

Critical Investigations to Revealed Preference Theory

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Quite oddly, it is extremely seldom in the literature for economists to concern with the critical investigation to revealed preference theory during its seventy-five years history since 1938 when Samuelson first proposed it, so that even if a critical glance at the initial notion of revealed preference can bring us surprising findings which will be enough to reverse the traditional view on this theory.

Consider two commodity bundles q^1 and q^2

$$q^1 = (q_1^1, \dots, q_n^1),$$

$$q^2 = (q_1^2, \dots, q_n^2),$$

and their prevailing price set p

$$p = (p_1, \dots, p_n).$$

Denote their budgets

$$pq^1 = \sum_{i=1}^n p_i q_i^1;$$

$$pq^2 = \sum_{i=1}^n p_i q_i^2.$$

Samuelson (1938) wrote in his seminal paper firstly to introduce the revealed preference concept like this:

Suppose now that one bought q^1 . If $pq^2 \leq pq^1$, it means that he could have purchased q^2 , but he did not choose to do so. That is, q^1 was selected over q^2 . We may say that $pq^2 \leq pq^1$ implies q^1 is preferred to q^2 .

In this way, “ q^1 is preferred to q^2 ”, a subjective utility judgment, can be revealed by $pq^2 \leq pq^1$, an objective budget measure, called “revealed preference”. Although revealed preference theory has been developed by Houthakker (1950), Afriat (1967), and Varian (1982), it has never left such a base.

From a positive point of view, in the situation supposed by Samuelson, if one bought q^1 and $pq^2 \leq pq^1$, there are virtually two possible interpretations for such a choice. One is

q^2 was preferred to q^1 but the consumer behaved irrationally;

and another

q^1 was preferred to q^2 and the consumer behaved rationally.

Samuelson’s revealed preference concept was specially addressed only following the latter interpretation but omitting the former. Where, “rationally” merely refers to “one behaves following his preference”, while “irrationally” merely refers to “one behaves following some motivations other than his preference”. “Rationally” and “irrationally” are distinguished here by nothing but “preference”, all are normal behaviors usually. A complete notion of revealed preference implies the above two interpretations.

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If we observe in experimental or empirical data that q^1 was bought and $pq^2 \leq pq^1$, we cannot certainly conclude “ q^1 was preferred to q^2 ” as Samuelson had told to us, unless the “rational” consumer is accepted as a priori; while it possibly means that q^2 was preferred to q^1 but the consumer behaved following some motivations other than his preference, i.e. so-called “irrationally”. Namely, the positive revealed preference interpretation for the observed event “a consumer bought q^1 and $pq^2 \leq pq^1$ ” always asserts that the consumer irrationalized or rationalized the data—a completely meaningless conclusion! It must imply that Weak and Strong Axioms of Revealed Preference (Samuelson, 1938; Houthakker, 1950) are meaningless too. This is the explanation naturally implied in revealed preference notion if we view it as a general positive rationale. Unless one accepts the “rational” consumer as a priori in a normative theory, he cannot conclude as Samuelson did. Samuelson’s inference is doubtlessly incomplete in the positive logics and had previously rooted a fatal hidden trouble in revealed preference theory such that it became a congenitally deficient system since it was born.

In fact, the utility maximization is the important one but not all of a consumer’s motivations for deciding his purchase choice. Utility maximization theory does not found itself upon the uniqueness of utility maximization motivation. Other motivations empirically exist and cannot be omitted for their considerable proportion in choice behaviors. For example, exhausting the whole budget is one of usual motivations easily observed in experimental and empirical processes, and to do so, from time to time a subject or a consumer at least partially neglects his preference. Such a consumer will be referred to as “a purely exhausting consumer”, who is one of “irrationally behaves” mentioned in the complete notion of revealed preference.

A purely exhausting consumer will very possibly buy q^1 but q^2 is preferred to q^1 in the situation of $pq^2 \leq pq^1$. Quite a few such cases had been observed in my probing experiments conducted in 2010, and it had to be taken into account in my experimental design for testing utility maximization, in which a special program was used to discriminate the purely exhausting subjects (see Subsections 2.1.1 and 2.2.2 in He, 2012b).

In the verification for the utility maximization hypothesis in revealed preference theory, the utility maximization is converted to the revealed preference concept relying on the following definition:

A utility function $u(x)$ rationalizes a set of observations (p^i, x^i) , $i=1, \dots, n$, if $u(x^i) \geq u(x)$ for all x such that $p^i x^i \geq p^i x$. (Varian, 1982)

As has been discussed above, such a definition also suffers from mixing “rational” and “irrational” behaviors in $p^i x^i \geq p^i x$, so that, for example, any purely exhausting consumer could worm his way into the “rational” consumers in the discrimination $p^i x^i \geq p^i x$. Namely, if a set of market empirical data is interpreted as “rationalized” in revealed preference theory, it could be resulted by the “irrational” purely exhausting motivation but uncertainly by the utility maximization. In other words, one only motivated from exhausting his budget will yield the false “rationalized” data to deceive revealed preference researchers. Hence, all empirical verifications using market statistical information like Varian (1982)’s analysis on post-war consumption data are invalid or, at least, highly uncertain. Varian concluded in his paper:

“Most existing sets of aggregate consumption data are post-war data, and this period has been characterized by small changes in relative prices and large changes in income. Hence, each year has been revealed preferred to the previous years in the sense that it has typically been possible in a given

year to purchase the consumption bundles of each of the previous years. Hence no 'revealed preference' cycles can occur and the data are consistent with the maximization hypothesis. This observation implies that those studies which have rejected the preference maximization using conventional parametric techniques are rejecting only their particular choice of parametric form." (Varian, 1982)

That is completely wrong. It is "revealed preference cyclical consistency" approach (Afriat, 1967; Diewert, 1973; Varian, 1982, 1983) to pave a wider gateway to utility maximization especially for "irrational" consumers, including but not restricted to the purely exhausting consumers, who had very likely sneaked into the post-war aggregate consumption data such that the empirical verification for utility maximization in those data had become impossible. It is just the failure of revealed preference theory.

The utility maximization motivation co-exists or, even, competes with other motivations in consumer choice behaviors. In the non-satiated consumption categories, in which the consumption quantities increase as the budget becomes larger, and with small changes in relative prices and large changes in increasing income, we unlikely exclude such a possibility that consumers' purely exhausting motivation will occupy or be mixed into a considerable proportion comparing to the number of utility maximization motivation. That is, the time-series aggregate consumption data, like those used in Varian's study mentioned, were perhaps yielded considerably together with the consumers' purely exhausting motivation. The verification based on those mixed data has to be thought perhaps neither a support nor a negation and considerably irrelevant to utility maximization hypothesis. Unfortunately, all such cases will be always judged as "rationalized" without discrimination in revealed preference theory. In the positive sense, the judgment of "each year has been revealed preferred to the previous years in the sense that it has typically been possible in a given year to purchase the consumption bundles of each of the previous years" is wrong.

The experimental and empirical verification approach developed by Afriat (1967) and Varian (1982, 1983) completely relies upon the observation to choice behaviors at different budget levels. It more easily appeal subjects to purchase by following or mixing the purely exhausting motivation in his decisions in experimental studies. Hence, the purely exhausting motivation may be more disastrous to this approach. On the other hand, if consumers or subjects thoroughly or partially follow the purely exhausting motivation and other "irrational" motivations, their behavioral effect will be thoroughly or partially irrelevant to utility maximization, then their deceiving effect cannot be ruled out by revealed preference verification approach, but can be deleted by those using parametric models to verify utility maximization because the deceiving result will be judged as violating utility maximization.

Comparing to those using parametric models, revealed preference verification approach is not only without superiority to reduce purely exhausting motivation in an experimental study, but also without superiority to rule out the deceiving result in an experimental or empirical study. The fatal fault of revealed preference verification approach is just its nonparametric character.

Not only the purely exhausting motivation, other "irrational" behaviors are also usual in choice behaviors. For example, the emotion utility judgment is another "trouble" haunting revealed preference theory for its irregular character, especially when it is mixed together with the purely exhausting motivation. The emotion utility judgment had been also avoided in the measurement of utility scales in He, 2012a. All so-called "irrational" behaviors will trouble a revealed preference description somehow in unexpected cases, during unexpected times, via unexpected ways, with unexpected forms, and by

unexpected results. The most disastrous for revealed preference theory is that all those hidden troubles will be always unknown by researchers but contribute false positive descriptions to mislead them. Revealed preference theory has no any immunity against the hoodwinking from “irrational” behaviors, and will be harassed always by the everlasting suspicion that whether or not the “irrational” behaviors have duped us. That is, if a data set agrees with a parametric utility maximization model, e.g. Klein-Rubin utility function and LES, it will mean that consumers rationalize the data set; but in contrast, if a data set agrees with a revealed preference maximization model, e.g. cyclical consistency, it will only meaninglessly mean that consumers rationalize or irrationalize the data set. Essentially, the experimental test of utility maximization is to examine whether the utility maximization is one among those motivating consumers’ choice behaviors. In such a task, revealed preference theory is certainly incompetent.

In summary, only as a standard normative system revealed preference theory is possibly valid, and as a positive description it is certainly meaningless. All current experimental or empirical verifications based on Samuelson’s revealed preference concept are doomed to be invalid (e.g. Varian, 1982, 1983, 2009).

The fatal defect implied in revealed preference theory is the absence of direct analysis on the attributes of preference or utility. The difference between perception utility and emotion utility must involve the empirical natures of utility itself. As a positive description it is unsuccessful to escape from the subjective utility measure by introducing revealed preference interpretation. To overcome the fault exposed in revealed preference concept, a positive consumer behavior theory must directly look into the subjective utility or preference measure itself.

The above discussion just aims at the revealed preference theory rather than whole ordinal utility theory (e.g. Hicks and Allen, 1934; Hicks, 1939). The concept of utility indifference is a subjective measure presented in the latter. Some earlier experimental studies following the traditional ordinal utility concept did explore this issue by measuring the subjective indifference judgment for some commodity bundles (e.g. Thurstone, 1931; MacCrimmon and Toda, 1969). Nonetheless, they could not deliver clear experimental evidences to confirm the indifference curve sufficiently satisfying all three strict standards convexity, diminishing, and non-intersecting for determining a utility maximization measure in subjects’ performances. Today, the ordinal utility maximization has still remained neither tested nor falsified, or, even, neither testable nor falsifiable. In behavioral economics, elicitation effect, preference reversal, and etc (e.g., Fredrick and Fischhoff, 1998; Slovic and Lichtenstein, 1983) imposed some restrictions on the ordinal utility concept but are not a thorough negation to it.

There are three utility concepts: cardinal utility, ordinal utility, and revealed preference. They are essentially different as the description of actual psychological processes, and cannot be replaced from each other by treating them only as some mathematical contexts.

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