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The Possible Trinity: Optimal Interest Rate, Exchange Rate, and Taxes on Capital Flows in a DSGE Model for a Small Open Economy

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Abstract

A traditional way of thinking about the exchange rate (XR) regime and capital account openness has been framed in terms of the 'impossible trinity' or 'trilemma', in which policymakers can only have 2 of 3 possible outcomes: open capital markets, monetary independence and pegged XRs. This paper is an extension of Escude (A DSGE Model for a SOE with Systematic Interest and Foreign Exchange Policies in Which Policymakers Exploit the Risk Premium for Stabilization Purposes, 2013), which focused on interest rate and XR policies, since it introduces the third vertex of the 'trinity' in the form of taxes on private foreign debt. These affect the risk-adjusted uncovered interest parity equation and hence influence the SOE's international financial flows. A useful way to illustrate the range of policy alternatives is to associate them with the faces of a triangle. Each of 3 possible government intervention policies taken individually (in the domestic currency bond market, in the FX market, and in the foreign currency bonds market) corresponds to one of the vertices of the triangle, each of the 3 possible pairs of intervention policies corresponds to one of its 3 edges, and the 3 simultaneous intervention policies taken jointly correspond to its interior. This paper shows that this interior, or 'possible trinity' is quite generally not only possible but optimal, since the CB obtains a lower loss when it implements a policy with all three interventions.

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Keywords DSGE models; small open economy; monetary and exchange rate policy; capital controls; optimal policy

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1 Introduction

Foreign trade and capital flows constitute the two fundamental links between the open economy and world markets. And both are affected by a host of government policies, including interest rate and exchange rate policies as well as different possible interventions in the free flow of financial assets. A traditional way of thinking about the exchange rate regime and capital account openness has been framed in terms of the 'impossible trinity' or 'trilemma'. According to this view, policymakers can only have two of three possible outcomes: open capital markets, monetary independence and pegged exchange rates (see Bordo 2003). Historically, during the gold standard period there predominated open capital markets and fixed exchange rates but monetary independence was lacking. Later, during the Bretton Woods period pegged exchange rates and monetary independence became possible since important capital controls prevailed. During the post-Bretton Woods period free capital mobility was again introduced, leading countries to a difficult choice between pegged exchange rates (with the consequent loss of monetary independence) and floating exchange rates (with monetary independence). Emerging market economies that chose to have pegged exchange rates tended to have periodic debt, currency and banking crises. Hence, for some time a 'bipolar view' prevailed that with free international capital markets most countries had to choose between very hard pegs and free floats. However, in many instances even very hard pegs, such as Argentina's Convertibility (that lasted 10 years), led to severe triple crises.

The 2008-9 financial meltdown has awakened new interest in these topics. There is a renewed interest in active FX reserves management and greater concern for financial stability due to the grave macroeconomic risks that financial meltdowns generate. Possibly because the main developed economies were hit hard by the crisis, there has tended to be a more receptive approach to topics that were until recently frowned upon, including FX market intervention, soft capital controls in normal times and more invasive practices during the management of crises. Even the IMF is tending to accept that under some circumstances capital controls may be useful and even necessary. Ostry et al. (2010), for example, point out that capital controls can help to address financial stability concerns, at least when there are insufficient prudential tools available. Obstfeld et al. (2008)

try to explain why there has been such a dramatic rise in global international reserve holdings (as a fraction of world GDP) during the post-Bretton Woods era. They argue that reserve accumulation is an important tool for managing domestic financial instability as well as exchange rates in a world in which financial globalization has ballooned and the domestic banking sector needs protection against different possible sources of drains (flights from currency or deposits) by means of the central bank's role as a lender of last resort. It is these concerns, much more than the traditional trade-related needs that led to such important accumulations of international reserves.

Fratzscher (2012) investigates the motives for the use of capital controls. He uses a broad set of macroeconomic and financial variables for 79 countries during the 1984–2009 period to assess which of four possible motives for the use of capital controls are most important (objectives related to FX policy, capital flow management, ensuring financial stability or general macroeconomic policy). He finds that FX policy management has been a central motive for the use of capital controls. In particular, 'countries with a high level of capital controls and countries actively raising existing controls are those that tend to have undervalued exchange rates and a high degree of exchange rate volatility.' He also finds that choices concerning capital flow restrictions, especially over the past decade, have been largely motivated by concerns about an overheating of the domestic economy. Recent empirical research has found that most countries, instead of choosing two of the three policy options of the 'trinity', actually choose a middle ground where the three options are used without extremes. Aizenman et al. (2010), for example, find that many countries choose to have a managed exchange rate with limited financial autonomy and controlled financial integration (see also Aizenman 2012).

At a theoretical level, Farhi and Werning (2012) study capital control policy in a standard open economy model with fixed exchange rates, building on Clarida et al. (2002) and Galí and Monacelli (2005, 2008). They use a non-monetary and non-stochastic model in which there is no endogenous risk premium to study the optimal use of capital controls in response to different shocks (productivity, export demand, terms of trade, foreign interest rates and exogenous risk premia) under different pricing assumptions (flexible prices, rigid prices, one-period in advance price fixing and Calvo price setting). They conclude that capital controls are more effective the more closed is the economy, and that they are particularly powerful in responding to fluctuations in the exogenous risk premium demanded by foreign investors.

At least since the heyday of the Mundell-Fleming model, modeling the intermediate ground between a firmly pegged (or fixed) exchange rate and a freely floating exchange rate has been wanting. Until very recently the difficulties involved in setting up a workable framework had not been surpassed, even with such modern developments in macroeconomic modeling such as the rational expectations revolution and DSGE modeling. In practical research, however, the analysis of the middle ground policies have not faced substantial difficulties. For example, in IMF Article 4 reports on emerging market economies, it has been traditional to study the developments in the balance sheets of the main institutional sectors (private and public financial institutions, Central Bank (CB) and Treasury) in order to obtain insight into actual exchange and interest rate policies and their consequences. In analytical macro modeling, however, there has been resistance to explicitly modeling the financial stocks and flows of the main sectors included in the model that would make it possible to represent such policy middle ground as managed exchange rates.

My research in the past few years has been mainly focused in this direction (Escudé 2006, 2007, 2009, 2013) and has led to a workable framework in small open economy DSGE modeling where policymakers may use two policy rules in order to determine operational targets for the nominal interest rate as well as the rate of nominal currency depreciation (or, alternatively, the CB international reserves–Escudé 2006). In Escudé (2013) I show the functioning of the framework in a relatively small DSGE model that, except for a few additions, is a standard monetary New Keynesian model. The additions are basically 1) an ad hoc risk premium function that is positively dependent on household foreign debt[1], 2) an ad hoc long-run target for the CB international reserves ratio (to GDP), 3) CB issued domestic currency bonds that are used for sterilization, 4) a careful formulation of the CB budget constraint along with the assumption that there is an institutional arrangement whereby the CB transfers (finances) any financial quasi-fiscal surplus (deficit) to (with) the Treasury, thereby maintaining a constant net

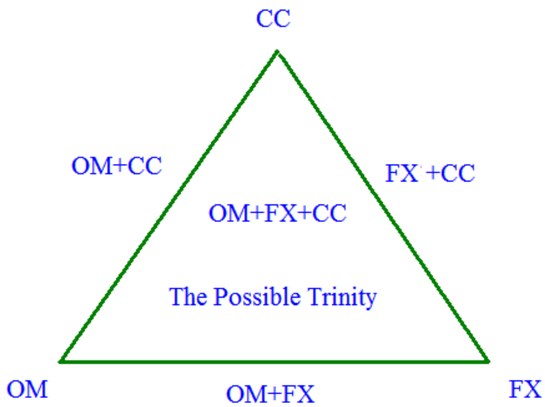
[1] In some of my previous papers it was banks that obtained funds abroad and hence faced these risk premia.

worth and a balance sheet structure that only changes during the transition, 5) a second policy rule where there is an operational target for the rate of nominal currency depreciation that may respond to the same variables (or gaps) as the policy rule for the nominal interest rate and, additionally, the gap between the CB international reserves ratio and its long run target, 6) the assignment of explicit instruments for the interventions in the domestic currency bond market (sales or purchases to achieve the operational target for the nominal interest rate) and the FX market (sales and purchases of international reserves to achieve the operational target for the rate of nominal currency depreciation).

In this framework, the CB always satisfies private sector money demand and has a long run target for the inflation rate (and hence for the rate of nominal depreciation). The role of the CB balance sheet equation is simply to determine the stock of domestic currency bonds that the CB must have in its liabilities at the end of the quarter as a result of its interventions in both markets. The concept of 'sterilized' FX market intervention is avoided because it implicitly subordinates exchange rate policies to interest rate policies. In principle, both policies are (equally) important and the sterilization of any unwanted monetary effects of the combined interventions in the two markets is reflected in the quarterly changes in the stock of CB bonds. The basic result in Escudé (2013) is that, leaving aside implementation costs (which remain unmodeled), it is optimal for policymakers to use the two policy rules, and consequently two operational targets and instruments. This is very intuitive since any one of the 'corner' policies is obtained by introducing an additional constraint: either abstain from intervening in the FX market, which implies keeping the CB reserves constant, or abstain from intervening in the domestic currency bond market, which implies keeping the CB domestic currency bonds outstanding constant.

The present paper is a natural extension of Escudé (2013) since it introduces the third vertex of the 'trinity' in the form of taxes on private foreign debt. These affect the risk-adjusted uncovered interest parity equation and hence influence the SOE's international financial flows. A useful way to illustrate the range of policy alternatives is to associate them with the faces of an isosceles triangle (as in Figure 1 below). Each of three possible government intervention policies taken individually (in the domestic currency bond market, in the foreign currency market, and in the foreign currency bonds market) corresponds to one of the vertices of the triangle, each of the three possible pairs of intervention policies correspond to one of the three edges of the triangle, and the three simultaneous intervention policies taken jointly correspond to the triangle's interior. This paper shows that this interior, or 'possible trinity' is quite generally not only possible but optimal, since the CB obtains a lower loss when it implements a policy with three interventions. As in the parent paper, any of the boundary regimes are obtained by introducing additional constraints to the policymakers' problem when a linear-quadratic optimal control framework is used. In the parent paper there were 2 corner policies and an interior policy that combined them, and they are represented by the base of the triangle in Figure 1. In the present paper there are 6 border policies and an interior policy that combines the three possible individual interventions and these policies are also represented in Figure 1. To implement any of the 3 edge regimes, the instrument that corresponds to the opposite vertex must be kept constant. To implement any of the 3 vertex regimes, the instruments that correspond to opposite 2 vertices must be kept constant. The 3 instruments are 1) the stock of domestic currency bonds in the (liability side of the) CB's balance sheet, 2) the stock of FX reserves in the (asset side of the) CB's balance sheet, and 3) the size of the tax or tax/subsidy scheme on household foreign currency liabilities. In this paper there are two alternative implementations for the third form of intervention: either a tax on the stock of household foreign debt or a tax (subsidy) on the increase (reduction) of household foreign debt. Obviously, there is a certain asymmetry since the first two instruments correspond to actual financial instruments that have a market in which the CB operates in order to obtain a desired operational target for the interest rate or the rate of nominal depreciation, whereas in the third case the instrument takes the form of a tax that must be collected (or subsidy that must be bestowed). Because most of the model is exactly as in Escudé (2013), only the extensions are detailed in the text, leaving the exposition of most of the model as well as the full set of model equations for the Appendix.

Figure 1
The 3 Interventions
OM: Open Market operations with CB domestic currency bonds
FX: Foreign eXchange market intervention
CC: (Capital Control) tax on household foreign currency bonds



The rest of the paper has the following structure. Section 2 contains the household decision problem and how it is affected by the two new forms of intervention considered. Section 3 shows the calibrations used for the model parameters and details the calibrations that are directly related to the risk premium function that foreign investors are assumed to use to determine the interest rate they demand. Section 4 specifies the alternative interest rate, exchange rate, and capital control policies that are available. Section 5 shows how the model works and illustrates the effects of the different policy regimes on the variability of the main target variables in 3 alternative frameworks available in Dynare for model solution: a) simple policy rules, b) optimal simple policy rules, and c) optimal policy under commitment and full information. Finally, Section 6 has the conclusions, Appendix A shows the parts of the model which are left out in the main body of the paper, and Appendix B lists all the model equations. Additionally, two Dynare model files are available, one for each of the two possible implementations of the tax on foreign debt.

2 The model

2.1 Households

An infinitely lived representative household consumes a CES bundle of domestic and imported goods (C_t) and holds financial wealth in the form of cash (M_t) and domestic currency denominated one period nominal bonds issued by the CB (B_t) that pay a nominal interest rate i_t and are considered riskless. The household also issues one period foreign currency bonds (D_t) abroad that pay a nominal foreign currency interest rate i_t^D. It is assumed that foreign investors are only willing to hold the SOE's foreign currency bonds if they receive a risk premium $\tau_D\left(\gamma_t^D, \gamma_t^R\right)$ over the international riskless rate i_t^* which, as a function, is exogenously given (since the Rest of the World -RW- is not modeled). This function varies directly with the SOE's aggregate foreign debt to GDP ratio γ_t^D and inversely with the CB's international reserves ratio γ_t^R (both defined below). There is also an exogenous stochastic and time-varying component ϕ_t^* of the total wedge $(1+i_t^D/1+i_t^*)$ between the (foreign currency) gross interest rates that apply to the SOE and to the RW. ϕ_t^* can represent general liquidity conditions in the international capital market and/or an exogenous component of the risk-premium. The foreign currency gross interest rate households face is hence:

$$1+i_t^D = (1+i_t^*)\phi_t^*\tau_D\left(\gamma_t^D, \gamma_t^R\right), \tag{1}$$

where $\tau_D(.)$ is increasing and convex ($\tau_D > 1$, $\tau'_{D,\gamma^D} > 0$ and $\tau''_{D,\gamma^D} > 0$) in γ_t^D, and decreasing ($\tau'_{D,\gamma^R} < 0$) in γ_t^R. The real exchange rate (RER), real foreign debt and international CB reserves (in terms of foreign prices), and corresponding foreign debt and CB reserves ratios to GDP are:

$$e_t \equiv \frac{S_t P_t^*}{P_t}, \quad d_t \equiv \frac{D_t}{P_t^*}, \quad r_t \equiv \frac{R_t}{P_t^*}, \quad \gamma_t^D = \frac{S_t D_t}{P_t Y_t} = \frac{e_t d_t}{Y_t}, \quad \gamma_t^R = \frac{S_t R_t}{P_t Y_t} = \frac{e_t r_t}{Y_t} \quad (2)$$

where R_t is the CB international reserves, S_t is the nominal exchange rate, P_t is the domestic goods price index, P_t^* is the price index of the goods the SOE imports, and Y_t is GDP.

A second exogenous function $\tau_M(.)$ represents gross transactions costs, and introduces the convenience of using cash. The household holds cash M_t to economize on transaction costs because in order to purchase the consumption bundle C_t it must spend $\tau_M(.)P_t^C C_t$ where P_t^C is the price index of the consumption bundle. The function $\tau_M\left(\gamma_t^M\right)$ is assumed to be a decreasing and convex function ($\tau_M > 1$, $\tau_M' < 0$, $\tau_M'' > 0$) of the cash/consumption ratio γ_t^M:

$$\gamma_t^M \equiv \frac{M_t}{P_t^C C_t} = \frac{m_t}{p_t^C C_t}, \quad (3)$$

where p_t^C is the relative price of consumption goods and m_t is real cash:

$$p_t^C \equiv \frac{P_t^C}{P_t}, \qquad m_t \equiv \frac{M_t}{P_t}. \quad (4)$$

The representative household maximizes an inter-temporal utility function which is additively separable in (constant relative risk aversion subutility functions of) goods C_t and labor N_t:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma^C} C_{t+j}^{1-\sigma^C} - \frac{\xi}{1+\sigma^N} N_{t+j}{}^{1+\sigma^N} \right\}, \quad (5)$$

where β is the intertemporal discount factor, σ^C, and σ^N are the constant relative risk aversion coefficients for goods and labor, respectively, and ξ is a parameter. The household budget constraint in period t is:

$$\begin{aligned} \tau_M\left(\gamma_t^M\right) P_t^C C_t + M_t + B_t - S_t D_t \quad &= \quad W_t N_t + \Pi_t - Tax_t - Tax_t^{DCol} \quad (6) \\ &\quad +M_{t-1} + (1+i_{t-1}) B_{t-1} - (1+i_{t-1}^D) S_t D_{t-1} \end{aligned}$$

where i_t is the nominal interest rate that CB bonds B_t pay each quarter, W_t is the nominal wage rate, Π_t is nominal profits, Tax_t is nominal lump sum taxes net of lump sum transfers and Tax_t^{DCol} is the government **collection** from a tax (or subsidy) related to the household's foreign debt. The latter is the main innovation in this paper with respect to Escudé (2013): here the government implements either a tax or a tax/subsidy scheme to influence capital flows from/to the RW. Let tax_t^D be a tax **rate** related to household foreign debt (i.e., foreign currency liabilities of domestic residents that are assets of residents in the RW). Two different foreign debt related concepts are considered: the first is simply a **tax** on the **level** of household foreign debt (and in this case I use tax_t^D for notation); the second is more complicated, since it is a **tax** on **increases in the level** of foreign debt and, symmetrically, a **subsidy** on foreign debt **cancellations** (and in this case I use the notation $taxsub_t^D$ to distinguish the fact that in this case it is also a subsidy rate). In the terminology used below, increases (reductions) in the level of foreign debt are referred to as capital inflows (outflows). This should pose no ambiguity as, for simplicity, households in this paper have no access to foreign assets. Consequently, there are two different possible forms for Tax_t^{DCol} in the household (nominal and real, respectively) budget constraint of period t:

Form 1 (level):

$$\begin{aligned} Tax_t^{DCol} &= tax_t^D S_t D_t, \\ tax_t^{DCol} &= tax_t^D e_t d_t. \end{aligned} \tag{7}$$

Form 2 (change in level):

$$\begin{aligned} Tax_t^{DCol} &= taxsub_t^D S_t (D_t - D_{t-1}), \\ tax_t^{DCol} &= taxsub_t^D e_t \left(d_t - \frac{d_{t-1}}{\pi_t^*} \right), \end{aligned} \tag{8}$$

where imported goods inflation π_t^* is defined as

$$\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}.$$

Introducing (1) in (6) and dividing by P_t, the real budget constraint is:

$$\tau_M\left(\gamma_t^M\right)p_t^C C_t + m_t + b_t - e_t d_t = w_t N_t + \frac{\Pi_t}{P_t} - tax_t - tax_t^{DCol} \tag{9}$$
$$+\frac{m_{t-1}}{\pi_t} + (1+i_{t-1})\frac{b_{t-1}}{\pi_t} - (1+i_{t-1}^*)\phi_{t-1}^*\tau_D\left(\gamma_{t-1}^D,\gamma_{t-1}^R\right)e_t\frac{d_{t-1}}{\pi_t^*},$$

where tax_t^{DCol} can adopt one of the two possible forms ((7) or (8)), and real CB bonds, the real wage, real lump sum taxes, and domestic inflation, are defined as:

$$b_t \equiv \frac{B_t}{P_t}, \qquad w_t \equiv \frac{W_t}{P_t}, \qquad tax_t \equiv \frac{Tax_t}{P_t}, \qquad \pi_t \equiv \frac{P_t}{P_{t-1}}.$$

The tax_t^{DCol} term in (9) is the only change in the household real budget constraint with respect to the parent model in Escudé (2013). To simplify, it is assumed that there is no imported goods inflation in the non-stochastic steady state (NSS) ($\pi^* = 1$). In the case of $taxsub_t^D$, if a shock makes the household foreign debt d_t temporarily increase and at some point it begins to decrease (until it again reaches its long run or non-stochastic steady state (NSS) value), the government first collects the (distortionary) tax during some time and at some point begins to return it as a subsidy. This makes the deficit closing lump-sum tax decline during the initial phase, and increase during the second phase to compensate for the subsidy. It is assumed in this paper that in the NSS tax^D or $taxsub^D$ are positive and less than unity.[2]

The household chooses the sequence $\left\{C_{t+j}, m_{t+j}, b_{t+j}, d_{t+j}, N_{t+j}\right\}_{j=0,\ldots,\infty}$ that maximizes (5) subject to its sequence of budget constraints (9) (and initial values

[2] The special case $tax^D = 0$ or $taxsub_t^D = 0$ is the model in Escudé (2013).

for the predetermined variables). In the $taxsub_t^D$ case, the Lagrangian is hence:

$$
\begin{aligned}
&E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{C_{t+j}^{1-\sigma^C}}{1-\sigma^C} - \xi \frac{{N_{t+j}}^{1+\sigma^N}}{1+\sigma^N} + \lambda_{t+j} \left\{ w_{t+j}N_{t+j} + \frac{\Pi_{t+j}}{P_{t+j}} + \frac{m_{t-1+j}}{\pi_{t+j}} \right.\right. \\
&+ (1+i_{t-1+j}) \frac{b_{t-1+j}}{\pi_{t+j}} - \tau_M \left(\frac{m_{t+j}}{p_{t+j}^C C_{t+j}} \right) p_{t+j}^C C_{t+j} - m_{t+j} - b_{t+j} \\
&- \left[(1+i_{t-1+j}^*) \phi_{t-1+j}^* \tau_D \left(\frac{e_{t-1+j}d_{t-1+j}}{Y_{t-1+j}}, \frac{e_{t-1+j}r_{t-1+j}}{Y_{t-1+j}} \right) - taxsub_{t+j}^D \right] \\
&\left.\left. \times e_{t+j} \frac{d_{t-1+j}}{\pi_{t+j}^*} + \left(1 - taxsub_{t+j}^D\right) e_{t+j} d_{t+j} - tax_{t+j} \right\} \right\},
\end{aligned}
$$

where (9) has been rearranged by gathering the terms in d_t and d_{t-1}, respectively (after inserting (8)). $\beta^j \lambda_{t+j}$ are the Lagrange multipliers, and can be interpreted as the marginal utility of real income.[3] In the simple tax_t^D case the only change is that (apart from substituting tax_t^D for $taxsub_t^D$) the tax term within the square bracket disappears.

For both forms of tax on foreign debt, the first order conditions for an optimum for variables C, m, b, and N, are exactly the same as in the parent paper:

$$
\begin{aligned}
C_t &: \quad C_t^{-\sigma^C} = \lambda_t p_t^C \varphi_M \left(m_t / p_t^C C_t \right) && (10) \\
m_t &: \quad \lambda_t \left[1 + \tau_M' \left(m_t / p_t^C C_t \right) \right] = \beta E_t \left(\lambda_{t+1} / \pi_{t+1} \right) && (11) \\
b_t &: \quad \lambda_t = \beta \left(1 + i_t \right) E_t \left(\lambda_{t+1} / \pi_{t+1} \right) && (12) \\
N_t &: \quad \xi N_t^{\sigma^N} = \lambda_t w_t && (13)
\end{aligned}
$$

Only the first order condition for d_t is affected by the introduction of the control on capital flows and it differs for the two cases:

Form 1 (level):

$$
\lambda_t \left(1 - tax_t^D\right) e_t = \beta (1+i_t^*) \phi_t^* E_t \left\{ \frac{\lambda_{t+1} e_{t+1}}{\pi_{t+1}^*} \left[\varphi_D \left(\frac{e_t d_t}{Y_t}, \frac{e_t r_t}{Y_t} \right) \right] \right\} \qquad (14)
$$

[3] A no-Ponzi game condition is implicit and yields the transversality condition $\lim_{t\to\infty} \beta^t d_t = 0$ that prevents households from incurring in Ponzi games.

Form 2 (change in level):

$$\lambda_t\left(1-taxsub_t^D\right)e_t \tag{15}$$
$$=\beta(1+i_t^*)\phi_t^* E_t\left\{\frac{\lambda_{t+1}e_{t+1}}{\pi_{t+1}^*}\left[\varphi_D\left(\frac{e_t d_t}{Y_t},\frac{e_t r_t}{Y_t}\right)-taxsub_{t+1}^D\right]\right\}$$

In (10) and the last two expressions, the auxiliary functions φ_M and φ_D that have been introduced for convenience are defined as:

$$\begin{aligned}\varphi_D\left(\gamma_t^D,\gamma_t^R\right) &\equiv \tau_D\left(\gamma_t^D,\gamma_t^R\right)+\gamma_t^D\tau'_{D,\gamma^D}\left(\gamma_t^D,\gamma_t^R\right),\\ \varphi_M\left(\gamma_t^M\right) &\equiv \tau_M\left(\gamma_t^M\right)-\gamma_t^M\tau'_M\left(\gamma_t^M\right),\end{aligned} \tag{16}$$

where $\tau'_{D,\gamma^D}\left(\gamma_t^D,\gamma_t^R\right)$ represents the partial derivative of τ_D with respect to γ_t^D. As in Escudé (2013), combining (11) and (12) gives the demand function for cash:

$$m_t=\mathscr{L}\left(1+i_t\right)p_t^C C_t, \tag{17}$$

where $\mathscr{L}(.)$ is defined as:

$$\mathscr{L}\left(1+i_t\right)\equiv\left(-\tau'_M\right)^{-1}\left(1-\frac{1}{1+i_t}\right), \tag{18}$$

and is strictly decreasing, since $\mathscr{L}'\left(1+i_t\right)=\left[-\tau''_M(\mathscr{L}\left(1+i_t\right))\left(1+i_t\right)^2\right]^{-1}<0$. Under the assumption that the CB always satisfies cash demand, (17) is henceforth the ‘cash market clearing condition’.

Using (10) to eliminate λ_t from (12) and (13) yields the Euler equation and the household’s labor supply, respectively:

$$\frac{C_t^{-\sigma^C}}{\varphi_M\left(m_t/p_t^C C_t\right)}=\beta\left(1+i_t\right)E_t\left(\frac{C_{t+1}^{-\sigma^C}}{\varphi_M\left(m_{t+1}/p_{t+1}^C C_{t+1}\right)}\frac{1}{\pi_{t+1}^C}\right), \tag{19}$$

$$N_t=\left(\frac{w_t}{\xi p_t^C C_t^{\sigma^C}\varphi_M\left(m_t/p_t^C C_t\right)}\right)^{\frac{1}{\sigma^N}}, \tag{20}$$

where in the first of these $\pi_t^C \equiv P_t^C/P_{t-1}^C$ is the gross rate of inflation of the basket of consumption goods and the identity $p_t^C/p_{t-1}^C = \pi_t^C/\pi_t$ is used.

Finally, the definition of the RER in (2) gives the identity $e_t/e_{t-1} = \delta_t \pi_t^*/\pi_t$, where $\delta_t \equiv S_t/S_{t-1}$ is the rate of nominal depreciation of the domestic currency. Hence, (15) may be written as:

$$1 = \beta(1+i_t^*)\phi_t^* E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right) \left(\frac{\varphi_D\left(\gamma_t^D, \gamma_t^R\right) - taxsub_{t+1}^D}{1 - taxsub_t^D} \delta_{t+1} \right) \right\}. \quad (21)$$

Eliminating β using (12) yields:

$$\begin{aligned} & (1+i_t)E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right) \\ = \quad & (1+i_t^*)\phi_t^* E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right) \left(\frac{\varphi_D\left(e_t d_t/Y_t, e_t r_t/Y_t\right) - taxsub_{t+1}^D}{1 - taxsub_t^D} \delta_{t+1} \right) \right\}. \end{aligned}$$

Using the fact that the expected value of the product of two random variables is the product of the expected values plus the covariance of the two variables, gives

$$\begin{aligned} & (1+i_t)E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right) \\ = \quad & (1+i_t^*)\phi_t^* \left\{ \begin{array}{l} E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right) E_t \left(\frac{\varphi_D(e_t d_t/Y_t, e_t r_t/Y_t) - taxsub_{t+1}^D}{1 - taxsub_t^D} \delta_{t+1} \right) \\ + Cov_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}}, \frac{\varphi_D(e_t d_t/Y_t, e_t r_t/Y_t) - taxsub_{t+1}^D}{1 - taxsub_t^D} \delta_{t+1} \right) \end{array} \right\}. \end{aligned}$$

Therefore, to a first order approximation the covariance term can be ignored and the risk-adjusted UIP equation is simply:

$$\begin{aligned} 1 + i_t &= (1+i_t^*)\phi_t^* E_t \left(\frac{\varphi_D\left(e_t d_t/Y_t, e_t r_t/Y_t\right) - taxsub_{t+1}^D}{1 - taxsub_t^D} \delta_{t+1} \right) \quad (22) \\ &= (1+i_t^*)\phi_t^* E_t \left[\left(1 + \frac{\overline{\varphi}_D\left(e_t d_t/Y_t, e_t r_t/Y_t\right) - \Delta taxsub_{t+1}^D}{1 - taxsub_t^D} \right) \delta_{t+1} \right] \end{aligned}$$

where in the second equality $\varphi_D(.) \equiv 1 + \overline{\varphi}_D(.)$ is used. Notice that an increase in $taxsub_t^D$ has the effect of increasing the domestic interest rate (*ceteris paribus*), while an expected increase in the next period has the opposite effect. Hence, if $taxsub_t^D$ increases initially and is subsequently expected to fall, both have the effect of increasing the domestic interest rate (*ceteris paribus*). In the case of the simple tax on the level of debt, to a first order approximation the UIP equation is:

$$\begin{aligned} 1+i_t &= (1+i_t^*)\phi_t^* \left(\frac{\varphi_D\left(e_t d_t/Y_t, e_t r_t/Y_t\right)}{1-tax_t^D} \right) E_t \delta_{t+1} \\ &= (1+i_t^*)\phi_t^* \left(1 + \frac{\overline{\varphi}_D\left(e_t d_t/Y_t, e_t r_t/Y_t\right) + tax_t^D}{1-tax_t^D} \right) E_t \delta_{t+1}. \end{aligned} \tag{23}$$

2.2 The public sector

The public sector includes the Government and the CB. The CB issues currency (M_t) and domestic currency bonds (B_t), and holds international reserves (R_t) in the form of foreign currency denominated riskless bonds issued by the RW. The CB supplies whatever amount of cash is demanded by households, and can influence these supplies by changing R_t or B_t, i.e. intervening in the foreign exchange market or in the domestic currency bond market. It is assumed that CB bonds are only held by domestic residents and that the CB transfers its quasi-fiscal surplus to (or has its quasi-fiscal deficit financed by) the Government each period, maintaining its net worth at zero each period.[4] Hence, the CB balance, for all t, is:

$$m_t + b_t = e_t r_t. \tag{24}$$

The Government spends on goods, receives the quasi-fiscal surplus (or finances the quasi-fiscal deficit) of the CB, and collects taxes. It is assumed that fiscal policy consists of an exogenous autoregressive path for real government expenditures as a (gross) fraction (G_t) of private consumption $\tau_M(.)\,p_t^C C_t$, collecting the tax on private capital flows, and collecting whatever lump-sum taxes are needed to balance the budget each period. The Public Sector real flow budget constraint is hence:

$$tax_t = (G_t - 1)\,\tau_M\left(m_t/p_t^C C_t\right) p_t^C C_t - q f_t - tax_t^{DCol}, \tag{25}$$

[4]See Escudé (2013) for more details.

where the real quasi-fiscal surplus includes interests on CB assets and capital gains or losses on CB international reserves:

$$qf_t = \left[\left(1+i^*_{t-1}\right) - 1/\delta_t\right] e_t \frac{r_{t-1}}{\pi^*_t} - \left[\left(1+i_{t-1}\right) - 1\right] \frac{b_{t-1}}{\pi_t}, \tag{26}$$

and the real domestic currency value of the tax collection related to capital flows is either (7) or (8).

Since the description of the rest of the model is exactly the same as in Escudé (2013) it is relegated to Appendix A.

2.3 Functional forms for auxiliary functions

The functional forms used for the endogenous risk premium and transaction costs functions are the same as in Escudé (2013):

$$\tau_D\left(\gamma^D_t, \gamma^R_t\right) \equiv \tau^D_t = 1 + \frac{\alpha_1}{1 - \alpha_2\gamma^D_t + \alpha_3\gamma^R_t}, \qquad \alpha_1, \alpha_2, \alpha_3 > 0, \tag{27}$$

$$\tau_M\left(\gamma^M_t\right) \equiv \tau^M_t = 1 + \frac{\beta_1}{\left(1+\beta_2\gamma^M_t\right)^{\beta_3}}, \qquad \beta_1, \beta_2, \beta_3 > 0, \tag{28}$$

which, according to definitions (16), imply:

$$\begin{aligned}\varphi_D\left(\gamma^D_t, \gamma^R_t\right) &\equiv \varphi^D_t = 1 + \left(\tau^D_t - 1\right)\left(1 + \frac{\alpha_2\gamma^D_t}{1 - \alpha_2\gamma^D_t + \alpha_3\gamma^R_t}\right), \\ \varphi_M\left(\gamma^M_t\right) &\equiv \varphi^M_t = 1 + \left(\tau^M_t - 1\right)\left(1 + \beta_3\frac{\beta_2\gamma^M_t}{1+\beta_2\gamma^M_t}\right).\end{aligned} \tag{29}$$

For convenience, define the respective **net** functions as:

$$\begin{aligned}\overline{\tau}_D(.) &= \tau_D(.) - 1, \qquad \overline{\varphi}_D(.) = \varphi_D(.) - 1 \\ \overline{\tau}_M(.) &= \tau_M(.) - 1, \qquad \overline{\varphi}_M(.) = \varphi_M(.) - 1.\end{aligned} \tag{30}$$

The partial elasticities of $\overline{\tau}_D$ and $\overline{\tau}_M$ (used below in calibrations) are, respectively:

$$\varepsilon_{\overline{\tau}_{D,1,t}} = \frac{\alpha_2\gamma^D_t}{1 - \alpha_2\gamma^D_t + \alpha_3\gamma^R_t}, \qquad \varepsilon_{\overline{\tau}_{D,2,t}} = \frac{-\alpha_3\gamma^R_t}{1 - \alpha_2\gamma^D_t + \alpha_3\gamma^R_t} \tag{31}$$

$$\varepsilon_{\overline{\tau}_{M,t}} = \beta_3\frac{\beta_2\gamma^M_t}{1+\beta_2\gamma^M_t}. \tag{32}$$

Finally, the liquidity preference function (18) that results from (28) is:

$$\frac{m_t}{p_t^C C_t} \equiv \gamma_t^M = \mathscr{L}(1+i_t) \equiv \frac{1}{\beta_2}\left[\left(\frac{\beta_1\beta_2\beta_3}{1-\frac{1}{1+i_t}}\right)^{\frac{1}{\beta_3+1}} - 1\right].$$

3 Calibration of parameters and the non-stochastic steady state

In this section the calibrated parameters that are used in the exercises below are shown and the calibration procedure used is only detailed inasmuch as it differs from that of the parent paper. Since the only expansion in this paper is that there is either a tax or a tax/subsidy scheme related to foreign debt (even in the NSS), the rest of the calibrations are the same as in Escudé (2013), which the interested reader can consult. It is convenient to stress that, although Argentine data has been used for some of the calibrations, the main objective has been to have a calibrated SOE economy similar in many respects to some of those most cited in the literature (e.g., Galí and Monacelli 2005 and De Paoli 2006) but endowed with the innovations that allow for the systematic and simultaneous use of interest and nominal depreciation policy rules.

The following are immediately obtained from the NSS versions of various equations:[5]

$$er/Y = \gamma^R, \quad \pi = \pi^C = \pi^T, \quad \delta = \pi^T/\pi^*, \quad \pi^* = \pi^{*X} = p^* = 1, \quad 1+i = \pi^T/\beta. \tag{33}$$

Table 1 summarizes the calibrated values of the main model parameters, along with some comparisons with parameter values used in other SOE models, and the calibrated NSS values of some of the endogenous variables (or ratios of endogenous variables).[6]

[5] See the complete set of equations in Appendix B.

[6] 'E.S.' denotes 'elasticity of substitution', G_M stands for Galí and Monacelli (2005), and De P for De Paoli (2006).

Table 1

Parameters		**This paper**	**G-M**	**De P**
β	Intertemporal discount factor	0.99	0.99	0.99
σ^C	Relative risk aversion for goods	1.5	1	1
σ^N	Relative risk aversion for labor	0.5	3	0.47
α	Probability of not adjusting price	0.66	0.75	0.66
θ	E.S. between domestic goods	6	6	10
θ^C	E.S. domestic vs. imported goods	1.5	1	3
a_D	Coef. for share of domestic goods	0.86	0.6	0.6
b^A	Coef. in prod. function for commodities	0.5	1	
$\varepsilon_{\overline{\tau}_{D,1}}$	Elasticity of risk function $\overline{\tau}_D(ed/Y, er/Y)$	10		
$\varepsilon_{\overline{\tau}_{D,2}}$	Elasticity of risk function $\overline{\tau}_D(ed/Y, er/Y)$	0		
$\varepsilon_{\mathscr{L}}$	Elasticity of $\mathscr{L}(1+i)$	1.02		
NSS values of endogenous variables or ratios				
Y	GDP	1.443		
G	Gov. Expend. to private consumption	1.19		
π^T	Inflation target	1.015		
γ^D	Household foreign debt to GDP	0.5		
γ^R	CB foreign reserves to GDP	0.13		
m/Y	Household cash to GDP	0.08		
π^*	RW export goods inflation	1		
$1+i^*$	RW interest rate	$1.03^{0.25}$		
ϕ^*	RW exogenous liquidity/risk premium	$1.005^{0.25}$		

The standard errors and persistence parameters used for the six shock variables are given in Table 2. They were calibrated taking into account the available time series for Argentina and the RW during the period 1994.1–2009.2: public consumption to GDP in the case of σ^G, imported and exported goods inflation as they conform Argentina's XTT, in the cases of σ^{π^*} and $\sigma^{\pi^{**}}$, Libor 3 months in the case of σ^{i^*}, and balance of payments information on private sector foreign debts and interest payments as well as the author's calculation of the spread over Libor 3 months, in the case of σ^{ϕ^*}. The only cases in which the standard deviations were taken exactly according to the data are the cases of σ^{i^*}, σ^{π}, and $\sigma^{\pi^{**}}$.

Table 2
Calibration of shock variables

Standard deviations		Persistence parameters	
σ^{ε}	0.01	ρ^{ε}	0.8
σ^{G}	0.03	ρ^{G}	0.85
σ^{i^*}	0.0046	ρ^{i^*}	0.7
σ^{ϕ^*}	0.05	ρ^{ϕ^*}	0.3
σ^{π^*}	0.0295	ρ^{π^*}	0.2
$\sigma^{\pi^{**}}$	0.0424	$\rho^{\pi^{**}}$	0.41
		$\rho^{\pi^{*XN}}$	0.81

The rest of the standard deviations were calibrated taking both the data (except for σ^{ε}) and the resulting theoretical standard deviation and variance decomposition for GDP with a baseline calibration of (38) and (39): $h_1 = 0.8$, $h_2 = 0.8$, $k_4 = -0.8$, and the rest of the coefficients equal to zero. This implied diminishing the observed standard deviation of G (from 0.054 in a simple AR(1) estimation from which the value for the persistence parameter ρ^G was taken), which seemed to weigh too heavily in the volatility of Y, and increasing the standard deviation of ϕ^* (from 0.0034), which seemed not to weigh enough.

Aside from the introduction of the new simple policy rule ((40) below) and the tax equation (either (7) or (8)), the only equation that changes with respect to the previous paper is the Risk-adjusted UIP ((22) or (23)) which, using (29) and (33) and manipulating, gives at the NSS:

Tax on level of debt:

$$\overline{\varphi}_D = \left(1 - tax^D\right) \frac{1}{\beta\left(1+i^*\right)\left(\phi^*/\pi^*\right)} - 1 \tag{34}$$

Tax on change of debt:

$$\overline{\varphi}_D = \left(1 - taxsub^D\right) \left(\frac{1}{\beta\left(1+i^*\right)\left(\phi^*/\pi^*\right)} - 1\right). \tag{35}$$

The risk function parameters α_1, α_2 and α_3 in (27) need to be calibrated. First, note that (31) gives directly

$$\varepsilon_{\overline{\tau}_{D,1}} = \alpha_2 \gamma^D \left(1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}\right) \tag{36}$$

$$\varepsilon_{\overline{\tau}_{D,2}} = -\alpha_3 \gamma^R \left(1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}\right) \tag{37}$$

which, given calibrated values for the elasticities and the great ratios, yield:

$$\begin{aligned} \alpha_2 &= \frac{1}{\gamma^D} \frac{\varepsilon_{\overline{\tau}_{D,1}}}{1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}} \\ \alpha_3 &= \frac{1}{\gamma^R} \frac{-\varepsilon_{\overline{\tau}_{D,2}}}{1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}}. \end{aligned}$$

Equations (36) and (37) also imply

$$\begin{aligned} 1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}} &= \frac{1}{1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R}, \\ 1 + \varepsilon_{\overline{\tau}_{D,1}} &= \frac{1 + \alpha_3 \gamma^R}{1 - \alpha_2 \gamma^D + \alpha_3 \gamma^R}, \end{aligned}$$

from which

$$\overline{\varphi}_D = \frac{\alpha_1 \left(1 + \alpha_3 \gamma_t^R\right)}{\left(1 - \alpha_2 \gamma_t^D + \alpha_3 \gamma_t^R\right)^2} = \alpha_1 \left(1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}\right) \left(1 + \varepsilon_{\overline{\tau}_{D,1}}\right).$$

Therefore, using either (34) or (35) the value of α_1 takes two different forms, according to the assumption on the capital control tax:

Tax on level of debt:

$$\alpha_1 = \frac{\left(1 - tax^D\right) \left(\frac{1}{\beta(1+i^*)(\phi^*/\pi^*)}\right) - 1}{\left(1 + \varepsilon_{\overline{\tau}_{D,1}}\right) \left(1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}\right)}$$

Tax on change of debt:

$$\alpha_1 = \frac{\left(1 - taxsub^D\right) \left(\frac{1}{\beta(1+i^*)(\phi^*/\pi^*)} - 1\right)}{\left(1 + \varepsilon_{\overline{\tau}_{D,1}}\right) \left(1 + \varepsilon_{\overline{\tau}_{D,1}} + \varepsilon_{\overline{\tau}_{D,2}}\right)}.$$

In order to be able to reasonably compare the two alternative forms of capital controls, their NSS values can be calibrated so that the non-tax variables have the same NSS values. In particular, this requires that the risk premium have the same value in the NSS. Looking at (34) and (35), a little algebra shows that for this to be the case the following relation between tax^D and $taxsub^D$ must hold:

$$tax^D = taxsub^D \left[1 - \beta \left(1 + i^*\right) \left(\phi^* / \pi^*\right)\right].$$

4 Interest rate, exchange rate, and capital control policies

As in Escudé (2013), in this paper the CB uses to stabilize the SOE's macroeconomy either I) simple policy rules or II) optimal control under commitment and full information. The simple rules may be Ia) simple and with exogenous coefficients, or Ib) simple and with optimal coefficients. In case Ia), the simple interest rate rule is a feedback rule, and the simple rules for nominal depreciation and the tax/subsidy on capital flows may or may not involve feedback. In case Ib), the CB is assumed to obtain the values of the coefficients in the policy rules by minimizing a weighted average of the squared deviations of certain target (endogenous) variables from their NSS values. When the CB uses II) (i.e., optimal policies under commitment and full information), the simple policy rules disappear and the CB obtains the trajectories for the intermediate targets (nominal interest rate, nominal rate of depreciation) and the tax rate (or tax/subsidy rate) by minimizing an expected discounted intertemporal quadratic loss function of the target variables.

In Escudé (2013) it was shown that when the target variables are the inflation rate, GDP, and the RER, it is 'always' better to use two policy rules instead of one of the two 'corner' regimes of Floating Exchange Rate -FER- or Pegged Exchange Rate -PER- (which can equivalently be called Floating Interest Rate -FIR- regime). In the FER regime the CB abstains from intervening in the FX market and has an intermediate target for the nominal interest rate, while in the PER regime the CB abstains from intervening in the domestic currency bond market and has an intermediate target for the rate of nominal depreciation. In the Managed Exchange Rate -MER- regime, on the other hand, there are two simple rules: one for the nominal rate of interest and an another for the rate of nominal currency depreciation. It turned out that it was ‘always’ better two use the MER regime, in the sense that the CB obtains lower losses under this regime for any set of CB preferences for inflation, GDP, or RER stabilization. The reason for this gain in using two rules is that the CB can thus better exploit private capital flows for its stabilization purposes, given the fact that these flows are (mainly) jointly determined by the risk-adjusted Uncovered Interest Parity condition (UIP) and its policy rules. Determining both ends of the UIP equation by means of the operational targets in the two policy rules has a crucial effect on the foreign-debt to GDP ratio, which is assumed to determine the risk assessment of foreign investors and hence the wedge between the domestic interest rate and the expected rate of depreciation of the currency.

This paper starts from that point and explores the effects of adding an additional policy rule: one that involves the particular form of ‘capital control’ device (tax_t^D or $taxsub_t^D$) introduced in the previous sections. As shown above, a tax on foreign debt or a tax/subsidy scheme on private capital flows becomes an integral part of the risk-adjusted UIP equation. Hence, if the CB has a policy rule for determining the level of these taxes or taxes/subsidies, it has an additional instrument that can affect private capital flows. With the same methods used in the previous paper, this paper shows that it is generally optimal to use 3 policy rules instead of either 2 or 1, and for the same reasons. Hence, the Trinity of interest rate, exchange rate, and capital control policies in the SOE is not only Possible, but is also Optimal.[7]

In the MER regime, the CB, through its regular and systematic interventions in the domestic currency bond (or ‘money’) and foreign exchange markets, aims for the achievement of two operational targets: one for the interbank interest rate i_t, and another for the rate of nominal depreciation δ_t. When there are simple policy rules, the CB can uses operational targets for i_t and δ_t that respond to deviations of the consumption inflation rate (π_t^C), GDP (Y_t) and/or the RER (e_t) from their respective NSS levels. The rate of nominal depreciation can additionally respond to deviations of the CB’s international reserves (IRs) to GDP ratio from a long run target (γ^R). In this paper, there is an additional policy rule that determines the tax

[7] However, it is shown below that in the Ramsey case it is only marginally better to use the 3 control variables instead of only the interest rate and tax/subsidy rate.

or tax/subsidy related to foreign debt, which in principle can respond to the same three basic target variables as the preceding rules. But it has been deemed preferable to be a little more specific by making it also respond to the deviations in the exogenous risk/liquidity shock ϕ_t^*, given that this shock variable directly affects the UIP equation and that such shocks are empirically important for emerging market economies. There may also be history dependence (or inertia) in any of the feedback rules through the presence of the lagged operational target variable. Hence, the simple rules in the case of tax_t^D are the following:[8]

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{h_0}\left(\frac{\pi_t^C}{\pi_t^T}\right)^{h_1}\left(\frac{Y_t}{Y}\right)^{h_2}\left(\frac{e_t}{e}\right)^{h_3} \tag{38}$$

$$\frac{\delta_t}{\delta} = \left(\frac{\delta_{t-1}}{\delta}\right)^{k_0}\left(\frac{\pi_t^C}{\pi_t^T}\right)^{k_1}\left(\frac{Y_t}{Y}\right)^{k_2}\left(\frac{e_t}{e}\right)^{k_3}\left(\frac{\gamma_t^R}{\gamma^R}\right)^{k_4} \tag{39}$$

$$\frac{tax_t^D}{tax^D} = \left(\frac{tax_{t-1}^D}{tax^D}\right)^{j_0}\left(\frac{\pi_t^C}{\pi_t^T}\right)^{j_1}\left(\frac{Y_t}{Y}\right)^{j_2}\left(\frac{e_t}{e}\right)^{j_3}\left(\frac{\phi_t^*}{\phi^*}\right)^{j_4}. \tag{40}$$

Any one (or any two) of these simple rules can be replaced by a corresponding equation that simply maintains a corresponding endogenous variable at its NSS value. As in Escudé (2013), in the case of the first two of these policy rules the instrument (b_t and r_t) that the CB uses (at high frequency) to achieve the respective operational target (for i_t or δ_t) are endogenous variables. Hence, when the CB abstains from intervening in the bond or FX market it keeps the corresponding instrument constant. Specifically, when there is a FER regime, the second of the above simple rules must be replaced by an equation that keeps the stock of CB foreign currency reserves constant at the NSS level ($r_t = r$). And when there is a PER regime, the first of the above simple rules is replaced by an equation that keeps the stock of CB domestic currency bonds constant at the NSS level ($b_t = b$). In the present paper, there is the additional possibility that the government (assumed to generally coordinate with the CB) abstain from actively using capital controls. In that case, the third simple policy rule above is replaced by the equation

[8] Variables without a time subscript denote NSS values. In the tax/subsidy case simply replace tax_t^D with $taxsub_t^D$.

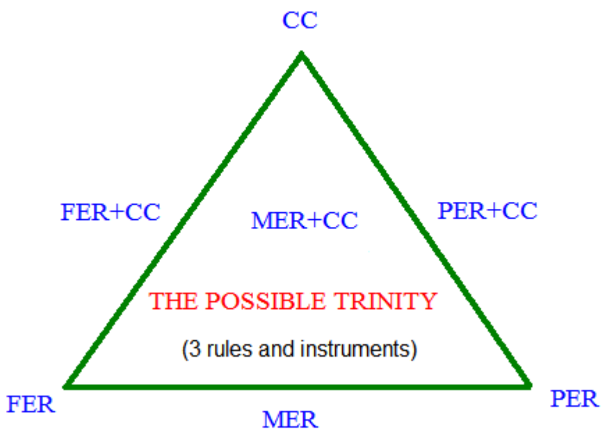
that keeps tax_t^D (or, $taxsub_t^D$, which for succinctness is not repeated below) at its NSS level. Hence, the three possible substituting equations are, respectively:

$$b_t = b, \qquad r_t = r, \qquad tax_t^D = tax^D. \tag{41}$$

As Figure 2 illustrates, there are seven possible 'policy regimes', corresponding to the seven 'faces' of the triangle (or 2-simplex). The three *vertices* (0-faces) of the triangle represent the 'pure' policies in which there is only one rule, and hence the other two are replaced by their substitutes. The three *edges* (1-faces) of the triangle represent the three policy regimes in which two of the rules operate and the third is replaced by its substitute. And the *interior* (2-face) of the triangle represents the case in which all three rules are used (the Possible Trinity). This latter policy regime is denominated Managed Exchange Rate with Capital Control (MER+CC).

The bottom edge of the triangle (including its two vertices) represents the three policy regimes studied in Escudé (2013) (in which there were no capital controls). These three regimes keep tax_t^D constant at its NSS level (which in the parent paper was not defined but in this paper may be zero or positive). The MER regime uses the first two of the above simple policy rules and replaces the third policy rule by $tax_t^D = tax^D$. Parting from the MER regime, the Floating Exchange Rate regime (FER) additionally replaces the second policy rule by $r_t = r$, and the Pegged Exchange Rate regime (PER) instead additionally replaces the first policy rule by $b_t = b$. The upper left edge of the triangle (FER+CC regime) adds the capital control rule to the interest rate rule, and the upper right edge of the triangle (PER+CC regime) adds the capital control rule to the nominal rate of depreciation rule.

Figure 2
The 7 interest rate, depreciation rate, and capital control regimes
vertices: 1 rule and instrument
edges: 2 rules and instruments
interior: 3 rules and instruments



Operational targets for individual rules:

FER - Floating XR regime:	interest rate
PER - Pegged XR regime:	depreciation rate
MER - Managed XR regime:	interest and depreciation rates
CC - Capital Control regime:	tax on foreign debt or tax/subsidy on change of foreign debt

The top vertex of the triangle is the policy which only uses the capital control rule and keeps the two usual instruments constant at their NSS levels. It may come as a surprise to many that such a policy rule easily makes the model satisfy the Blanchard-Kahn conditions for stability and determinacy. In fact, using the calibrations detailed in Escudé (2013) and section 3 above, and starting from the baseline simple policy rule defined in the second column of Table 3, each coefficient can vary individually within the (very wide) intervals given by the third column without impairing the Blanchard-Kahn conditions:[9]

So far, simple policy rules have been considered, whether their coefficients are exogenously given or optimal in the sense that they represent the minimum of an ad-hoc CB loss function. The latter case is handled by means of the 'osr' (optimal simple rule) Dynare command. In the case of optimal policy under commitment there are no simple policy rules, the corresponding equations disappear, and hence there are more endogenous variables than system equations. The paths for the endogenous variables that lack an equation (the set or a non-empty subset of the three intermediate targets) are obtained as solutions to the optimal control problem in which the CB minimizes the expected discounted sum of all (present and) future losses. This case is handled by means of the 'ramsey' Dynare command. The appropriate combination of 'instruments' (i.e., the control variables whose paths are obtained as optimum for the optimal control problem) must be chosen, and the corresponding substitute equation(s) must be introduced for those of the three possible 'instruments' variables that are not used as such. For example, for solving the model for the MER+CC regime under 'ramsey' using Dynare, the option 'instruments=(ii,delta,taxsubD)' for the 'ramsey' command must be used and the three simple rules are simply eliminated (with no substitute equation). But for the remaining 6 policy regimes at least one of these instruments is not used. In particular, for the FER+CC regime the option to use is 'instruments=(ii,taxsubD)', the first and third simple rules must be eliminated (with no substitute equation) and the second policy rule must be replaced by $r_t = r$. Analogously, for the PER+CC regime, the option to use is 'instruments=(delta,taxsubD)', the second and third simple rules must be eliminated (with no substitute equation) and the first policy rule must be replaced by $b_t = b$. As an example of the three policy regimes in which only one instrument is used let us take the case of the CC regime. In this case, the option to use is 'instruments=(taxsubD)', the third simple rule is eliminated (with no substitute equation) and the first two policy rules are substituted by $b_t = b$ and $r_t = r$.

[9] Only values of the j_k up to 100 in absolute value are reported, but the negative values can be much higher in aboslute value.

Table 3
CC simple policy rule
Individual coefficient BK stability ranges

Coefficient	Baseline value	Stability range
j_0	1.5	-100 to -1.85 $\cup$ 1.001 to 100
j_1	-1.5	-100 to 33
j_2	-1.5	-100 to 7
j_3	-1.5	-100 to 12
j_4	0.0	-100 to 100

It should be clear that in the case of the Ramsey problem, the optimal policy under any one of the six 'boundary' regimes cannot dominate the optimal rule under the MER+CC regime due to the fact that in any of the latter the government imposes at least one additional restriction on itself ('ties its hands'), hence relinquishing its use of one or more of its potential 'control' variables and using instead one or more of the possible substitute equations. For the same reason, the optimal loss for a 'vertex' policy (one of the vertices of the triangle) cannot be greater than the optimal loss for an 'edge' policy (one of the sides of the triangle) that has that vertex as one of its extremes. Hence, there is a clear hierarchy here: the optimal loss for the MER+CC regime is less than or equal to the optimal loss of the MER, FER+CC, or PER+CC regimes (its edges), the optimal loss of the MER regime is less than or equal to the optimal loss of the FER or PER regimes (its vertices), the optimal loss of the FER+CC regime is less than or equal to the optimal loss of the FER or CC regimes (its vertices), and the optimal loss of the PER+CC regime is less than or equal to the optimal loss of the PER or CC regimes (its vertices). What is of interest is the extent to which an additional instrument reduces the loss, the ranking of the losses within the three edges and within the three vertices, and how the relative losses vary with different CB preferences (or 'styles').

Table 4
MER regime with simple policy rules

Simple policy rules				Results		
Coefficient values				Variable	Mean	Std. Dev.
h_0	1.3	k_0	-0.2	piC	1.015	0.0091
h_1	2.1	k_1	-0.4	Y	1.443	0.0748
h_2	-0.01	k_2	0.1	e	0.5951	0.0404
h_3	0.05	k_3	-0.3	ii	1.0253	0.0157
		k_4	-0.1	delta	1.015	0.0675
				d	1.2124	0.323
				gammaD	0.5	0.1173
				varphiD	1.0013	0.006
				taxsubD	0.1	0

5 The role of a tax/subsidy in capital inflows/outflows in stabilization

5.1 Preliminary illustration of the effects of introducing capital controls through simple policy rules

First, let us illustrate how the introduction of a variable tax/subsidy scheme on capital flows can achieve stabilization objectives, by which is meant a reduction in the standard deviation (s.d.) of certain target variables. Let us assume that initially there is a MER regime with the simple policy rules defined in the first four columns of Table 4. Running the model gives the s.d. shown in the last column for some of the typical target variables (π_t^C, Y_t, e_t), intermediate target variables (i_t, δ_t), and three variables related to household foreign debt (d, γ^D, φ^D). Since taxsubD is not used as an instrument in the MER regime its s.d. is zero.

The model was then run using a MER+CC regime which has the same two policy rules as in the MER regime and an additional simple policy rule for the tax/subsidy scheme shown in columns 5 and 6 of Table 5. this table shows that the s.d. for consumption inflation has been reduced by 37.4%, whereas the s.d. for GDP and the RER have both increased by 5.2%.

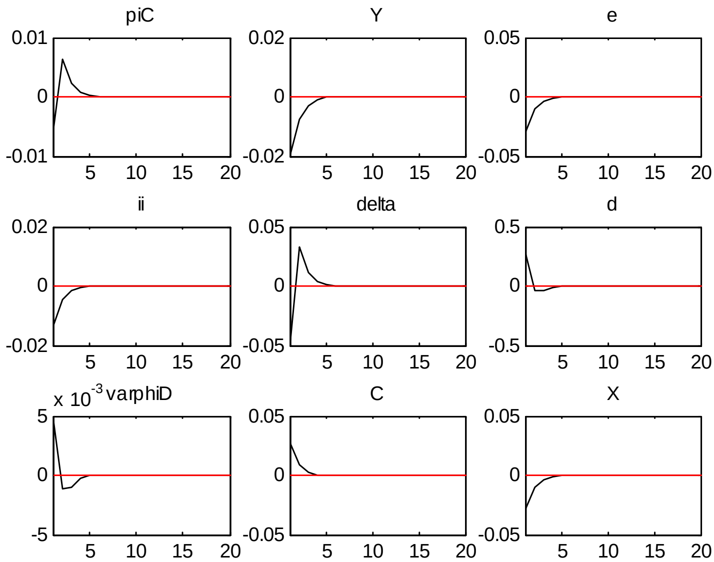
Table 5
MER+CC regime with simple policy rules

Simple policy rules						Results			% Ch.
Coefficient values						Variable	Mean	Std. Dev.	vs. MER
h_0	1.3	k_0	-0.2	j_0	0.5	piC	1.015	0.0057	-37.4%
h_1	2.1	k_1	-0.4	j_1	-0.2	Y	1.443	0.0787	+5.2%
h_2	-0.01	k_2	0.1	j_2	0.0	e	0.5951	0.0425	+5.2%
h_3	0.05	k_3	-0.3	j_3	0.0	ii	1.0253	0.0121	-22.9%
		k_4	-0.1	j_4	-0.03	delta	1.015	0.0499	-26.1%
						d	1.2124	0.4603	+42.5%
						gammaD	0.5	0.186	+58.6%
						varphiD	1.0013	0.0095	+58.3%
						taxsubD	0.1	0.1694	+∞%

The s.d. for the two operational targets in the MER regime (the nominal interest rate and the rate of nominal depreciation) have been reduced by 22.9% and 26.1%, respectively. Of course, this has been achieved by increasing the s.d. for taxsubD infinitely (since it was null under the MER regime and now it is positive), and the s.d. for d, γ^D and φ^D by 42.5%, 58.6% and 58.3%, respectively.

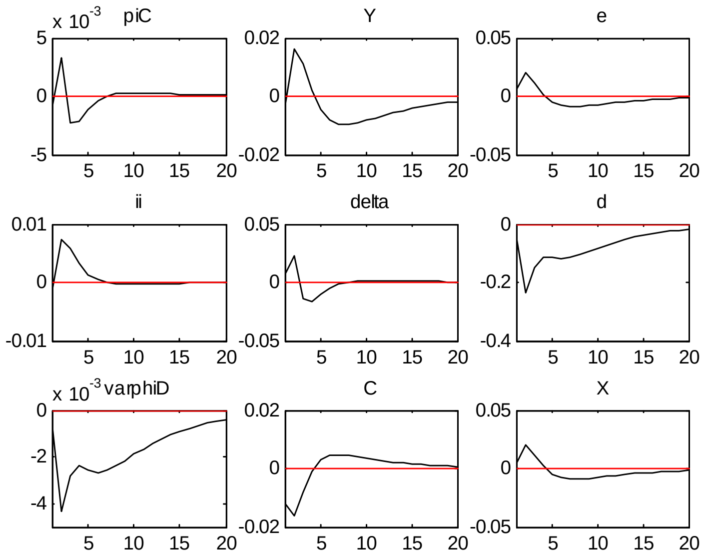
Figures 3 and 4 show the Impulse Response Functions (IRFs) corresponding to a surprise reduction in the exogenous risk/liquidity premium ϕ^*. Under the MER regime, the liquidity shock induces households to take advantage of the cheaper funds and thereby increase their foreign debt on impact and increase their consumption. However, the shock also generates real appreciation, making exports fall. This negative effect predominates over the increase in consumption, so GDP falls. Figure 4 shows the IRFs after the tax/subsidy scheme has been introduced (and hence there is a MER+CC regime). The behavior of households is seen to be quite different in the initial quarters. Instead of initially increasing their foreign debt they reduce it (thereby obtaining a subsidy) and instead of increasing their consumption, they reduce it. Instead of a real appreciation, there is now an initial real depreciation, increasing exports. The latter effect neutralizes the fall in consumption, so there is no initial effect on GDP but subsequently it rises for a few quarters, since consumption recovers faster than exports start to fall.

Figure 3
Negative shock to ϕ^*
MER regime



In essence, the introduction of the tax/subsidy scheme generates during the initial quarters a substitution of expensive foreign funds for cheap government funds (that pay no interest). It can be concluded that governments that use a MER regime with the coefficients shown above in a SOE prone to significant RW liquidity shocks and have a stronger preference for stabilizing consumption inflation than GDP or the RER have something to gain by introducing a tax/subsidy scheme on capital flows.

Figure 4
Negative shock to ϕ^*
MER+CC regime



This exercise is only an illustration of how the tax/subsidy scheme can change the dynamic paths of variables that typically interest policymakers most. One should bear in mind that the IRFs illustrate the effect of the introduction of the tax/subsidy scheme on the (deterministic) dynamics of the model when there is a shock to the exogenous risk/liquidity premium, whereas the standard deviations of the endogenous variables shown in Tables 4 and 5 above illustrate the stochastic properties of the whole set of shock variables. Table 6 below show the variance decompositions of the variables corresponding to the two exercises. They show that the four really significant shocks in the model are those that hit public expenditures (G), the exogenous risk/liquidity premium (ϕ^*-phiStar), and the inflation rates for imports (π^*-piStar) and exports (π^{**}-piStarX), whereas the shocks on productivity (ε-epsilon) and the world interest rate (i^*-iStar) individually account for at most 5.7% of the variances of the target variables.

Table 6
Variance decomposition (in percent)

VARIANCE DECOMPOSITION (in percent)						
MER regime						
	eps_epsilon	eps_G	eps_iStar	eps_phiStar	eps_piStar	eps_piStarX
piC	0.20	0.24	1.18	88.72	0.73	8.93
Y	5.73	68.35	0.36	7.76	7.72	10.09
e	0.53	0.82	2.33	57.41	4.00	34.91
ii	1.07	0.97	1.66	82.26	0.91	13.13
delta	0.09	0.15	1.33	72.07	14.74	11.62
d	1.12	5.61	2.59	68.52	5.65	16.51
gammaD	2.39	1.69	2.57	65.68	12.78	14.89
varphiD	2.39	1.69	2.57	65.68	12.78	14.89
taxsubD	0.24	9.08	1.41	67.95	6.35	14.97
MER+CC regime						
	eps_epsilon	eps_G	eps_iStar	eps_phiStar	eps_piStar	eps_piStarX
piC	0.51	0.62	3.00	71.35	1.85	22.68
Y	5.17	61.74	0.32	16.67	6.98	9.12
e	0.48	0.74	2.10	61.58	3.61	31.49
ii	1.78	1.63	2.77	70.41	1.51	21.90
delta	0.17	0.27	2.42	49.05	26.90	21.19
d	0.55	2.76	1.28	84.53	2.78	8.10
gammaD	0.95	0.67	1.02	86.37	5.08	5.90
varphiD	0.95	0.67	1.02	86.37	5.08	5.90
taxsubD	0.00	0.00	0.00	100.00	0.00	0.00

5.2 The relative efficiency of the different policy regimes with optimal simple rules

In order to implement optimal simple policy rules, in this subsection policymakers are assumed to minimize an ad hoc loss function that is a weighted average of the variances of certain target variables and the change in the intermediate target variables $(\Delta i_t, \Delta\delta_t)$. This operationalizes our assumption that policymakers have preferences for the relative stabilization of target variables which are defined by the weights they attach to the variance of the different potential target variables (inflation, GDP, RER). In order to additionally reflect a preference for the avoidance of too much policy activism, the **variance of the changes** in the interest rate and the nominal depreciation rate are also included in the loss function. Specifically, it is assumed that, given the weights ω_k, policymakers search for the coefficients in the simple policy rules h_i, k_i, j_i that minimize the loss function within the curly brackets below, that is simply a weighted average of the variances of certain variables.

$$\arg\min_{h_i,k_i,j_i} \left\{ \begin{array}{c} \omega_\pi Var\left(\pi_t^C\right) + \omega_Y Var\left(Y_t\right) + \omega_e Var\left(e_t\right) \\ +\omega_{\Delta i} Var\left(\Delta i_t\right) + \omega_{\Delta\delta} Var\left(\Delta\delta_t\right) \end{array} \right\} =$$

$$\arg\min_{h_i,k_i,j_i} \lim_{\beta\to 1} E_0 \sum_{t=1}^{\infty} (1-\beta)\beta^t \left\{ \begin{array}{c} \omega_\pi \left(\pi_t^C - \pi^T\right)^2 + \omega_Y \left(Y_t - Y\right)^2 \\ +\omega_e \left(e_t - e\right)^2 + \omega_{\Delta i} \left(\Delta i_t\right)^2 + \omega_{\Delta\delta} \left(\Delta\delta_t\right)^2 \end{array} \right\}.$$

Notice that in addition to the usual terms (with weights ω_π, ω_Y, $\omega_{\Delta i}$), this loss function also allows for CB preferences with respect to the variances of the RER and the changes in the rate of nominal depreciation (with weights ω_e, $\omega_{\Delta\delta}$, respectively). Four different CB styles (A-D) are defined in Table 7 according to the combinations of weights in each. All these styles are defined using the same weights for the variances of the changes in each of the operational targets (50) because it is the different preferences with respect to the variances of the target variables that is the main concern here. Also, zeros have been avoided by giving a weight of 1 to target variables with very little importance. Hence, these CB preferences can be summarized by saying that in style A only inflation matters, in style B only GDP matters, both inflation and GDP matter equally in style C, and the RER matters as much as inflation and GDP in style D.

Table 7
Central Bank Styles

Weights	A	B	C	D
ω_π	100	1	100	100
ω_Y	1	100	100	100
ω_e	1	1	1	100
$\omega_{\Delta i}$	50	50	50	50
$\omega_{\Delta\delta}$	50	50	50	50

The ‘osr’ (‘optimal simple rules’) command in Dynare calculates the loss for an initial set of values for the coefficients h_i, k_i, j_i, and then follows an algorithm that searches for lower losses by changing the values of these coefficients. There are several options for the algorithm. The second has been used, which is the default option. For simplicity, the model was run assuming that the risk premium does not respond to the CB international reserves ($\varepsilon_{\overline{\tau}_{D,2}} = 0$). Also, only the exercise for the tax/subsidy case was done in its entirety, since this procedure is very time consuming and the differences with the tax case did not seem too significant or particularly interesting. It should be mentioned that there is no guarantee that the optimal coefficients found in any run are a global optimum: different initial values can and usually do lead to different final coefficients and (locally) optimal losses. Hence, in all cases several different initial sets of values (at least 5 and sometimes as many as 10) have been used, and the set that achieved the lowest loss was chosen. In some cases, this implied obtaining extremely high absolute values for some of the coefficients. Nevertheless, the lowest loss obtained in any of the runs, along with the corresponding policy rule coefficients, is reported in the tables below.

Table 8 below reports the losses thus obtained for each regime and each CB style, as well as the losses relative to the MER+CC regime and the ranking of the regimes for each CB style. For all the CB styles, the lowest loss was obtained with the MER+CC and the second lowest loss was obtained with the MER regime, with an increase in loss as little as 5% (for style A) and as high as 40% (for style B). The third and fourth lowest losses, however, vary with the CB style. Style A gave the PER+CC regime as third in the ranking, and the FER+CC regime fourth. This order is inverted for styles B, C, and D. Hence, giving up either the interest rate or the nominal depreciation rule, implies increases in loss of at least 50% and as high as 570%. For all styles the simple CC regime (in which the only policy rule is the tax/subsidy scheme) obtained the fifth highest loss. The last two places in the ranking are obtained by the FER and PER regimes: the FER regime is last in the ranking for styles A, C and D whereas the PER regime is last for style B. Table 9 shows the optimal coefficients in the simple policy rules obtained for each regime and style.

Table 8
Optimal Simple Rules ('osr')

CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
	Loss						
A	0.012	0.080	0.039	0.012	0.271	0.234	0.115
B	0.047	0.160	0.198	0.066	0.615	0.710	0.326
C	0.123	0.183	0.214	0.150	0.813	0.792	0.556
D	0.173	0.257	0.296	0.231	0.944	0.923	0.578
	Relative loss						
A	1.00	6.7	3.3	1.05	22.7	19.6	9.7
B	1.00	3.4	4.3	1.4	13.2	15.3	7.0
C	1.00	1.5	1.7	1.2	6.6	6.5	4.5
D	1.00	1.5	1.7	1.3	5.5	5.3	3.3
	Ranking						
A	1	4	3	2	7	6	5
B	1	3	4	2	6	7	5
C	1	3	4	2	7	6	5
D	1	3	4	2	7	6	5

To obtain some intuition on why the losses are lower in the MER+CC regime than in any of the six 'boundary regimes', let us take the log-linear approximations of the simple policy rules equations and the UIP equation:

$$
\begin{aligned}
\widehat{i}_t &= h_0\widehat{i}_{t-1} + h_1\widehat{\pi}_t^C + h_2\widehat{Y}_t + h_3\widehat{e}_t \\
\widehat{\delta}_t &= k_0\widehat{\delta}_{t-1} + k_1\widehat{\pi}_t^C + k_2\widehat{Y}_t + k_3\widehat{e}_t + k_4\left(\widehat{r}_t + \widehat{e}_t - \widehat{Y}_t\right) \\
ta\widehat{xsub}_t^D &= j_0 ta\widehat{xsub}_{t-1}^D + j_1\widehat{\pi}_t^C + j_2\widehat{Y}_t + j_3\widehat{e}_t + j_4\widehat{\phi}_t^* \\
\widehat{i}_t &= \widehat{i}_t^* + \widehat{\phi}_t^* + E_t\widehat{\delta}_{t+1} + a_1\left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t\right) + a_2a_3 ta\widehat{xsub}_t^D \\
&\quad -a_2\left(E_t ta\widehat{xsub}_{t+1}^D - ta\widehat{xsub}_t^D\right)
\end{aligned}
$$

$$
\begin{aligned}
a_1 &= \frac{\varphi_D\left(\gamma^D\right)}{\varphi_D\left(\gamma^D\right) - tax^{KF}} \frac{2\alpha_1}{\left(1-\alpha_2\gamma^D\right)^2 + \alpha_1} \frac{\alpha_2\gamma^D}{1-\alpha_2\gamma^D} \equiv \varepsilon_D^{\varphi} \\
a_2 &= \frac{tax^{KF}}{\varphi_D\left(\gamma^D\right) - tax^{KF}}, \qquad a_3 = \frac{\varphi_D\left(\gamma^D\right) - 1}{1 - tax^{KF}}
\end{aligned}
$$

Table 9
Optimal Simple Rules ('osr')

OPTIMAL VALUE OF THE PARAMETERS				
	CB style			
	A	B	C	D
MER+CC				
h_0	1.61	3.50	1.16	25.41
h_1	1.46	-2.82	-0.20	-10.54
h_2	-0.02	-10.37	-0.06	-35.33
h_3	-0.04	0.36	0.03	4.27
k_0	-0.29	-0.19	0.43	-0.11
k_1	-0.20	0.19	0.11	0.01
k_2	-0.14	-1.51	-1.78	-0.81
k_3	-0.16	-0.18	-0.26	-0.99
k_4	-0.001	-0.003	-0.003	-0.006
j_0	-0.03	0.00	0.00	0.01
j_1	-0.21	-0.19	-0.20	-0.43
j_2	0.01	0.07	-0.05	0.29
j_3	-0.03	-0.05	-0.03	0.34
j_4	-0.01	-0.02	-0.02	-0.02
FER+CC				
h_0	414.13	86.72	211.13	259.87
h_1	415.82	16.39	-77.08	-3.68
h_2	29.53	-128.77	-347.02	-321.35
h_3	121.68	-47.11	90.40	-50.94
j_0	0.07	-0.23	0.38	-0.01
j_1	-19.56	-31.43	-47.77	-33.28
j_2	-4.83	-1.11	-102.73	0.69
j_3	-86.47	-43.75	-56.57	-146.92
j_4	-0.02	-0.01	-0.03	-0.02
PER+CC				
k_0	0.71	0.48	0.48	21.55
k_1	-0.47	-0.18	-0.17	-77.55
k_2	-0.61	-0.97	-0.99	192.64
k_3	-0.06	-0.12	-0.11	36.95
k_4	0.00	0.00	0.00	46.81
j_0	0.17	-0.03	-0.03	-1.00
j_1	-0.21	-0.21	-0.21	26.46
j_2	0.00	0.50	0.50	146.95
j_3	0.00	0.00	0.01	-63.28
j_4	-0.02	-0.01	-0.01	0.01

Table 9 (cont.)
Optimal Simple Rules ('osr')

OPTIMAL VALUE OF THE PARAMETERS				
	CB style			
	A	B	C	D
	MER			
h_0	1.13	0.88	13.53	5.52
h_1	1.34	0.34	-3.22	-2.63
h_2	0.00	1.14	-7.19	-4.18
h_3	0.00	-0.09	0.88	1.68
k_0	0.16	0.43	0.08	0.08
k_1	-0.07	0.03	1.03	0.32
k_2	-0.17	-3.50	-0.93	-2.31
k_3	-0.11	0.05	-0.22	-1.35
k_4	-0.001	0.000	-0.002	-0.006
	FER			
h_0	760.43	1.69	7611.81	5771.43
h_1	8632.73	-0.36	42873.50	32799.90
h_2	-311.79	-3.28	17829.70	12930.20
h_3	2822.09	-1.03	6780.07	6346.12
	PER			
k_0	-0.58	-0.63	-0.67	-0.63
k_1	-1.51	-2.58	-2.83	-1.95
k_2	-0.15	-3.16	-1.93	-1.64
k_3	-0.32	0.07	0.23	-0.13
k_4	0.06	1.52	0.52	-0.02
	CC			
j_0	6.51	1.48	208.93	1.30
j_1	-9148.44	-713.61	-16688.20	-457.81
j_2	-872.94	-334.66	-70288.90	-219.35
j_3	-1189.48	105.25	-6001.14	25.90
j_4	-0.26	-0.08	-6.34	-0.07

Leading the second and third equations and eliminating $\widehat{i}_t$, $E_t\widehat{\delta}_{t+1}$, and $E_t t\widehat{ax}^{\Delta d}_{t+1}$ from the fourth gives the following:

$$\varepsilon^{\varphi}_{D}\left(\widehat{d}_t+\widehat{e}_t-\widehat{Y}_t\right)+\widehat{i}^*_t+\widehat{\phi}^*_t=\left[h_0\widehat{i}_{t-1}-k_0\widehat{\delta}_t+(j_0-1-a_3)\,a_2ta\widehat{x}sub^D_t\right]$$
$$+\left[h_1\widehat{\pi}^C_t-(k_1-a_2j_1)\,E_t\widehat{\pi}^C_{t+1}\right]+\left[h_2\widehat{Y}_t-(k_2-a_2j_2)\,E_t\widehat{Y}_{t+1}\right]$$
$$+\left[h_3\widehat{e}_t-(k_3-a_2j_3)\,E_t\widehat{e}_{t+1}\right]-k_4\left(E_t\widehat{r}_{t+1}+E_t\widehat{e}_{t+1}-E_t\widehat{Y}_{t+1}\right)+a_2j_4E_t\widehat{\phi}^*_{t+1}.$$

On the l.h.s. is the log-linear deviation (from the NSS) of the UIP risk/liquidity premium. On the r.h.s. is a complex term that exclusively depends on the log-linear deviations of the (contemporary and expected) potential target variables the CB may use for its policy rules (including the CB international reserves) and the policy instruments. All the subterms on the r.h.s. are multiplied by a policy rule coefficient.[10] The CB policy rules have the effect of modifying the effects that the UIP risk/liquidity premium has on some important variables as a response to shocks. By making some of these coefficients equal to zero, the constraints that the respective 6 ‘boundary’ regimes impose imply that the CB has less leeway to affect international capital flows in the direction that may help it stabilize the economy according to its style (or preferences). For example, under the FER regime, in which all the k_s and j_s as well as $ta\widehat{xsub}_t^D$ are zero, the equation reduces to

$$\varepsilon_D^{\varphi}\left(\widehat{d}_t+\widehat{e}_t-\widehat{Y}_t\right)+\widehat{i}_t^*+\widehat{\phi}_t^*=h_0\widehat{i}_{t-1}+h_1\widehat{\pi}_t^C+h_2\widehat{Y}_t+h_3\widehat{e}_t.$$

Clearly, with such zero constraints policymakers have less ability to influence the debt ratio (and hence foreign debt and capital flows) in the direction convenient for stabilizing the target variables than in the general case.

5.3 The relative efficiency of the different policy regimes with optimal policy under commitment

In this subsection Dynare’s ‘ramsey’ command is used to obtain the optimal policy under commitment, i.e., the paths for the control variables (in the sense of optimal control theory, but in our terminology the intermediate targets) that yield the minimum expected value, conditional on the information at $t=t_0$, of a discounted ad hoc loss function:

$$\mathscr{L}_{t_0}=E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}L_t, \tag{42}$$

where L_t is given by:

$$\begin{aligned} L_t \;=\;& \omega_\pi\left(\pi_t^C-\pi^T\right)^2+\omega_Y\left(Y_t-Y\right)^2+\omega_e\left(e_t-e\right)^2+\omega_r\left(r_t-r\right)^2 \\ &+\omega_{\Delta i}\left(\Delta i_t\right)^2+\omega_{\Delta\delta}\left(\Delta\delta_t\right)^2+\omega_{taxsubD}\left(taxsub_t^D-taxsub^D\right)^2, \end{aligned} \tag{43}$$

[10] Notice that the only one which is not multipled by a policy rule coefficient is $ta\widehat{xsub}_t^D$ which we have chosen not to replace by its constituents solely for avoiding an even longer expression.

given initial values for the predetermined values, and subject to all the non-policy model equations. Notice that two extra terms have been added in the period loss function. The first one implies a preference (measured by ω_r) for CB international reserves stability. The reason for this is that, under a Ramsey optimal policy, system stability requires that $\omega_r > 0$. Since there is no specific interest here in evaluating the effects of preferences for reserves stability, it has simply been assumed that $\omega_r = 1$ (a very low preference) and (aside from this addition) the same definition of CB styles as in the previous section has been maintained. There is also a term that implies a penalty (measured by $\omega_{taxsubD}$) for using the tax/subsidy on capital flows. The reason for this is that if $\omega_{taxsubD} = 0$ it is necessary to increase the policymakers' discount factor above the assumed household intertemporal discount factor $\beta = .99$, to, say, planner_discount=0.999, to achieve convergence. Also, if $\omega_{taxsubD} = 0$, an excessively intense (and perhaps quite unrealistic) use of this instrument is sometimes obtained as well as a consequent excessive inertia in some of the endogenous variables.[11] Nevertheless, to see what the unrestrained use of this policy instrument and a much more restrained use imply in the ranking of regimes, the losses for $\omega_{taxsubD} = 0$ and $\omega_{taxsubD} = 10$ are also reported below.

Table 10 below reports the losses, the relative losses, and the ranking of the alternative regimes for each CB style. This time, however, there is no ambiguity with respect to the optimum, since the linear-quadratic framework ensures that a global optimum is achieved in each of the 28 cases.

[11] If instead of a quadratic penalty for deviations from the NSS one proceeds as with the two operational targets and assumes a preference for small *changes* in the tax/subsidy, the model delivers a unit root.

Table 10a
Optimal policy under commitment
$\omega_{taxsubD}$=1, planner_discount=0.99

CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
Loss							
A	81.8	81.9	82.3	119.9	121.0	120.9	135.9
B	37.5	37.6	39.1	112.0	114.5	117.3	347.8
C	150.7	150.8	151.9	378.1	388.1	388.8	429.8
D	205.8	206.0	206.4	394.5	405.2	405.7	437.1
Relative loss							
A	1	1.0003	1.005	1.465	1.478	1.477	1.660
B	1	1.0026	1.043	2.991	3.058	3.132	9.286
C	1	1.0007	1.008	2.509	2.575	2.580	2.852
D	1	1.0009	1.003	1.917	1.969	1.971	2.123
Ranking							
A	1	2	3	4	6	5	7
B	1	2	3	4	5	6	7
C	1	2	3	4	5	6	7
D	1	2	3	4	5	6	7

Table 10b
Optimal policy under commitment
$\omega_{taxsubD}$=0, planner_discount=0.999

CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
Loss							
A	84.3	84.3	85.5	272.5	277.6	275.5	182.2
B	34.0	34.0	37.4	330.1	339.1	341.1	448.2
C	203.5	203.5	205.3	800.2	826.0	821.9	645.3
D	326.3	326.3	326.9	932.8	971.4	965.8	695.7
Relative loss							
A	1	1.00000003	1.014	3.23	3.29	3.27	2.16
B	1	1.00000019	1.100	9.72	9.98	10.04	13.19
C	1	1.00001572	1.009	3.93	4.06	4.04	3.17
D	1	1.00001935	1.002	2.86	2.98	2.96	2.13
Ranking							
A	1	2	3	5	7	6	4
B	1	2	3	4	5	6	7
C	1	2	3	5	7	6	4
D	1	2	3	5	7	6	4

Table 10c
Optimal policy under commitment
$\omega_{taxsubD}$=10, planner_discount=0.99

CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
Loss							
A	93.3	93.4	93.6	119.9	121.0	120.9	151.8
B	65.6	66.1	67.8	112.0	114.5	117.3	396.5
C	207.0	207.8	208.8	378.1	388.1	388.8	480.0
D	237.8	238.8	239.3	394.5	405.2	405.7	487.6
Relative loss							
A	1	1.001	1.003	1.286	1.297	1.296	1.63
B	1	1.008	1.034	1.71	1.75	1.79	6.04
C	1	1.004	1.009	1.83	1.87	1.88	2.32
D	1	1.004	1.006	1.66	1.70	1.71	2.05
Ranking							
A	1	2	3	4	6	5	7
B	1	2	3	4	5	6	7
C	1	2	3	4	5	6	7
D	1	2	3	4	5	6	7

As expected, the MER+CC regime always dominates the six 'boundary' regimes. In this case, however, the loss with the FER+CC regime is always less than 1% above the loss with the MER+CC regime. Hence, although the FER+CC regime is clearly second in rank, adding as control variable the rate of nominal depreciation when there one is already using the nominal interest rate and the tax/subsidy scheme does not significantly increase the realization of stabilization objectives. Also, the PER+CC regime is always third in rank. In this case, the loss under CB styles A, C, and D are always less than 1.4% higher than with the MER+CC regime. But the increase in loss is more than 3% higher with CB style B and as much as 10% higher in the unrealistic case in which no penalty is imposed on the use of the tax/subsidy ($\omega_{taxsubD}=\mathbf{0}$). The MER regime (where policymakers abstain from using the tax/subsidy on capital flows) is at least 29% more costly than in the MER+CC regime and as much as 870% higher (for style C and $\omega_{taxsubD}=\mathbf{0}$). The three 'vertex' regimes (FER, PER, CC) have losses that are much higher than in the MER+CC regime (ranging from 30% to 1200% higher). The MER regime is fourth in ranking for all CB styles for $\omega_{taxsubD}=\mathbf{1}$ and **10**, but for $\omega_{taxsubD}=\mathbf{0}$ it is only fourth for CB style B (only GDP matters) and is fifth for the rest of the styles, being replaced in the fourth position by regime CC. The ranking among the three 'vertex' regimes vary from style to style. For $\omega_{taxsubD} = \mathbf{1}$ and **10** the CC regime is always last in the ranking, but for $\omega_{taxsubD} = \mathbf{0}$ it is only in the last position for CB style B and the FER regime is last for the other styles. More generally, in the loss comparison between an edge and its opposite vertex (see Figure 2), in all the cases reported the loss in the FER+CC regime is less then the loss in the PER regime and the loss in the PER+CC regime is less than the loss in the FER regime. However, as already mentioned, in the loss comparisons between the MER and CC regimes, while the loss of the MER regime is lower for $\omega_{taxsubD} = \mathbf{1}$ and **10**, it is higher in the $\omega_{taxsubD} = \mathbf{0}$ case for all styles except B.

Table 11 below reports the same exercises for the case in which there is a tax on the level of debt (no subsidy). Most of what has been said about the tax/subsidy scheme is valid also for this case. For this reason, only the main differences will be mentioned. First, note that when $\omega_{taxD} = \mathbf{0}$ all the regimes in which there is a tax on foreign debt (and hence CC appears in the regime nomenclature) the losses are exactly the same as in the tax/subsidy scheme. This means that when there is no penalty the same path for debt related tax collection can be achieved with either form of implementation. Hence, only for the MER, FER, and PER regimes are there any differences in loss, and they are quite minor (less then 0.66% in absolute value), with no change in the ranking. In the vertex versus opposite edge comparison, even for $\omega_{taxD} = \mathbf{1}$, or $\omega_{taxD} = \mathbf{10}$ there are styles in which the CC regime has a lower loss than the MER regime.

Table 11a
Optimal policy under commitment
ω_{taxD}= 1,planner_discount=0.99

Loss							
CB Style	Regime						
	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	76.6	76.6	77.3	119.6	120.4	120.4	128.5
B	21.0	21.0	23.5	113.1	115.2	117.8	255.1
C	137.3	137.3	138.6	380.5	388.9	389.5	380.0
D	198.6	198.6	199.1	396.8	405.8	406.2	388.2
Relative loss							
CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	1	1.00001	1.009	1.561	1.572	1.571	1.677
B	1	1.00006	1.121	5.384	5.487	5.611	12.148
C	1	1.00002	1.010	2.772	2.833	2.837	2.768
D	1	1.00001	1.002	1.998	2.043	2.045	1.954
Ranking							
CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	1	2	3	4	6	5	7
B	1	2	3	4	5	6	7
C	1	2	3	5	6	7	4
D	1	2	3	5	6	7	4

Table 11b
Optimal policy under commitment
ω_{taxD}= 0,planner_discount=0.999

Loss							
CB Style	Regime						
	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	84.3	84.3	85.5	271.6	275.8	274.0	182.2
B	34.0	34.0	37.4	331.6	339.0	341.0	448.2
C	203.5	203.5	205.3	803.5	825.0	821.4	645.3
D	326.3	326.3	326.9	936.3	968.9	964.0	695.7
Relative loss							
CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	1	1.00000003	1.014	3.22	3.27	3.25	2.16
B	1	1.00000019	1.100	9.76	9.98	10.04	13.19
C	1	1.00001572	1.009	3.95	4.05	4.04	3.17
D	1	1.00001935	1.002	2.87	2.97	2.95	2.13
Ranking							
CB Style	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	1	2	3	5	7	6	4
B	1	2	3	4	5	6	7
C	1	2	3	5	7	6	4
D	1	2	3	5	7	6	4

Table 11c
Optimal policy under commitment
ω_{taxD}= 10,planner_discount=0.99

	Loss						
CB Style	Regime						
	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	85.3	85.4	85.9	119.6	120.4	120.4	133.1
B	36.8	36.9	39.3	113.1	115.2	117.8	269.3
C	170.4	170.5	172.0	380.5	388.9	389.5	386.9
D	219.4	219.5	220.1	396.8	405.8	406.2	394.8
	Relative loss						
	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	1	1.0004	1.007	1.401	1.411	1.410	1.56
B	1	1.0018	1.069	3.07	3.13	3.20	7.31
C	1	1.0008	1.010	2.23	2.28	2.29	2.27
D	1	1.0004	1.003	1.81	1.850	1.852	1.80
	Ranking						
	MER+CC	FER+CC	PER+CC	MER	FER	PER	CC
A	1	2	3	4	6	5	7
B	1	2	3	4	5	6	7
C	1	2	3	4	6	7	5
D	1	2	3	5	6	7	4

6 Conclusion

The parent paper Escudé (2013) tries to bridge the gap between the fact that many central banks systematically intervene in the foreign exchange market and the absence of any generally accepted model for the representation of this practice. It presents a model and a policy framework in which the CB can simultaneously intervene in the foreign exchange and domestic currency bond markets, varying its outstanding bond liabilities and reserve assets in order to achieve two operational targets: one for the interest rate and another for the rate of nominal depreciation. To make this possible, the DSGE model includes financial variables and institutional practices ('nuts and bolts' of central banking) that are left out of the modeling when only the usual 'corner' policy regimes of a pure float or a pure peg are considered, but cannot be left out when trying build a more general model. In the present paper I have extended this policy framework to include the third corner of the traditional 'impossible trinity' by including the use of a tax on household foreign currency liabilities or a tax/subsidy scheme that taxes increases in such liabilities and subsidizes their reductions. In the present context, however, there is a 'possible trinity' since all three forms of intervention can be used simultaneously. Furthermore, when used in the optimal control framework this 'trinity' is not only possible but also optimal in the sense that any of the 6 boundary regimes (the 3 edges and the 3 vertices of the policy triangle) cannot attain a lower loss than the simultaneous use of 3 instruments and 3 (operational) targets due to the obvious fact that they are derived from adding one or two additional constraints that reflect the abstention from using one or two of the 3 possible instruments (interventions).

As in the parent paper, three frameworks are considered: 1) simple policy rules, 2) optimal simple policy rules, where the coefficients of the simple rules are derived by minimizing an ad hoc loss function that is a weighted average of the variances of the target variables (inflation, GDP and the RER), 3) optimal policy under commitment and full information, where policymakers minimize an ad hoc discounted intertemporal loss function related to the same target variables. The main interest in 2) and 3) is in seeing whether the capital controls can achieve a significant reduction in loss to the already potent simultaneous interest and exchange rate policies, and in obtaining the ranking in the losses obtained for the 7 possible policy regimes (the use of all 3 policies and the 6 additional possibilities derived from eliminating either one or two of these policies).

Two forms of (soft) capital controls are considered: 1) a tax on the level of household foreign debt, and 2) a tax/subsidy scheme in which increases in household foreign debt are taxed and reductions are subsidized. The results show that the losses obtained are not very different for the two forms of capital controls. A point of interest lies in gauging the extent to which forfeiting one or two of the possible instruments reduces the loss for different central bank preferences (defined by the weights in the ad hoc loss function). The results show that increases in the loss function due to forfeiting intervention in the FX market or in the CB bond market (starting from the 'possible trinity' are much higher in the case of optimal simple rules than in the case of optimal policy under commitment. But since there is no assurance that a global minimum is reached when using optimal simple policy rules with Dynare, the results for optimal policy under commitment are a welcome complement. In the latter context it was found that there is hardly any additional loss in forfeiting FX policy (and letting the exchange rate float) when starting from the use of the 3 instruments. The increase in loss is significantly higher but not very high when it is the interest rate policy that is eliminated (and the interest rate is allowed to float). In most cases, doing away with the capital controls implies a large increase in loss. A surprising result is that, depending on the CB preferences, the Managed Exchange Rate regime may be superior or inferior to the simple Capital Control (CC) regime in which both interest rate and FX policies are eliminated and only the CC rule stabilizes the economy.

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Appendix A: The rest of the model

A1 Firms

The production side of the economy is exactly as in Escudé (2013). There is perfect competition in the production (or bundling) of final domestic output Q_t, using as inputs the output of a continuum of monopolistically competitive firms. A representative final domestic output firm uses the following CES technology:

$$Q_t = \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{44}$$

where $Q_t(i)$ is the output of the intermediate domestic good i and θ is the elasticity of substitution between any two varieties of goods. Profit maximization yields the demand for each type of domestic good as an input and the domestic goods price index:

$$Q_t(i) = Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}, \qquad P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{45}$$

The production function of each firm is proportional to its use of labor: $Q_t(i) = \varepsilon_t N_t(i)$, where ε_t is an industry-wide transitory productivity shock. Hence, each firm's real marginal cost (in terms of domestic goods) is $mc_t = w_t/\varepsilon_t$. Since $N_t(i)$

is firm i's labor demand, using (45) and integrating yields aggregate labor demand:

$$N_t^D = \int_0^1 N_t(i)di = \int_0^1 \frac{Q_t(i)}{\varepsilon_t}di = \frac{1}{\varepsilon_t}\int_0^1 Q_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta}di = \frac{Q_t}{\varepsilon_t}\Delta_t \tag{46}$$

where Δ_t is a measure of price dispersion at period t defined as:

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di \geq 1.$$

Equating labor supply (20) and demand (46) gives the labor market equilibrium real wage:

$$w_t = \xi\left(\frac{Q_t}{\varepsilon_t}\Delta_t\right)^{\sigma^N} p_t^C C_t^{\sigma^C}\varphi_M\left(m_t/p_t^C C_t\right). \tag{47}$$

Firms make pricing decisions taking the aggregate price and quantity indexes as parametric. Every period, each firm has a probability $1-\alpha$ of being able to set the optimum price for its specific type of good. The firms that can't optimize must leave the same price they had last period. The pricing problem of firms that get to optimize is:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j} Q_{t+j}(i)\left\{\frac{P_t(i)}{P_{t+j}} - mc_{t+j}\right\} \tag{48}$$

subject to the demand they will face until they can again optimize:

$$Q_{t+j}(i) = Q_{t+j}\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\theta}. \tag{49}$$

where the pricing kernel of firms is:

$$\Lambda_{t,t+j} \equiv \beta^j \frac{p_t^C C_t^{\sigma^C}}{p_{t+j}^C C_{t+j}^{\sigma^C}}. \tag{50}$$

In Escudé (2013) it is shows that the firm's first order condition is:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta} \left\{ \widetilde{p}_t \frac{P_t}{P_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\}. \tag{51}$$

where $\widetilde{p}_t \equiv \widetilde{P}_t / P_t$ is the relative price of firms that optimize with respect to the general price level. Also, the price index in (45) implies the following law of motion for the aggregate domestic goods price index:

$$P_t^{1-\theta} = \alpha \left(P_{t-1}\right)^{1-\theta} + (1-\alpha)\widetilde{P}_t^{1-\theta}. \tag{52}$$

Dividing through by $P_{t-1}^{1-\theta}$ and rearranging yields the relative price of optimizers as an increasing function of the inflation rate:

$$\widetilde{p}_t = \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{-1}{\theta-1}}. \tag{53}$$

The Phillips equation (51) can be expressed in recursive form by defining:

$$\begin{aligned} \Gamma_t &= E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \left(Q_{t+j} / \left(p_{t+j}^C C_{t+j}^{\sigma^C}\right)\right) \left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} \qquad (54) \\ \Psi_t &= \frac{\theta}{\theta-1} E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \left(Q_{t+j} / \left(p_{t+j}^C C_{t+j}^{\sigma^C}\right)\right) \left(\frac{P_{t+j}}{P_t}\right)^{\theta} mc_{t+j} \end{aligned}$$

and transforming (51) to three equations that lack infinite sums (see Escudé (2013)):

$$\begin{aligned} \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{-1}{\theta-1}} \Gamma_t &= \Psi_t, \\ \Gamma_t &= \left(Q_t / \left(p_t^C C_t^{\sigma^C}\right)\right) + \beta\alpha E_t \pi_{t+1}^{\theta-1} \Gamma_{t+1}, \\ \Psi_t &= \frac{\theta}{\theta-1} \left(Q_t / \left(p_t^C C_t^{\sigma^C}\right)\right) mc_t + \beta\alpha E_t \pi_{t+1}^{\theta} \Psi_{t+1}. \end{aligned}$$

Since Δ_t is an additional variable in the model, there is need for an additional equation. The following is a recursive equation for the dynamics of this variable (see Escudé (2013)):

$$\Delta_t = \alpha \pi_t^{\theta} \Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha \pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}. \tag{55}$$

A2 GDP, Foreign trade, and the balance of payments

Imported and Domestic goods

The Law of One Price is assumed to hold. Hence, the domestic price of (the aggregate of) imported goods is simply $P_t^N = S_t P_t^*$. Hence, the relative price of imported to domestic goods P_t^N/P_t is simply the RER (defined in (2)). The consumption index used in the household optimization problem is a constant elasticity of substitution (CES) aggregate consumption index of domestic $\left(C_t^D\right)$ and imported $\left(C_t^N\right)$ goods:

$$C_t = \left(a_D{}^{\frac{1}{\theta^C}}\left(C_t^D\right)^{\frac{\theta^C-1}{\theta^C}} + a_N{}^{\frac{1}{\theta^C}}\left(C_t^N\right)^{\frac{\theta^C-1}{\theta^C}}\right)^{\frac{\theta^C}{\theta^C-1}}, \qquad a_D + a_N = 1, \tag{56}$$

where C_t^D and C_t^N are CES aggregates of an infinity of domestic and imported varieties of goods, respectively, each produced by a monopolist under monopolistic competition, and where $\theta^C (\geq 0)$ is the elasticity of substitution between domestic and imported goods. C_t^D, for example, is:

$$C_t^D = \left(\int_0^1 C_t^D(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{57}$$

where θ is the elasticity of substitution between varieties of domestic goods in household expenditure. a_D and $a_N = 1 - a_D$ are directly related to the shares of domestic and imported consumption in total consumption expenditures. It is assumed that there is a bias for domestic goods, i.e., $a_D > 1/2 > a_N$, and that $\theta^C > 1$.

Minimization of total consumption expenditure $P_t^C C_t = P_t C_t^D + S_t P_t^* C_t^N$ subject to (56) for a given C_t, yields the following relations:

$$P_t = P_t^C \left(\frac{C_t^D}{a_D C_t}\right)^{-\frac{1}{\theta^C}}, \qquad S_t P_t^* = P_t^C \left(\frac{C_t^N}{a_N C_t}\right)^{-\frac{1}{\theta^C}}. \tag{58}$$

Introducing these in (56) yields the consumption price index:

$$P_t^C = \left(a_D \left(P_t\right)^{1-\theta^C} + a_N \left(S_t P_t^*\right)^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}}, \tag{59}$$

and dividing through by P_t yields a relation between the relative prices of consumption and imported goods and the RER:

$$p_t^C = \left(a_D + \left(1-a_D\right) e_t^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}}. \tag{60}$$

Relations (58) can used to eliminate C_t^D and C_t^N:

$$\begin{aligned} C_t^D &= a_D \left(p_t^C\right)^{\theta^C} C_t \\ C_t^N &= \left(1-a_D\right) \left(\frac{p_t^C}{e_t}\right)^{\theta^C} C_t. \end{aligned}$$

And the shares of domestic and imported goods in total expenditure can be expressed in terms of e_t and p_t^C as:

$$\begin{aligned} \frac{P_t C_t^D}{P_t^C C_t} &= a_D \left(p_t^C\right)^{\theta^C - 1} = \frac{a_D}{a_D + \left(1-a_D\right) e_t^{1-\theta^C}}, \\ \frac{S_t P_t^* C_t^N}{P_t^C C_t} &= \left(1-a_D\right) \left(\frac{p_t^C}{e_t}\right)^{\theta^C - 1} = \frac{\left(1-a_D\right) e_t^{1-\theta^C}}{a_D + \left(1-a_D\right) e_t^{1-\theta^C}}. \end{aligned} \tag{61}$$

Export firms

Firms in the export sector use domestic goods and the composite of goods that defines GDP. I assume that the export good is a single homogenous primary good (a commodity). Firms in this sector sell their output in the international market at the foreign currency price P_t^{*X}. They are price takers in factor and product markets. The price of primary goods in terms of the domestic currency is merely the exogenous international price multiplied by the nominal exchange rate: $S_t P_t^{*X}$. Let the production function employed by firms in the export sector be the following:

$$X_t^* = \left(Q_t^X\right)^{b^A} Y_t^{1-b^A}, \qquad 0 < b^A < 1, \tag{62}$$

where Q_t^X is the amount of domestic goods used as input in the export sector and Y_t is real GDP. These firms maximize profit $S_t P_t^{*X} X_t^* - P_t Q_t^X$ subject to (62). In terms of domestic goods, they maximize:

$$\frac{\Pi_t^X}{P_t} = e_t p_t^* \left(Q_t^X\right)^{b^A} Y_t^{1-b^A} - Q_t^X$$

where the SOE's external terms of trade (XTT) is defined as $p_t^* \equiv P_t^{*X}/P_t^*$, where P_t^* is the price index of the foreign currency price of the SOE's imports. Notice that the XTT is a ratio of two price indexes determined in the RW. Hence, the follow identity relates the rates of foreign inflation of exported and imported goods to the XTT (giving the dynamics of the XTT):

$$\frac{p_t^*}{p_{t-1}^*} = \frac{\pi_t^{*X}}{\pi_t^*}, \qquad \text{where} \quad \pi_t^{*X} \equiv \frac{P_t^{*X}}{P_{t-1}^{*X}}.$$

The first order condition for profit maximization yields the export sector's (factor) demand for domestic goods:

$$Q_t^X = \left(b^A e_t p_t^*\right)^{\frac{1}{1-b^A}} Y_t. \tag{63}$$

Inserting this in (62) shows that the real value of exports in terms of domestic goods is:

$$X_t = \frac{S_t P_t^{*X} X_t^*}{P_t} = e_t p_t^* X_t^* = e_t p_t^* \left(b^A e_t p_t^*\right)^{\frac{b^A}{1-b^A}} Y_t = \kappa_X \left(e_t p_t^*\right)^{b_X} Y_t, \tag{64}$$

where $b_X \equiv \left(1-b^A\right)^{-1}$ and $\kappa_X \equiv \left(b^A\right)^{b^A/\left(1-b^A\right)}$ are introduced for simplicity of notation.

Domestic output, GDP, and the balance of payments

Government expenditure is assumed to be a time-varying and stochastic fraction $\overline{G}_t$ of private consumption expenditure. Define the gross government expenditure fraction as: $G_t \equiv 1 + \overline{G}_t$. For simplicity, it is assumed that the government must pay the same transaction costs as the private sector when it purchases domestic and foreign goods. Hence, using (61) and (64), GDP in terms of domestic goods is:

$$Y_t = a_D \tau_M \left(\gamma_t^M\right) G_t \left(p_t^C\right)^{\theta^C} C_t + X_t. \tag{65}$$

In the domestic goods market, the output of firms Q_t must satisfy final demand from households (including the resources for transactions), the government, and the export sector:[12]

$$Q_t = a_D \tau_M \left(\gamma_t^M\right) G_t \left(p_t^C\right)^{\theta^C} C_t + Q_t^X = Y_t - \left(1 - b^A\right) X_t. \tag{66}$$

Inserting $Y_t = w_t N_t + \Pi_t / P_t$ in the household budget constraint (9) and consolidating the household, CB and government budget constraints yields the balance of payments equation (first equation below), where CA_t is the current account and TB_t is the trade balance:

$$\begin{aligned}
r_t - d_t &= CA_t + r_{t-1} - d_{t-1} \\
CA_t &= \left(\frac{1 + i_{t-1}^*}{\pi_t^*} - 1\right) r_{t-1} - \left[\frac{1 + i_{t-1}^*}{\pi_t^*} \phi_{t-1}^* \tau_D \left(\frac{e_{t-1} d_{t-1}}{Y_{t-1}}\right) - 1\right] d_{t-1} + TB_t, \\
TB_t &= \frac{1}{a_D e_t} \left[\left(p_t^C\right)^{1-\theta^C} X_t - (1 - a_D)\, e_t^{1-\theta^C} Y_t\right].
\end{aligned}$$

[12]Notice that intermediate output used up in the export sector (63) can be written as:

$$Q_t^X = b^A X_t.$$

Appendix B: The system of nonlinear equations

In this section I put together the model non-policy equations. Hence, there are three more endogenous variables than equations. In the case of simple policy rules, there are also the three additional equations extracted from (38)-(41) according to the regime. And in the case of optimal policy under commitment, except for the MER+CC regime, at least one of the equations in (41) must be used (the ones that are not used as control variables).

Risk-adjusted uncovered interest parity

$$
\begin{aligned}
1+i_t &= (1+i_t^*)\phi_t^* E_t\left(\frac{\varphi_t^D - taxsub_{t+1}^D}{1-taxsub_t^D}\delta_{t+1}\right) \\
&\quad \text{or} \\
1+i_t &= (1+i_t^*)\phi_t^*\left(\frac{\varphi_t^D}{1-tax_t^D}\right)E_t\delta_{t+1}
\end{aligned}
$$

Consumption Euler

$$
\frac{C_t^{-\sigma^C}}{\varphi_t^M} = \beta\left(1+i_t\right)E_t\left(\frac{C_{t+1}^{-\sigma^C}}{\varphi_{t+1}^M}\frac{1}{\pi_{t+1}^C}\right)
$$

Phillips equations

$$
\begin{aligned}
\Gamma_t &= \frac{Q_t}{p_t^C C_t^{\sigma^C}} + \beta\alpha E_t \pi_{t+1}^{\theta-1}\Gamma_{t+1}, \\
\Psi_t &= \frac{\theta}{\theta-1}\frac{Q_t}{p_t^C C_t^{\sigma^C}} mc_t + \beta\alpha E_t \pi_{t+1}^{\theta}\Psi_{t+1}, \\
\Psi_t &= \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{-1}{\theta-1}}\Gamma_t
\end{aligned}
$$

Price dispersion

$$
\Delta_t = \alpha\pi_t^{\theta}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}
$$

Balance of Payments

$$
\begin{aligned}
r_t - d_t &= CA_t + r_{t-1} - d_{t-1}, \\
CA_t &= \left(\frac{1+i^*_{t-1}}{\pi^*_t} - 1\right) r_{t-1} - \left(\frac{1+i^*_{t-1}}{\pi^*_t}\phi^*_{t-1}\tau^D_{t-1} - 1\right) d_{t-1} + TB_t, \\
TB_t &= \frac{1}{a_D e_t}\left[\left(p^C_t\right)^{1-\theta^C} X_t - (1-a_D)\, e_t^{1-\theta^C} Y_t\right], \\
X_t &= \kappa_X \left(e_t p^*_t\right)^{b_X} Y_t
\end{aligned}
$$

Real marginal cost, real wage and hours worked

$$
\begin{aligned}
mc_t &= \frac{w_t}{\varepsilon_t}, \\
w_t &= \xi\, p^C_t C_t^{\sigma^C} \varphi^M_t N_t^{\sigma^N}, \\
N_t &= (Q_t/\varepsilon_t)\,\Delta_t
\end{aligned}
$$

Domestic goods market clearing, GDP, and consumption relative price

$$
\begin{aligned}
Q_t &= Y_t - \left(1-b^A\right) X_t, \\
Y_t &= a_D \tau^M_t G_t \left(p^C_t\right)^{\theta^C} C_t + X_t, \\
p^C_t &= \left(a_D + (1-a_D)\, e_t^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}}
\end{aligned}
$$

Money market clearing and CB balance sheet

$$
\begin{aligned}
m_t &= \frac{p^C_t C_t}{\beta_2}\left[\left(\frac{\beta_1\beta_2\beta_3}{1-\frac{1}{1+i_t}}\right)^{\frac{1}{\beta_3+1}} - 1\right], \\
b_t &= e_t r_t - m_t
\end{aligned}
$$

Identities

$$
\frac{\pi^C_t}{\pi_t} = \frac{p^C_t}{p^C_{t-1}}, \qquad \frac{e_t}{e_{t-1}} = \frac{\delta_t \pi^*_t}{\pi_t}, \qquad \frac{p^*_t}{p^*_{t-1}} = \frac{\pi^{*X}_t}{\pi^*_t}
$$

Great ratios

$$
\gamma^M_t = \frac{m_t}{p^C_t C_t}, \qquad \gamma^D_t = \frac{e_t d_t}{Y_t}, \qquad \gamma^R_t = \frac{e_t r_t}{Y_t}
$$

Risk premium, transaction costs, and corresponding auxiliary variables

$$\begin{aligned}
\tau_t^D &= 1+\frac{\alpha_1}{1-\alpha_2\gamma_t^D+\alpha_3\gamma_t^R}, \quad \varphi_t^D = 1+\left(\tau_t^D-1\right)\left(1+\frac{\alpha_2\gamma_t^D}{1-\alpha_2\gamma_t^D+\alpha_3\gamma_t^R}\right) \\
\tau_t^M &= 1+\frac{\beta_1}{\left(1+\beta_2\gamma_t^M\right)^{\beta_3}}, \qquad \varphi_t^M = 1+\left(\tau_t^M-1\right)\left(1+\beta_3\frac{\beta_2\gamma_t^M}{1+\beta_2\gamma_t^M}\right)
\end{aligned}$$

Taxes

$$\begin{aligned}
tax_t &= \left(G_t-1\right)\tau_t^M p_t^C C_t - qf_t - tax_t^{DCol} \\
qf_t &= \left[\left(1+i_{t-1}^*\right)-1/\delta_t\right] e_t \frac{r_{t-1}}{\pi_t^*} - \left[\left(1+i_{t-1}\right)-1\right]\frac{b_{t-1}}{\pi_t} \\
tax_t^{DCol} &= tax_t^{KF} e_t\left(d_t-\frac{d_{t-1}}{\pi_t^*}\right) \qquad \text{or} \qquad tax_t^{DCol} = tax_t^D e_t d_t
\end{aligned}$$

Autoregressive shock variables

$$\begin{aligned}
\varepsilon_t &= \left(\varepsilon_{t-1}\right)^{\rho^{\varepsilon}} \exp\left(\sigma^{\varepsilon}\varepsilon_t^{\varepsilon}\right) \\
G_t &= \left(G_{t-1}\right)^{\rho^G} G^{1-\rho^G} \exp\left(\sigma^G\varepsilon_t^G\right) \\
1+i_t^* &= \left(1+i_{t-1}^*\right)^{\rho^{i^*}} \left(1+i^*\right)^{1-\rho^{i^*}} \exp\left(\sigma^{i^*}\varepsilon_t^{i^*}\right) \\
\phi_t^* &= \left(\phi_{t-1}^*\right)^{\rho^{\phi^*}} \left(\phi^*\right)^{1-\rho^{\phi^*}} \exp\left(\sigma^{\phi^*}\varepsilon_t^{\phi^*}\right) \\
\pi_t^{*X} &= \left(\pi_{t-1}^{*X}\right)^{\rho^{\pi^{**}}} \left(\pi^{*X}\right)^{1-\rho^{\pi**}} \exp\left(\sigma^{\pi^{**}}\varepsilon_t^{\pi^{**}}\right) \\
\pi_t^* &= \left(\pi_{t-1}^*\right)^{\rho^{\pi^*}} \left(\pi^*\right)^{1-\rho^{\pi*}} \left(p_{t-1}^*\right)^{\alpha_{\pi^*}} \exp\left(\sigma^{\pi^*}\varepsilon_t^{\pi^*}\right).
\end{aligned}$$

In the implementation in Dynare, the 6 shock variables have been expressed in logs. For example, the first of the shock equations is $\varepsilon_t = \rho^{\varepsilon}\varepsilon_{t-1}+\sigma^{\varepsilon}\varepsilon_t^{\varepsilon}$ (where changing the name of the variable to e.g. $l\varepsilon_t$ has been avoided) and every time ε_t appears in the (non-shock) model equations above it is replaced by $\exp(\varepsilon_t)$ to eliminate the log transformation. Additionally, in Dynare the standard deviation of the stochastic shocks is defined in the 'shocks' block. For example, in the mod file the first shock equation is actually $\varepsilon_t = \rho^{\varepsilon}\varepsilon_{t-1}+\varepsilon_t^{\varepsilon}$.

Because leads and lags of variables are forbidden within the Dynare 'planner_objective' command, in the case of optimal policies under commitment it is necessary to introduce two new variables and equations:

$$di_t = i_t - i_{t-1}, \qquad d\delta_t = \delta_t - \delta_{t-1}.$$

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