

# A Directional-Change Event Approach for Studying Financial Time Series

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**Abstract** Financial markets witness high levels of activity at certain times but remain calm at others. This makes the flow of physical time discontinuous. Therefore, to use physical time scales for studying financial time series runs the risk of missing important activities. An alternative approach is to use an event-based time scale that captures periodic activities in the market. In this paper, the authors use a special type of event, called a directional-change event, and show its usefulness in capturing periodic market activities. The study confirms that the length of the price-curve coastline, as defined by directional-change events, turns out to be a long one.

Special Issue

[New Approaches in Quantitative Modeling of Financial Markets](#)

**JEL** G10

**Keywords** Directional-change event; intrinsic time; high-frequency finance; foreign exchange market; time-series analysis

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**Citation** Monira Aloud, Edward Tsang, Richard Olsen, and Alexandre Dupuis (2012). A Directional-Change Event Approach for Studying Financial Time Series. *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 6, 2012-36. <http://dx.doi.org/10.5018/economics-ejournal.ja.2012-36>

## 1 Introduction

The Foreign Exchange (FX) market is open 24 hours a day, 7 days a week. Trading activities in FX market vary at different times of the day. For instance, on the announcement of political or economic news, there tends to be a sharp rise in market trading activity in response to the news, even during weekends when trading activity has a tendency to decline. Consequently, financial time series are unevenly spaced, which makes the flow of physical time discontinuous. Hence, it is an essential to redefine financial time series beyond the notion of physical time changes.

To model the non-seasonal fluctuations in terms of volatility, new time scales were introduced by Mandelbrot and Taylor (1967), Allais (1974), Stock (1988) and Muller et al. (1995). These new time scales are called “intrinsic time” as the time is determined by the time series itself and replaces the notion of physical time scale. Mandelbrot and Taylor (1967) introduced a “transaction clock” that ticks at every worldwide transaction. Allais (1974) proposed a psychological time scale in which the distinction to physical time corresponds to a transformation of the time scale. Stock (1988) introduced a time scale defined by the speed of the aggregated market activity or information flows. Muller et al. (1995) proposed intrinsic time defined as the cumulated sum of a market activity variable which is a statistical measure of volatility. Muller et al. (1995) successfully applied intrinsic time scale to a FX forecasting model.

Guillaume et al. (1997) introduce the directional-change event approach where intrinsic time is defined by directional-change events, and events are characterised by a fixed threshold of different sizes. The directional-change event approach defines time in price time series, eliminating any irrelevant details of price evolution. Guillaume et al. (1997) describe the price evolution by the frequency of directional-change events over a sampling period which provides an alternative measure of the risk. Later in 2011, using statistical analysis based on the directional-change event approach, Glattfelder et al. (2011) discover 12 new empirical scaling laws related to foreign exchange data series across 13 currency exchange rates. These 12 scaling laws enhance our understanding of the behaviour of prices in financial markets, giving us new insights. Bisig et al. (2012) define the so-called Scale of Market Quakes (SMQ), designed based on the directional-change event approach.

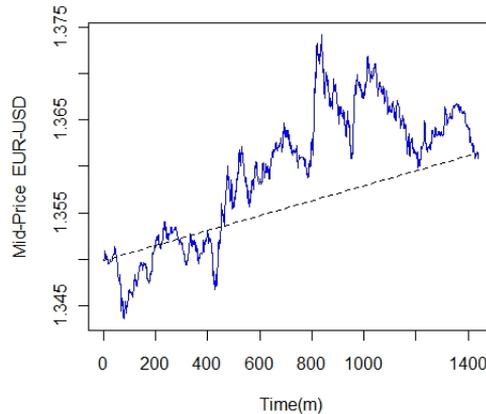
The purpose of SMQ is to quantify the FX market activity on a continuous basis at major economic and political events announcements. Kablan and Ng (2011) developed a new method of capturing volatility using the directional-change event approach.

In this paper we show the significance of the directional-change event approach for studying and analyzing financial time series. This is done using a four year data sample of high frequency tick-by-tick market data for EUR/USD and EUR/CHF, spanning January 1, 2006 to December 31, 2009. Our study shows that the length of the price-curve coastline defined by a directional-change event approach is longer than the length of the price-curve coastline defined by fixed time intervals (physical time).

The remainder of the paper is organised as follows. Section 2 describes the concept of intrinsic time combined by showing empirically the limitations of a physical time scale for studying price time series. Section 3 gives a definition of the directional-change event approach. Data and empirical results, with regard to the directional-change event approach, are presented in Section 4. Section 5 offers a measurement of the length of EUR/USD price-curve coastline. This paper is concluded with Section 6.

## **2 Intrinsic Time**

The majority of traditional approaches to observe price movements in financial time series are based on physical time changes Guillaume et al. (1997). These are the changes that occur in a time series, ranging from seconds through hourly to daily changes, in which the flow of periodic fixed patterns is discontinuous. In addition, as a result of the availability of high-frequency data, in which data, by its nature is non-homogeneous in time, it has become increasingly difficult and challenging to observe price movements through the use of physical time Dacorogna et al. (2001). Figures 1 and 2 show price activities for EUR/USD on January 7, 2009. From both graphs, it is clear that significant patterns of the trading activity are ignored when considering the time series of prices based on daily and hourly changes (physical time). By way of illustration, in Figure 1 and Figure 2, the price movement from midnight, hour 0, to hour 13 and 57 minutes represents an investment opportunity

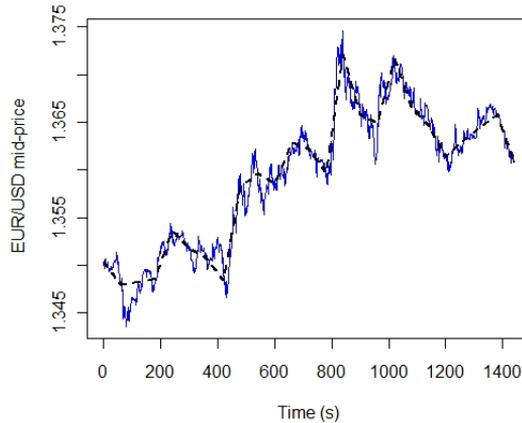


**Figure 1:** Price activities for EUR/USD on January 7, 2009. The straight line shows the gradient of the price changes for that day. The graph shows a major part of the activity is overlooked. So, chopping off the time series of prices will eliminate many significant price activities.

to sell, with the price rising by 1.8%. This was overlooked in both the end of that day's return and the hourly return.

Physical time fails to clearly capture the important activity of price movements in a significant manner, since the variety of trading activity obviously depends on the time of day. Using physical time to detect periodic patterns in time series, maps a variety of patterns with different magnitudes, which makes the flow of physical time discontinuous. Computational efforts are required to analyse and study price time series, which results in obvious costs associated with the study process. To deal with this vital issue, alternative solutions have been proposed. Among them is the study of financial time series using intrinsic time Guillaume et al. (1997), which provides a reliable solution.

Intrinsic time adopts an event-based system in contrast to physical time which adopts a point-based system. In intrinsic time, time is defined by events. An event is characterized by a fixed threshold  $\lambda$  and is defined as the absolute price change between two local extremal values exceeding a given threshold  $\lambda$  Glattfelder et al.



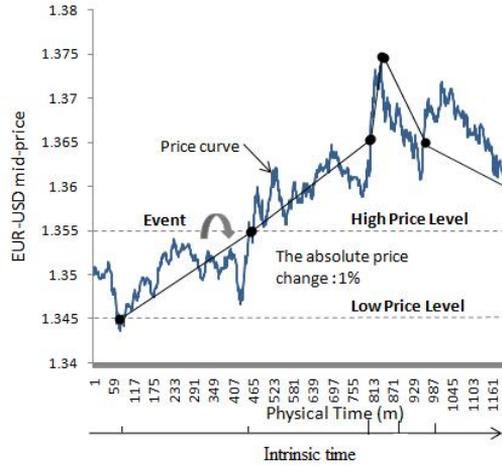
**Figure 2:** Price activities for EUR/USD on January 7, 2009. The dashed lines show hourly return for that day. Even with hourly return, some activity is still overlooked.

(2011). In contrast, intrinsic time is inhomogeneous in time seeing that time triggers only at periodic events, independent of the notion of physical time. Figure 3 shows EUR/USD price activities on the 7, January 2009 sample onto a reduced set of four sequential events defined by a threshold  $\lambda = 1\%$ . Physical time is homogenous which means time scales equally spaced on any chosen time scale.

### 3 Directional-Change Event Approach

#### 3.1 Directional-Change (DC) Event

According to Tsang (2010) a directional-change (DC) event can take one of the two forms—a downturn event or an upturn event. A downward run is a period between a downturn event and the next upturn event, while an upward run is a period between an upturn event and the next downturn event. A downturn event terminates an upward run, and starts a downward run, whereas an upturn event terminates a downward run and starts an upward run.



**Figure 3:** The graph shows a 24-hour EUR/USD mid-price sample and sequential events (solid lines) defined by a threshold  $\lambda=1\%$ . Intrinsic time triggers only at periodic events whereas physical time ticks equally across different patterns of different magnitudes in the price curve.

During a downward run, the last low price  $p_L$  is continuously updated to the minimum of (a) the current market price  $p(t)$  and (b) the last low price  $p_L$ . Similarity, during an upward run, the last high price  $p_H$  is continuously updated to the maximum of (a) the current market price  $p(t)$  and (b) the last high price  $p_H$  Tsang (2010). At the beginning of the sequence, the last high price  $p_H$  and last low price  $p_L$  are set to the initial market price  $p(t_0)$  at the beginning of the sequence.

In an upward run, a downturn event is an event when the absolute price change between the current market price  $p(t)$  and the last high price  $p_H$  is lower than a given threshold  $\Delta x_{DC}$ :

$$p(t) \leq p_H \times (1 - \Delta x_{DC}) \quad (1)$$

The starting point of a downturn event is a downturn point which is the point at which the price last peaked ( $p_H$ ). The end of a downturn event is a downturn directional-change point which is the point at which the price has dropped from the last downturn point by the threshold  $\Delta x_{DC}$ .

In a downward run, an upturn event is an event when the absolute price change between the current market price  $p(t)$  and the last low price  $p_L$  is higher than a given threshold  $\Delta x_{DC}$ :

$$p(t) \geq p_L \times (1 + \Delta x_{DC}) \quad (2)$$

The starting point of an upturn Event is an upturn point which is the point at which the price last troughed ( $p_L$ ). The end of an upturn event is an upturn directional-change point which is the point at which the price has risen from the last upturn point by the threshold  $\Delta x_{DC}$ .

### 3.2 Overshoot (OS) Event

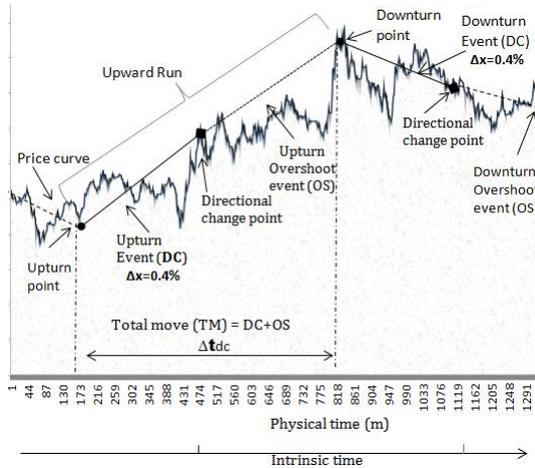
A directional-change (DC) event—a downturn event or an upturn event—is usually followed by a price overshoot event rather than an opposite directional-change event direction Glattfelder et al. (2011). The overshoot (OS) event represents the time interval of price movement beyond the directional-change event. An overshoot event can take one of two forms—a downturn event or an upturn event. An upturn overshoot event is a period between the previous upturn DC event and the starting point of the next downturn DC event. Similarly, a downturn overshoot event is a period between the previous downturn DC event and the starting point of the next upturn DC event. Figure 4 illustrates how the price curve is composed of directional-change and overshoot events. Algorithm 1 illustrates how to define directional-change and overshoot events during a time period  $T$ .

### 3.3 Total Move (TM)

A total price movement  $\Delta x_{TM}$  between two local extremal price values (minimum and maximum price) is decomposed into a directional-change (DC) event and an overshoot (OS) event Glattfelder et al. (2011). A total price movement  $\Delta x_{TM}$  is defined by:

$$\Delta x_{TM} = \Delta x_{DC} + \Delta x_{OS} \quad (3)$$

where  $\Delta x_{DC}$  is the size of a directional-change event while  $\Delta x_{OS}$  is the size of an overshoot event.



**Figure 4:** Price movements within 24 hours are shaped by a directional-change point (diamond) with a threshold  $\Delta x_{DC} = 0.4\%$ . A total price movement between two local values (minimum and maximum price) is decomposed into directional- change (solid lines) and an overshoot (dashed lines) events.

In a downward run, a downturn event is usually followed by a downturn overshoot event, which is ended by the next upturn event. In an upward run, an upturn event is usually followed by an upturn overshoot event, which is ended by the next downturn event Tsang (2010). So the directional-change event approach defines price time series as a sequence of

$\dots \rightarrow$  *downturn event*  $\rightarrow$   
*downturn overshoot event*  $\rightarrow$   
*upturn event*  $\rightarrow$   
*upturn overshoot event*  $\rightarrow$   
*downturn event*  $\rightarrow \dots$

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**Algorithm 1** Defining directional-change (DC) and overshoot (OS) events

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Require: initialise variables (event is Upturn Event,  $p^h = p^l = p(t_0)$ ,  $\Delta x_{dc}$  (Fixed)  $\geq 0$ ,  $t_0^{dc} = t_1^{dc} = t_0^{os} = t_1^{os} = t_0$ )

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1: if event is Upturn Event then
2:   if  $p(t) \leq p^h * (1 - \Delta x_{dc})$  then
3:     event  $\leftarrow$  Downturn Event
4:      $p^l \leftarrow p(t)$ 
5:      $t_1^{dc} \leftarrow t$  // End time for a Downturn Event
6:      $t_0^{os} \leftarrow t + 1$  // Start time for a Downward Overshoot Event
7:   else
8:     if  $(p^h < p(t))$  then
9:        $p^h \leftarrow p(t)$ 
10:       $t_0^{dc} \leftarrow t$  // Start time for a Downturn Event
11:       $t_1^{os} \leftarrow t - 1$  // End time for an Upward Overshoot Event
12:    end if
13:  end if
14: else
15:  if  $p(t) \geq p^l * (1 + \Delta x_{dc})$  then
16:    event  $\leftarrow$  Upturn Event
17:     $p^h \leftarrow p(t)$ 
18:     $t_1^{dc} \leftarrow t$  // End time for an Upturn Event
19:     $t_0^{os} \leftarrow t + 1$  // Start time for an Upward Overshoot Event
20:  else
21:    if  $(p^l > p(t))$  then
22:       $p^l \leftarrow p(t)$ 
23:       $t_0^{dc} \leftarrow t$  // Start time for an Upturn Event
24:       $t_1^{os} \leftarrow t - 1$  // End time for a Downward Overshoot Event
25:    end if
26:  end if
27: end if

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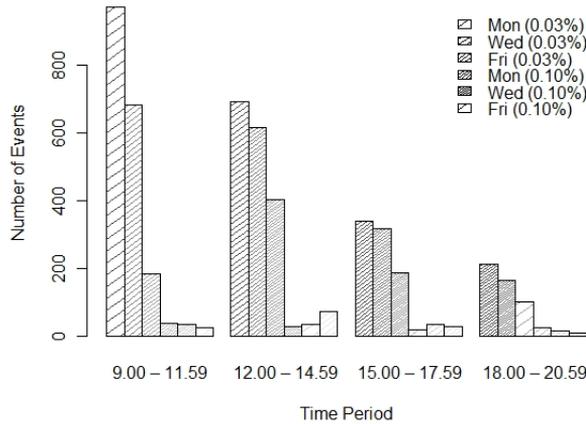
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## 4 Spectral Analysis of Tick Data

### 4.1 The Dataset

We use a high-frequency dataset composed of two currency pairs (EUR/USD and EUR/CHF) spanning a four year period, from January 1, 2006 to December 31, 2009. The dataset includes a bid, an ask price of a currency pair at a timestamp. Throughout the paper, the following definition of mid-price is used

$$p(t) = \frac{(b_t + a_t)}{2} \quad (4)$$



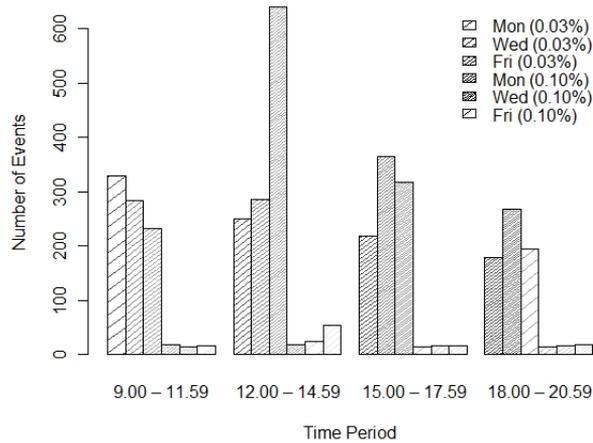
**Figure 5:** Number of events of 0.03% and 0.10% magnitude in different periods of the 5<sup>th</sup> (Monday), 7<sup>th</sup> (Wednesday) and 9<sup>th</sup> (Friday) January 2009 in EUR/USD mid-price time series.

where  $p(t)$  is the mid-price of a currency pair at time  $t$ ,  $b_t$  is the bid price at time  $t$  and  $a_t$  is the ask price at time  $t$ . We aim to use this high-frequency dataset to show the significance of the directional-change event approach in capturing periodic market activities.

## 4.2 Event Time Scales

Directional-change events are significant events as they capture periodic major change in a price time series in which the magnitude of an event is defined by the observer. Under physical time, we divide time into periods of equal length which has the drawback of missing significant events in a price time series. However, different time periods in a price time series may contain a different number of events of different magnitude, which demonstrate that price evolution is separate from physical time changes.

Figures 5 and 6 report the number of events of 0.03% and 0.10% magnitude in different time periods of the 5<sup>th</sup>, 7<sup>th</sup> and 9<sup>th</sup> January 2009 in EUR/USD and



**Figure 6:** Number of events of 0.03% and 0.10% magnitude in different periods of the 5<sup>th</sup> (Monday), 7<sup>th</sup> (Wednesday) and 9<sup>th</sup> (Friday) January 2009 in EUR/CHF mid-price time series.

EUR/CHF mid-price time series. For EUR/USD, we can observe from the results in Figure 5, that the period between 9:00–11:59 on the 5<sup>th</sup> (Monday) January contained 971 events with a threshold of 0.03%, while for the same period on the 7<sup>th</sup> (Wednesday) January there were 683 events. On the 7<sup>th</sup> (Wednesday) January there were 34 events for three periods of time with a 0.10% threshold, while this was not the case on the 5<sup>th</sup> (Monday) January. Obviously, the reported results in Figures 5 and 6 highlight two important observations: (a) for different currency pairs the same periods of time with the same threshold size on different days may contain a different number of events, and (b) with the same threshold size, some periods on the same day have more events than others. These two observations indicate that events of different magnitude are independent of physical-time changes.

## 5 The Price-Curve Coastline

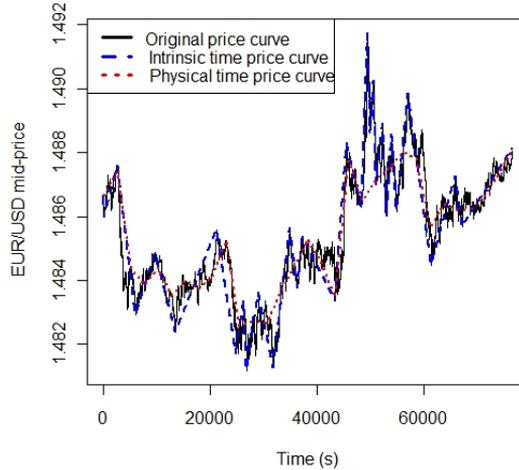
Glattfelder et al. discovered scaling laws that give an estimation of the length of the price-curve coastline Glattfelder et al. (2011). They found that the length of the coastline defined by intrinsic time is long. In this section, we have developed a new method for estimating the length of the price-curve coastline based on the price distance between fixed points. The measurement is defined by two different concepts: intrinsic time (directional-change events) and physical time changes (fixed time intervals), with the aim of assessing their performance in summarizing price movements. Assuming perfect foresight, the length of the price-curve coastline over a defined time period  $T$ , represents the profit potential.

The measurement of the length of the price-curve coastline over a time period  $T$ , as defined by intrinsic time, is the average upwards and downwards price moves, considering the number of events. Both the upwards and downwards price movements are defined by a fixed threshold size  $\Delta x_{DC}$ . The downward price movement is the total price move from a downturn point to the next upturn point. In another word, the downward price movement is the absolute difference between  $p_i$  and  $p_{i+1}$ , where  $p_i$  is the price of the  $i$ -th downturn turning point and  $p_{i+1}$  is the price of the next upturn point. Similarly, the upward price movement is the total price move from an upturn point to the next downturn point. Under intrinsic time, the length of the price-curve coastline  $C(\Delta x_{DC})$  is measured by

$$c(\Delta x_{DC}) = \frac{1}{N(\Delta x_{DC})} \sum_{i=1}^{N(\Delta x_{DC})} |p_i - p_{i+1}| \quad (5)$$

where  $\Delta x_{DC}$  is a fixed threshold (%),  $N(\Delta x_{DC})$  is the total number of events on which the length of the price-curve coastline is measured. Hence, the number of events is determined by the threshold size,  $\Delta x_{DC}$ , that we use.  $p_i$  is the price of the  $i$ -th turning point, whether upturn or downturn point.

In contrast, the measurement of the length of the price-curve coastline defined by physical time changes at fixed time intervals is the average price movement between fixed points over a time period  $T$ , in which the time interval between



**Figure 7:** EUR/USD mid-prices time series defined by physical time and intrinsic time. The sampling period is over November 5<sup>th</sup>, 2009. For intrinsic time, the number of events is 30 determined by a threshold size of 0.1%.

these fixed points is equivalent. Under physical time, the length of the price-curve coastline  $C(t)$  is measured by

$$c(t) = \frac{1}{n} \sum_{i=1}^n |p_i - p_{i+1}| \quad (6)$$

where  $p_i$  is the price at point  $i$  and  $n$  is the total number of fixed points which is equal to the number of events used in Equation 5. For a fair comparison, we use the same number of points in physical time and intrinsic time.

Figure 7 shows EUR/USD mid-price activities over November 5<sup>th</sup>, 2009. The graph contrasts the price changes defined by physical time and by intrinsic time using the same number of points. The lines in the graph summarize the price movements using intrinsic time and physical time (at fixed time intervals). The price changes under intrinsic time are represented in the graph by dashed lines

lining up the upturn/downturn points in terms of directional-change events. From Figure 7, it is obvious that significant events of trading activity are ignored when we consider changes over time based on fixed time intervals (physical time changes). By way of illustration, the price peak at 1.4917 was overlooked on the price curve based on physical time changes. Therefore, the length of the coastline defined by physical time (0.01%) is shorter than the coastline defined by intrinsic time (0.10%) which highlights the significance of intrinsic time.

It is important to be acquainted with how well intrinsic time and physical time lines fit prices in a time series. This allows the evaluation of their performance in terms of which is finest at describing the price changes in a time series. We measure the total error,  $E_T$ , of the length of the price-curve coastlines under both physical time and intrinsic time (Figure 7). The total error,  $E_T$ , is a measure of the overall error of intrinsic time and physical time in summarizing price movement over a time period  $T$ , and is defined by

$$E_T = \sum_{t=0}^n (\Delta x_t)^2 \quad (7)$$

where  $n$  is the total number of time scales over a time period  $T$ ,  $(\Delta x_t)^2$  is the square difference between two prices in (i) the price curve and (ii) in whether in the physical time or intrinsic time line calculated for the same time scale,  $t$ . We found that the measurement of the total error in terms of intrinsic time (0.03) is shorter than that for physical time (0.04) (Figure 7). This result indicates that summarizing price movements using intrinsic time gives a better description of price changes than does using physical time with fixed intervals.

The reported results in Table 1 and 2 compare the average lengths of the annualized EUR/USD and EUR/CHF coastline measured by (a) physical time scales (evenly spaced in time—fixed time interval), and (b) intrinsic time (unevenly spaced in time). The sampling period covers the four years from 2006 to 2009. We annualize the results by dividing them by 4, the number of years in our data sample.

The results in Table 1 and 2 draw attention to the fact that the length of the coastline depends completely on the frequency of changes in the price. Also, these reported results highlight the importance of considering events in studying the

Threshold	$\langle N(\text{points}) \rangle$	$c(\Delta x_{DC})$	$c(t)$
0.01%	5.46E+05	0.04 %	0.01 %
0.10%	19E+04	0.47 %	0.08 %
1%	7.78E+02	3.15 %	0.37 %
3%	1.89E+02	7 %	1.16 %

**Table 1:** The measurement of the length of EUR/USD price-curve coastline over four years from 2006 to 2009, as defined by intrinsic time (directional-change events)  $c(\Delta x_{DC})$  and physical time changes (fixed intervals)  $c(t)$ . The average number of fixed points is denoted by  $\langle N(\text{points}) \rangle$ .

Threshold	$\langle N(\text{points}) \rangle$	$c(\Delta x_{DC})$	$c(t)$
0.01%	6.99E+05	0.04 %	0.01 %
0.10%	7.20E+03	0.56 %	0.07 %
1%	4.10E+02	3.11 %	0.29 %
3%	3.00E+01	5.87 %	1.12 %

**Table 2:** The measurement of the length of EUR/CHF price-curve coastline over four years from 2006 to 2009, as defined by intrinsic time (directional-change events)  $c(\Delta x_{DC})$  and physical time changes (fixed intervals)  $c(t)$ . The average number of fixed points is denoted by  $\langle N(\text{points}) \rangle$ .

price-curve, rather than physical time changes due to the long coastline of price changes based on intrinsic time. In all the thresholds that we have tested, the length of the price-curve coastline defined by physical time is shorter than the coastline defined by intrinsic time. Intrinsic time enables the analyst to capture the short term market dynamics involved by presenting significant information and a clear picture of price behaviour, based on the observer's expectations of the market. In addition, it reduces the complexity of real-world price time series given the small number of price points for evaluation. The computational costs (which include the cost of evaluating data) must not be ignored. Thus, it should be part of the criteria when it comes to deciding on an approach for studying price time series.

## 6 Conclusion

The directional-change event approach can gradually improve our understanding of the dynamic behaviour of financial markets in a simplified way, beyond what physical time at fixed time intervals can achieve. This approach allows the understanding of the price time series as an event-based process independent of the notion of physical time which allows the detection of periodic patterns.

This paper illustrates the definition of the directional-change event approach in conjunction with the motivation behind introducing this approach, its advantages and significance, and the working mechanisms of this approach. The spectral analysis shows that the size of the threshold determines the number of events in a price time series. An increase in the size of the threshold,  $\Delta x_{DC}$ , means a decrease in the number of existence events in the price time series. Additionally, the reported results in the paper indicate that with the same threshold size the same periods of time on different days may contain a different number of events. Also, with the same threshold value, some periods on the same day may have more events than others. The measurement of the length of the price-curve coastline shows a long coastline of price changes defined by intrinsic time confirming similar results in the study by Glattfelder et al. (2011).

The directional-change event approach opens doors to many research directions on financial markets. One obvious direction is to extend the catalogue of stylised facts by studying the dynamic behaviour of markets based on the directional-change event approach. Another important research direction is to model a trading strategy based on directional-change events. This would allow traders to capture the short term market dynamics involved by the detection of major periodic patterns in time series.

We believe that the directional-change event approach has the potential to act as a robust foundation for studying financial time series, and could lead to new findings about financial behaviour in various situations.

**Acknowledgements:** The authors would like to thank OANDA Corporation for their help and for providing the FX market data. This work was supported in part by the Olsen Ltd.

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