yielding an instantaneous gross real rate of return ρ. Capital goods and consumption goods consist of the same physical stuff and can be costlessly and instantaneously transformed into each other. Capital depreciates as the constant instantaneous rate $\delta \geq 0$. The real wage earned at time *v* by a household born at time *s* is denoted $\overline{w}(s,v)$ and the lump-sum tax paid (lump-sum transfer payment received if negative) at time *v* by a household born at time *s* is $\overline{\tau}(s,v)$. Labor supply is inelastic and scaled to 1.

Competition ensures that pecuniary rates of return on bonds and capital are equalized. With money yielding positive utility, there can be no equilibrium with a negative nominal interest rate. Let $r(t)$ be the instantaneous risk-free real interest rate on non-monetary financial instruments and $\pi(t) = \frac{\dot{P}(t)}{P(t)}$ the instantaneous rate of inflation. It follows that condition (3) has to hold in any equilibrium in which both money and bonds are held, and that condition (4) has to hold in any equilibrium in which both bonds and capital are held.

$$i(t) \geq i^M(t) \tag{3}$$

$$\rho(t) - \delta(t) = r(t) = i(t) - \pi(t) \tag{4}$$

For some purposes it would have been interesting to disaggregate base money into its three components. Let *M* be the aggregate stock of nominal base money, *N* the stock of currency, R^r the stock of required reserves and R^e the stock of excess reserves. The nominal interest rates on the three components of base money are i^N, i^r and i^e respectively. Normally, we would expect that $i^N = 0$. With $M \equiv N + R^r + R^e$ and the average interest rate on base money i^M defined as $i^M \equiv \frac{i^N N + i^r R^r + i^e R^e}{M}$ (a weighed arithmetic average of the interest rates on the three components, with the weight given by each component's share in the aggregate base money stock). The reason this is not pursued here is expositional simplicity. To bring reserves into the model would require modeling a banking sector. The alternative, of adding the three components separately into the household's utility function would not add any new insight and look messy, if not silly. Assuming the three components are perfect substitutes in the household utility function makes for notational simplicity but results in corner solutions unless $i^N = i^r = i^e = i^M$. This is effectively the assumption we have to make if our set-up is to accommodate multiple base money components.

The instantaneous budget identity of a household born at time $s \le t$ that has survived till period t is:

$$\dot{\bar{k}}(s,v) + \frac{\dot{\bar{m}}(s,v) + \dot{\bar{b}}(s,v)}{P(v)} \equiv$$

$$\left(\rho(v) - \delta + \lambda\right)\bar{k}(s,v) + \left(i(t) + \lambda\right)\frac{\bar{b}(s,v)}{P(v)} + \left(\lambda + i^M(t)\right)\frac{\bar{m}(s,v)}{P(v)} + \bar{w}(s,v) - \bar{\tau}(s,v) - \bar{c}(s,v)$$

(5)[7]

The real value of total non-human wealth (or financial wealth) at time v of a household born at time s is

$$\bar{a}(s,v) \equiv \bar{k}(s,v) + \frac{\bar{m}(s,v) + \bar{b}(s,v)}{P(v)} \tag{6}$$

The flow budget identity (5) can, using (4) and (6) be written as:

$$\dot{\bar{a}}(s,v) \equiv \left(r(v) + \lambda\right)\bar{a}(s,v) - \left(i(v) - i^M(v)\right)\frac{\bar{m}(s,v)}{P(v)} + \bar{w}(s,v) - \bar{\tau}(s,v) - \bar{c}(s,v) \tag{7}$$

The no-Ponzi finance solvency constraint for the household is that the present discounted value of its terminal financial wealth be non-negative in the limit as the time horizon goes to infinity:

$$\lim_{v\to\infty} \bar{a}(s,v) e^{-\int_t^v (r(u)+\lambda)} du \ge 0$$

[7] The notational convention is that $\dot{\bar{k}}(s,v) \equiv \dfrac{\partial \bar{k}(s,v)}{\partial v}$.

Because the instantaneous utility function is increasing in both consumption and the stock of real money balances, the solvency constraint will bind:

$$\lim_{v\to\infty} \overline{a}(s,v)e^{-\int_t^v (r(u)+\lambda)}du = 0 \tag{8}$$

Note that base money is viewed as an asset by the holder (the household). The terminal net financial wealth whose present discounted value (NPV) must be non-negative includes the household's stock of base money.

The optimality conditions of the household's choice problem imply the following decision rules for the household:

$$\overline{c}(s,t) = (1-\alpha)(\theta+\lambda)\left(\overline{a}(s,t)+\overline{h}(s,t)\right) \tag{9}$$

$$\overline{h}(s,t) = \int_t^\infty \left(\overline{w}(s,t)-\overline{\tau}(s,t)\right)e^{-\int_t^v (r(u)+\lambda)du}dv \tag{10}$$

$$\frac{\overline{m}(s,t)}{P(t)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{i(t)}\overline{c}(s,t) \tag{11}$$

The net present discounted value of household after-tax labor income, $\overline{h}(s,t)$, will be referred to as human capital. A shorter life expectancy (a higher value of λ) raises the marginal propensity to consume out of comprehensive wealth, $a + h$

2.1.2 Aggregation

We assume that there is a constant and age-independent instantaneous birth rate $\beta \geq 0$. The size of the cohort born at time t is normalized to $\beta e^{(\beta-\lambda)t}$. The size of the surviving cohort at time t which was born at time $s \leq t$ is therefore $\beta e^{(\beta-\lambda)s}e^{-\lambda(t-s)}$. Total population at time t is therefore given, for $\beta > 0$ by $\beta e^{-\lambda t}\int_{-\infty}^t e^{\beta s}ds = e^{(\beta-\lambda)t}$. For the case $\beta = 0$ we set the size of the population at $t = 0$ to equal 1, so population size at time t is again $e^{(\beta-\lambda)t} = e^{-\lambda t}$. For any individual household variable $\overline{x}(s,t)$, we define the corresponding population aggregate $X(t)$ as follows:

$$X(\mathrm{t}) = \beta e^{-\lambda t}\int_{-\infty}^{t}\overline{x}(s,t)e^{\beta s}ds \quad \text{if } \beta > 0$$
$$= \overline{x}(0,t)e^{-\lambda t} \qquad\qquad \text{if } \beta = 0$$

We assume that each household earns the same wage and pays the same taxes, regardless of age:

$$\overline{w}(s,t) = \overline{w}(t)$$
$$\overline{\tau}(s,t) = \overline{\tau}(t)$$

It follows that each household, regardless of age, has the same human capital:

$$\overline{h}(s,t) = h(t)$$

Finally, there are neither voluntary nor involuntary bequests in this model, so

$$\overline{a}(s,\mathrm{s}) = 0 \tag{12}$$

By brute-force aggregation, if follows that aggregate consumption is determined as follows:

$$C(t) = (1-\alpha)(\theta+\lambda)\big(A(t)+H(t)\big) \tag{13}$$

$$\frac{M(t)}{P(t)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{i\big(t\big)}C(t) \tag{14}$$

$$\dot{A}(t) \equiv r(t)A(t) - \big(i(t)-i^{M}(t)\big)\frac{M(t)}{P(t)} + W(t) - T(t) - C(t) \tag{15}$$

$$H(t) = \int_{t}^{\infty}\big(W(v)-T(v)\big)e^{-\int_{t}^{v}(r(u)+\beta)du}dv \tag{16}$$

$$A(t) = K(t) + \frac{M(t)+B(t)}{P(t)} \tag{17}$$

For future reference, the solvency constraint of the aggregate household sector is

$$\begin{aligned}&\lim_{v\to\infty} A(v)e^{-\int_t^v r(u)\mathrm{du}} = 0\\ &\text{or}\\ &\lim_{v\to\infty}\left(K(v)+\frac{M(v)+B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} = 0\end{aligned} \tag{18}$$

Comparing the aggregate household financial wealth dynamics equation (15), with the individual *surviving* household financial wealth dynamics equation (7) shows that the return on the annuities, λA is missing from the aggregate dynamics. This is as it should be, because $\lambda A(t)$ is both the extra returns over and above the risk-free rate earned by all surviving households at time *t* and the amount of wealth paid to the annuities sellers by the (estates of the) fraction λ of the population that dies at time *t*.

Comparing the aggregate human capital equation (16) – describing the human capital of all generations currently alive but not of those yet to be born – and the individual surviving household's human capital equation (10), we note that if the households alive at time *t* were to discount *all* future after-tax labor income at the individually appropriate, annuity premium-augmented rate of return $r+\lambda$, they would fail to allow for the fact that the labor force to whom that after-tax labor income accrues includes the surviving members of generations born after time *t*. In the absence of the institution of "inherited slavery", those currently alive cannot claim the labor income of the future surviving members of generations as yet unborn. Population and labor force grow at the proportional rate $\beta-\lambda$, so the appropriate discount rate applied to the future aggregate streams of labor income is $r+\beta$.

2.2 The State

The State whose budget identity and solvency constraint we model is the consolidated general government (the Treasury in what follows) and Central Bank. Let G denote real public spending on goods and services (exhaustive public spending, current and or capital). The State's budget identity and solvency constraint are given in equation (19) and (20) respectively.

$$\frac{\dot{M}(t)+\dot{B}(t)}{P(t)} \equiv i(t)\frac{B(t)}{P(t)}+i^{M}(t)\frac{M(t)}{P(t)}+G(t)-T(t) \tag{19}$$

Because of the irredeemability of base money, the solvency constraint of the State requires that the present discounted value of its terminal net *non-monetary* liabilities be non-positive, not that the present discounted value of its terminal net financial liabilities be non-positive.

$$\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} \leq 0 \tag{20}$$

Equation (20) is the natural way to formalize the familiar notion that fiat base money is an asset (wealth) to the holder (the owner – households in this simple model) but does not constitute in any meaningful sense a liability to the issuer (the 'borrower' – the State or the Central Bank as an agent of the State). The owner of a \$20 dollar Federal Reserve Note may find comfort in the fact that "This note is legal tender for all debts, public and private", but she has no claim on the Federal Reserve, now or ever, other than for an amount of Federal Reserve Notes adding up to \$20 in value. UK currency notes worth £X carry the proud inscription "… *promise to pay the bearer the sum of £X"* but this merely means that the Bank of England will pay out the face value of any genuine Bank of England note no matter how old. The promise to pay stands good for all time but simply means that the Bank will always be willing to exchange one (old, faded) £10 Bank of England note for one (new, crisp) £ 10 Bank of England note (or even for two £ 5 Bank of England notes). Because it promises only money in exchange for money, this 'promise to pay' is, in fact, a statement of the irredeemable nature of Bank of England notes.

I believe that the irredeemability property of fiat currency – that it is an asset to the holder but not a liability of the issuer – extends also to the other component of base money (commercial bank reserves held with the Central Bank), but the simple theoretical model does not depend on this and does not make this distinction.
Equation (20) implies that

$$\frac{M(t)+B(t)}{P(t)} \equiv \int_t^{\infty}\left(T(v)-G(v)+\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)+B(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{21}$$

Because of the irredeemability of base money (reflected in equation (20)) the intertemporal budget constraint of the State is

$$\frac{M(t)+B(t)}{P(t)} \geq \int_t^{\infty}\left(T(v)-G(v)+\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du} \tag{22}$$

Substituting the intertemporal budget constraint of the State into the aggregate consumption function (13), using (16) and (17), and rearranging yields:

$$C(t) \geq (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^{\infty}\left(W(v)-G(v)e^{\beta(v-t)}\right)e^{-\int_t^v (r(u)+\beta)du}dv \\ -\int_t^{\infty}T(v)e^{-\int_t^v (r(u)+\beta)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\frac{1}{P(t)}\left(\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right) \end{array}\right] \tag{23}$$
[8]

From integration by parts it follows that

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[8] Note that $\int_t^{\infty}\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}dv+\lim_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}$
$=\frac{1}{P(t)}\left(\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right)$

$$\int_t^\infty \left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv + \lim_{v\to\infty} M(v)e^{-\int_t^v i(u)du}$$
$$= \int_t^\infty \left(\dot{M}(v)-i^M(v)M(v)\right)e^{-\int_t^v i(u)du}dv + M(t) \tag{24}$$

It follows that (23) can be rewritten as:

$$C(t) \geq (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^\infty \left(W(v)-G(v)e^{\beta(v-t)}\right)e^{-\int_t^v (r(u)+\beta)du}dv \\ -\int_t^\infty T(v)e^{-\int_t^v (r(u)+\beta)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\frac{1}{P(t)}\left(M(t)+\int_t^\infty \left(\dot{M}(v)-i^M(v)M(v)\right)e^{-\int_t^v i(u)du}dv\right) \end{array}\right] \tag{25}$$

2.3 Debt Neutrality

When the birth rate is zero, the consumption function is equivalent to the consumption function of the representative agent model. From the perspective of pure fiscal stabilization policy – a cut in lump-sum taxes today accompanied by a credible commitment to an increase in future taxes equal in net present value to the up-front tax cut, will not boost household demand. With $\beta > 0$, an up-front tax cut and the credible announcement of a future increase in taxes of equal net present discounted value when discounted at the riskless rate *r* boosts the human capital of those currently alive because some of the deferred taxes will fall on as yet unborn generations. With $\beta = 0$ the wedge between the government's discount rate for future taxes, *r*, and the effective discount rate of the private sector for future taxes, $r+\beta$ disappears, and Ricardian equivalence or debt neutrality prevails. With $\beta = 0$, the aggregate consumption function (25) becomes

$$C(t) \geq (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^\infty \big(W(v)-G(v)\big)e^{-\int_t^v r(u)du}dv \\ +\dfrac{1}{P(t)}\left(\begin{array}{l} \int_t^\infty \big(i(v)-i^M(v)\big)M(v)e^{-\int_t^v i(u)du}dv \\ + \lim\limits_{v\to\infty} M(v)e^{-\int_t^v i(u)du} \end{array}\right) \end{array}\right] \tag{26}$$

or, equivalently

$$C(t) \geq (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^\infty \big(W(v)-G(v)\big)e^{-\int_t^v r(u)du}dv \\ \\ +\dfrac{1}{P(t)}\left(\begin{array}{l} M(t) \\ +\int_t^\infty \big(\dot{M}(v)-i^M(v)M(v)\big)e^{-\int_t^v i(u)du}dv \end{array}\right) \end{array}\right] \tag{27}$$

Lump-sum taxes disappear from the aggregate consumption function once the intertemporal budget constraint of the State is used to substitute out the initial values of the private sector's holdings of monetary and non-monetary sovereign debt. The first line on the RHS of equations (26) and (27) shows the result, familiar from non-monetary representative agents models that the bite taken out of private comprehensive wealth by the government is measured by the net present discounted value of future exhaustive public spending.

2.4 Helicopter money with debt neutrality

Assume until further notice that equation (3) holds as a strict inequality and base money is pecuniary-rate-of-return-dominated by bonds as a store of value. Even in a representative agent model with debt neutrality/Ricardian equivalence, monetary injections will boost private consumption demand, holding constant the sequences of current and future spending on real goods and services $\{G(v); v \geq t\}$, prices, wages and interest rates. The path of lump-sum taxes and of non-monetary debt is irrelevant with $\beta = 0$, as long as the State satisfies its intertemporal budget constraint (22).

It is immediately obvious from equations (26) and (27) that, holding constant the sequence of current and future real exhaustive public spending constant, monetary injections will always boost consumption demand, as long as the price level $P(t)$ is positive. We can think of monetary injections, holding constant the path of current and future exhaustive public spending, as being introduced either through lump-sum transfer payments, *T*, or by purchasing non-monetary debt (sovereign bonds) from the private sector (QE or quantitative easing). If the State, starting at time *t*, increases the stock of base money by buying back non-monetary public debt from the public, say with $\dot{M}(v) = -\dot{B}(v) > 0$ for $t \le v \le t'$, $t' > t$, it is clear from the intertemporal budget constraint of the State, equation (21) that, provided the interest rate on bonds exceeds the interest rate on base money, that is, equation (3) holds as a strict inequality (and holding constant the current and future paths of the price level and of all interest rates), the State will have to raise the NPV of future public spending minus taxes to satisfy its intertemporal budget constraint. Permanent open market purchases of non-monetary public debt by the Central Bank (irreversible QE) are deferred helicopter money: future taxes will be cut and/or future public spending will have be raised if the State is to satisfy its intertemporal budget constraint.[9]

2.5 The creditor state

Remember that equation (20) does not have to hold with strict equality. The same holds for equations (22), (23), (25), (26) and (27). Consider the case

[9]Indeed, the State could choose to become a net non-monetary creditor to the private sector, with *B < 0*. The State's solvency constraint after all only requires the NPV of its terminal stock of *non-monetary* debt to be non-positive (equation (19)). It could be strictly negative in equilibrium, as long as the household sector satisfies its solvency constraint, that the NPV of the terminal value of its financial assets $K + \frac{M+B}{P}$ is non-negative.

$\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} < 0$, where the State is a net (non-monetary) creditor to the private sector, even in the very long run. In the simplest case of an endowment economy ($\bar{k}(s,t) = K(t) \equiv 0$), the aggregate household solvency constraint (18) implies $-\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} = \lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} > 0$ or $-\lim_{v\to\infty} B(v)e^{-\int_t^v i(u)\mathrm{du}} = \lim_{v\to\infty} M(v)e^{-\int_t^v i(u)\mathrm{du}} > 0$. The state is a permanent creditor to the household sector, something it can do when the long-run growth rate of fiat base money is at least as high as the long-run nominal interest rate, since $\lim_{v\to\infty} M(v)e^{-\int_t^v i(u)\mathrm{du}} > 0$ requires $\lim_{v\to\infty}\frac{\dot{M}(v)}{M(v)} \geq \lim_{v\to\infty} i(v) > 0$. From the government's intertemporal budget constraint (22) it is clear that the fiscal space created by $\lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} > 0$ can be used to cut future taxes or increase future public spending.

Consider what is perhaps the normal case, when, in the long run, the State grows the nominal stock of fiat base money at a proportional rate strictly below the instantaneous risk-free nominal interest rate, that is, $\lim_{v\to\infty} M(v)e^{-\int_t^v i(u)du} = 0$. In the representative agent case ($\beta = 0$) the consumption function becomes

$$C(t) \geq (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^\infty \left(W(v) - G(v)\right)e^{-\int_t^v r(u)du}dv \\ +\frac{1}{P(t)}\left(\int_t^\infty \left(i(v) - i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv\right) \end{array}\right]$$

The State can boost demand by monetary injections, for given sequences of exhaustive public spending, the general price level and interest rates. A larger future money supply will, ceteris paribus, provided *M(v) > 0*, and $i(v) > i^M(v)$, *v*

$\geq t$, as is assumed until further notice, increase the comprehensive wealth or permanent income of the household sector by boosting the NPV of the interest bills saved by borrowing through the issuance of base money rather than through non-monetary debt if, as we assume here, base money is pecuniary-rate-of-return dominated.

The same conclusion stares one in the face even more clearly when we use the equivalent expression for the seigniorage blessings of monetary issuance, shown in equation (27). The wealth-creating effect of seigniorage is the outstanding stock of base money plus the NPV of future base money issuance: $\frac{1}{P(t)}\left(M(t)+\int_t^{\infty}\left(\frac{\dot{M}(v)}{M(v)}-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv\right)$. Again this can be made arbitrarily large for given sequences of *G, P* and *I* by increasing the stock of base money at a proportional rate greater than the own interest rate on base money.

2.6 Helicopter money in a liquidity trap

Consider an economy stuck in the ultimate liquidity trap with the nominal interest rate on bonds equal to the own interest rate on base money forever. With $i(v)=i^M(v), v\geq t$, monetary injections lose none of their potency. Sure, the NPV of the current and future interest saved by issuing base money rather than non-monetary securities (bonds) is zero: $\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv=0$ when $i(v)=i^M(v), v\geq t$. But the NPV of the terminal stock of base money can be made anything the State (the monetary authority) wants it to be in a permanent liquidity trap. Assume the initial date is $t = 0$.

$$\lim_{\ell\to\infty}M(\ell)e^{-\int_t^{\ell} i(u)du}=\lim_{\ell\to\infty}M(\ell)e^{-\int_t^{\ell} i^M(u)du}=M(t)\lim_{\ell\to\infty}e^{\int_t^{\ell}\left(\frac{\dot{M}(u)}{M(u)}-i^M(u)\right)du}$$

when $i(v)=i^M(v), v\geq t$. From equation (24), the alternative expression for the wealth represented by the seigniorage monopoly of the State in a permanent liquidity trap is: $M(t)+\lim_{\ell\to\infty}\int_t^{\ell}\left(\frac{\dot{M}(v)}{M(v)}-i^M(v)\right)M(v)e^{-\int_t^v i^M(u)du}dv$. This again shows that the authorities can use helicopter money to boost consumer demand even in the severest of all conceivable liquidity traps. What they have to do is set a growth rate for the nominal stock of base money, for some finite period of time (forever is possible, but would result in an infinite increase in wealth), that is greater than the own interest rate on base money:

$$\frac{\dot{M}(v)}{M(v)}>i^M(v), t\le t_1\le v\le t_2,\, t_2>t_1$$

What this means is that a fiat money economy where the State controls the issuance of fiat money, a liquidity trap is a choice, not a necessity. Most general equilibrium completions of a model with the consumption function used in this paper will have the property that if, in a perpetual liquidity trap or effective lower bound (ELB) equilibrium, real demand is boosted by a sufficiently large magnitude, the permanent liquidity trap vanishes.

Equations (26) or (27) (or their more general versions without Ricardian equivalence) make it clear that it is also possible for the State to boost public spending on real goods and services, current or capital, and avoid any negative impact of the anticipation of higher future taxes on demand by monetizing the resulting public sector deficits.

2.7 Helicopter money without Ricardian equivalence

The way helicopter money affects household demand is the same in the overlapping generations model (the Yaari-Blanchard model with $\beta>0$) as in the representative agent model ($\beta=0$). A comparison of equations (23) and (25) with equations (26) and (27) shows that the comprehensive wealth term in the aggregate consumption function is augmented by base money issuance to the tune of $\int_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du}$ or, equivalently,

$$M(t)+\int_{t}^{\infty}\left(\dot{M}(v)-i^{M}(v)\right)e^{-\int_{t}^{v}i(u)du}dv$$

It is clear from the model without Ricardian equivalence that permanent monetary base expansions of a given magnitude in NPV terms will now have different effects when they are implemented through up-front lump-sum transfer payments/tax cuts than through up-front QE (open market purchases of sovereign bonds) followed by deferred transfer payments or tax cuts. Because the deferred tax cuts will in part be enjoyed by generations not yet born today, the 'up-front QE and deferred transfer payment boost' version will be less expansionary, for a given NPV of base money issuance, than the version with the up-front transfer payment boost.

3 Some further Considerations

3.1 Fiat base money is special

In this model unbacked fiat base money is unique for two reasons. First, it performs liquidity or transactions functions that cause it to be willingly held by private agents despite being pecuniary-rate-of-return-dominated. Currency carries a pecuniary rate of return of zero and required or excess reserves carry nominal rates of interest (or own rates of interest) that are almost always below those yielded by other safe nominally denominated assets with the same (short) maturity, like domestic-currency denominated sovereign debt: $i^M < i$. In exceptional circumstances, at the effective lower bound or ELB, nominal interest rates on safe monetary assets can get close to or even equal those on currency and Central Bank reserves: $i \approx i^M$. It is hard to think of circumstances where safe non-monetary debt yields a consistently lower pecuniary rate of return than base money or even than Central Bank reserves: $i^M > i$. Unless Central Banks engage in risky enterprises (acting as a de-facto agricultural development bank, as was the case in Peru under the first Garcia government, purchasing outright at better than fair prices debt issued by a sovereign at high risk of default, or accepting such debt as collateral for loans to banks that would likely default themselves were the sovereign to default, as the ECB has done in the case of Greece) Central Banking is a profitable business. Yet the opposite (effectively that $i^M > i$) is assumed by Fergus Cumming in a recent blogpost (Cumming 2015) in which he aims to set the tale straight as regards helicopter money but ends up with a series of bizarre propositions (loss of control of the balance sheet and loss of control of inflation to name but two) that are all stand or fall with the assumption that Central Banking is unprofitable (at least incrementally) – the rate of return on Central Bank assets is less than the rate of return on Central Bank reserves. Fortunately, this will not be the case when a Central Bank is managed properly. Not only currency but also Central Bank reserves are likely pay an interest rate below the rate of return on the Central Bank's assets.

I shoe-horned this uniqueness of base money into the model by having base money as an argument in the household's direct utility function. This is not very satisfactory. The only justification is simplicity and the robustness of the results of the paper to using other mechanisms for making fiat base money a superior asset (money in the production function, cash–in-advance or legal restrictions. What makes something (or some class of objects) desirable because of its unique transactions-facilitating properties differs in the many different approaches that have been adopted for generating a willingness to hold something that is pecuniary-rate-of-return-dominated as a store of value. It is the outcome of a collective, decentralized social choice. It may help if something is granted legal tender status by the State, but this not a necessary condition. Should fiat base money issued by the State lose this unique advantages it has in facilitating transactions, it will have to pay interest at the same rate as the other safe, liquid financial assets – bonds in this model, or it will not be held voluntarily by private agents. We are in the Wallace (1981, 1990) world of the Modigliani-Miller theorem for open market operations. The net present discounted value of future interest saved is, of course zero in this case. However, if the monetary asset is irredeemable, the NPV of the terminal base money stock would still be net wealth. For this to be positive, the growth rate of the nominal stock of base money would have to be at least equal to the nominal rate of interest (on both base money and other safe assets) in the long run. In the liquidity trap case, with a the safe rate on non-monetary assets equal to the interest rate on base money forever, a helicopter money drop would still be effective in boosting household consumption demand, even though a helicopter bond drop would not be.

3.2 Fiat base money is net wealth

Fiat base money is net wealth for the consolidated private sector and State sector. Despite fiat money technically being inside money and an inside asset (issued by one economic agent and held by another), fiat base money behaviorally or effectively is like nature's bounty: an asset and wealth to the owner but not a claim on or liability of the issuer.

Indeed, looking at the Ricardian version of the aggregate consumption function in equation (26) or (27), note that the term $\frac{1}{P(t)}\left(M(t)+\int_t^\infty \left(\dot{M}(v)-i^M(v)\right)e^{-\int_t^v i(u)du}dv \right)$ could equally well represent true 'outside assets', like intrinsically worthless pet rocks or Rai, the stone money used on the Isle of Yap. The stock of rare bits of rock deposited on earth by meteorites, say, could be represented by *M(t)* and the net present value of future meteorite deposits could be could be $\int_t^\infty \left(\dot{M}(v)-i^M(v)\right)e^{-\int_t^v i(u)du}dv$. It would be hard to rationalise a non-zero own rate of interest on pet rocks or rai, but if (intrinsically worthless) rabbits were a socially preferred currency, a positive own rate of interest would be quite plausible. With some slight modifications, almost intrinsically worthless commodities like gold and intrinsically worthless virtual media of exchange like Bitcoin could also fit into our consumption function. Both are, of course, costly to produce or 'mine'. Helicopter drops of Rai, gold or Bitcoin would not share with fiat base money the property that they are issued by the State and can be used to fund the State. They don't roll off the printing presses but are gifts from nature (Rai and gold) and from human ingenuity (in the case of Bitcoin).

3.3 When should a helicopter money drop be preferred to a bond-financed fiscal stimulus?

When there is no Ricardian equivalence, aggregate demand can be stimulated through sovereign bond-financed tax cuts (or through higher exhaustive public spending) as well as through helicopter money. Which method one prefers depends on how the model of the economy is completed and on policy preferences. The formal model of this note is not well suited to deal with problems like sovereign default risk or inflation risk, but richer models that permit a meaningful discussion of these issues would likely have the property that if (1) the sovereign has a high stock of non-monetary net debt outstanding and (2) there are political limits to its current and future capacity to raise taxes or cut public spending, adding to the stock of non-monetary debt through further sovereign bond issuance could raise sovereign default risk. That would call for monetary financing as the preferred funding method for a fiscal stimulus. The case for monetary financing would be stronger if inflation is below target and if one or more key financial markets are illiquid.

If the public finances are healthy (low sovereign debt and deficit, considerable political scope for cutting public spending or raising taxes) and inflation is above-target, using sovereign bonds to fund a stimulus would make sense.

In the current economic conditions faced by the euro area, Japan and, to a slightly lesser degree, by the US and the UK, with question marks behind the sustainability of the public finances and with inflation well below target, monetizing a fiscal stimulus would seem to be the obvious first choice.

3.4 The institutional implementation of helicopter money drops

In most contemporary advanced economies, the issuance of fiat base money (often with legal tender status) is performed by an agency of the State, the Central Bank, that has some degree of operational independence (and in a few cases even a measure of target independence) in the design and implementation of monetary policy. Some Central Banks can act as fiscal agents for the State (central government or federal Treasury/Ministry of Finance) but none that we know of can act openly as fiscal principals. Central Banks typically transfer their profits (over and above what they want to add to reserves or provisions) to their beneficial owner, the central government or federal Treasury.[10] Specifically, Central Banks

[10] The European Central Bank (ECB) is unique in that its shareholders are the national Central Banks (NCBs) of the 28 (as of May 2014) European Union member States. The profits of the ECB are distributed to the 18 (as of May 2014) NCBs of the EU member States that are also members of the euro area.

cannot levy taxes, make transfer payments or pay overt subsidies to other domestic economic entities, nor can they engage in exhaustive public spending other than what is inevitably involved in the running of the Central Bank (payroll, capital expenditure on buildings and equipment, supplies, utilities etc.). The fact that many Central Banks have engaged in large-scale quasi-fiscal interventions, most recently during and after the North-Atlantic financial crisis of 2007–2008, does not change the basic legal and institutional reality that a Central Bank cannot implement helicopter money on its own.

Cooperation and coordination between the Central Bank and the Treasury is required for the real-world implementation of helicopter money drops. In practice, to implement the temporary fiscal stimulus permanently/irreversibly financed through the issuance of fiat base money that is closest to the original Friedman helicopter money parable – a lump-sum transfer payment households permanently funded through base money issuance –, the following coordinated fiscal-monetary actions would take place. There would be a one-off cash transfer to all eligible households by the Treasury. The Treasury funds these payments by selling Treasury debt to the Central Bank, which credits the account held by the Treasury with the Central Bank (which is not normally counted as part of the monetary base but constitutes a non-monetary liability of the Central Bank). As the Treasury pays out the cash to the eligible households, the Treasury's account with the Central Bank is drawn down. The monetary base increases because the transfer payment to the households either ends up as increased cash/currency held by households, corporates or banks or as increased bank reserves held with the Central Bank. A virtually identical story can be told if instead of a transfer payment to the household sector, the Treasury were to engage in a program of exhaustive current or capital expenditure.

3.5 The irrelevance of the cancellation of Treasury debt held by the Central Bank.

From a fundamental economic perspective, it makes no difference whether the Central Bank cancels the sovereign bonds it buys (as proposed e.g. by Turner 2013) or holds them indefinitely (rolling them over as they mature). This is because the Treasury is the beneficial owner of the Central Bank. The Treasury therefore receives the Central Bank's profits and is responsible for its losses. Their accounts (including balance sheets and P&L account) therefore can be – or indeed ought to be – consolidated to get a proper perspective on the flow of funds and balance sheet accounts that matter. The only reason to prefer cancellation of sovereign debt held by the Central Bank over the Central Bank holding the sovereign debt permanently is that cancellation may be seen as a more credible commitment device.

The disaggregated period (instantaneous) budget identity, the intertemporal budget identity and the solvency constraint of the Treasury are given in equations (28), (29) and (30). Those of the Central Bank are given in equations (31), (32) and (33). As before, B stands for Treasury debt held outside the Central Bank. B^{cb} denotes Treasury debt held by the Central Bank. T is the real value of taxes paid by the private sector, T^{cb} is the real value of payments made by the Central Bank to the Treasury. The Central Bank is extremely frugal and does not spend on real goods and services. To keep things simple, we consider a closed economy: the Central Bank does not hold any foreign exchange reserves.

$$\frac{\dot{B}(t)+\dot{B}^{cb}(t)}{P(t)} \equiv i(t)\left(\frac{B(t)+B^{cb}(t)}{P(t)}\right)+G(t)-T(t)-T^{cb}(t) \tag{28}$$

$$\begin{aligned}\frac{B(t)+B^{cb}(t)}{P(t)} \equiv & \int_t^\infty \left(T(v)+T^{cb}(t)-G(v)\right)e^{-\int_t^v r(u)du}dv \\ & +\lim_{v\to\infty}\left(\frac{B(v)+B^{cb}(t)}{P(v)}\right)e^{-\int_t^v r(u)du}\end{aligned} \tag{29}$$

$$\lim_{v\to\infty}\left(\frac{B(v)+B^{cb}(t)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} \leq 0 \tag{30}$$

$$\frac{\dot{M}(t)-\dot{B}^{cb}(t)}{P(t)} \equiv T^{cb}(t)-i(t)\frac{B^{cb}(t)}{P(t)}+i^M(t)\frac{M(t)}{P(t)} \tag{31}$$

$$\frac{M(t)-B^{cb}(t)}{P(t)} \equiv \int_t^{\infty}\left(-T^{cb}(v)+\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)-B^{cb}(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{32}$$

$$\lim_{v\to\infty}\left(\frac{B^{cb}(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} \geq 0 \tag{33}$$

The Treasury's intertemporal budget identity and solvency constraint imply the Treasury's intertemporal budget constraint:

$$\frac{B(t)+B^{cb}(t)}{P(t)} \leq \int_t^{\infty}\left(T(v)+T^{cb}(t)-G(v)\right)e^{-\int_t^v r(u)du}dv \tag{34}$$

The Central Bank's intertemporal budget identity and solvency constraint, which recognizes the irredeemability of fiat base money implies the Central Bank's intertemporal budget constraint, which we can write as the requirement that the Comprehensive Net Worth (Comprehensive Capital or Comprehensive Equity) of the Central Bank, Ω_t be non-negative, that is

$$\Omega_t = \frac{B^{cb}(t)-M(t)}{P(t)} - \int_t^{\infty}\left(T^{cb}(v)-\left(i(v)-i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \geq 0 \tag{35}$$

The Treasury, as the beneficial owner of the Central Bank, receives all its profits and absorbs all its losses (not necessarily when they are earned/incurred), subject to the constraint that the Comprehensive Net Worth, Ω_t of the Central Bank be equal to some non-negative value $\bar{\Omega}_t \geq 0$ at all times. The sequence of current and future net payments of the Central Bank to the Treasury therefore satisfies:

$$\int_t^{\infty} T^{cb}(v)e^{-\int_t^v r(u)du}dv =$$

$$-\bar{\Omega}_t + \frac{B^{cb}(t) - M(t)}{P(t)}$$

$$+\int_t^{\infty}\left(\left(i(v) - i^M(v)\right)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{36}$$

or equivalently:

$$\int_t^{\infty} T^{cb}(v)e^{-\int_t^v r(u)du}dv = -\bar{\Omega}_t + \frac{B^{cb}(t)}{P(t)} + \int_t^{\infty}\left(\frac{\dot{M}(v) - i^M(v)M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv$$

A very simple rule that satisfies (36) provided the initial ($t = 0$, say) value of the Central Bank's comprehensive net worth equals $\bar{\Omega}_0 \geq 0$, would be for the Central Bank to have a continuously balanced budget:

$$T^{cb}(t) = i(t)\frac{B^{cb}(t)}{P(t)} - i^M(t)\frac{M(t)}{P(t)} \tag{37}$$

which implies that

$$\dot{M}(t) = \dot{B}^{cb}(t) \tag{38}$$

Briefly, it does not matter whether the Central Bank today cancels an amount $B^{cb}(t)$ of debt owed to it by the Treasury and as a result does not pay out as profits to the Treasury an infinite future stream of Central Bank profits $\left\{i(v)B^{cb}(t); v \geq t\right\}$ (whose NPV is, of course, $B^{cb}(t)$), or whether it keeps its existing holdings of Treasury debt on its books and pays out as profits to the Treasury an infinite stream of future profits $\left\{i(v)B^{cb}(t), v \geq t\right\}$. This equivalence of the Central Bank cancelling forgiving) a given amount of Treasury debt versus holding it forever (rolling it over when required) is, of course, consistent with the consolidated intertemporal budget constraint of the Central Bank and the Treasury in equations (21) and (22), in which the Central Bank's holdings of Treasury debt are absent.

3.6 Helicopter money drops and the ECB

Matters are slightly more complicated for the ECB, whose equity is held by the national Central Banks (NCBs) of the member States that are part of the euro area. Each NCB has its national Central Bank as its beneficial owner. Cancelling an amount $B_i^{cb}(t)$ of sovereign debt of euro area member state *i* (which has an equity stake η_i in the ECB), represents ultimately a wealth transfer of $(1-\eta_i)B_i^{cb}(t)$ to the Treasury of member State *i* from the Treasuries of all other member States. Holding $B_i^{cb}(t)$ indefinitely on the balance sheet of the ECB would result in an infinite stream of profits $\left\{i(v)\eta_i B_i^{cb}(t), v \geq t\right\}$ to the NCB of country *i* and thus ultimately to the Treasury of country *i* and $\left\{i(v)(1-\eta_i)B_i^{cb}(t), v \geq t\right\}$ to the NCBs of the remaining euro area member States and thus ultimate to their national Treasuries..

This real-world implementation of helicopter money drops is legal and easily implemented everywhere except in the euro area. Article 123.1 of the Treaty on the Functioning of the European Union States:

> *"Overdraft facilities or any other type of credit facility with the European Central Bank or with the Central Banks of the Member States (hereinafter referred to as 'national Central Banks') in favour of Union institutions, bodies, offices or agencies, central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of Member States shall be prohibited, as shall the purchase directly from them by the European Central Bank or national Central Banks of debt instruments."*[11]

This clause has commonly been interpreted as ruling out the financing of government deficits in the euro area through government debt sales to the ECB (or to the national Central Banks (NCBs) of the Eurosystem) and their monetization by the Eurosystem. Unless this can be fudged by the Eurosystem purchasing the sovereign debt in the secondary markets (as it did under the Securities Markets Programme and proposes to do under the Outright Monetary Transactions programme (should it ever be activated)), Article 123.1 deprives the euro area of

11 http://www.lisbon-treaty.org/wcm/the-lisbon-treaty/treaty-on-the-functioning-of-the-european-union-and-comments/part-3-union-policies-and-internal-actions/title-viii-economic-and-monetary-policy/chapter-1-economic-policy/391-article-123.html

the one policy instrument – a temporary fiscal stimulus permanently funded by and monetized by the Central Bank – that is guaranteed to prevent or cure deflation, "lowflation" or secular stagnation. It is time for Article 123 to be scrapped in its entirety if the euro area does not wish to face an unnecessary risk of falling into any of these traps.

4 Conclusion

4.1 The two advantages of fiat base money: zero or below-market nominal interest rates and irredeemability.

The fiat base money analyzed in this paper can be produced at zero marginal cost by the State (much like paper currency or bank reserves with the Central Bank in the real world). Currency is willingly held by households and firms at a zero nominal interest rate. Banks are either required to hold reserves (as with required reserves) or hold reserves with the Central Bank voluntarily (as with excess reserves), even when alternative non-monetary stores of value with higher nominal interest rates are available. As a funding instrument for the State, base money has two things going for it, compared to interest-bearing non-monetary debt. First, the State saves each period (instant in the continuous time model) the interest bill it would have paid had it issued bonds instead of base money. Second, even if the safe nominal rate of interest on non-monetary instruments is at the effective lower bound, and even if it is confidently expected to be at the ELB forever, money is a more attractive funding instrument for the State because it is irredeemable. Fiat base money is net wealth to the private sector in the sense that its current stock plus the NPV of net future issuance (net of any interest paid on the outstanding stock of base money) is a component of the comprehensive wealth of the household sector.

4.2 Helicopter money drops always boost demand

A permanent helicopter drop of irredeemable fiat base money boosts demand both when Ricardian equivalence does not hold and when it holds. It makes secular stagnation (see Summers 2013 and Buiteret et al. 2014) a policy choice, not something driven by circumstances beyond national policy makers' control. It boosts demand when nominal risk-free interest rates are above the interest rate on base money and when they equal it – and even in a pure liquidity trap when nominal interest rates are at the ELB forever. A helicopter money drop always boosts demand when the price of money is positive and Central Banking is a profitable business.[12]

Acknowledgement The views and opinions expressed are those of the author alone and should not be attributed to Citigroup or to any other organization the author is associated with. I would like to thank Larry Summers from prodding me to write a short note setting out the essence of the helicopter money argument.

[12] In dynamic general equilibrium with flexible nominal prices, there always exists an equilibrium with a zero price of money in all periods and all States of nature – the barter equilibrium or non-monetary equilibrium. Obviously, helicopter money drops won't boost demand in such an equilibrium.

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