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Evolutionary Model of the Bank Size Distribution

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Abstract

An evolutionary model of the bank size distribution is presented based on the exchange and creation of deposit money. In agreement with empirical results the derived size distribution is lognormal with a power law tail. The theory is based on the idea that the size distribution is the result of the competition between banks for permanent deposit money. The exchange of deposits causes a preferential growth of banks with a fitness that is determined by the competitive advantage to attract permanent deposits. While growth rate fluctuations are responsible for the lognormal part of the size distribution, treating the mean growth rate of banks as small, large banks benefit from economies of scale generating the Pareto tail.

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1 Introduction

Extensive empirical and theoretical investigations have been carried out to understand the bank size distribution and its dynamics.[1] The empirical bank size distribution has been shown to exhibit a highly skewed shape. It is explained similar to the firm size distribution by Gibrat's law of proportionate effects (Gibrat 1913). Gibrat's law suggests that bank growth is driven by unsystematic random factors such that the bank growth rates are uncorrelated. The multiplicative growth process generates a lognormal size distribution. The validity of Gibrat's law is of major interest for the size distribution (Tschoegl 1983, Benito 2008). Moreover, during the last decades the banking industry has experienced significant changes. Liberalization and deregulation processes and the technological progress declined the number of institutions in many countries, mainly by mergers and acquisitions (Berger et al. 1993, Berger et al. 1999). Despite temporal variations, however, the general shape of the bank size distribution can be always described by a lognormal distribution with a Pareto tail (Janicki and Prescott 2006, Benito 2008).

The aim of this paper is to derive the bank size distribution from the growth dynamics of banks, which is essentially determined by the exchange and expansion of deposit money. The model takes the creation of fiat money, the fast exchange of deposits due to economic activity and the attraction of permanent deposits by banks into account. Also restructuring processes in the banking system induced by the exit and entrance of banks are included. For the case of a polypoly banking system the presented dynamic model establishes a bank size distribution that is in agreement with empirical studies. The evolutionary model is established in Section 2 and discussed in Section 3.

2 The Model

We want to characterize the size of a bank by its deposits. The size of the *i-th* bank at time step t is denoted $S_i(t)$ and corresponds to the total amount of deposit money

[1] Alhadeff and Alhadeff (1964), Rhoades and Yeats (1974), Yeats et al. (1975), Tschoegl (1983), Wilson and Williams (2000), Ennis (2001), Goddard et al. (2002), Goddard et al. (2004), Janicki and Prescott (2006), Benito (2008).

of a bank. In a closed banking system the total amount of deposits is determined by the sum over all banks:

$$\tilde{S}(t) = \sum_{i=1}^{n(t)} S_i(t) \tag{1}$$

where $n(t)>>1$ is the total number of banks. We want to consider a sufficiently long time interval Δt for the evolution of the banking system starting at t_0 and ending at t_1.[2] The total amount of deposit money at t_1 is denoted M. In order to establish a continuous model, the size of a bank is scaled by this large quantity:

$$s_i(t) = \frac{S_i(t)}{M} \tag{2}$$

such that $s_i(t)$ can be treated as a real positive number. The total amount of scaled deposit money becomes $\tilde{s} = \Sigma s_i$, while $\tilde{s}(t) \leq 1$.

In order to model the dynamics of bank deposits, four processes changing the amount of deposit money are taken into account:

(i) The core activity of a bank is granting loans. Banks lend out money from current deposits and generate new fiat money leaving a certain percentage as a minimum reserve s_i^0. The amount of money that can be created by a bank is therefore proportional to the amount of current deposits. The growth of deposit money of the *i-th* bank by the generation of fiat money has the form:

$$\frac{ds_i(t)}{dt} \sim \alpha_i(t) s_i(t) \tag{3}$$

The money growth rate α_i can be written as:

$$\frac{ds_i(t)}{dt} \sim y_i(t) = y_i^{in}(t) - y_i^{out}(t) \tag{4}$$

where α_i' is the generation rate of deposit money by granting loans and $\alpha_i'' s_i$ is the total backflow of money by repaying loans.

[2] It is assumed that the time interval is sufficient to approach the established bank size distribution.

(ii) Due to economic activity there is a fast flow of money between banks. This flow is characterized by an exchange rate y_i. The size of the *i-th* bank is governed by the balance:

$$\frac{ds_i(t)}{dt} \sim y_i(t) = y_i^{in}(t) - y_i^{out}(t) \tag{5}$$

where $y_i^{in} \geq 0$ is the inflow rate and $y_i^{out} \geq 0$ the outflow rate of money. The balance relation suggests that a positive y_i is related to an effective inflow and a negative to an effective outflow of money.

(iii) In order to increase the ability to lend out money, banks try to attract money for a longer time period by offering interests and advantages for their customers. These deposits are bonded to a bank and termed permanent deposits. The success of the migration of permanent deposits is taken into account by an additional growth term:

$$\frac{ds_i(t)}{dt} \sim \eta_i(t)s_i(t) \tag{6}$$

with the effective growth rate η_i.

(iv) Restructuring processes by the entry and exit of banks change the number of banks *n(t)* and leads also to a shift of permanent deposits. In particular mergers and acquisitions increase the amount of permanent deposits of the surviving banks. The growth of the *i-th* bank caused by restructuring of the banking system is taken into account by the growth term:

$$\frac{ds_i(t)}{dt} \sim \beta_i(t)s_i(t) \tag{7}$$

where β_i is the corresponding growth rate.

Since these processes can be regarded as independent, the time evolution of deposit money of a bank can be approximated by:

$$\frac{ds_i(t)}{dt} = \alpha_i(t)s_i(t) + \beta_i(t)s_i(t) + \eta_i(t)s_i(t) + y_i(t) \tag{8}$$

The processes can be distinguished into exchange processes (ii, iii, iv and the growth process i) of bank deposits. For the exchange processes the total amount of deposit money must be constant, thus:[3]

$$\sum_{i=1}^{n}\left(\beta_i(t)s_i(t)+\eta_i(t)s_i(t)+y_i(t)\right)=0 \tag{9}$$

The money creation process is associated with a growth of the total amount of deposits by:

$$\sum_{i=1}^{n}\frac{ds_i(t)}{dt}=\sum_{i=1}^{n}\alpha_i(t)s_i(t)=\langle\alpha(t)\rangle\tilde{s}(t) \tag{10}$$

while $<\alpha(t)>$ is the mean money growth rate.[4]

2.1 Economic Activity

The fast exchange of money due to economic activity is treated in a first approximation as a random process with a total exchange rate $\Sigma y_i^{out} = \Sigma y_i^{in} = D'\tilde{s}$. The chance that money flows into respectively out of a bank is in a random process proportional to the size of a bank. Therefore the outflow of money can be approximated by:

$$y_i^{out}(t)=D's_i(t)+\zeta_i^{out}(t) \tag{11}$$

while the inflow has the form:

$$y_i^{in}(t)=D's_i(t)+\zeta_i^{in}(t) \tag{12}$$

where ζ_i^{in} and ζ_i^{out} are fluctuating terms. We obtain for the size evolution of the *i-th* bank:

[3] This relation implies that the entrance and exit of banks is not accompanied with a change of the total amount of deposit money.

[4] Brackets are used to indicate the average over deposits.

$$y_i(t) = \zeta_i(t) \tag{13}$$

with $\zeta_i = \zeta_i^{in} - \zeta_i^{out}$. The average exchange of money between banks due to economic activity cancels out. The total amount of permanent deposits of a bank suffers, however, from fluctuations as a result of this fast money exchange process.[5]

2.2 The Evolutionary Dynamics

We want to continue by considering the evolution of permanent deposits, evolving slower on the considered time scale. Neglecting the fluctuation contribution ζ_i we obtain from Eq. (8):

$$\frac{ds_i(t)}{dt} = \alpha_i(t)s_i(t) + f_i(t)s_i(t) \tag{14}$$

where we introduced:

$$f_i(t) = \beta_i(t) + \eta_i(t) \tag{15}$$

In the evaluation of the bank size evolution, we have to take the condition into account that the growth of the total amount of deposit money is governed by Eq. (10). This constraint can be satisfied by adding a free parameter ξ to Eq. (14), such that:

$$\frac{ds_i(t)}{dt} = \left(\alpha_i(t) + f_i(t) + \xi\right)s_i(t) \tag{16}$$

Applying Eq. (10) yields:

$$\frac{d\tilde{s}(t)}{dt} = \left(\langle\alpha(t)\rangle + \langle f(t)\rangle + \xi\right)\tilde{s}(t) = \langle\alpha(t)\rangle\tilde{s}(t) \tag{17}$$

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5 Note that money exchange implies also the transformation of deposit money into cash money and vice versa. This exchange process has, however, no impact on the result of the model as long as fluctuations of the deposits δs caused by economic activity are small compared to the reserves $\delta s << s^0$. Only in the case of a "bank run", where the outflow of deposits and hence the need for cash money is much larger than the inflow, this condition is violated. A bank run as a result of a collapse of the banking system is not included in the model.

and we obtain for the free parameter:

$$\xi = -\langle f \rangle = -\frac{1}{\tilde{s}} \sum_{i=1}^{n} f_i s_i \tag{18}$$

With this relationship the growth dynamics of banks is determined by:

$$\frac{ds_i(t)}{dt} = \alpha_i(t)s_i(t) + \left(f_i(t) - \langle f(t) \rangle\right)s_i(t) \tag{19}$$

The presented model suggests therefore that the time evolution of banks is governed on the one hand by the creation of deposit money proportional to the money growth rate α_i. The second term in Eq. (19), however, is a replicator term. It expresses a preferential growth process caused by the competition between banks for permanent deposits. The replicator dynamics is determined by the parameter f which is usually termed as fitness. Hence, the rate f_i is a bank fitness characterizing the ability to attract permanent money. The replicator dynamics suggests that banks with a higher than the mean fitness attract a higher amount of deposit money and can increase their size in time at the expense of banks with lower fitness. This can be done either by attracting deposits from competitors or by mergers and acquisitions.

For further use we introduce the fitness advantage of the *i-th* bank by:

$$\Delta f_i(t) = f_i(t) - \langle f(t) \rangle \tag{20}$$

Eq. (19) turns into:

$$\frac{ds_i(t)}{dt} = \alpha_i(t)s_i(t) + \Delta f_i(t)s_i(t) \tag{21}$$

2.3 The Bank Size Evolution

A key process of bank growth is the competition for permanent deposits, because it limits the ability to generate fiat money. It is determined by the capability to attract permanent deposits characterized in this model by the fitness advantage Δf. The varying success of banks in this competition can be taken into account by regarding the fitness advantage as a fluctuating variable. We can write the fitness advantage as:[6]

$$\Delta f(t) = \langle \Delta f \rangle_t + \delta f(t) \tag{22}$$

where the first term represents a time averaged fitness advantage of a bank and the second term indicates fitness fluctuations.[7]

Also the effective money growth rate $\alpha(t)$ is not constant but fluctuates under the impact of varying loan granting. The money growth rate can be written as the sum of a time averaged mean growth rate $<\alpha>_t$ and growth rate fluctuations $\delta\alpha(t)$ as:

$$\alpha(t) = \langle \alpha \rangle_t + \delta\alpha(t) \tag{23}$$

The growth dynamics of the deposits of a bank given by Eq. (21) turns into:

$$\frac{ds(t)}{dt} = \theta s(t) + \rho(t)s(t) + \zeta(t) \tag{24}$$

where the fluctuating variable $\rho(t)$ is characterized by money growth rate variations and fitness fluctuations:

$$\rho(t) = \delta\alpha(t) + \delta f(t) \tag{25}$$

and the mean growth rate of a bank is given by:

$$\theta = \langle \alpha \rangle_t + \langle \Delta f \rangle_t \tag{26}$$

As a first approximation the fluctuating function $\rho(t)$ is treated as an independent, identical distributed (iid), random variable with mean value and time correlation:

$$\begin{aligned} &\langle \rho(t) \rangle_t = 0 \\ &\langle \rho(t), \rho(t') \rangle_t = 2D\delta(t-t') \end{aligned} \tag{27}$$

[6] Brackets with index t indicate a time average over the long time interval Δt.

[7] For brevity the index i is omitted.

while D is a white noise amplitude. This assumption implies that short term correlations relax sufficiently fast that $\rho(t)$ can be treated as uncorrelated.[8]

The time evolution of a bank given by Eq. (24) is governed by the mean growth rate θ and multiplicative respectively additive growth fluctuations. We can further regard the multiplicative growth fluctuations as dominant compared to additive fluctuations and neglect $\zeta(t)$ in the remainder of the model.

Concerning the evolution of the banking system we have to distinguish between two cases. If the mean growth rate of the banks are much larger than growth rate fluctuations $\theta >> \rho$, it is shown in Appendix A that banks suffer in the run of time from a replacement process. In this process banks with a higher mean growth rate replace those with lower mean growth rate. As a consequence the banking system reduces in the stationary state to a monopoly. Since we are interested in the bank size distribution we confine here to the case that the banking system can be regarded as a polypoly. In this case the mean growth rate must be small compared to growth rate fluctuations $\theta << \rho$. Therefore we assume that the mean growth rate of a bank has a small magnitude:

$$\theta \sim \varepsilon \tag{28}$$

with $\varepsilon << 1$.

2.4 The Bank Size Distribution

The bank size distribution $P(s)$ is determined by the probability to find the size of a bank s_i in the interval s and $s+ds$. The size distribution is governed by the dynamics of the banks given by Eq. (24). Taking advantage from Eq. (28), the growth process depends on the size of a bank. For small banks $s(t) \leq \varepsilon$ the first term in Eq. (24) is of the order ε^2 and can be neglected compared to the second term. The growth dynamics becomes for small banks:

$$\frac{ds(t)}{dt} \cong \rho(t)s(t) \tag{29}$$

[8] Note that variations of the fitness of a large bank have necessarily an impact on the fitness of all other banks for short time periods. The time scale is regarded to be chosen such that these short term correlations can be neglected.

This relation describes a multiplicative stochastic growth process and is equivalent to Gibrat's law of proportionate effects. Note that it is a consequence of the competition between banks for permanent deposit money. With Eq. (27) the central limit theorem suggests that the size distribution for small banks ($s \leq \varepsilon$) is given by a lognormal probability distribution of the form (Sornette 2006):

$$P(s,t) = \frac{1}{\sqrt{2\pi t}\,\omega s} \exp\left(-\frac{\left(\ln(s/s') - ut\right)^2}{2\omega^2 t} \right) \tag{30}$$

where u and ω are free parameters and s/s' is the bank size scaled by the size at t_0.

For large banks with $s > \varepsilon$, however, also the first term in Eq. (24) has to be taken into account. This relation can be interpreted as a generalized Langevin equation (Richmond and Solomon 2000). It yields after sufficiently long time a stationary size distribution of the form (Appendix B):

$$P(s) \sim \frac{1}{s^{\left(1+\frac{\langle\theta\rangle}{D}\right)}} \tag{31}$$

The bank size distribution can be described for large banks by a power law (Pareto) distribution which can be related to Zipf's law (Saichev et al. 2011). The evolutionary model suggests therefore that the size distribution of banks counted in deposits is generally a lognormal distribution with a power law tail. The Pareto tail is caused by the small mean growth contribution. If $<\theta>=0$, the Pareto tail disappears and the size distribution is dominated by the lognormal contribution.[9]

Note that the stationary bank size distribution is the Pareto distribution, but due to the small mean growth rate, small banks approach this stationary distribution much slower than large banks. Therefore the size distribution remains separated into two parts even over long time periods.

[9] This case occurs when the mean money growth rate disappears. Since a considerable contribution to the growth of large banks is due to money creation, they suffer therefore much more from a stagnant or even decreasing mean growth of fiat money than smaller banks.

3 Conclusion

The presented evolutionary model derives a bank size distribution that is in agreement with empirical results. It takes the fundamental processes of a banking system into account, the exchange and creation of money. In this theory the size distribution is on the one hand the result of the competition between banks for permanent deposit money. It is described by a preferential growth process determined by a bank fitness. The varying success in the competition is captured by treating the bank fitness as a fluctuating variable. Together with money growth fluctuations it generates the lognormal part of the bank size distribution. On the other hand also a small mean growth contribution comes into play, causing a so-called preferential attachment process (Newman 2005). Large banks benefit more from this small mean growth rate than smaller banks. This effect can be interpreted as economies of scale. They are the origin of the power law tail in the bank size distribution.

The presented model suggests that the Pareto tail is governed by the Pareto exponent $1+\langle\theta\rangle/D$. An increasing exponent indicates a more evenly distributed Pareto tail (Newman 2005), which is the case when the mean growth rate of money dominates over deposit fluctuations.[10] The power law tail becomes, however, more uneven when the bank evolution is suffered from increased growth rate fluctuations. It indicates a more intense competition between banks for permanent deposits. In particular mergers may lead to large fluctuations of permanent deposits and hence to an increasing skewness of the bank size distribution, accompanied with a concentration of banks in the Pareto tail. This effect has been found in empirical studies (Janicki and Prescott 2006, Benito 2008).

The key idea of the presented theory is that the bank size distribution is mainly influenced by the competition between banks for deposits. Competition is the origin of Gibrat's law of proportionate effects in similarity to a previous paper establishing the firm size distribution (Kaldasch 2012). In difference to banks, however, firms cannot generate money, but collect money in the reproduction process. While banks compete for permanent deposits, firms compete for sales.

[10] In this case is $\langle\theta\rangle \gg D$. Zipf's law is retained if the mean money growth rate and growth rate fluctuations of deposits are of the same order $\langle\theta\rangle \approx D$.

Their size is therefore determined by the fitness of their goods in the sales process. The model suggests that a lognormal size distribution with a power law tail must always occur when elements of an economic system growth with a small mean growth rate and large growth rate fluctuations. Since size distributions similar to the bank size distribution can be often found in social systems (Bouchaud and Mezard 2000, Gabaix 2008, Malevergne et al. 2013) it can be expected that its origin can be explained analogous to the presented model.

Appendix A

We want to discuss the case that the mean growth rates of the banks dominate over growth rate fluctuations $\theta >> \rho$. In this case fluctuations can be neglected and using Eq. (24) the evolution of the *i-th* bank becomes:

$$\frac{ds_i(t)}{dt} = \theta_i s_i(t) \tag{A1}$$

The growth of the total deposits is determined by:

$$\frac{d\tilde{s}}{dt} = \langle \theta \rangle \tilde{s} \tag{A2}$$

where the mean growth rate over all banks is $<\theta>$. The key variable of this consideration is the market share of the *i-th* bank m_i, defined as:

$$m_i(t) = s_i(t) / \tilde{s}(t) \tag{A3}$$

Taking the time derivative of this relation we obtain:

$$\frac{ds_i(t)}{dt} = \frac{dm_i(t)}{dt}\tilde{s}(t) + \frac{d\tilde{s}(t)}{dt} m_i(t) \tag{A4}$$

Using Eq. (A1) and Eq. (A2) this relation can be rearranged to:

$$\frac{dm_i(t)}{dt} = \left(\theta_i - \langle \theta \rangle\right) m_i(t) \tag{A5}$$

which is a replicator equation for the market shares. It can be also written as:

$$\frac{d\ln(m_i(t))}{dt} = \theta_i - \langle\theta\rangle \tag{A6}$$

In order to derive the market share evolution of the banks we diminish a second bank with index j from Eq. (A6) and obtain:

$$\frac{d\ln(m_i(t))}{dt} - \frac{d\ln(m_j(t))}{dt} = \theta_i - \theta_j = \Delta\theta_{ij} \tag{A7}$$

where $\Delta\theta_{ij}$ represents a constant relative fitness advantage. The relation between the two market shares becomes:

$$\frac{m_i(t)}{m_j(t)} = \frac{m_i(t_0)}{m_j(t_0)}\exp(\Delta\theta_{ij}t) \tag{A8}$$

With the identity:

$$m_i(t) = \frac{m_i(t)}{\sum_j m_j(t)} = \frac{1}{1+\sum_{j\neq i}\frac{m_j(t)}{m_i(t)}} \tag{A9}$$

we formally obtain for the time evolution of the *i-th* market share:

$$m_i(t) \cong \frac{1}{1+\sum_{j\neq i} e^{-\Delta\theta_{ij}t+\kappa_{ij}}} \tag{A10}$$

with appropriate coefficients κ_{ij}. For the two bank case Eq. (A10) turns into the well-known Fisher-Pry replacement relation:

$$m_i(t) = \frac{1}{1+e^{-\Delta\theta_{ij}t+\kappa_{ij}}}; m_j(t) = \frac{1}{1+e^{-\Delta\theta_{ji}t+\kappa_{ji}}} \tag{A11}$$

For a constant $\Delta\theta_{ij}$ the model suggests therefore that banks replace each other in the run of time, such that banks with higher $\Delta\theta_{ij}$ replace those with lower fitness advantage.

The stationary solution of Eq. (A5) is determined by $dm_i/dt=0$. This is the case when either $m_i=0$ or $\theta_i=\langle\theta\rangle$. Since $\Sigma m_i=1$, there must be at least one bank with $m_i\neq 0$, say the bank with index $i=1$. The only solution for the stationary state is:

$$\begin{aligned} m_1(t) &= 1 \quad \theta_1 = \langle \theta \rangle \\ m_i(t) &= 0 \quad \theta_i \neq \langle \theta \rangle \end{aligned} \tag{A12}$$

The stationary state of the banking system is characterized therefore by a monopoly bank.

Appendix B

Eq. (24) with $s>\varepsilon$ represents a generalized Langevin equation (Richmond and Solomon 2000). It has the form:

$$\frac{ds}{dt} = F(s) + \rho G(s) \tag{B1}$$

with $F(s)=\theta s$ and $G(s)=s$. This multiplicative stochastic relation that can be transformed into a relation with additive noise by introducing the functions $h(s)$ and $V(s)$ according to:

$$\frac{dh(s)}{dt} = \frac{1}{G(s)}\frac{ds}{dt} \tag{B2}$$

and

$$-\frac{dV(s)}{dh(s)} = \frac{F(s)}{G(s)} \tag{B3}$$

Inserting these relations in Eq. (B1) we obtain the usual Langevin equation:

$$\frac{dh}{dt} = -\frac{dV}{dh} + \rho \tag{B4}$$

For uncorrelated fluctuations suggested by Eq. (27), this relation describes a random walk of h in the potential V. For a sufficiently long time the probability distribution for h becomes (Sornette 2006):

$$B(h)dh = \frac{1}{\mathrm{N'}}\exp\left(-\frac{V(H)}{D}\right)dH \tag{B5}$$

where *N'* is a normalization constant. In terms of the original variable, we get:

$$P(s)ds = B(h)dh = \frac{1}{\mathrm{N'}}\exp\left(-\frac{1}{D}\int \frac{F(s')}{G(s')^2}ds'\right)\frac{ds}{G(s)} \tag{B6}$$

which yields with the corresponding functions for *G(s)* and *F(s)* a power law distribution of the form:

$$P(s) \sim \frac{1}{s^{\left(1+\frac{\langle\theta\rangle}{D}\right)}} \tag{B7}$$

References

Alhadeff, D. A., and Alhadeff, C .P. (1964). Growth of Large Banks 1930–1960. *Review of Economics and Statistics*, 46: 356–363. http://www.jstor.org/stable/1924043

Berger, A. N., Hunter W. L., Timme S. G. (1993). The Efficiency of Financial Institutions: A Review and Preview of Research Past, Present and Future. *Journal of Banking and Finance*, 17: 221–249. http://www.sciencedirect.com/science/article/pii/037842669390030H

Berger, A. N., Demsetz R., Strahan P. H. (1999). The Consolidation of the Financial Services Industry: Causes, Consequences and Implications for the Future. *Journal of Banking and Finance* 24: 135–194. http://ideas.repec.org/a/eee/jbfina/v23y1999i2-4p135-194.html

Benito E. (2008). Size Growth and Bank Dynamics. Documentos de Trabajo N.º 0801. Banco de Espana, Madrid. http://ideas.repec.org/p/bde/wpaper/0801.html

Bouchaud, J., and Mezard M. (2000). Wealth Condensation in a Simple Model of Economy. *Physica A*, 282: 536–545. http://lptms.u-psud.fr/membres/mezard/Pdf/00_BM_PA.pdf

Ennis, H. M. (2001). On the Size Distribution of Banks. *Economic Quarterly* 87, 1–25. http://ideas.repec.org/a/fip/fedreq/y2001ifallp1-25.html

Gabaix, X. (2008). Power Laws in Economics and Finance. *Annual Review of Economics*, 1: 255–294. http://ideas.repec.org/a/anr/reveco/v1y2009p255-294.html

Gibrat, R. (1913). Les Inegalites Economiques, *Sirey, Paris*.

Goddard, J. A., Mckillop D. G., and Wilson J. O. S. (2002). The Growth of US Credit Unions. *Journal of Banking and Finance*, 26: 2327–2356. http://www.sciencedirect.com/science/article/pii/S0378426601002035

Goddard, J. A, Molineux P., and Wilson J. O. S. (2004). Dynamics of Growth and Profitability in Banking. *Journal of Money, Credit and Banking*, 36: 1069–1090. http://www.jstor.org/stable/3839101

Janicki H.P., and Prescott E.S. (2006). Changes in the Size Distribution of U.S. Banks: 1960–2005. *Economic Quarterly* 94: 291–316. http://ideas.repec.org/a/fip/fedreq/y2006ifallp291-316nv.92no.4.html

Kaldasch, J. (2012). Evolutionary Model of the Growth and Size of Firms. *Physica A*, 391: 3751–3769. http://www.sciencedirect.com/science/article/pii/S0378437112000532

Malevergne Y., Saichev A. and Sornette D. (2013) Zipf's Law and Maximum Sustainable Growth, *Journal of Economic Dynamics and Control*, 37: 1195–1212. http://ideas.repec.org/a/eee/dyncon/v37y2013i6p1195-1212.html

Newman, M. E. J. (2005). Power Laws, Pareto Distributions and Zipf's Law. *Contemporary Physics*, 46: 323–51. http://arxiv.org/abs/cond-mat/0412004

Rhoades, S. A., and Yeats A. J. (1974). Growth, Consolidation and Mergers in Banking. *Journal of Finance*, 29: 1397–1405. http://www.jstor.org/stable/2978545

Richmond, P., and Solomon, S. (2000). Power Laws Are Boltzmann Laws in Disguise, e-print: cond-mat/0010222. http://arxiv.org/abs/cond-mat/0010222

Saichev A., Sornette, D., and Malevergne, Y. (2011). *Theory of Zipf's Law and Beyond*. Springer-Verlag Berlin.

Sornette D. (2006). *Critical Phenomena in Natural Sciences.* Springer-Verlag, Berlin.

Tschoegl, A. E. (1983). Size, Growth and Transnationality among the World's Largest Banks. *Journal of Business*, 56: 187–201. http://www.jstor.org/stable/2352823

Wilson, J. O. S., and Williams J. (2000). The Size and Growth of Banks: Evidence from Four European Countries. *Applied Economics*, 32: 1101–1109. http://ideas.repec.org/a/taf/applec/v32y2000i9p1101-1109.html

Yeats, A. J., Irons E. D., and Rhoades S. A. (1975). An Analysis of New Bank Growth. *Journal of Business*, 48: 199–203. http://www.jstor.org/stable/2352483