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The Influence of the Specification of Climate Change Damages on the Social Cost of Carbon

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Abstract Drawing upon climate change damage specifications previously proposed in the literature that the authors have calibrated to a common level of damages at 2.5°C, the authors examine the effect upon the social cost of carbon (SCC) of varying damage specifications in a DICE-like integrated assessment model. They find that SCC estimates are highly sensitive to uncertainty in extrapolating damages to high temperatures at moderate-to-high levels of risk aversion, but only modestly so at low levels of risk aversion. While in the absence of risk aversion, all of the SCC estimates but one agree within a factor of two, with a moderate level of risk aversion included, the differences among estimates grow greatly. For example, one composite damage specification, combining elements of different literature-derived specifications and roughly taking into account calibration uncertainty, yields SCC values 32% higher than the standard quadratic DICE damage function in the absence of risk aversion. With a coefficient of relative risk aversion of 1.4, however, the same uncertain specification yields SCC values almost triple those of the standard function. The authors conclude that failure to consider damages uncertainty and risk aversion jointly can lead to significant underestimation of the SCC.

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1 Introduction

The social cost of carbon (SCC) is a monetized estimate of the change in expected social welfare that results from a marginal change in carbon dioxide (CO_2) emissions. More precisely, it is defined as the ratio of the change in expected social welfare from a unit of emissions to the change in expected social welfare from a unit of material consumption in the period of emissions. Its evaluation requires an integrated assessment model (IAM) that couples together a global economic model and a model of the physical climate system. A core component of this coupling is a specification of damages that translates physical climate outcomes (in practice, primarily changes in temperature) into effects on human welfare.

Ideally, damage specifications should be as comprehensive as possible and consistent with the best available results from detailed assessments of vulnerabilities, impacts and adaptation. Because Monte Carlo simulations to estimate the SCC will necessarily sample some low-probability, high-climate sensitivity[1] states of the world, they should also give meaningful results under these extreme conditions. The first pair of objectives is addressed through the calibration of damages within an IAM; the last objective (and the primary focus of this paper) is addressed through the choice of functional forms used to extrapolate damages beyond the calibration range.

The Dyanmic Integrated model of Climate and the Economy (DICE) IAM (Nordhaus and Boyer, 2000; Nordhaus, 2007), as one example, is calibrated against estimates of damages at 2.5°C warming to agriculture (Darwin et al., 1995), coastal infrastructure (Yohe and Schlesinger, 1998), and health (Murray et al., 1996). It also includes ad hoc estimates of impacts on energy demand, ecosystems, and settlements (Nordhaus, 2007), as well as estimates based on an expert elicitation study of expected damages resulting from potential climate catastrophes (Nordhaus, 1994). In total, DICE estimates non-catastrophic damages equal to 0.6% of GDP

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[1] Climate sensitivity characterizes the long-term global mean temperature response to a change in CO_2 concentration or other radiative forcings; it is frequently quoted in terms of the equilibrium warming associated with a doubling of atmospheric CO_2 concentrations. The Intergovernmental Panel on Climate Change's Fourth Assessment Report (Forster et al., 2007) estimates that climate sensitivity is most likely 3° per CO_2 doubling, with a 67% confidence range of 1.5–4.5° and a difficult to characterize upper tail.

and expected catastrophic damages equal to 1.2% of GDP at 2.5°C warming (Nordhaus, 2007). Adaptation is incorporated indirectly, via the assumptions underlying the different sectoral studies used for calibration. Damages are extrapolated beyond the 2.5°C calibration point by assuming an approximately quadratic relationship between temperature and fraction of GDP lost.

As another example, the Climate Framework for Uncertainty, Negotiation and Distribution (FUND) IAM (Anthoff and Tol, 2010; Tol, 2002) includes sector- and region-specific damage functions for agriculture (calibrated against five computable general equilibrium model evaluations published between 1992 and 1996), forestry (calibrated against Perez-Garcia et al., 1997, and Sohngen et al., 2001), water resources and energy consumption (calibrated against Downing et al., 1995, 1996), sea level rise (calibrated primarily against Fankhauser, 1995, for protection costs and Kattenberg et al., 1995, for sea level rise projections), the "warm glow" effect of ecosystem loss (Tol, 2002), diarrhea (calibrated against World Health Organization data), vector-borne diseases (calibrated against four studies from 1995-1997), cardiovascular disease (calibrated against World Health Organization data), and tropical and extratropical storm damage (calibrated against a World Meteorological Organization statement and Toya and Skidmore, 2007). These damages are functions of a combination of temperature, rate of temperature change, CO_2 concentration and adaptive capacity (indexed by per capita income). For typical scenarios, they total about 0.9% to 1.6% of GDP at 2.5°C warming (Warren et al., 2006). (Note that FUND does not include potential catastrophic impacts.) The functional form used to extrapolate damages to higher temperatures varies between sectors.

The Intergovernmental Panel on Climate Change's Fourth Assessment Report notes that "on balance, the current generation of aggregate estimates in the literature is more likely than not to understate the actual costs of climate change" (Schneider et al., 2007). Consistent with this observation, the U.S. government's recent SCC analysis (Interagency Working Group on the Social Cost of Carbon, 2010) noted the incomplete treatment of non-catastrophic damages, potential catastrophic damages, inter-sectoral interactions, and inter-regional interactions in the cost-benefit IAMs (DICE, FUND and the Policy Analysis for the Greenhouse Effect [PAGE] model) that they used for SCC estimation.[2] These omissions likely all serve to lower damage projections. The U.S. government analysis also noted that adaptation is obscurely treated in some of the IAMs, which could either raise or lower damage projections, depending on the implicit assumptions. (See Kopp and Mignone, 2012, for further discussion of these limitations.)

Uncertainty arises both in calibration and extrapolation of damages, as well as elsewhere in the IAM (e.g., in baseline socio-economic projections and in the physical climate model). The more comprehensively uncertainty is taken into account in the estimation of the SCC, the more sensitive the final values will be to the level of risk aversion. The U.S. government analysis assumed zero risk aversion, while different versions of DICE use a coefficient of relative risk aversion of either 2.0 or 1.5. (See Kopp and Mignone, 2012, and Kousky et al., 2011, for further discussion.)

While there are more than three hundred published estimates of the SCC – and over two hundred in the last decade alone – more than three-quarters have been produced by just three research groups, the same three that authored the IAMs employed in the U.S. government analysis (see Tol, 2011, for a review). The damage functions in these IAMs are calibrated against impact studies that are frequently dated and conducted for low levels of warming, generally in the range of 2–3°C. In this paper, however, our focus is not on the details of calibration, but on the extrapolation of impacts to higher temperatures.

Accepting for the sake of this analysis the 2.5°C calibration of expected damages from DICE, we use a modified implementation of DICE to evaluate, under different levels of risk aversion, the sensitivity of SCC estimates to the extrapolation of damages. We first review a range of alternate damage specifications for DICE-like models, some deterministic and some uncertain, that have been employed in the peer-reviewed literature. Next, we consider the effect on SCC estimates of

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[2] We distinguish cost-benefit IAMs like DICE from other, more disaggregated IAMs (e.g., the Global Change Assessment Model [GCAM], the Integrated Global System Modeling framework [IGSM], and the Integrated Model to Assess the Global Environment [IMAGE]) that have more detailed representations of the costs of mitigation but have not traditionally included economic estimates of climate change damages (and thus the economic benefits of climate mitigation). These more disaggregated IAMs are used for assessing the costs of mitigation policies but, since they do not include monetized damages, cannot be used for estimating the SCC.

substituting these specifications for the default DICE quadratic damage function at four different levels of risk aversion and two different emissions scenarios, while keeping damages at the 2.5°C calibration point constant. We also consider new, composite, uncertain damage specifications that combine characteristics of the literature-derived specifications. We then return to examine the sensitivity of our results to changes in expected damages at the calibration point. We find that SCC estimates are highly sensitive to uncertainty in extrapolating damages to high temperatures at moderate-to-high levels of risk aversion, but only modestly so at low levels of risk aversion – a result that emphasizes the importance of jointly considering damages uncertainty and risk aversion.

2 Taxonomy of damage specifications

The damage specifications that appear in peer-reviewed literature can be characterized based on a number of factors. Damages can affect output, utility, and/or capital, and they can be either unbounded or bounded through a rational or exponential mapping. Most specifications represent damages as functions of temperature increase over preindustrial levels, while others take into account additional climate parameters, such as rate of warming or absolute CO_2 concentration. Rate terms reflect damages to which society can adapt when warming slows, while certain biogeochemical impacts (notably, CO_2 fertilization and ocean acidification) are controlled by concentration rather than level of warming. The damage functions are typically power functions of these parameters. Some damage functions attempt to treat uncertain catastrophic damages in an explicit, stochastic fashion, whereas others fold their expected impacts into deterministic terms. IAMs can also take into account damages to environmental goods that do not become more abundant with increasing material consumption, either by including within the damage function terms that increase with material consumption, or through a modification of the production or utility functions that effectively achieves the same end. Income elasticity of damages can also reflect greater adaptive capacity at higher income levels.

The damage functions evaluated in this study are summarized in Table 1 and discussed below. Table 1 also summarizes the abbreviations used to refer to these damage functions in the remainder of the paper. Figures 1 and 2 show damages as a function of temperature with different specifications; Figures 3 and 4 provide an alternate perspective, showing the implications for consumption in the reference scenario of different damage functions.

In this paper, we focus on damage specifications that can be implemented in a one-region, DICE-type model, in order to highlight the sensitivity of SCC results to choices about functional form. Table 2 provides parallel descriptors for damage specifications that appear in cost-benefit IAMs other than DICE. In the context of multi-region models like PAGE and FUND, other authors (e.g., Hope, 2008; Anthoff et al., 2009) have dealt with questions related to how to combine social costs experienced in different regions with different levels of vulnerability; addressing these issues in the context of this paper would expand the scope too broadly. Like DICE and PAGE and unlike FUND, we do not model separate economic sectors with separate damage functions; such a choice would similarly obscure our focus, which is on how different choices about extrapolation affect the SCC. Nonetheless, the core elements of the functional forms used in all the sectors in FUND do appear – albeit within in a single sector – in the specifications we examine.

2.1 The nature of damages

In principle, damages from climate change can negatively impact three different terms in an economic model: utility U, output Y, and capital K. Damages to utility represent negative impacts to the ability of individuals to benefit from material consumption; damages to output represent negative impacts on the ability to make productive use of capital; and damages to capital are simply that. (Fankhauser and Tol (2005) describe a similar division, between non-market impacts, market impacts, and accelerated capital depreciation; they also note the possibility of damages to population via health impacts.)

Following the standard DICE formalism but keeping the savings rate fixed, we employ a Solow production model. In the absence of climate damages,

$$Y = AL^{1-\gamma}K^{\gamma} \tag{1}$$

$$\dot{K} = sY - Kd \tag{2}$$

Table 1: Key characteristics of damage functions assessed

	Source	Function of	Functional form	Uncertainty	Bounding mapping	Applied to
D	DICE 2007	Temperature	aT^2	No	Rational	Output
We	Weitzman (2009b)	Temperature	aT^2	No	Exponential	Output
L	Lempert et al. (2000)	Temperature, Rate	$aT^2 + b(T - \bar{T}_{30})^3$	No	Rational	Output
SP	Sterner and Persson (2008)	Temperature	aT^2	No	Rational	Output, Eco-Goods
Wa	Weitzman (2009a)	Temperature, Consumption	aCT^2	No	Rational	Utility
Ad	FUND-inspired	Temperature, Consumption	$aC^{-0.4}T^2$	No	Rational	Output
K	Keller et al. (2004)	Temperature	aT^2 at $T < T_c$; $aT^2 + x$ at $T \geq T_c$	Threshold damages	Rational	Output
AL	Azar and Lindgren (2003)	Temperature.	aT^2 or bT^4	Functional form	Rational	Output
ASB	Ackerman et al. (2010)	Temperature	aT^x	Exponent	Rational	Output

Table 1: Key characteristics of damage functions assessed (continued)

	Source	Function of	Functional form	Uncertainty	Bounding mapping	Applied to
Xa	This study	Temperature	See text	Threshold damages	Rational	Output
Xaa	This study	Temperature, Consumption	See text	Threshold damages	Rational	Output
Xb	This study	Temperature, Consumption	as **Xaa**	Threshold damages	Rational	Capital, Output, Eco-Goods
Xau	This study	Temperature	See text	See text	Rational	Output
Xaau	This study	Temperature, Consumption	See text	See text	Rational	Output
Xbu	This study	Temperature, Consumption	as **Xaau**	See text	Rational	Capital, Output, Eco-Goods

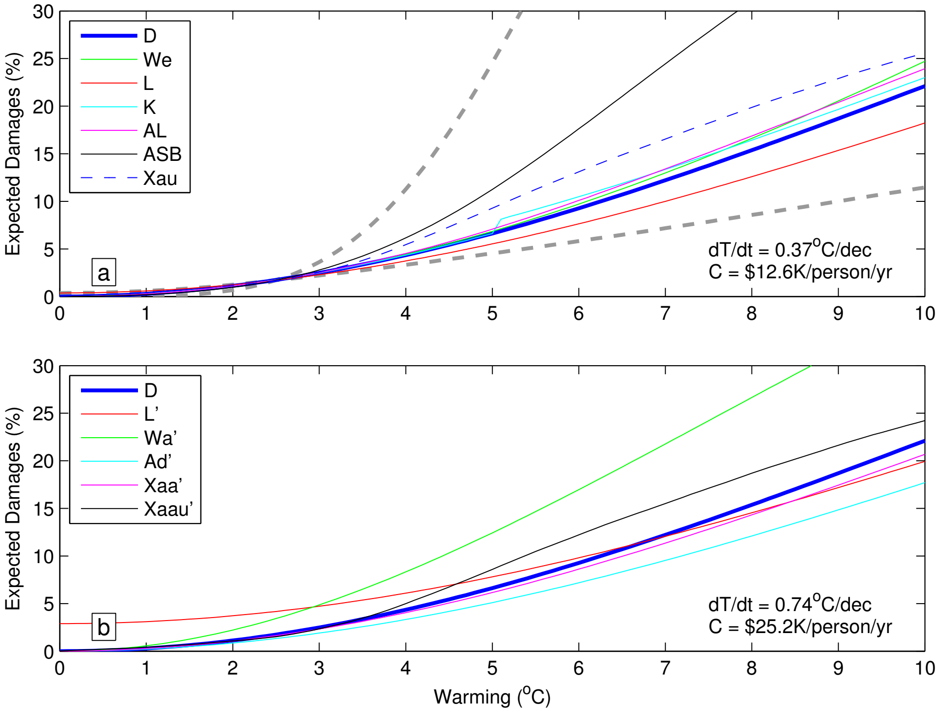

Figure 1: Expected climate damages for different damage functions, calibrated such that they all agree with the default DICE damage function for 2.5°C warming. Damage functions are defined in Table 1. In (a), rate of temperature increase and level of material consumption are set at the calibration levels (the values achieved at 2.5°C warming in the reference scenario with damages **D** and 3°C/CO_2 doubling climate sensitivity); in (b), they are set to twice these levels. In (a), heavy dashed grey lines indicates the lowest 5th percentile value and the highest 95th percentile value across all damage functions, excluding **Xau**, **Xaau** and **Xbu**. In (a), **SP**, **Wa**, **Ad**, **Xa**, **Xaa** and **Xb** match **D** and so are not shown. In (b), only **D** and functions with rate or consumption sensitivity are shown. Specifications are labeled with primes in the legend to highlight that they are shown for non-calibration levels of warming rate and consumption. **Xb** and **Xbu** match **Xaa** and **Xaau**, respectively.

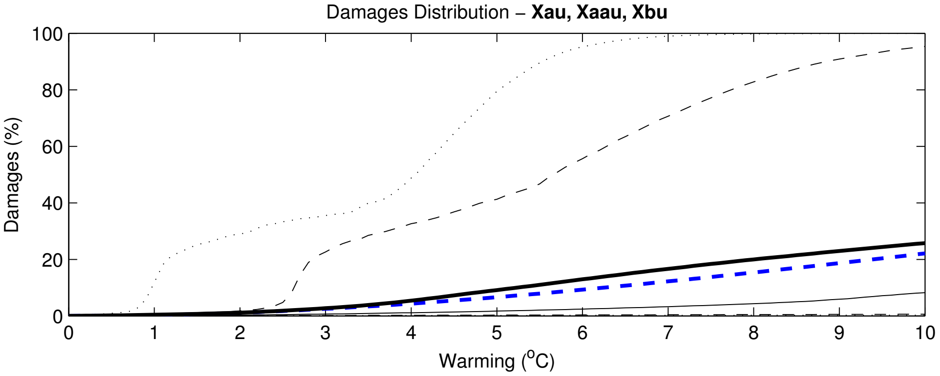

Figure 2: Distribution of damages for **Xau**, **Xaau**, and **Xbu** at the calibration level of consumption. (Damages are identical for all three specifications.) The heavy black line indicates expected damages, while the thin line indicates median damages, dashes lines indicates the 5th and 95th percentile, and dotted lines the 1st and 99th percentiles. (The difference between expected and median damages arises from the skewness of the distribution.) The jaggedness of the dotted line reflects numerical noise. The dashed blue line, for reference, indicates the standard DICE damage function.

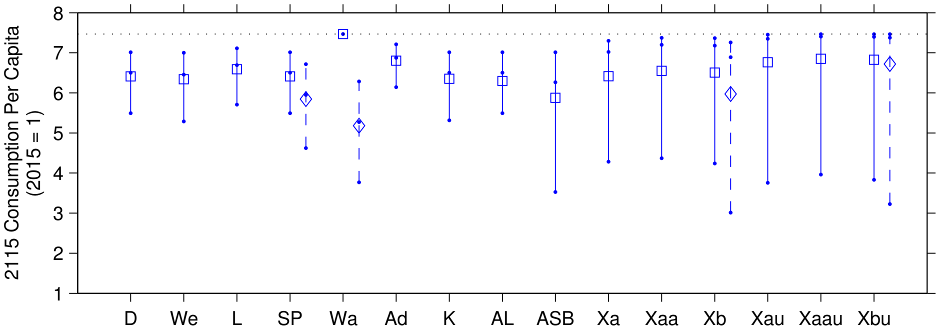

Figure 3: Consumption per capita in 2115 for the reference scenario with different damage specifications. Squares indicate expected changes, while dots connected by lines indicates 5th, 50th, and 95th percentiles. When ecological goods (**SP**, **Xb**, **Xbu**) or a modified utility function (**Wa**) are used, diamonds and dashed lines indicate effective consumption per capita. In the absence of climate damages, per capita consumption in 2115 is 7.5 times per capita consumption in 2015 (denoted by dotted horizontal line).

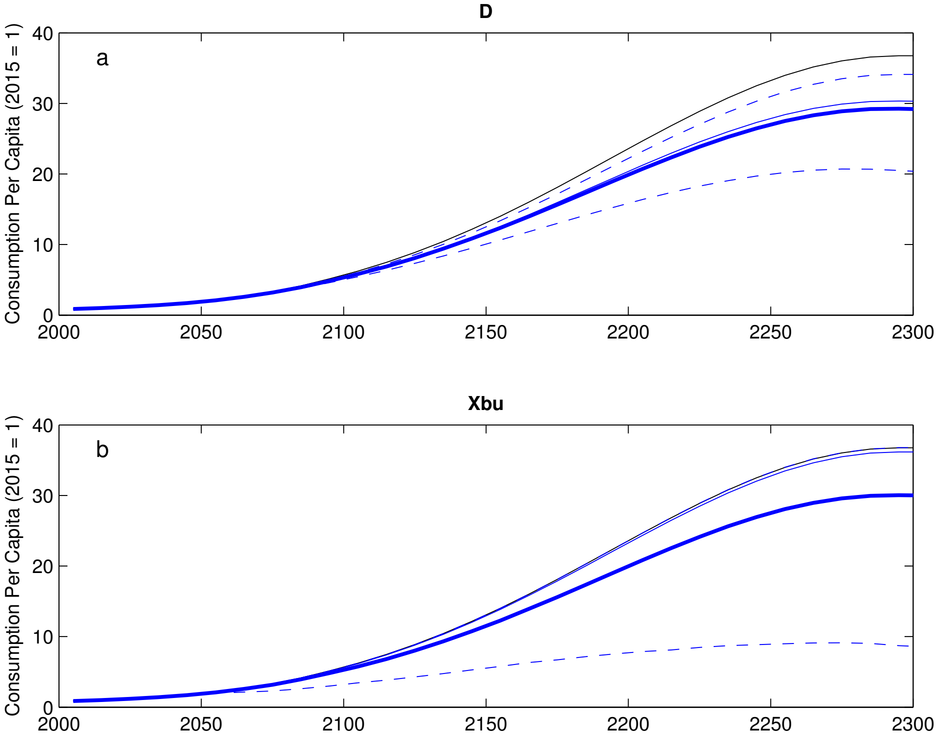

Figure 4: Changes in consumption per capita for damage specifications (a) **D** and (b) **Xbu**. Black line indicates the reference path. Heavy line represents expected values, light solid line indicates median values, and dashed lines indicate 5th and 95th percentiles. In (b), the 95th percentile path is visually indistinguishable from the reference path.

Table 2: Key characteristics of damages in other major cost-benefit IAMs

IAM	**Function of**	**Functional form**	**Uncertainty**	**Bounding mapping**	**Applied to**
FUND	Temperature, Rate, CO_2 concentration, Income	Varies by sector	No	By sector	Output
PAGE	Temperature	aT^x at $T < X$; $aT^x + y$ at $T \geq X$	Exponent and uncertain threshold damages	No	Output
MERGE	Temperature	aT^2	No	No	Output
CETA-M	Rate	$a\dot{T}^b$	No	No	Output

$$
\begin{aligned}
C &= (1-s)Y/L && (3)\\
U_C &= \frac{C^{1-\eta}}{1-\eta} && (4)
\end{aligned}
$$

where C denotes per capita material consumption, U_C represents the utility of a representative agent based upon his or her material consumption, A denotes total factor productivity, L represents population, γ is the elasticity of capital in the production function, s is the savings rate, d is the depreciation rate, and η is the elasticity of the marginal utility of consumption. Note that climate change impacts on labor productivity, another possible pathway for damages, would manifest as decreases in A and are therefore equivalent to impacts on output in this formalism. (For our numerical calculations, we keep s fixed at 20% of output and, following DICE, set $\gamma = 0.3$ and $d = 10\%$ per year. The time series of A and L are exogenously specified by the scenario.)

As discussed at greater length in section 2.4, Sterner and Persson (2008) introduce a utility function for a representative agent that is dependent upon both material consumption and consumption of ecological goods. Effectively, their model can be viewed as substituting effective consumption per capita C_{eff} for the standard material consumption per capita C in the utility function:

$$
\begin{aligned}
C_{\text{eff}} &= [(1-\beta)C^{1-1/\sigma} + \beta E^{1-1/\sigma}]^{\sigma/(\sigma-1)} && (5)\\
U &= \frac{C_{\text{eff}}^{1-\eta}}{1-\eta} && (6)
\end{aligned}
$$

where E is the per capita consumption of ecological goods, β is the fraction of utility accounted for by consumption of ecological goods, and σ is the elasticity of substitution between material and ecological goods. The following discussion employs this formalism for generality.

We represent damage functions as the composite of two functions, a function D with an arbitrary form and a function Ω that can map D into a bounded interval. Early versions of DICE (e.g., Nordhaus, 1992) used D directly as the damage function; that is, $\Omega(D) = D$, so that output net of damages was given by

$$
Y' = Y \times \Big(1 - \Omega\big(D(T)\big)\Big) = Y \times D(T). \qquad (7)
$$

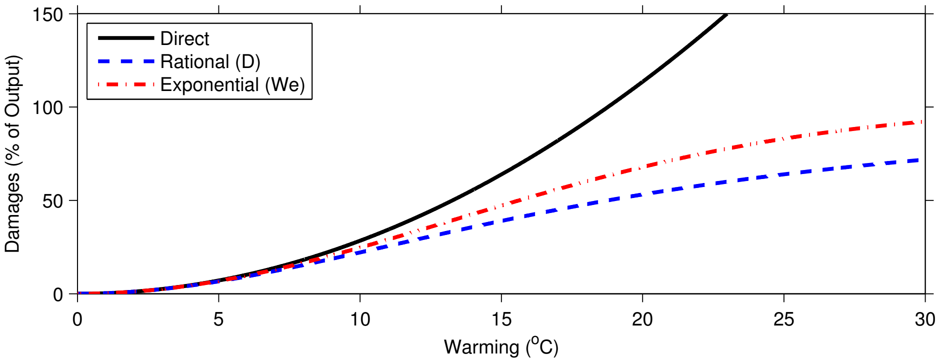

Figure 5: A polynomial damage function ($D(T) = aT^b$, $b = 2$), with a direct mapping ($\Omega(D) = D$), a rational mapping ($\Omega(D) = 1 - 1/(1+D)$), and an exponential mapping ($\Omega(D) = 1 - e^{-D}$). Early versions of DICE used a direct mapping; current versions use a rational mapping, as do most of the damage specifications in this analysis. Weitzman (2009b) suggested an exponential mapping. The rational mapping shown here matches our damage function **D**; the exponential mapping matches our damage function **We**.

This representation can at sufficiently high temperatures give rise to negative net output. To avoid this consequence, Nordhaus and Boyer (2000) bound damages at 100% of GDP by the rational mapping

$$\Omega(D) = 1 - 1/(1+D). \tag{8}$$

Weitzman (2009b) (our damage function **We**) suggests bounding damages using an exponential mapping

$$\Omega(D) = 1 - \exp(-D) \tag{9}$$

rather than a rational mapping. This mapping matches the standard DICE mapping at low-to-moderate temperatures but approaches 100% of GDP more rapidly at higher temperatures (Figure 5). As indicated in Table 1, all of the damage specifications in our analysis except **We** employ a rational bounding mapping.

Let f_U, f_Y, f_K represent the initial fractional contribution of damages to utility, output and capital to reducing utility ($f_U + f_Y + f_K = 1$). Then damages Ω can be distributed:

$$\begin{aligned}
K' &= (1-\Omega)^{f_K/\gamma} \times K && (10)\\
Y^\dagger &= AL^{1-\gamma}K'^{\gamma} = (1-\Omega)^{f_K} \times AL^{1-\gamma}K^{\gamma} = (1-\Omega)^{f_K} \times Y && (11)\\
Y' &= (1-\Omega)^{f_Y} \times Y^\dagger = (1-\Omega)^{f_K+f_Y} \times Y && (12)\\
E' &= (1-\Omega)^{f_K+f_Y} \times E_0 && (13)\\
C^\dagger &= (1-s)Y'/L = (1-\Omega)^{f_K+f_Y} \times C && (14)\\
C^\dagger_{\text{eff}} &= [(1-\beta)C^{\dagger 1-1/\sigma} + \beta E'^{1-1/\sigma}]^{\sigma/(\sigma-1)} && (15)\\
&= (1-\Omega)^{f_K+f_Y} C_{\text{eff}} && (16)\\
C'_{\text{eff}} &= (1-\Omega)^{f_U} \times C^\dagger_{\text{eff}} = (1-\Omega) \times C_{\text{eff}} && (17)
\end{aligned}$$

where E_0 is the supply of ecological goods in the absence of climate change damages and K', Y', E', and C'_{eff} represent respectively capital, output, consumption of ecological goods, and effective consumption after adjustment for damages.

Even if calibrated thus, so that the immediate impact on utility is the same, damages to utility, output, and capital have moderately different long-term implications. Damages to utility impact well-being but not the growth of the material economy. Damages to output leave current capital untouched but reduce investment and therefore future capital and output. Damages to capital will produce the same investment reduction as damages to output, while also impacting current capital. Consider the effect of a short but very severe shock to each of these factors: a severe shock to utility will temporarily make people very unhappy; a severe shock to output will cause a recoverable depression; and a severe shock to capital will require a protracted period of rebuilding. For the same damage function, the SCC will therefore be higher the greater the proportion of damages accruing to capital, and lower the greater the proportion accruing to utility, as can be seen in Figure 6. In this example, using the standard DICE damage function, a constant climate sensitivity of 3°/CO_2 doubling and a flat 3% discount rate, damages applied purely to output yield an SCC in 2015 of \$33/tonne CO_2, damages applied purely to utility yield an SCC of \$25 ton/CO_2, and damages applied purely to capital yield an SCC of \$41/tonne CO_2.

Most of the damage specifications in the literature impact output, while Weitzman (2009a) simplifies his discussion by focusing on utility damages, and the

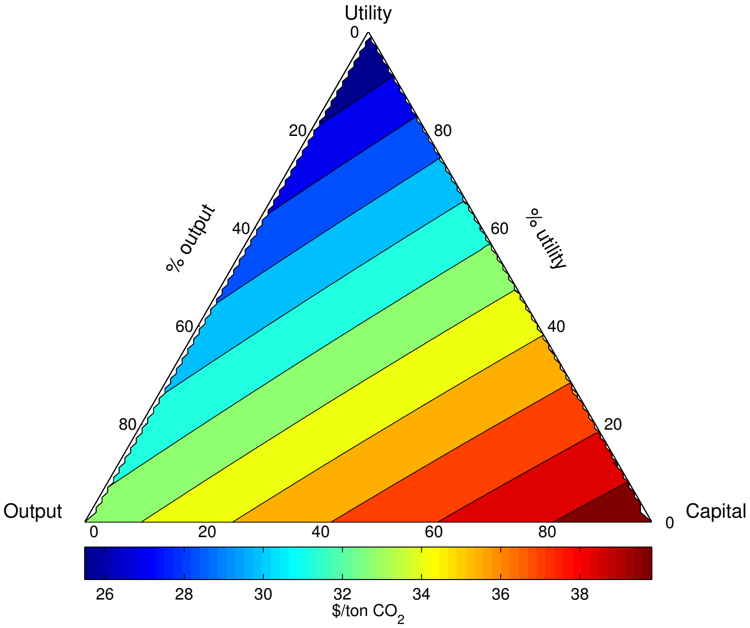

Figure 6: 2015 social cost of carbon, calculated at a flat 3% discount rate off of the reference scenario, using the standard DICE damage function **D** applied in different proportions to capital, output, and utility. Most models apply damages strictly to output (bottom left corner). Pure capital damages and pure utility damages are shown in the bottom right corner and the top, respectively. (The ragged edges are a computational artifact.)

ecological damages in the Sterner-Persson model similarly appear in the utility function. Fankhauser and Tol (2005) construct a simple climate-economic model for the purpose of examining the multiple pathways by which climate change can cause economic damages; this paper is the only example we are aware of examining direct capital impacts in a standard climate change IAM.

2.2 Polynomial functions of temperature

As noted in the introduction, climate damages in DICE are a quadratic function of global average temperature increase over preindustrial levels ($D(T) = aT^b$,

$b = 2$). This expression for D is employed in our damage specification **D** with a rational mapping (i.e., **D** is the standard DICE damage function); our damage specification **We**, following Weitzman (2009b)'s suggestion, uses the same D with an exponential mapping.

A number of IAMs adopt similar polynomial forms to DICE. The Model for Estimating the Regional and Global Effects (MERGE) (Manne et al., 1995) adopts the DICE quadratic function for market damages and applies a separate willingness to pay (WTP) function for non-market damages. PAGE (Hope, 2006) models economic, non-economic, and catastrophic ("discontinuity") damages separately for eight geographic regions, all as power functions in a form also similar to DICE, but with the exponent treated as uncertain, ranging from 1 to 3, instead of being fixed at 2 as in DICE.

Ackerman et al. (2010) (our damage specification **ASB**) introduces greater uncertainty in the damages exponent by modifying DICE, treating b as a random variable with a triangular distribution with mode 2, minimum 1, and maximum 5. Along similar lines, Mastrandrea and Schneider (2001) adjust b through iteration with a simple model of the Atlantic Meridional Overturning Circulation (AMOC) to incorporate the expected damages associated with decline or collapse of AMOC.

2.3 Polynomial functions of temperature and rate of warming

A number of functions are dependent on the trajectory of temperature increase. Damages in the Carbon Emissions Trajectory Assessment (CETA) model (Peck and Teisberg, 1992), for instance, depend on the decadal rate of temperature change. Lempert et al. (2000) (our damage specification **L**) develop a damage function intended to capture the impacts of climate variability. Their function includes a term with the five-year running average of temperature for variability to which society and ecosystems can adapt on the timescale of several years, and another term with 30-year running averages for variability to which society and ecosystems adapt over longer timescales:

$$D(T) = a_1 \overline{T}_5^{b_1} + a_2 (T - \overline{T}_5)^{b_2} + a_3 (T - \overline{T}_{30})^{b_3} \tag{18}$$

where $\overline{T}_5$ and $\overline{T}_{30}$ are five- and thirty-year running averages of temperature. (Since our model has ten-year time steps, $T = \overline{T}_5$, so we set a_2 to zero in our implementation of this function.)

2.4 Polynomial functions of temperature and consumption or income

Damage functions are typically expressed as a fractional loss of output, but some work has explored damage specifications in which effective damages at a given temperature increase more rapidly than output. These specifications represent damages to environmental goods (for example, clean air and water or ecological diversity) for which material goods are imperfectly substitutable. As material consumption increases, the relative price of the fixed supply of environmental goods also increases, and so the welfare impacts of damages to this supply become more severe. As mentioned previously, Sterner and Persson (2008) model these imperfectly substitutable goods directly. Following their numerical example, our damage specification **SP** sets $\beta = 0.1$ and $\sigma = 0.5$ in equation 5.

Weitzman (2009a) presents an "additive" specification of damages, which he contrasts with the standard "multiplicative" specification. His terminology derives from the appearance of the utility function with the two different specifications when η is equal to 2. In equation 4, with $\eta = 2$ and all damages applied to utility,

$$U_C' \sim \left(C \times (1 - \Omega)\right)^{-1}. \tag{19}$$

If Ω is a rational mapping,

$$U_C' \sim 1/C \times (1 + D); \tag{20}$$

this is what Weitzman (2009a) calls a multiplicative specification. The additive specification is given by

$$U_C' \sim 1/C + D; \tag{21}$$

this is equivalent to

$$C' \sim \frac{C}{1 + DC}, \tag{22}$$

to

$$\Omega = \frac{1}{1+DC}, \tag{23}$$

and to the standard formalism if $D' = DC$ is substituted for D. The additive specification (our damage specification **Wa**) is thus equivalent to making relative damages a function of material consumption as well as temperature:

$$D(C,T) = aCT^b. \tag{24}$$

(We employ $b = 2$ for **Wa**). Weitzman argues that this alternative form is as plausible as its standard "multiplicative" equivalent on a priori grounds. He also shows that shows that the Sterner-Persson utility function possesses the same properties as a utility function with both additive and multiplicative damage terms.

Note that – due to the relative price effects Sterner and Persson (2008) discuss – the introduction of multiple, non-substitutable utility-enhancing goods has important implications for SCC calculations. The SCC is defined as the ratio of the change in expected welfare from a unit of emissions to the change in expected welfare from a unit of material consumption in the period of emissions. If E is declining over time – or even increasing slower than C – the relative value of a unit of material consumption is declining over time. This effect will appear in the denominator of the SCC, but it will appear more strongly in the numerator, where a stream of damages is being inflicted on a world with lower utility than a world where C and E were perfectly substitutable. For values of η less than one, lower utility worlds make a smaller contribution to welfare than higher utility worlds, so the net effect will be to decrease the social cost of carbon. In the calculations below where we employ a Sterner-Persson utility function, we therefore present results calculated in terms of equivalent first-period material consumption: essentially, converting damages from current dollars to constant dollars. In constant dollars, incorporating an imperfectly substitutable, non-increasing good will increase the SCC. The conversion to constant dollars is given by:

$$\begin{aligned} C_{\text{equiv}} &= \left(\frac{C_{\text{eff}}^{1-1/\sigma} - \beta E_0^{1-1/\sigma}}{1-\beta}\right)^{\sigma/(\sigma-1)} \quad (25) \\ &= \left(C^{1-1/\sigma} + \frac{\beta}{1-\beta}\left(E^{1-1/\sigma} - E_0^{1-1/\sigma}\right)\right)^{\sigma/(\sigma-1)}. \quad (26) \end{aligned}$$

(For reference, we also report current dollar values without this conversion in Tables 4 and 5.)

As noted in the introduction, many of the damage functions in FUND include per capita income as a term, in order to reflect greater adaptive capacity in higher income societies (Anthoff and Tol, 2010). Some sectoral damages in FUND are modulated by terms such as $(y/y_0)^{\varepsilon}$, where y is per capita GDP, y_0 is per capita GDP in a reference year, and ε is an income elasticity. Since in our formalism consumption is a fixed fraction of output, this can be viewed as equivalent to a damage function of the form

$$D(C,T) = aC^{\varepsilon}T^{b}, \tag{27}$$

where $\varepsilon < 0$ reflects increasing adaptive capacity in higher consumption societies and $\varepsilon > 0$ reflects effects of the sort discussed by Sterner and Persson and by Weitzman. In FUND, ε ranges from –2 for health impacts, to between –0.2 and –0.3 for water, energy, and agricultural impacts, to 0 for sea level rise impacts. Based on a weighted average of income elasticities across sectors in FUND, we construct damage specification **Ad** with $\varepsilon = -0.4$ (and the DICE standard $b = 2$).

2.5 Uncertainty in functional form

Several damages specifications use uncertainty in functional form to represent more explicitly impacts from low-probability catastrophic events. For example, PAGE (Hope, 2006) allows for a discontinuity that causes damages equal to 5–20% of GDP. The probability of such a discontinuity increases at a linear rate with increasing temperature. Keller et al. (2004) (our damage specification **K**) model damages from the collapse of the AMOC, assumed to vary uniformly between 0 and 3 percent of global GDP, by adding a term for damages above a certain threshold.

Other modelers represent states of the world that experience catastrophic damages with higher values for the exponent b in the damage function. Azar and Lindgren (2003) (our damage specification **AL**) model two uncertain future states of the world: a high-probability state in which damages follow the quadratic form of DICE, and a low-probability but catastrophic state in which damages are a quartic function of global average surface temperature change. Similarly, as mentioned previously, Ackerman et al. (2010) (**ASB**) treat the exponent b of temperature in the damage function as uncertain; higher values of b cause damages to increase more rapidly with temperature and represent states of the world more subject to catastrophic climate change.

2.6 Disaggregation of damages

We introduce six new damage specifications that combine elements of the literature specifications discussed above to progressively disaggregate the standard DICE damage function.

Whereas the standard DICE damage function treats catastrophic damages through their expected value, specification **Xa** follows **K** and PAGE in separating the damage function into a deterministic gradual damages term and an uncertain threshold damages term:

$$D(T) = a_1 m(T) T^b + a_2 \Big/ \Big(1 + \exp(a_3(T_c - T))\Big) \tag{28}$$

where $m(T)$ is a multiplier equal to 1 at low-to-moderate values of T, T_c is the uncertain temperature threshold for a catastrophe, and a_3 (set equal to 10) controls the warming range over which the catastrophe occurs. To calibrate, we note that Nordhaus (2007) estimates that risk-neutral expected damages from catastrophic climate change are responsible for 66% of total damages (0.61% of GDP) at the 2.5°C calibration point and that Nordhaus and Boyer (2000) defines a catastrophe as causing 30% loss of global GDP indefinitely. Using these constraints to set a_1 and a_2, we then calibrate the distribution of T_c and the value of $m(T)$ so that expected damages match their standard DICE values.

Damage specification **Xaa** builds upon **Xa** by allowing adaptation to gradual impacts; as in **Ad**, the income elasticity of damages ε is set equal to –0.4. Damage specification **Xb** distributes the damages in **Xaa** between output, capital, and Sterner-Persson-type environmental goods, as described in section 2.1. Nordhaus (2007) attributes about one-quarter of non-catastrophic damages at 2.5°C to his "ecosystems and settlements" sector; we assume that about two-thirds of those damages are to environmental goods, and therefore infer that about 15% of total consumption is accounted for by environmental goods (i.e,. we set β in equation 5

Table 3: Uncertain Parameters in **Xau**, **Xaau**, and Xbu

Parameter	**Mean**	**Median**	**Range**	**Distribution**	**Uncertain in**
Climate sensitivity (°C/CO$_2$ doubling)	3.5	3	1.6–8.2	Roe-Baker	all
Damage exponent b	2	1.7	0.53–5.3	Log normal	**Xau**, **Xaau**, **Xbu**
Expected damages at 2.5°C (%)	1.77	1.25	0.25–6.4	Log normal	**Xau**, **Xaau**, **Xbu**
Damage associated with catastrophe (%)	30	30	15–45	Triangular	**Xau**, **Xaau**, **Xbu**
Threshold for catastrophe (°C)	61	12.4	1.94–497	inferred	**Xau**, **Xaau**, **Xbu**
Adaptation income elasticity ε	-0.4	0.4	-0.8–0	Triangular	**Xaau**, **Xbu**
Eco-goods elasticity of substitution σ	0.75	0.75	0.5–1.0	Triangular	**Xbu**
Eco-goods share of consumption β	0.15	0.15	0–0.3	Triangular	**Xbu**
Capital share of damages f_k	0.15	0.15	0–0.3	Triangular	**Xbu**

Ranges show full range for triangular distributions and 2.5th to 97.5th quantile for others.

to 0.15). Assuming that another third of this sector constitutes damages to capital, and that one third of damages due to coastal impacts are also to capital, we infer that another 15% of damages occur via capital losses. Again following Sterner and Persson's numerical example, we set σ in equation 5 to 0.5.

Damage specifications **Xau**, **Xaau** and **Xbu** (Figure 2) add uncertainty to, respectively, **Xa**, **Xaa** and **Xb**. Distributions of key parameters for these specifications are summarized in Table 3. We let b be log normally distributed with a mean of 2.0 and a geometric standard deviation of 1.4 (i.e., a 95% uncertainty range spanning a factor of two). We let expected damages at 2.5°C be log normally distributed with a mean of 1.77% of global GDP (matching DICE) and a geometric standard deviation of 2. We keep gradual damages at one-third of expected damages. The damage resulting from a catastrophe is triangularly distributed, with a mean of 30% of global GDP and a range of 15% to 45%. For **Xaau** and **Xbu**, the income elasticity of damages is triangularly distributed with a mean of –0.4 and a range of –0.8 to 0. For **Xbu**, the elasticity of environmental goods in the utility function is triangularly distributed between 0.5 and 1.0, with a mean of 0.75. Both the share of environmental goods in the utility function and the fraction of damages accruing through capital impacts are triangularly distributed with means of 0.15 and ranges of 0 to 0.3.

These damage specifications are intended to be illustrative, and the choices of variance are admittedly somewhat arbitrary – indeed, necessarily so, given the absence of any basis in the literature for calibrating them. Lacking other guideposts, we have chosen to rely primarily on factors of two. As an intuitive check of the reasonableness of our choices, we can examine their implications for consumption paths. The expected consumption path for **Xbu** (Figure 4) is very close to that calculated using **D**; the median consumption path for **Xbu** is higher than for **D** and the 5th percentile is considerably lower, reflecting the explicit representation of low-probability, high-impact states of the world through both the catastrophic damages term and the uncertain exponent. In **Xaau** and **Xbu**, there is an ˜1.8% probability in the reference scenario that our 24th century descendants will have lower equivalent material consumption per capita than we do, and an ˜0.8% chance that they will be eking out a subsistence existence, with a equivalent material consumption per capita of less than $2/person/day. It is our judgement that this distribution is a reasonable guess.

3 Methodology

3.1 Modeling framework

We employ a matDICE, a MATLAB-based implementation of DICE that we have written and optimized for Monte Carlo analysis. (The source code is available in the supplementary material.) We run the model in ten-year time steps from 2005 to 2305. For our reference scenario, we employ the same MiniCAM-based three-century reference scenario used as one of the scenarios in the US government social cost of carbon analysis (Interagency Working Group on the Social Cost of Carbon, 2010). To examine the effect of baseline emissions on the SCC, we also employ

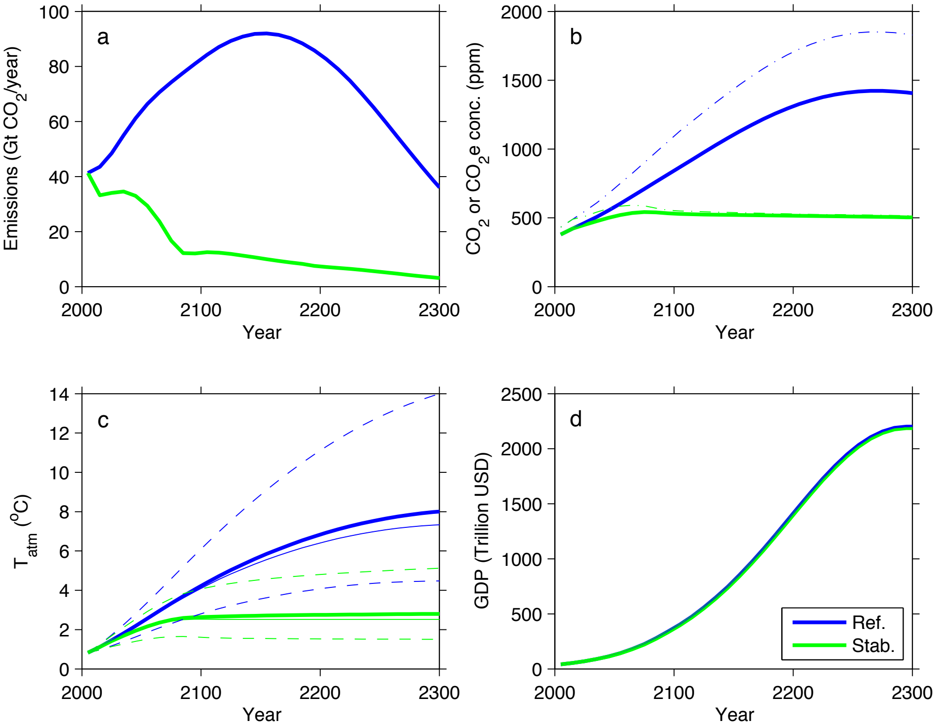

Figure 7: Reference (blue) and stabilization (green) scenarios used for the SCC calculations in this paper, showing (a) CO_2 emissions, (b) CO_2 (solid) and CO_2e (dotted-dashed) concentrations, (c) global mean warming, and (d) GDP. Dashed lines indicate 5th and 95th percentile values for temperature, while the thin solid lines indicate median projections and the heavy solid lines indicate expected values. Projections shown here exclude the deleterious effects of climate damages on GDP and thus emissions. These effects are taken into account when calculating the SCC.

a stabilization scenario calculated to yield a 50% chance of limiting warming to 2.5°C (Figure 7).[3]

We replace the default DICE damage function with the damage functions listed in Table 1, calibrated such that all the functions are in agreement on climate damages at 2.5°C. Rate- or consumption-dependent functions are calibrated at the same warming rate (0.37°/decade) and per capita consumption level ($12.6 thousand/person/year) as is reached in the reference scenario with the standard DICE damage specification **D** and a climate sensitivity of 3°C per CO_2 doubling. For our primary calculations, we set the calibration point to agree with the default DICE 2007 calibration (1.77% of GDP loss at 2.5°C). We employ a lower bound to effective consumption of $500/person/year. (See the discussion in Weitzman, 2009b, on bounding damages with a parameter akin to the value of a statistical life.)[4]

Following the US government analysis, we treat climate sensitivity as an uncertain parameter with a Roe and Baker (2007) distribution, truncated at 10°C per CO_2 doubling and calibrated such that the median value is 3°C per CO_2 doubling and the 67% range is approximately 2 to 4.5°C per CO_2 doubling. We take one thousand samples evenly from the distribution, and we use Latin hypercube sampling when considering more than one uncertain parameter. (For specifications **Xau**, **Xaau** and **Xbu**, with a larger number of random variables, we use five thousand samples).

For the reference scenario, the standard DICE damage function **D**, and a flat 3% discount rate, our model yields a 2010 SCC of $37/tonne CO_2. This value which can be compared to the value of $29/tonne calculated with DICE for the same year,

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[3] In addition to the modifications to the damage specifications, we have also modified matDICE to allow for mitigation of land use CO_2 emissions and non-CO_2 emissions. Non-CO_2 emissions are represented in DICE by a single non-CO_2 forcing value; in matDICE, mitigating CO_2 emissions also proportionally reduces the non-CO_2 forcing, with a lag time of one time step (ten years).

[4] As a sensitivity test, we ran **Xbu** with $\eta = 1.4$ and two alternative lower bounds to effective consumption, $125/year and $2000/year. With a $500 lower bound, the SCC is $155/tonne; with the two alternative bounds, it is $168/tonne and $132/tonne, respectively. At $\eta = 0$, alternate choices resulted in negligible changes to the SCC (<1%), as would be expected given the minimal contribution of low output worlds to expected welfare at low value of η. Our default value of $500/year is consistent with the World Bank definition of extreme poverty, $1.25/person/day in 2005 US dollars

scenario, and discount rate in the U.S. government analysis. The slight difference is largely attributable to a change in the calibration of the climate and carbon cycle transfer coefficients between DICE 2007, used in the U.S. government analysis, and DICE 2010, whose values we employ.

3.2 Discounting and risk aversion

The social cost of carbon (called the "certainty-equivalent SCC" by Newbold et al., 2010) is defined as the change in expected welfare from a unit emission of carbon dioxide in a given year, normalized to change in expected welfare from a unit of consumption in the same year. Note that it is distinct from "deterministic SCC" values, which are ratios of the corresponding changes in welfare, conditioned upon a specific state of the world. It makes sense to talk about a probability distribution for the latter, but not for the former. As shown by Newbold et al. (2010), the expected deterministic SCC is equal to the SCC when the denominator is known with certainty (e.g., in the first time step), but not generally. In the presentation of our results, we show both the SCC and the distribution of deterministic SCC values.

In the isoelastic utility function employed by DICE and other IAMs, the elasticity of the marginal utility of consumption η serves as both the coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution. The value of η and the pure rate of time preference ρ are related to the discount rate r and the growth rate of (effective) per capita consumption g via Ramsey's rule:[5]

$$1+r=(1+\rho)(1+g)^{\eta} \tag{29}$$

Increasing risk aversion while holding the pure rate of time preference ρ constant will therefore also increase the discount rate.

Rather than disaggregating discount rates into values of η and ρ along lines of equation 29, the U.S. government SCC analysis used flat discount rates of 2.5%,

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[5] This expression is frequently written as $r=\rho+\eta g$. This alternative form is an approximation when applied in the context of discrete time models like this one, where discount factors are of the form $(1+r)^{-t}$. It is strictly true for continuous times models, where discount factors are of the form $\exp(-rt)$.

3% and 5% per year. Because the growth rates in that analysis's reference scenarios (as in our scenarios) decline to zero between 2100 and 2300, the choice of flat discount rates is strictly consistent only with $\eta = 0$. (This choice is consistent with the U.S. government analysis's risk neutrality, but inconsistent with its description of a plausible range for η as being between 0.5 and 4.)

In our analysis, we vary η and ρ such that, in the reference scenario in the absence of climate change damages, the average discount rate r over 2015–2115 remains fixed at the U.S. government analysis's middle value of 3% per year. We view this 3% value as an "observed" risk-free interest rate specified by the scenario, with which any decomposition into η and ρ must be consistent. (We do not address the separate scenario-design question of whether the 3% rate is a reasonable choice.) We choose four different values of η (0, 1.0, 1.4 and 2.0), which, in light of the average 2.03% annual growth rate between 2015 and 2115, lead to selection of values of ρ of 3.0%, 0.95%, 0.14% and –1.1% per year, respectively.[6]

These choices allow us to investigate the implications for the SCC of different values of the elasticity of the marginal utility of consumption while retaining consistency with the initial risk-free interest rates specified in the U.S. government analysis. Within the constraints of the isoelastic utility function, however, it is not readily possible to isolate the two different effects of increasing η – increasing risk aversion and increasing the weight placed on damages in the slower growing 22nd and 23rd centuries. Caution should therefore be used in attributing the differences between SCCs calculated at different values of η solely to risk aversion; for a single pair of η and ρ, however, the differences in SCCs for damage functions giving rise to nearly the same expected consumption path with different levels of uncertainty (e.g., **D** vs. **Xa** vs. **Xau**) can be so attributed.

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[6] The negative value of ρ associated with $\eta = 2$ is necessary for consistency with the scenario-specified interest rate. We do not address here the question of whether negative values of pure rate of time preference are reasonable; see Loewenstein and Prelec (1991) and Atkinson et al. (2009) for behavioral evidence that it can be under some circumstances. Nevertheless, to avoid this issue, we choose in our discussion to focus on $\eta = 1.4$ and $\rho = 0.14\%$ as our illustrative moderate-risk aversion set of preference.

4 Results and discussion

The consequences of changing the damages specification depend strongly on η (Table 4, Figure 8). For the risk neutral case ($\eta = 0$), specifications with similar expected damages (Figure 1) yield similar results. Thus **D**, **We**, **K**, **L** and **Xa** cluster in the range of \$34–\$39/tonne CO_2 emitted in 2015. Because of the modestly (**AL**, **Xau**) or highly (**ASB**) uncertain exponent in their damage functions, **AL**, **Xau** and **ASB** are associated with higher SCC values of \$42, \$44, and \$62/tonne, respectively. **Wa** is also high, at \$49/tonne, reflecting the increasing severity of damages under this specification as material consumption increases. As noted previously, due to relative price effects, employing a Sterner-Persson damage function decreases the SCC (\$26 and \$23/tonne for **SP** and **Xb**). When these SCCs are corrected for price effects, they roughly double. The incorporation of adaptation through a negative income elasticity of damages modestly reduces the SCC (\$30 for **Ad** vs. \$37 for **D**, \$33 for **Xaa** vs. \$35 for **Xa**, and \$42 for **Xaau** vs. \$44 for **Xau**). (In all the specifications in our analysis, the costs of adaptation are implicitly included in the damages.)

When risk aversion is taken into account, the SCC depends not only on expected damages but also on the variability of damages. Since we keep the average discount rate over 2015–2115 constant while increasing risk aversion, SCC values from deterministic damage functions with an approximately quadratic form increase only modestly. This increase is due in part to the lowering of the discount rate in the 22nd and 23rd century, when the reference consumption growth rate slows, and in part to the risk associated with an uncertain climate sensitivity. Damages with **D**, for example, increase by about 40% as η goes from 0 to 1.4. The increase in damages with **Ad**, which lets adaptive capacity increase with income and so exhibits a smaller spread, is less (22%). The exponentially-mapped specification **We** exhibits a slightly larger response (increasing by 49%) than **D**, because of the greater damages it yields in high climate sensitivity states of the world. Although **Wa** is also deterministic, it responds more severely because of its high damages at high levels of material consumption, increasing by 165% to \$130/tonne.

SCCs associated with uncertain damage specifications also grow rapidly with risk aversion (e.g., by 63%, 114% and 120% for **Xa**, **ASB** and **AL**). The degree of this effect depends upon the variability of damages. The most uncertain damage

Table 4: SCC values and quantiles of the deterministic SCC for different levels of risk aversion and pure rate of time preference in the reference scenario

	η = 0, ρ = 3%				η = 1.0, ρ = 0.95%				η = 1.4, ρ = 0.14%				η = 2.0, ρ = -1.1%			
	SCC	**50th**	**5th**	**95th**	**SCC**	**50th**	**5th**	**95th**	**SCC**	**50th**	**5th**	**95th**	**SCC**	**50th**	**5th**	**95th**
D	$37	$33	$16	$68	$45	$39	$18	$93	$52	$44	$20	$114	$73	$57	$24	$183
We	$39	$35	$16	$75	$49	$41	$18	$108	$58	$46	$20	$138	$86	$61	$24	$247
L	$36	$32	$14	$73	$43	$37	$15	$93	$48	$40	$17	$109	$63	$50	$20	$159
SP	$49	$46	$23	$86	$70	$62	$29	$137	$89	$76	$33	$194	$172	$128	$49	$467
SP*	$26	$24	$12	$46	$45	$39	$18	$91	$65	$54	$24	$146	$172	$128	$49	$467
Wa	$49	$47	$24	$81	$85	$77	$36	$161	$130	$111	$49	$279	$394	$276	$97	$1,152
Ad	$30	$27	$14	$55	$34	$30	$15	$67	$36	$32	$15	$75	$43	$36	$17	$97
K	$39	$35	$16	$72	$48	$41	$18	$100	$55	$46	$20	$123	$77	$60	$24	$193
AL	$42	$33	$16	$68	$61	$39	$18	$93	$90	$44	$20	$114	$384	$57	$24	$183
ASB	$63	$45	$16	$185	$98	$57	$18	$357	$139	$67	$19	$574	$420	$94	$23	$1,924
Xa	$35	$15	$6	$149	$48	$18	$7	$192	$57	$20	$7	$218	$84	$26	$8	$278
Xaa	$33	$11	$5	$148	$44	$13	$5	$187	$52	$14	$5	$210	$73	$16	$6	$252
Xb	$47	$20	$8	$184	$76	$26	$9	$261	$104	$30	$10	$308	$219	$42	$13	$913
Xb*	$23	$8	$4	$119	$47	$14	$6	$205	$73	$19	$7	$265	$219	$42	$13	$913
Xau	$44	$8	$1	$210	$82	$9	$1	$358	$147	$10	$1	$551	$1,008	$13	$1	$1,772
Xaau	$42	$7	$1	$207	$76	$7	$1	$317	$130	$8	$1	$421	$718	$9	$1	$890
Xbu	$48	$9	$1	$231	$90	$10	$1	$387	$151	$10	$1	$541	$723	$12	$1	$1,350
Xbu*	$37	$6	$1	$194	$84	$8	$1	$362	$155	$10	$1	$542	$777	$15	$2	$1,700

* Sterner-Persson utilities using current rather than constant relative prices.
All values in 2007 US dollars per metric tonne CO_2 emitted in 2015.
η and ρ chosen to maintain an average discount rate of 3% per year over 2015–2115.

Table 5: SCC values and quantiles of the deterministic SCC for different levels of risk aversion and pure rate of time preference in the stabilization scenario

	$\eta = 0, \rho = 3\%$				$\eta = 1.0, \rho = 0.95\%$				$\eta = 1.4, \rho = 0.14\%$				$\eta = 2.0, \rho = -1.1\%$			
	SCC	**50th**	**5th**	**95th**	**SCC**	**50th**	**5th**	**95th**	**SCC**	**50th**	**5th**	**95th**	**SCC**	**50th**	**5th**	**95th**
D	$34	$30	$14	$69	$41	$35	$16	$88	$46	$39	$17	$102	$60	$48	$21	$144
We	$35	$31	$14	$71	$42	$36	$16	$92	$47	$39	$17	$107	$62	$49	$21	$152
L	$33	$28	$12	$69	$38	$32	$13	$84	$42	$35	$14	$95	$53	$42	$17	$127
SP	$52	$46	$21	$105	$72	$61	$26	$159	$89	$72	$30	$208	$143	$108	$43	$377
SP*	$24	$22	$11	$44	$40	$35	$16	$85	$58	$48	$21	$132	$143	$108	$43	$377
Wa	$64	$58	$26	$123	$107	$91	$37	$234	$150	$122	$48	$358	$316	$228	$83	$890
Ad	$27	$24	$12	$51	$30	$26	$13	$59	$32	$28	$13	$65	$37	$32	$15	$79
K	$35	$30	$14	$70	$42	$35	$16	$91	$47	$39	$17	$109	$63	$48	$21	$170
AL	$42	$30	$14	$69	$54	$35	$16	$88	$64	$39	$17	$102	$97	$48	$21	$144
ASB	$50	$34	$13	$148	$63	$39	$14	$204	$73	$44	$16	$245	$105	$55	$19	$369
Xa	$32	$11	$5	$113	$42	$13	$6	$176	$49	$14	$6	$205	$68	$17	$7	$278
Xaa	$29	$9	$4	$111	$38	$9	$4	$174	$45	$10	$5	$202	$60	$11	$5	$274
Xb	$44	$15	$7	$201	$68	$18	$8	$288	$89	$20	$9	$344	$163	$26	$11	$501
Xb*	$20	$7	$3	$71	$41	$10	$5	$166	$61	$13	$6	$285	$163	$26	$11	$501
Xau	$39	$6	$1	$221	$61	$7	$1	$319	$90	$8	$1	$376	$434	$10	$2	$551
Xaau	$36	$5	$1	$204	$55	$6	$1	$289	$84	$6	$1	$349	$689	$7	$1	$489
Xbu	$43	$7	$1	$249	$69	$7	$1	$370	$104	$8	$1	$465	$506	$9	$2	$643
Xbu*	$31	$5	$1	$187	$62	$6	$1	$332	$104	$8	$1	$451	$556	$12	$2	$757

* Sterner-Persson utilities using current rather than constant relative prices.

All values in 2007 US dollars per metric tonne CO_2 emitted in 2015.

η and ρ chosen to maintain an average discount rate of 3% per year over 2015–2115.

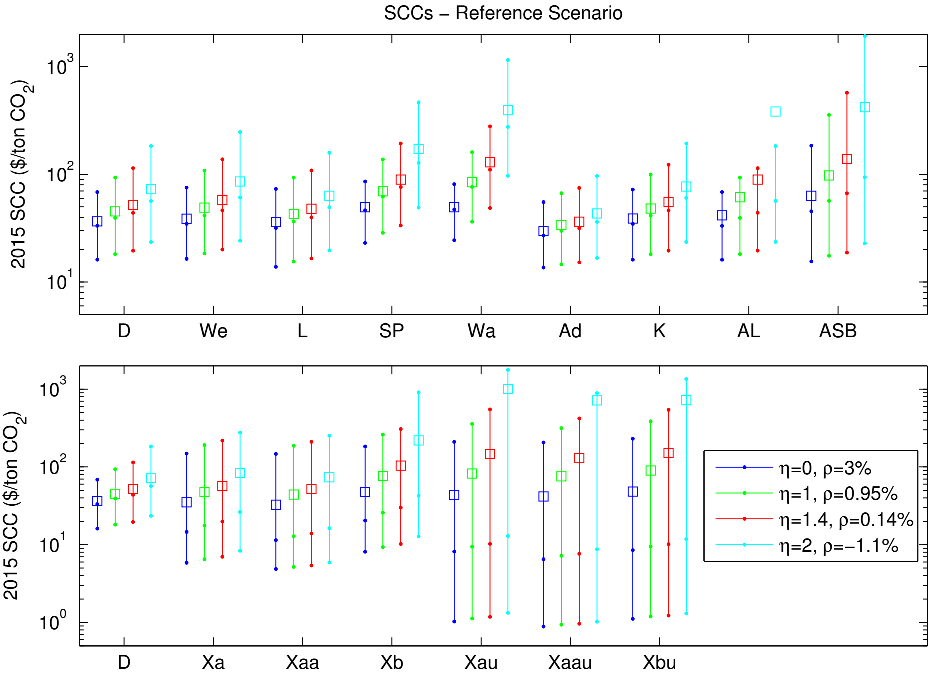

Figure 8: 2015 SCC values calculated using different damage functions and different disaggregations of a 3% per year average discount rate. Squares indicate the SCC, while dots connected by lines indicates 5th, 50th, and 95th percentiles of the deterministic SCC.

functions (**Xau**, **Xaau** and **Xbu**) exhibit this effect most dramatically, with the SCC more than tripling from a risk-neutral \$42-\$48/tonne with $\eta = 0$ to \$130-\$151/tonne at $\eta = 1.4$, as an increasing amount of weight is placed on the small number of states of the world in which the future is poorer than the present.

As η increases, the current-dollar SCC penalty associated with adopting a Sterner-Persson utility function also disappears. At $\eta = 1$, for example, current-dollar SCC values from **D** and **SP** (denoted **SP*** in the tables) are identical. This is because the future in the Sterner-Persson world is poorer in effective consumption terms; at $\eta = 0$, the future therefore contributes less to welfare, while at $\eta = 1$, this effect vanishes because marginal utility is proportional to utility. At $\eta > 1$, poorer

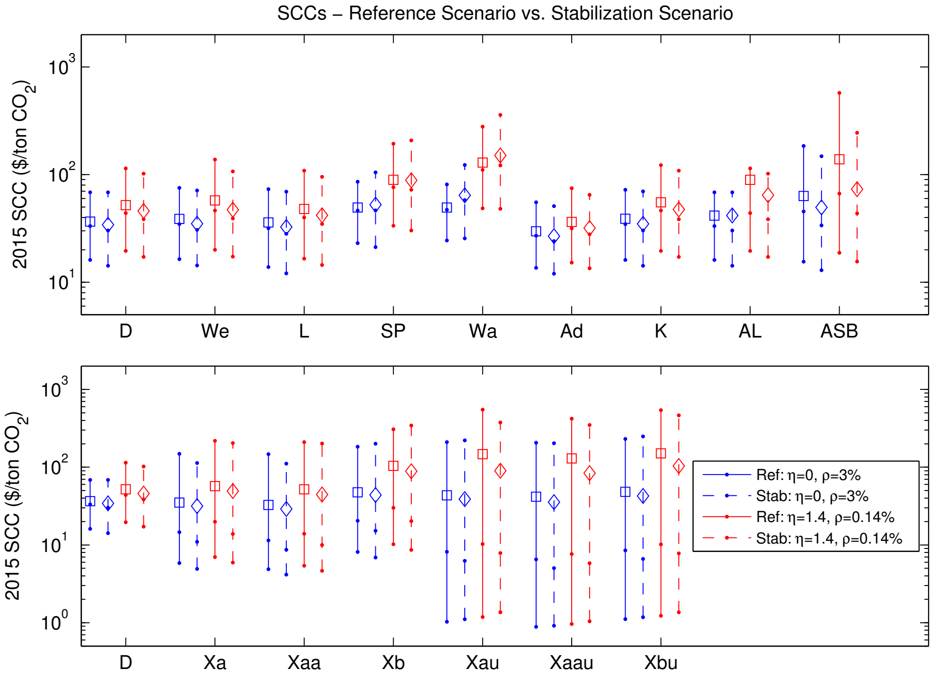

Figure 9: 2015 SCC values calculated using different damage functions and different disaggregations of a 3% per year discount rate. Solid lines indicate values for the reference scenario, while dashed lines indicate values for an emissions stabilization scenario with a 50% chance of limiting warming to 2.5°C.

states of the world contribute more than proportionally to expected social welfare, so current-dollar SCC values from **SP** exceed their counterparts from **D**.

In general, SCC values calculated off the stabilization scenario are moderately lower than SCC values calculated off the reference scenario (Table 5, Figure 9). Employing fat-tailed specifications like **ASB** ($73/tonne at $\eta = 1.4$, vs. $139/tonne with the reference scenario) magnifies this reduction, as for most climate sensitivities, the stabilization scenario avoids the high-temperature portion of the damage function, where the consequences of higher exponents are felt most strongly. **Wa** exhibits the converse effect: with this damage specification, the SCC is higher

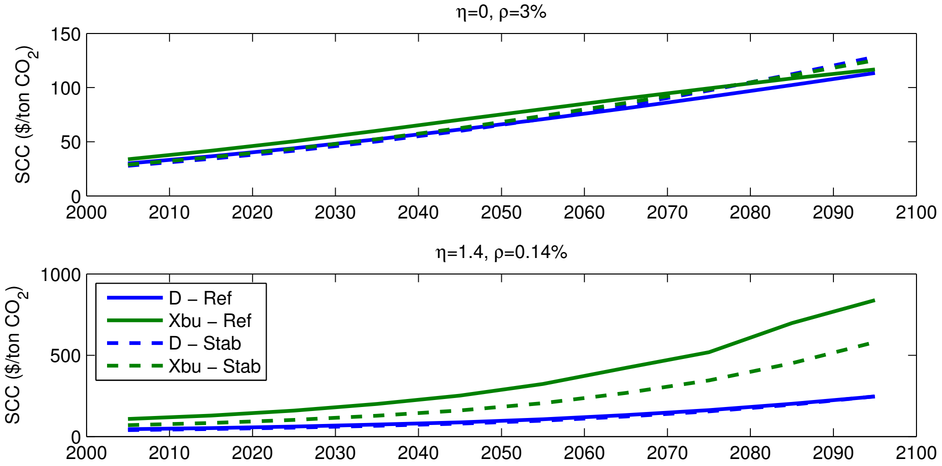

Figure 10: 21st century SCC values calculated using damage functions D (blue) and Xc (green) and different disaggregations of a 3% per year discount rate. Solid lines indicate values for the reference scenario, while dashed lines indicate values for an emissions stabilization scenario with a 50% chance of limiting warming to 2.5°C.

under the stabilization scenario than under the reference scenario ($150/tonne vs. $130/tonne at $\eta = 1.4$). Later in the 21st century, other damage specifications, including both **D** and **Xbu** in the risk neutral case, exhibit a similar phenomenon (Figure 10). These examples provide a concrete illustration of the concerns about non-convex damage functions raised by Baumol (1972) in the context of the social cost of pollution generically and by Kopp and Mignone (2012) in the context of the SCC. With non-convex damage functions, emissions and growing consumption in the reference scenario can carry temperature well over an inflection point beyond which the marginal damages associated with additional warming start decreasing. In this situation, applying the SCC calculated off the reference path as a Pigouvian tax will drive emissions toward a local optimum but not necessarily to the globally optimal level.

For a given damage function, adjusting damages at the 2.5°C calibration point downward has a roughly linear effect on the SCC (Figure 11). For most damage functions, cutting calibration damages in half decreases values by about 40%–

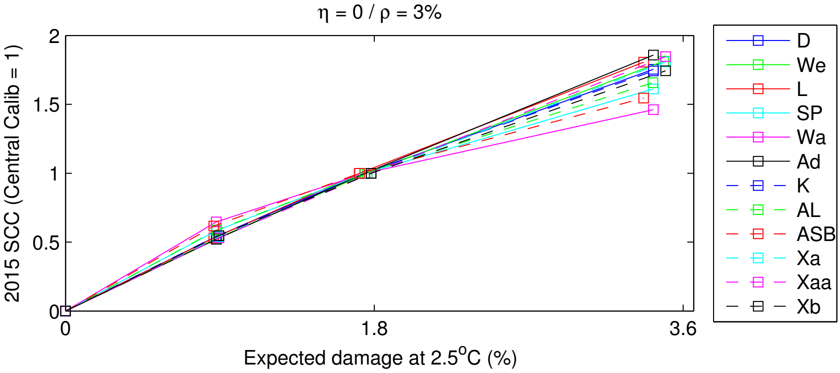

Figure 11: 2015 SCC values calculated using different damage functions and different disaggregations of a flat 3% per year discount rate, using a 2.5°C calibration level double or half the default DICE level. SCC values shown are normalized to the SCC for the central calibration point (1.8% damages at 2.5°C).

50%, while doubling them has a similar but slightly muted effect, increasing SCC values by about 50%–80%. This muting is due to the upper bound on the damage functions; higher calibration damages bring the damage functions closer to the bound, and thus into non-convexity, at lower temperatures. The muting effect can be seen most clearly with **Wa**, which, as noted previously, is well in the non-convex range of behavior under the central calibration. The SCC calculated under **Wa** decreases by about 30% in response to a halving of 2.5°C calibration damages and increases by about 30% in response to a doubling of calibration damages.

5 Conclusions and next steps

Our analysis highlights the importance of jointly considering risk aversion and uncertainty in damages when estimating the SCC. Incorporating uncertainty into damages while assuming risk neutrality will minimize the effects of this modification; incorporating risk aversion while using a certain damage function will do likewise.

For a certain damage specification but uncertain climate sensitivity, increasing risk aversion (while adjusting the pure rate of time preference to maintain exogenously specified average discount rates) modestly increases the SCC. For an uncertain damage specification – even one that yields the same expected damages – increasing risk aversion can greatly increase the SCC. Our composite uncertain damage specifications (**Xau**, **Xaau** and **Xbu**) – which yield nearly the same expected future consumption path as the standard DICE damage specification **D** but also acknowledge a small chance that climate change could make the future less well off than the world today – yield SCC values that go from being just 14–32% higher than the SCC calculated with the standard DICE damage specification **D** with no risk aversion to nearly triple it (150–190% higher) with a coefficient of relative risk aversion of 1.4.

Because we employed a standard isoelastic utility function, we could not isolate the effects of increasing risk aversion from the effects of a declining intertemporal elasticity of substitution. Fundamentally, this is because the isoelastic utility function requires the elasticity of intertemporal substitution and the level of risk aversion to be controlled by a single elasticity of the marginal utility of consumption (η). Yet empirical psychological work (e.g., Atkinson et al., 2009) indicates that individuals are not equally averse to risk and to intertemporal inequality; future work should therefore examine the joint sensitivity of the SCC to damage specification and risk aversion in models with utility functions that do not require these parameters to be coupled (Traeger, 2009).

In addition to the damage function's form, its calibration also matters. The risk-neutral SCC scales approximately linearly with damages at the calibration point. If the uncertainty in this calibration is large – as seems likely, given that FUND estimates non-catastrophic damages of about 0.9% to 1.6% of GDP at 2.5°C warming while DICE estimates non-catastrophic damages of 0.6%, and that both the likelihood of climate catastrophes and their economic consequences are poorly characterized – then both form and calibration can have effects on the SCC of similar magnitude.

Both calibration of the damage function and identification of a suitable form can be advanced through both empirical and modeling work. Emerging retrospective analyses (e.g., Lobell et al., 2011) can help characterize damages for 0.8°C of warming realized to date more accurately, while modeling economic impacts at levels of warming significantly higher than 2.5°C (e.g., New et al., 2011) can advance the construction of damage functions beyond the stage of fitting a curve to two points.

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