Vol. 5, 2011-13 | September 9, 2011 | http://dx.doi.org/10.5018/economics-ejournal.ja.2011-13

On Various Ways of Measuring Pro-Poor Growth

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Abstract This paper examines three possible approaches to pro-poor growth. The first one assumes that the poverty line remains constant in real terms over time. The second perspective examines the case where the poverty line is equal to half the median of the income distribution but assumes that such a poverty line is determined exogenously. Finally we also propose a third type of decomposition of the change in poverty, one which is obtained when the poverty line is assumed to be endogenous.

In addition, whatever the assumption made concerning the poverty line, we take both a relative and an absolute approach to inequality measurement when defining pro-poor growth. With a relative approach to pro-poor growth it is assumed that inequality does not to vary when all incomes are multiplied by a constant whereas, with an absolute approach to pro-poor growth, inequality is supposed not to vary when an equal sum is added to all incomes. The empirical illustration covers the period 1990–2006 in Israel and the analysis is based on the use of the FGT poverty index. It turns out that the assumptions made concerning the way the poverty line is defined and the choice between a relative and an absolute approach to pro-poor growth greatly affect the results. As a whole however growth was pro-rich in Israel during the 1990–2006 period.

Special Issue
The Measurement of Inequality and Well-Being: New Perspectives

JEL I32, O15
Keywords Inequality; Israel; pro-poor growth; Watts index

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Citation Joseph Deutsch and Jacques Silber (2011). On Various Ways of Measuring Pro-Poor Growth. *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 5, 2011-13. http://dx.doi.org/10.5018/economics-ejournal.ja.2011-13

1 Introduction

The concept of pro-poor growth has become very popular during the last decade. It reflects the idea that economic growth should affect all the segments of society and this is why the term "inclusive growth" is also often used. There are however various ways of understanding the term "pro-poor". Some would argue that growth is pro-poor when it raises the incomes of the poor. Others consider that growth can be labeled "pro-poor" only if it raises the incomes of poor proportionately more than it raises the average income in society (see, Kakwani et al., 2004, and Ravallion, 2004, for more details on these two approaches). Dollar and Kraay (2002) thus found, on the basis of a large cross-country data set, that the incomes of the individuals who belong to the two poorest deciles of the income distribution rise on average at the same rate as the mean income.

Another issue that should be stressed is that most studies of pro-poor growth take an anonymous approach in the sense that they are usually based on cross-sections and do not follow individuals over time, as would have been possible, had panel data been available (see, however Grimm, 2007, and Nissanov and Silber, 2009, for an approach to the topic that does not assume anonymity).

Finally because studies on pro-poor growth have generally looked at developing countries they took an absolute approach to the definition of the poverty line in the sense that they assumed a constant (in real terms) poverty line. When looking at poverty in developed countries one tends however to define the poverty line in relative terms, that is, to assume that it is equal to some percentage of the median or mean standardized income. In such a case the issue of pro-poor growth becomes clearly quite different.

The present paper aims precisely at checking whether growth was pro-poor in a country which is considered today as a developed country, Israel. We start (Section II) with a short review of the literature on pro-poor growth measurement. Then in a more methodological section (Section III) we make a distinction between three possible approaches to pro-poor growth analysis. The first one assumes that the poverty line remains constant in real terms over time (what was earlier labeled the absolute approach to poverty). The second perspective examines the case where the poverty line is equal to half the median of the income distribution (what was called previously the relative approach to poverty) but assumes that such a poverty line is determined exogenously. Since such a definition of the poverty line implies in fact that it will vary whenever there is growth or inequality change, we also propose a third type of decomposition of the change in poverty, the one which one obtains when the poverty line is assumed to be endogenous.

In addition, whatever the assumption made concerning the poverty line, we took both a relative and an absolute approach to inequality measurement when defining pro-poor growth. With a relative approach to pro-poor growth it is assumed that inequality does not to vary when all incomes are multiplied by a constant whereas, with an absolute approach to pro-poor growth, inequality is supposed not to vary when an equal sum is added to all incomes. Section IV is then devoted to an empirical illustration based on the annual Israeli income surveys, during the period 1990–2006. We first draw the Growth Incidence Curve (see, Ravallion and Chen, 2003) for the standardized net income, the latter being equal to the ratio of the household's net income over the square root of the number of individuals in the household (see, Buhman et al., 1988, for more details on such an approach to the issue of equivalence scales). Then we give the results of the various decompositions of the changes over time in the poverty index. As poverty index we selected the popular FGT index. Concluding comments are given in Section V.

2 On Various Ways of Measuring Pro-Poor Growth: A Short Review of the Literature

During the past ten years or so there have been various suggestions concerning the way one could check whether economic growth was in favor of the poor. The present section gives a quick survey of the different proposals that have appeared in the literature to measure "pro-poor growth".

Before reviewing these contributions a distinction should be made between an absolute and a relative approach to this topic. Thus some studies (see, Baulch and McCulloch, 2002, or Kakwani and Pernia, 2000) consider that growth will be pro-poor if poverty falls more than it would have fallen, had all incomes grown at the same rate. This is therefore a "relative approach" in the sense that a pro-poor growth requires that the incomes of the poor grow at a higher rate than those of the non-poor.

It is however also possible to take an "absolute approach" to poverty. In such a case growth will be assumed to be “pro-poor” if the standard living of the poor people has improved.

Whatever approach one selects it should be clear that the answer to the question “was growth “pro-poor”?” will depend on the measure of poverty that is selected and the poverty line that is adopted.

2.1 The Ravallion and Chen (2003) Definition of “Pro-poor” Growth

Ravallion and Chen (2003) have proposed an interesting tool to measure the impact of growth on poverty. They called it the “Growth Incidence Curve” (GIC) and it is defined as follows. On the horizontal axis plot the various percentiles of the income (or consumption) distribution[1]. As a consequence at the 50th percentile the Growth Incidence Curve will indicate the growth rate of the median income. Clearly if the curve is above the horizontal axis at all points up to some percentile $\tilde{p}$, we can conclude that poverty has fallen when it is measured via the headcount ratio and the poverty line is not greater than $\tilde{p}$ (see, Atkinson, 1987). Note that the area under the growth incidence curve up to the headcount ratio will give the total growth in incomes of the poor during the period under analysis. Ravallion and Chen (2003) have thus defined the “pro-poor growth rate” as the mean growth rate of the poor. They have also shown that it is equal to the change in the Watts poverty index per unit of time, divided by the headcount ratio. There is clearly a difference between this mean growth rate of the poor and the growth rate of the mean income (consumption) of the poor.

2.2 The Baulch and McCulloch (2002) Approach

“Pro-poor” growth may be also analyzed from a different angle. Since an index of poverty can usually be expressed as a function of the mean of the distribution of the variable on the basis of which this index is computed and of the Lorenz curve

[1] Naturally, if one works with data collected at the household level, the variable should be some standardized income or consumption level, the normalization depending on the equivalence scale that is chosen.

corresponding to this distribution, it is generally possible to decompose a change in poverty (in the poverty index) into elements measuring respectively the impact of the growth rate of the mean income (consumption), that of the changes in the distribution (variations in the degree of inequality of the distribution) and generally some interaction effect (see, for example, Datt and Ravallion, 1992). Then growth will be defined as "distribution neutral" (corresponding to a flat Growth Incidence Curve) if the redistribution component that was just mentioned is nil whereas it will be "pro-poor" if this redistribution component is negative. In other words Baulch and McCulloch (2002) derive their measure of pro-poor growth by comparing the actual distribution of income with the one that would have been observed, had there been no change in the distribution of incomes (that is, had growth been "distribution-neutral"). Note that Kakwani (2000) proposed a decomposition which does not include any interaction effect (see, Appendix A for more details)

2.3 The Kakwani and Pernia (2000) Approach

These authors defined first what they called the total poverty elasticity of growth, that is, the percentage change in poverty when the growth in the mean income (consumption) is equal to 1%. They then defined a second elasticity which measures the percentage change in poverty that is observed when the growth in mean income (consumption) is equal to 1% *and there is no change over time in relative inequality*.

For Kakwani and Pernia (2000) the Pro-Poor Growth index (*PPGI*) is equal to the ratio of these two elasticities and they concluded that growth is pro-poor if this ratio *PPGI* is greater than one.

Note that if there is negative growth, growth will be defined as pro-poor in relative terms if the relative loss in income from negative growth is smaller for the poor than for the non-poor, that is if the ratio $PPGI$ is smaller than one (see, Appendix A for more details on this approach).

2.4 The Approach of Kakwani and Son (2002)

It may be observed that the concept of $PPGI$ that was just defined does not take into account the actual level of growth that is observed. This is why Kakwani and Son (2002) have defined what they call the "poverty equivalent growth rate" $(PEGR)$. The $PEGR$ refers to the growth rate that would result in the same level of poverty reduction as the one actually observed, assuming there had been no change in inequality during the growth process.

Growth will therefore be assumed to be pro-poor if the $PEGR$ is higher than the actual growth rate. If the $PEGR$ is positive but smaller than the actual growth rate, it implies that growth is accompanied by an increase in inequality but a reduction in poverty is still observed. In such a case Kakwani et al. (2004) talk about a "trickle down" process where the poor receive proportionally less benefits from growth than the non-poor. Finally, if the $PEGR$ is negative, we have the case where positive economic growth leads to an increase in poverty.

2.5 The Approach of Son (2004)

Son (2004) defined what she called a poverty growth curve (PGC). It is defined as follows. Let $g(p)$ refer to the growth rate of the mean income (consumption) of the bottom p percent of the population. By plotting $g(p)$ on the vertical axis against p on the horizontal axis one obtains what Son (2003) called a Poverty Growth Curve.

It should be clear that if $g(p) \succ 0$ ($g(p) \prec 0$) for all p, poverty decreased (increased) during the period under examination.

If $g(p)$ is greater than the average growth rate for all $p \prec 100\%$, one can conclude that growth was pro-poor. If $g(p)$ is positive for all $p \prec 100\%$ but smaller than the average growth rate, one can then conclude that growth reduced poverty but during the period inequality increased. Such a situation could refer to what has been called a "trickle down growth", a situation where growth reduces poverty but the benefits of growth are smaller for the poor than for the non-poor. Finally if $g(p)$ is negative for all $p \prec 100\%$, we have a situation where the increase in inequality more than "compensates" growth so that the net effect of growth is to increase poverty, a situation which corresponds to what has been called "immiserizing growth".

One may wonder what difference there is between a Growth Incidence Curve (GIC) and a Poverty Growth Curve (PGC). As stressed by Son (2003) it can be shown that the GIC is derived from first-order stochastic dominance while the PGC is based on second-order stochastic dominance. Since second-order stochastic dominance is more likely to hold than first-order, the PGC should provide more conclusive results, although it is based on stronger assumptions.

Son (2004) emphasizes another potential advantage of the PGC. Since the GIC is based on individual data while the PGC implies estimating the growth rate of the mean income (consumption) up to the p^{th} percentile, the latter procedure is somehow less prone to measurement errors.

3 Methodological Considerations for the Empirical Implementation

While most of the studies mentioned previously used formulations based on a continuous approach to the topic, we prefer to work in discrete terms, among other reasons because we also want to check the existence of pro-poor growth over periods that are longer than a year. In what follows a distinction will be made between a relative and an absolute approach to pro-poor growth. Moreover we will also examine several cases, as far as the definition of the poverty line is concerned.

As is well known, a distinction has to be made between an absolute and a relative poverty line. An absolute poverty line is a threshold expressed in terms of a level of expenditures or income (in real terms) that covers basic needs. Such a poverty line will therefore not depend on the rate of economic growth and will not vary when there is an increase in living standards. A relative poverty line, on the contrary, depends on the rate of economic growth since usually it is defined as being equal to some percentage of the average or median income.

Both approaches can naturally be criticized. On one hand an absolute poverty line does not take into account the fact that what is viewed as basic needs varies over time and that people do make comparisons between their standard of living and that of others. On the other hand when a relative poverty line is adopted, poverty can never disappear and if there is economic growth together with an increase in income inequality, one may observe at the same time an increase in relative poverty and a decrease in absolute poverty.

But even if one adopts a relative poverty line so that the latter will vary over time, it can be considered as exogenous or assumed to be endogenously determined (e.g. when it is defined as being equal to half the median of the relevant income distribution). In the following subsection we examine the case of relative pro-poor growth, assuming the poverty line remains constant over time. The cases of absolute pro-poor growth when it is assumed that there is a constant poverty line and that where the poverty line varies over time are examined in Appendix C.

3.1 The Case of Relative Pro-poor Growth, Assuming the Poverty Line Remains Constant over Time

Let $\{x\} = \{x_1,...,x_n\}$ and $\{y\} = \{y_1,...,y_n\}$ represent the vector of incomes at times 0 and 1 and let $\theta(x)$ and $\theta(y)$ refer to the poverty index at times 0 and 1. Finally let $(\Delta\theta / \theta(x))$ refer to the relative change in the poverty index between times 0 and 1, with $\Delta(\theta) = \theta(y) - \theta(x)$.
Assuming no change in the poverty line z, the relative change $\Delta(\theta)/\theta$ in the poverty index will now be expressed as

$$\Delta(\theta)/\theta = g((\Delta\bar{x}/\bar{x}), \Delta I^R) \quad (1)$$

where $\bar{x}$ is the mean income of the distribution given by $\{x\}$, $\Delta\bar{x} = (\bar{y} - \bar{x})$ is the difference between the average income at times 0 and 1 and I^R refers to some relative measure of income inequality of the distribution given by (x).

Using the concept of Shapley decomposition (see, Shorrocks, 1999, Sastre and Trannoy, 2002, and Appendix B, for more details), $\Delta(\theta)/\theta$ may be written as

$$(\Delta\theta/\theta) = C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R) \quad (2)$$

where $C(\Delta\bar{x}/\bar{x})$ refers to the contribution of the relative change over time in the average income and $C(\Delta I^R)$ to the contribution of the change in relative inequality between time 0 and time 1.

The contribution $C(\Delta\bar{x}/\bar{x})$ may itself be expressed as

$$C(\Delta\bar{x}/\bar{x}) = (1/2)\{[(\Delta\theta/\theta)\,with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0) - (\Delta\theta/\theta)$$
$$with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0)]$$
$$+[(\Delta\theta/\theta)\,with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0)$$
$$-(\Delta\theta/\theta)\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)]\} \qquad (3)$$

Similarly the contribution $C(\Delta I^R)$ may be written as

$$C(\Delta I^R) = (1/2)\{[(\Delta\theta/\theta)\,with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0) - (\Delta\theta/\theta)$$
$$with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0)]$$
$$+[(\Delta\theta/\theta)\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0)$$
$$-(\Delta\theta/\theta)\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)]\} \qquad (4)$$

Combining (3) and (4) we observe that

$$C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R)$$
$$= [(\Delta\theta/\theta)\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0)] - [(\Delta\theta/\theta)$$
$$with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)] \qquad (5)$$
$$= [(\theta(\{y\}) - \theta(\{x\}))/\theta(\{x\})] - [(\theta(\{x\}) - \theta(\{x\}))/\theta(\{x\})]$$
$$= [(\theta(\{y\}) - \theta(\{x\}))/\theta(\{x\})]$$

Let us now define the expression $(\Delta\theta/\theta)\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0)$.
It is easy to derive that this expression may be also expressed as

$$(\theta(\{x(1+k)\}) - \theta(\{x\})/\theta(\{x\})$$

where $k = (\Delta\bar{x}/\bar{x})$ and $\theta(\{x(1+k)\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the original incomes multiplied by a factor k equal to the growth rate of the average income between times 0 and 1. It is should be clear that if all the incomes are multiplied by the same constant k, by definition relative inequality will have remained constant.

Similarly the expression $(\Delta\theta/\theta)\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0)$ may be written as $(\theta(\{(y/(1+k))\}) - \theta(\{x\})/\theta(\{x\})$, where $\theta(\{(y/(1+k))\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the incomes observed at time 1 divided by one plus the growth rate of the average income between time 0 and time 1.

We therefore end up with

$$\begin{aligned} C(\Delta\bar{x}/\bar{x}) = (1/2)\{&[[(\theta(\{y\}) - \theta(\{x\}))/\theta(\{x\})] \\ &-[(\theta(\{(y/(1+k))\}) - \theta(\{x\}))/\theta(\{x\})]] \\ &+[[(\theta(\{x(1+k)\}) - \theta(\{x\}))/\theta(\{x\})] \\ &-[[\theta(\{x\}) - \theta(\{x\})]]\} \\ = (1/2)\{&[(\theta(\{y\}) - \theta(\{(y/(1+k))\})/\theta(\{x\})] \\ &+[(\theta(\{x(1+k)\}) - \theta(\{x\}))/\theta(\{x\})]\} \end{aligned} \tag{6}$$

Similarly we can write that

$$\begin{aligned} C(\Delta I^R) = (1/2)\{&[[(\theta(\{y\}) - \theta(\{x\}))/\theta(x)] \\ &-[(\theta(\{x(1+k)\}) - \theta(\{x\}))/\theta(x)]] \\ &+[[(\theta(\{y/(1+k)\}) - \theta(\{x\}))/\theta(x)] \\ &-[(\theta(\{x\}) - \theta(\{x\}))/\theta(x)]]\} \\ = (1/2)\{&[(\theta(\{y\}) - \theta(\{x(1+k)\}))/\theta(x)] \\ &+[(\theta(\{y/(1+k)\}) - \theta(\{x\}))/\theta(x)]\} \end{aligned} \tag{7}$$

Combining (6) and (7) we observe again that, as expected,

$$C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R) = (\theta(\{y\}) - \theta(\{x\}))/\theta(x) \tag{8}$$

The actual relative change in the poverty index observed between times 0 and 1 will therefore be expressed as

$$(d\theta/\theta) = Gr^R + In^R \tag{9}$$

where

$$Gr^R = C(\Delta\bar{x}/\bar{x})/\theta \tag{10}$$

and

$$In^R = C(\Delta I^R)/\theta \qquad (11)$$

where θ, as before is the value of the poverty index at the original period 0.

Using (3), (4), and (5) we will now call ∂ the total poverty elasticity of growth, that is, the percentage change in poverty $(d\theta/\theta)$ when the growth in the mean income (consumption) is equal to 1%. Similarly, following Kakwani and Son (2008), call η^R the percentage change in poverty (Gr^R) that is observed when the growth in mean income (consumption) is equal to 1% *and there is no change over time in absolute inequality*.

Kakwani and Son's (2008) concept of Pro-Poor Growth index that was defined previously (see also Appendix A) can therefore be expressed as

$$PPGI^R = \frac{\delta}{\eta^R} \qquad (12)$$

Similarly their "poverty equivalent growth rate" will be derived as

$$PEGR^R = \gamma^R = (\frac{\delta}{\eta^R})\gamma = (PPGI^R) \times \gamma \qquad (13)$$

3.2 Measuring Absolute Pro-Poor Growth: The Case of a Poverty Line That Is Constant Over Time

This case is examined in Appendix C. The demonstration is quite similar to that given in the previous subsection.

3.3 The Case of a Poverty Line That Varies over Time

A distinction should be made here between the case where the variation in the poverty line is considered to be exogenous and that where it is endogenous. In both cases one has again the choice between a relative and an absolute approach to pro-poor growth. It should however be stressed that if the poverty line is determined endogenously some of axioms commonly used in the poverty literature should be stated more cautiously. For instance, if the poverty line is 50% of the median, then the monotonicity axiom allows only those increments/ reductions in the incomes of the poor that do not change the poverty line. A similar remark holds for the transfer axiom. Likewise, for the focus axiom to be satisfied only those changes in the incomes of the non-poor are allowed that do not change the poverty line. Note however that no such problem arises with the symmetry and population principles[2].

The detailed decomposition of the variations in poverty that are obtained when the poverty line varies over time is given in Deutsch and Silber (2009) as well as in Appendix C.

4 An Empirical Illustration: Relative Pro-Poor Growth in Israel during the Period 1990–2006

4.1 What Do the Growth Incidence and Poverty Growth Curves for the Period 1990–2006 Show?

The Growth Incidence Curve for the whole period 1990–2006 is given in Figure 1 for the standardized net income. One may observe that as a whole, during this period of 16 years, growth was pro-rich rather than pro-poor. This was however not true for the annual growth since years of pro-poor growth alternated with period of pro-rich growth, as will be apparent below when we will look at specific measures of pro-poor growth. Results are quite similar when drawing for this same period a Poverty Growth Curve for the standardized net incomes. At the exception of the 15 lowest percentiles, growth was, as a whole, pro-rich[3].

4.2 Looking at Changes in the Poverty Indices

4.2.1 The Case of a Poverty Line That Is Constant over Time

We defined as constant poverty line (in real terms), the poverty line that was observed at the middle of the period 1990–2006, that is in 1998, assuming it was then equal to half the median of the distribution of standardized net income in 1998.

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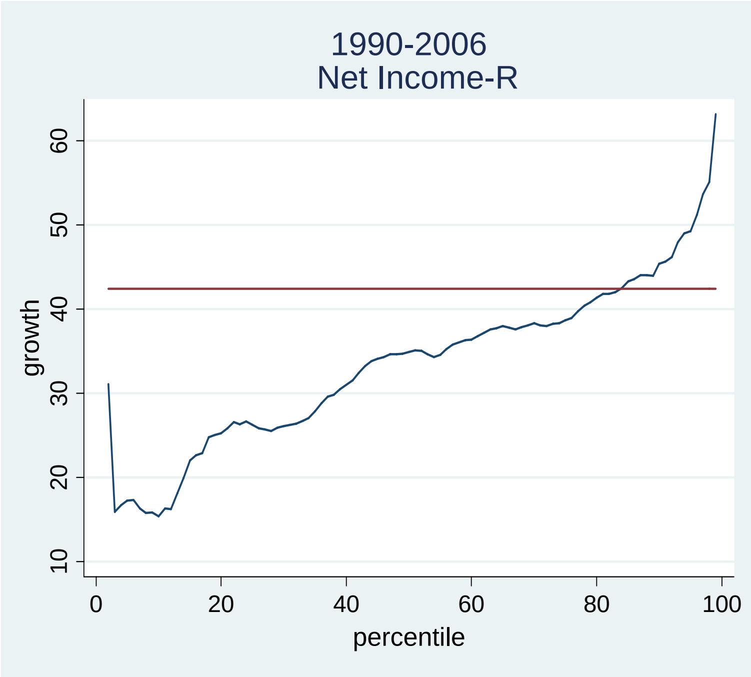
[2] We thank Satya Chakravarty for drawing our attention to these issues.

[3] Such a graph may be obtained upon request from the authors.

Figure 1: Growth Incidence Curve (GIC) in Israel for the Standardized Net Income (Period 1990–2006)



1-The Relative Approach to Pro-poor Growth

Table 1 gives first the annual percentage change in the FGT index when the parameter α is equal to 2 and when using standardized net incomes. This annual change is then broken down, using a Shapley type of decomposition, into two components. The first component shows what the percentage change in poverty would have been, had there been "pure" growth, that is, growth without change in relative inequality. The second component shows what the percentage change in poverty would have been, had there been no growth but only a change in relative inequality (the one actually observed). The results show periods where poverty increased and periods where it decreased. If we concentrate our attention on the last periods we observe that poverty increased during the periods 2001–2002 and 2003–2004, with, in particular, an extremely strong increase in 2001–2002 since poverty increased by almost 52% in one year according to the FGT index. The FGT index shows that in 2001–2002 about 40% of this increase was due to the negative "pure growth" (assuming no change in relative inequality).

During the last two periods (2004–2005 and 2005–2006) poverty decreased by 9.7% and 8.6%, according to the FGT index and this decrease was mainly a "pure growth" effect.

Table 1 gives also results for broader periods. We have divided the whole period 1990–2006 into three sub-periods: a first period (1990–2000) where, as a whole, the growth rate of the per capita GDP (derived from national accounts) was positive, a second period (2000–2003) according to which the growth rate of the per capita GDP was negative and a third period (2003–2006) where this growth rate was again positive. It appears that poverty decreased during the first and third period, increased during the second period and decreased over the whole period. However, whereas during the period of decrease in poverty, the main effect was that of pure growth, we should note that during the period 2000–2003 when poverty increased, this was rather the consequence of inequality change than of pure growth.

Using again a relative approach to pro-poor growth we computed in Table 2 the pro-poor growth index $PPGI$ and the poverty equivalent growth rate $PEGR$, on the basis of the FGT index for the standardized net incomes.

We observe that there were nine periods during which growth was pro-poor ($PPGI$ greater than one). The highest values of the $PPGI$ were observed in 1994–1995 ($PPGI$ = 3.765), 1997–1998 ($PPGI$ = 2.529) and 2001–2002 ($PPGI$ = 2.363). Note that, during the last year of observation (2005–2006), growth was clearly pro-poor ($PPGI$ = 1.214).

Table 2 gives also the Poverty Equivalent Growth Rate ($PEGR$) computed on an annual basis. It appears that, at least when poverty is measured via the FGT index, there were seven periods during which the $PEGR$ was higher than the average growth rate of the standardized net income. These periods were 1992–1993, 1994–1995, 1997–1998, 1998–1999, 1999–2000, 2000–2001 and 2005–2006.

Table 1: The Relative Approach to Pro-poor Growth, with a Constant Poverty Line Decomposition of the Actual Percentage Change in Poverty Indices into "Pure Growth" and "Pure Inequality Change" Components—the Case of Standardized Net Income

Period	Actual percentage change in the FGT poverty index (with the parameter α equal to 2)	Hypothetical percentage change in the FGT poverty index (with the parameter α equal to 2), assuming growth without inequality change	Hypothetical percentage change in FGT poverty index (with the parameter α equal to 2), assuming there was only a change in inequality and no growth
	Total	Gr	In
1990-1991	0.021	0.015	0.005
1991-1992	0.055	-0.081	0.136
1992-1993	-0.068	0.041	-0.109
1993-1994	-0.046	-0.201	0.156
1994-1995	-0.286	-0.076	-0.210
1995-1996	0.125	0.094	0.031
1996-1997	0.073	-0.213	0.286
1997-1998	-0.153	-0.060	-0.092
1998-1999	-0.182	-0.166	-0.016
1999-2000	-0.097	-0.072	-0.025
2000-2001	-0.085	-0.071	-0.014
2001-2002	0.516	0.218	0.298
2002-2003	-0.005	-0.018	0.013
2003-2004	0.101	-0.074	0.175
2004-2005	-0.097	-0.121	0.024
2005-2006	-0.086	-0.071	-0.015
1990-2000	-0.483	-0.539	0.056
2000-2003	0.380	0.108	0.272
2003-2006	-0.092	-0.263	0.171
1990-2006	-0.352	-0.719	0.367

Table 2: The Relative Approach to Pro-poor Growth, with a Constant Poverty Line
Annual Measures of Pro-poor Growth—the Case of Standardized Net Income

Period	Actual Poverty Elasticity of Growth (FGT index with the parameter α equal to 2)	Hypothetical Poverty Elasticity of Growth, assuming no change in inequality (FGT index with the parameter α equal to 2)	Pro-Poor Growth Index (FGT index with the parameter α equal to 2)	Poverty Equivalent Growth Rate (FGT index with the parameter α equal to 2)	Actual Growth Rate of Standardized Net Income
	δ	η	PPGI	PEGR	
1990-1991	-3.641	-2.687	1.355	-0.008	-0.006
1991-1992	1.843	-2.718	-0.678	-0.020	0.030
1992-1993	4.318	-2.631	-1.641	0.026	-0.016
1993-1994	-0.580	-2.561	0.227	0.018	0.079
1994-1995	-9.263	-2.460	3.765	0.116	0.031
1995-1996	-4.413	-3.325	1.327	-0.038	-0.028
1996-1997	0.954	-2.775	-0.344	-0.026	0.077
1997-1998	-6.170	-2.440	2.529	0.063	0.025
1998-1999	-2.683	-2.447	1.097	0.075	0.068
1999-2000	-3.596	-2.683	1.340	0.036	0.027
2000-2001	-3.189	-2.668	1.195	0.032	0.027
2001-2002	-7.905	-3.346	2.363	-0.154	-0.065
2002-2003	-0.731	-2.491	0.294	0.002	0.007
2003-2004	3.507	-2.559	-1.371	-0.039	0.029
2004-2005	-1.802	-2.247	0.802	0.043	0.054
2005-2006	-2.759	-2.272	1.214	0.038	0.031
1990-2000	-1.522	-1.699	0.896	0.284	
2000-2003	-11.364	-3.231	3.517	-0.118	
2003-2006	-0.777	-2.228	0.349	0.041	
1990-2006	-0.831	-1.697	0.490	0.207	

2-The Absolute Approach to Pro-poor Growth:

In Table 3 we decompose again the growth rate in poverty into two components, one reflecting "pure growth" and the other the impact of inequality change on poverty, but this time an absolute approach to inequality is taken. Let us first concentrate our attention on the years where poverty significantly increased, that

Table 3: The Absolute Approach to Pro-poor Growth, with a Constant Poverty Line Decomposition of the Actual Percentage Change in Poverty Indices into "Pure Growth" and "Pure Inequality Change" Components—the Case of Standardized Net Income

Period	Actual percentage change in the FGT poverty index (with the parameter α equal to 2)	Hypothetical percentage change in the FGT poverty index (with the parameter α equal to 2), assuming growth without inequality change	Hypothetical percentage change in FGT poverty index (with the parameter α equal to 2), assuming there was only a change in inequality and no growth
	Total	Gr	In
1990-1991	0.021	0.054	-0.033
1991-1992	0.055	-0.287	0.341
1992-1993	-0.068	0.147	-0.214
1993-1994	-0.046	-0.745	0.699
1994-1995	-0.286	-0.283	-0.003
1995-1996	0.125	0.346	-0.221
1996-1997	0.073	-0.850	0.923
1997-1998	-0.153	-0.251	0.099
1998-1999	-0.182	-0.724	0.541
1999-2000	-0.097	-0.322	0.225
2000-2001	-0.085	-0.326	0.241
2001-2002	0.516	1.032	-0.516
2002-2003	-0.005	-0.083	0.078
2003-2004	0.101	-0.348	0.449
2004-2005	-0.097	-0.607	0.510
2005-2006	-0.086	-0.368	0.282

is, 2001–2002 and eventually 2003–2004. In 2001–2002 it appears that "pure growth" had the main impact on poverty, its effect being actually much higher than the overall change in poverty. This was a year when the per capita standardized net income decreased by 6.5% (see, Table 2) and although the change in inequality would per se have led to a decrease in poverty this effect did not compensate the strong effect of the negative growth on poverty. Note that this is due to the fact that since growth was negative, richer people, in “dollar terms” lost more than poorer people so that an absolute approach to pro-poor growth should indicate that inequality change per se should have decreased poverty.

The picture is different in 2003–2004. There “pure growth” per se would have led to a decrease in poverty and inequality change to an increase. This was a year where the average standardized net income increased by 2.9% (see, Table 2) and clearly the absolute (in "dollar terms") increase in this income was higher for the rich than the poor so that inequality change per se would have led to an increase in poverty, and this effect of inequality change was in fact stronger than the pro-poor effect of “pure growth”.

If we now take a look at the year where poverty most decreased (in 1994–1995) we observe (see, Table 3) that in that year the decline in poverty was only the consequence of “pure growth”.

We have also computed the values of the measures $PPGI$ and $PEGR$ of pro-poor growth when an absolute approach to the topic is taken and when working with standardized net income[4]. It turns out that the Pro-Poor Growth Index ($PPGI$) was never greater than one and that the only years (1992–1993, 1995–1996 and 2001–2002) in which the Poverty Equivalent Growth Rate ($PEGR$) was higher than the actual growth rate (or less negative) were years where actual growth was negative.

4.2.2 The Case of a Poverty Line That Varies over Time but Is Exogenous:

1-The Relative Approach to Pro-poor Growth:

The results of this investigation are given in Table 4. Note first that once the poverty line is allowed to vary over time, it is less likely that poverty will decrease, as it did in the case of a constant poverty line. As mentioned previously

[4] The results may be obtained upon request from the authors.

Table 4: Decomposition of the Actual Percentage Change in Poverty Indices into Components Reflecting Respectively: “Pure Growth”, “Pure Inequality Change” and Variations in the (Exogenous) Poverty Line—the Case of Standardized Net Income and of a Relative Approach to Pro-poor Growth

Period	Actual percentage change in the FGT index	Hypothetical percentage change in the FGT index, assuming there was only growth	Hypothetical percentage change in the FGT index, assuming there was only a change in inequality	Hypothetical percentage change in the FGT index, assuming there was only a change in the poverty line
1990-2000	-0.020	-0.757	0.075	0.662
2000-2003	0.232	0.102	0.228	-0.099
2003-2006	0.138	-0.302	0.185	0.255
1990-2006	0.373	-1.108	0.542	0.939

we assumed that the poverty line was equal at times 0 and 1 to half the median of the standardized net income, although at this stage we still consider this poverty line as being exogenously determined. Table 4 shows thus that over the whole period 1990–2006 the FGT index increased by 37.3%. When we decompose these overall variations in poverty we also observe the important contribution of changes in the poverty line. If the only change that had taken place during the period 1990–2006 had been “pure growth” the FGT index would have decreased by 111%. On the other hand if there had been only a change in the poverty line (with no growth and inequality change), poverty, measured via the FGT index, would have increased by 94%. Finally the change in inequality would, ceteris paribus, have induced an increase of 54%. Table 4 indicates in fact that whatever the period examined, the change inequality would have led to an increase in poverty (assuming no growth and no change in the poverty line).

2-The Absolute Approach to Pro-poor Growth:

This case is examined in Table 5 which shows for the last three years of the period examined the decomposition of variations in the poverty indices in the case of both an absolute and a relative approach. The absolute approach is in fact not fit to analyze longer periods because, in periods of negative growth, the average dollar decrease in income may be big enough to bring the poorest individuals to a potential negative income, assuming there was only "pure growth". When comparing the absolute and relative approaches we observe for example during the period 2005–2006 where there was a decrease of 2.6% in the FGT poverty index, that, according to the relative approach, "pure growth" would per se have led to a 7.5% decrease in the FGT index while the "pure inequality change" would have led to an additional decrease of 1.4% in the FGT index. There was however a countervailing effect of the change in the poverty line. The absolute approach on the contrary indicates that the "pure growth" effect should, per se, have led to a decrease of 33.1% in the value of the FGT poverty index while the pure effect of (absolute) inequality change would have led to a 24.2% increase in poverty, most likely because the dollar increase in the incomes of the rich was much higher than the dollar change in the income of the poor. The effect of the variation in the poverty line is evidently the same as in that of the relative case. This illustration shows therefore very clearly how relevant a distinction between a relative and an absolute approach to pro-poor growth is.

Table 5: Decomposition of the Actual Percentage Change in Poverty Indices into Components Reflecting Respectively: "Pure Growth", "Pure Inequality Change" and Variations in the (Exogenous) Poverty Line—the Case of Standardized Net Income: Comparing the Relative and Absolute Approach to Pro-poor Growth (Annual Changes) for Selected Years

Approach selected	Period	Actual percentage change in the FGT index	Hypothetical percentage change in the FGT index, assuming there was only growth	Hypothetical percentage change in the FGT index, assuming there was only a change in inequality	Hypothetical percentage change in the FGT index, assuming there was only a change in the poverty line
Absolute	2003-2004	0.155	-0.331	0.423	0.064
Absolute	2004-2005	0.011	-0.571	0.474	0.108
Absolute	2005-2006	-0.026	-0.332	0.242	0.063
Relative	2003-2004	0.155	-0.076	0.168	0.064
Relative	2004-2005	0.011	-0.129	0.031	0.109
Relative	2005-2006	-0.026	-0.075	-0.014	0.064

4.2.3 The Case of a Poverty Line That Varies over Time but Is Endogenous:

1-The Relative Approach to Pro-poor Growth:

As mentioned previously, once the poverty line is defined as being equal to half the median of the distribution of net standardized income, we cannot consider a change over time in the poverty line as exogenous. It has to be the consequence of either "pure growth" or/and "inequality change". Once a change in the poverty line is considered as endogenous, we are, once again, left with a simple decomposition of the variation over time in the poverty index into a component reflecting the impact of "pure growth" and one that is the consequence of a "pure inequality change". The results of this type of analysis are given in Table 6. Note that if one takes a relative approach to pro-poor growth, it turns out that, when a change in the poverty line is considered as being endogenous, there is practically no "pure growth" effect. Such a result was expected because when there is no change in inequality and the poverty line is equal to half the median of the distribution, no important change in poverty should occur.

Table 6: Decomposition of the Actual Percentage Change in Poverty Indices into Components Reflecting Respectively "Pure Growth" and "Pure Inequality Change"—the Case of an Endogenous Poverty Line, Standardized Net Income and a Relative Approach to Pro-poor Growth (Annual Changes)

Period	Actual percentage change in the FGT index	Hypothetical percentage change in the FGT index, assuming there was only growth	Hypothetical percentage change in the FGT index, assuming there was only a change in inequality
1990-2000	-0.020	0.000	-0.020
2000-2003	0.232	-0.001	0.232
2003-2006	0.138	0.001	0.137
1990-2006	0.373	0.000	0.372

Table 7: Decomposition of the Actual Percentage Change in Poverty Indices into Components Reflecting Respectively "Pure Growth" and "Pure Inequality Change"—the Case of an Endogenous Poverty Line and Standardized Net Income: Comparing the Relative and Absolute Approaches for Annual Changes for Selected Years

Approach selected	Period	Actual percentage change in the FGT index	Hypothetical percentage change in the FGT index, assuming there was only growth	Hypothetical percentage change in the FGT index, assuming there was only a change in inequality
Absolute	2003-2004	0.155	-0.239	0.395
Absolute	2004-2005	0.011	-0.416	0.427
Absolute	2005-2006	-0.026	-0.241	0.215
Relative	2003-2004	0.155	0.001	0.154
Relative	2004-2005	0.011	0.000	0.010
Relative	2005-2006	-0.026	-0.001	-0.025

2-The Absolute Approach to Pro-poor Growth:

The results of this type of analysis are given in Table 7. Note first that if one takes an absolute approach to pro-poor growth, when a change in the poverty line is considered as being endogenous, there is now both a "pure growth" as well as a "pure inequality change" effect. Such a result was expected because equal additions to all incomes induce indeed a translation of the income distribution and hence of the median and since the poverty line is defined as being equal to half the median, the data will now reveal different amounts of poverty[5]. Let us take a look, for example, at the period 2003–2004 during which the FGT index rose by 15.5%. The relative approach to pro-poor growth indicates (see, Table 7) that this was only the consequence of inequality change while the absolute approach shows that

[5] This is simple to show in the case where poverty is measured via the headcount ratio. Assume, for simplicity, three incomes equal respectively to 100, 300 and 600. The poverty line will be equal to 150 and hence a third of the individuals will be poor. Assume now equal additions of 200 to all incomes. The new distribution is {300, 500, 800} and the new poverty line is 250, so that no individual is poor.

the "pure growth" effect should have led to a decrease in poverty but the impact of (absolute) inequality change was very strong and of opposite direction (this was a period of growth and hence the incomes of rich people increase, in dollar terms, more than those of poor people). So here again we see an important difference between what one may conclude on the basis of a relative and an absolute approach to inequality.

5 Concluding Comments

This paper attempted to check whether growth in Israel was pro-poor during the period 1990–2006. It used concepts that have appeared recently in the literature on pro-poor growth, such as that of Pro-Poor Growth Index and Poverty Equivalent Growth Rate. Three basic scenarios were examined. In the first one it was assumed that the poverty line was constant in real terms and hence did not vary over time. The second scenario supposed that the poverty line was equal to half the median of the relevant income distribution. It therefore varied over time but we still assumed that it could be considered as exogenous. The last case we examined was that in which the poverty line was still assumed to be equal to half the median but we considered this poverty line as endogenous. We each time checked whether the change in growth was mainly the consequence of "pure growth" or whether it was also influenced by changes in inequality. Under the second scenario we obviously had also a third possible impact, that of an exogenous change in poverty. Whatever the scenario under scrutiny, we always took first a relative approach to pro-poor growth, that is, one where inequality is assumed not to vary when all incomes are multiplied by a constant, second an absolute approach to pro-poor growth, that is, one where inequality is supposed not to change when an equal sum is added to all incomes. We analyzed the distribution of the standardized net income and selected as poverty measure the FGT index.

The following conclusions may be drawn. First it turns out that the assumptions made concerning the way the poverty line was defined and the choice between a relative and an absolute approach to pro-poor growth greatly affected the results. Second it turns out that during the period 1990–2006 as whole growth in Israel was pro-rich rather than pro-poor. This result is obtained when drawing Growth Incidence as well as Poverty Growth Curves. Such a conclusion is however not true for the annual growth since years of pro-poor growth alternated with period of pro-rich growth. Third, assuming a constant poverty line in real terms and taking a relative approach to poverty, it appears that the annual Poverty Equivalent Growth Rate ($PEGR$) was generally smaller than the average growth rate (but not always). For broader periods the $PEGR$ was however never greater than the growth rate of the average net standardized income. Fourth, assuming still a constant poverty line but taking an absolute approach to pro-poor growth, we observed that the main impact on poverty was generally that of "pure growth". Fifth, when it is assumed that the poverty line varies over time but is exogenous, and if one takes a relative approach to pro-poor growth, we observed a very important contribution of changes in the poverty line to the overall change in the FGT index. Finally, when it is assumed that the poverty line varies over time but is endogenous and if one takes a relative approach to pro-poor growth, it turns out, and this was expected, that there is practically no "pure growth" effect.

Acknowledgement: The authors acknowledge the financial support of the van Leer Jerusalem Institute and of the Adar Foundation of the Department of Economics at Bar-Ilan University.

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Appendix A: On Various Ways of Measuring Pro-Poor Growth

1) Kakwani's (2000) Proposal and the Approach of Baulch and McCulloch (2002):

Let θ be a poverty measure that is fully characterized by the poverty line *z*, the mean income μ and the Lorenz curve *L(p)*, so that

$$\theta = \theta(z, \mu, L(p)) \tag{A-1}$$

The proportional change $(d\theta / \theta)$ in poverty between times *t* and *t'* may then be expressed as

$$(d\theta / \theta) = Ln[\theta(z, \mu_{t'}, L_{t'}(p))] - Ln[\theta(z, \mu_t, L_t(p))] \tag{A-2}$$

where the subscripts refer to the time period (*t* or *t'*). It is assumed that there is no change over time in the poverty line *z*. Using the concept of Shapley decomposition[6] (see, Shorrocks, 1999, and Sastre and Trannoy, 2002, as well Appendix B, for more details on this decomposition) it can be shown that the relative change in poverty $(d\theta / \theta)$ may be expressed as the sum of two components, one, *Gr*, reflecting the impact of growth, inequality remaining constant, and the other, *In*, measuring the effect of a change in inequality, the mean income staying constant, that is

$$(d\theta / \theta) = Gr + In \tag{A-3}$$

where

$$\begin{aligned} Gr = (1/2)\{&[Ln(\theta(z, \mu_{t'}, L_t(p)))] - Ln\theta((z, \mu_t, L_t(p)))] \\ &+ [Ln(\theta(z, \mu_{t'}, L_{t'}(p)))] - Ln(\theta(z, \mu_t, L_{t'}(p)))]\} \end{aligned} \tag{A-4}$$

and

[6] Kakwani (2000) did not use explicitly the concept of Shapley decomposition but the decomposition he proposed amounts in fact to using the Shapley decomposition.

$$In = (1/2)\{[Ln(\theta(z, \mu_t, L_{t'}(p)))] - Ln(\theta(z, \mu_t, L_t(p)))] + [Ln(\theta(z, \mu_{t'}, L_{t'}(p)))] - Ln(\theta(z, \mu_{t'}, L_t(p)))]\} \quad \text{(A-5)}$$

The concept of “poverty bias of growth (PBG)” defined by Baulch and McCulloch (2002) may, in fact, be expressed as

$$PBG = -In \quad \text{(A-6)}$$

In other words Baulch and McCulloch (2002) derive their measure of pro-poor growth by comparing the actual distribution of income with the one that would have been observed, had there been no change in the distribution of incomes (that is, had growth been “distribution-neutral”).

2) The Kakwani and Pernia (2000) Approach:

Let ∂ be the total poverty elasticity of growth, that is, the percentage change in poverty $(d\theta/\theta)$ when the growth in the mean income (consumption) is equal to 1%. Similarly call η the percentage change in poverty (Gr) that is observed when the growth in mean income (consumption) is equal to 1% *and there is no change over time in relative inequality*. The measure η is also called the relative growth elasticity of poverty and it is clearly always negative.

Kakwani and Pernia (2000) have then defined the Pro-Poor Growth index (*PPGI*) as

$$PPGI = \frac{\delta}{\eta} \quad \text{(A-7)}$$

Clearly growth is pro-poor if *PPGI* is greater than one.

3) The Approach of Kakwani and Son (2002):

Call γ the actual growth rate (of the mean income) and γ^* the growth rate that would have been observed had there been no change in inequality. Under a distribution neutral growth scenario the relative change in poverty would hence been equal to $\eta\gamma^*$. We would like this hypothetical relative change in poverty to be equal to the one which was actually observed and is equal to $\delta\gamma$. It is then easy to conclude that if $\eta\gamma^* = \delta\gamma$, we must have

$$PEGR = \gamma^* = \left(\frac{\delta}{\eta}\right)\gamma = (PPGI) \times \gamma \quad \text{(A-8)}$$

Expression (A-8) implies that growth is pro-poor if γ^* is greater than γ.

4) The Approach of Son (2004):

Son (2004) defined the concept of poverty growth curve (PGC) and derived it from the link which exists between movements in the generalized Lorenz curve and changes in poverty. This connection is in fact a consequence of the relationship between stochastic dominance and poverty measurement that was put forth by Atkinson (1987).

a) The Concept of Poverty Growth Curve (PGC):

Let again μ represent the mean income (consumption) in the population and $L(p)$ refer to the height of the Lorenz curve (on the vertical axis) at the cumulative percentage p (horizontal axis). As is well known the Generalized Lorenz Curve is defined as the plot of $\mu L(p)$ on the vertical axis against that of the cumulative percentages p on the horizontal axis.

Consider now a general class of additive poverty measures defined as

$$\theta = \int_0^z P(z,x)\, f(x)\, dx \quad \text{(A-9)}$$

where $f(x)$ is the density function of income x and z is the poverty line. In addition let us assume that $(\partial P/\partial x) \prec 0$, $(\partial^2 P/\partial x^2) \succ 0$, $P(z,z)=0$ and $P(z,x)$ is a homogenous function of degree zero in z and x.

It can then be shown, on the basis of Atkinson's theorems (1987), that if $\Delta(\mu L(p)) \geq 0$ for all p, then $\Delta\theta \leq 0$ for all poverty lines and the class of poverty measures that has just been defined (poverty measures that are: *non-decreasing, anonymous and obey the principle of transfer*).

As stressed by Son (2003), it should be clear that if the generalized Lorenz curve shifts upward (downward), one can conclude that poverty decreased (increased). This result is the basis for the derivation by Son (2003) of the concept of poverty growth curves.

Let us before remember that the height of the Lorenz curve $L(p)$ may be expressed as

$$L(p) = \frac{\mu_p p}{\mu} \quad \text{(A-10)}$$

where evidently $L(p)$ refers to the share in total income (consumption) of the p percent poorest individuals in the population while μ_p is the mean income (consumption) of these p percent poorest individuals (μ, as before, represents the average income or consumption in the whole population).

Taking logarithms on both sides of (A-10) we then derive that

$$Ln(\mu_p) = Ln(\mu L(p)) - Ln(p) \quad \text{(A-11)}$$

If we now take the first difference in (A-11) we obtain (see, Son, 2003)

$$g(p) = \Delta Ln(\mu L(p)) \quad \text{(A-12)}$$

where $g(p) = \Delta Ln(\mu_p)$ is the growth rate of the mean income (consumption) of the bottom p percent of the population. By plotting $g(p)$ on the vertical axis against p on the horizontal axis one obtains what Son (2003) called a Poverty Growth Curve.

It should now be clear that if $g(p) \succ 0$ ($g(p) \prec 0$) for all p, poverty decreased (increased) during the period under examination.

Note that (A-12) may also be expressed as

$$g(p) = \gamma + \Delta Ln(L(p)) \quad \text{(A-13)}$$

where $\gamma = \Delta Ln(\mu)$ is the growth rate of the mean income (consumption) in the whole population.

Expression (A-13) clearly implies that if $g(p) \succ \gamma$ for all $p \prec 100\%$, growth was pro-poor since this implies that $\Delta Ln(L(p)) \succ 0$, that is the entire Lorenz curve shifted upward (inequality decreased). If $0 \prec g(p) \prec \gamma$ for all $p \prec 100\%$, we can conclude that growth reduced poverty but during the period inequality increased. Such a situation could refer to what has been called a "trickle down growth", a situation where growth reduces poverty but the benefits of growth are smaller for the poor than for the non-poor. Finally if $g(p) \prec 0$ for all $p \prec 100\%$ (assuming $g \succ 0$) we have a situation where the increase in inequality more than "compensates" growth so that the net effect of growth is to increase poverty, a situation which corresponds to what has been called "immiserizing growth".

b) Comparing Poverty Growth Curves and Growth Incidence Curves:

Call x_p the income (consumption) level of an individual that is located at the p^{th} percentile. Since it is well known (see, Kakwani, 1980) that the derivative $L'(p)$ of the Lorenz curve may be expressed as

$$L'(p) = (x_p / \mu) \qquad \text{(A-14)}$$

we derive that $x_p = \mu L(p)$. If we now take the logarithms of the latter expression and then its first difference we end up with

$$r(p) = \gamma + \Delta Ln(L'(p)) \qquad \text{(A-15)}$$

where $r(p)$ refers to the growth rate of the income (consumption) of the individual located at the p^{th} percentile. The plot of $r(p)$ on the vertical axis against that of the cumulative percentages p on the horizontal axis gives us precisely what Ravallion and Chen (2003) called the Growth Incidence Curve. The higher this curve is, the greater the reduction in poverty.

One may wonder what difference there is between a Growth Incidence Curve (GIC) and a Poverty Growth Curve (PGC). As stressed by Son (2003) and mentioned previously, the GIC is derived from first-order stochastic dominance while the PGC is based on second-order stochastic dominance. Since second-order stochastic dominance is more likely to hold than first-order, the PGC should provide more conclusive results (although, as stressed previously, it is based on stronger assumptions).

Son (2004) emphasizes another potential advantage of the PGC. The estimation of $r(p)$ is based on individual data while that of $g(p)$ implies estimating the growth rate of the mean income (consumption) up to the p^{th} percentile, a procedure which is somehow less prone to measurement errors.

c) Using the Poverty Growth Curve to Derive an Index of Pro-Poor Growth:

As explained previously, the higher the PGC, the greater the reduction in poverty. This is why Kakwani and Son (2006) proposed to use the area under the PGC as a measure of pro-poor growth. More precisely, integrating (A-3) on both sides, they defined a new pro-poor growth rate $g*$ as

$$g* = \int_0^1 g(p)dp = \gamma + \int_0^1 \Delta \ln L(p)dp \qquad \text{(A-16)}$$

It is well known that the Gini index G may be expressed as

$$G = 2\{\int_0^1 [p - L(p)]dp\} \qquad \text{(A-17)}$$

Let us now similarly define an inequality index $G*$ as

$$\ln(G*) = 2\{\int_0^1 [\ln(p) - \ln(L(p))]dp\} \qquad \text{(A-18)}$$

One may then prove (see, Kakwani and Son, 2006) that

$$g* = \gamma - (1/2)\Delta \ln(G*) \qquad \text{(A-19)}$$

Expression (A-19) implies that growth is pro-poor if $(1/2)\Delta \ln(G*) \prec 0$.

Appendix B: A Short Summary of the Concept of Shapley Decomposition

Let $F(a,b)$ be a function depending on two variables a and b. Such a function need not be linear. Although Chantreuil and Trannoy (1999) and Sastre and Trannoy (2002) limited their application of the Shapley value to the decomposition of income inequality, Shorrocks (1999) has shown that such a decomposition could be applied to any function.

The idea of the Shapley value is to consider all the possible sequences allowing us to eliminate the variables a and b. Let us start with the elimination of the variable a. This variable may be the first one or the second one to be eliminated. If it is eliminated first, the function $F(a,b)$ will become equal to $F(b)$ since the variable a has been eliminated so that in this case the contribution of a to the function $F(a,b)$ is equal to $F(a,b) - F(b)$. If the variable a is the second one to be eliminated the function F will then be equal to $F(a)$. Since both elimination sequences are possible and assuming the probability of these two sequences is the same, we may conclude that the contribution $C(a)$ of the variable a to the function $F(a,b)$ is equal to

$$C(a) = (1/2)[F(a,b) - F(b)] + (1/2)F(a) \tag{B-1}$$

Similarly one can prove that the contribution $C(b)$ of the variable b to the function $F(a,b)$ is

$$C(b) = (1/2)[F(a,b) - F(a)] + (1/2)F(b) \tag{B-2}$$

Combining (B-1) and (B-2) we observe that

$$C(a) + C(b) = F(a,b) \tag{B-3}$$

Appendix C: The Case of Absolute Pro-Poor Growth When the Poverty Line Is Constant and That Where the Poverty Line Varies over Time

I) The Cases of Absolute Pro-Poor Growth When the Poverty Line Is Constant over Time:

Using the notations of section III-A let us now express the absolute change $\Delta(\theta)$ in the poverty index as

$$\Delta(\theta) = f(\Delta\bar{x}, \Delta I^{A}) \tag{C-1}$$

where I^{A} refers to some absolute measure of income inequality and ΔI^{A} to the change in this measure of inequality.

Using again the concept of Shapley decomposition, $\Delta(\theta)$ may be written as

$$\Delta(\theta) = C(\Delta\bar{x}) + C(\Delta I^{A}) \tag{C-2}$$

where $C(\Delta\bar{x})$ refers to the contribution of the change over time in the average income and $C(\Delta I^{A})$ to the contribution of the change in absolute inequality between time 0 and time 1.

The contribution $C(\Delta\bar{x})$ may itself be expressed as

$$\begin{aligned} C(\Delta\bar{x}) = (1/2)\{&[\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^{A} \neq 0) - \Delta\theta(\Delta\bar{x} = 0; \Delta I^{A} \neq 0)] \\ &+ [\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^{A} = 0) - \Delta\theta(\Delta\bar{x} = 0; \Delta I^{A} = 0)]\} \end{aligned} \tag{C-3}$$

Similarly the contribution $C(\Delta I^{A})$ may be written as

$$\begin{aligned} C(\Delta I^{A}) = (1/2)\{&[\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^{A} \neq 0) - \Delta\theta(\Delta\bar{x} \neq 0; \Delta I^{A} = 0)] \\ &+ [\Delta\theta(\Delta\bar{x} = 0; \Delta I^{A} \neq 0) - \Delta\theta(\Delta\bar{x} = 0; \Delta I^{A} = 0)]\} \end{aligned} \tag{C-4}$$

Combining (C-3) and (C-4) we observe that

$$C(\Delta\bar{x})+C(\Delta I^{A})=[\Delta\theta(\Delta\bar{x}\neq 0;\Delta I^{A}\neq 0)] \quad \text{(C-5)}$$
$$-[\Delta\theta(\Delta\bar{x}=0;\Delta I^{A}=0)]$$
$$=[\theta(\{y\})-\theta(\{x\})]-[\theta(\{x\})-\theta(\{x\})]$$
$$=[\theta(\{y\})-\theta(\{x\})]$$

Let us now define the expression $\Delta\theta(\Delta\bar{x}\neq 0;\Delta I^{A}=0)$.

It is easy to derive that this expression may be also expressed as $\theta(\{x+\Delta\bar{x}\})-\theta(\{x\})$ where $\theta(\{x+\Delta\bar{x}\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the original incomes at time 0 plus an amount $\Delta\bar{x}=\bar{y}-\bar{x}$ assumed to have been added to every individual.

Similarly the expression $\Delta\theta(\Delta\bar{x}=0;\Delta I^{A}\neq 0)$ may be also written as $\theta(\{y-\Delta\bar{x}\})-\theta(\{x\})$ where $\theta(\{y-\Delta\bar{x}\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the difference between the incomes at time 1 and a sum $\Delta\bar{x}=\bar{y}-\bar{x}$ assumed to have been deducted from every individual.

We therefore end up with

$$C(\Delta\bar{x})=(1/2)\{[[\theta((y\})-\theta(\{x\})]-[\theta(\{y-\Delta\bar{x}\})-\theta(\{x\})]] \quad \text{(C-6)}$$
$$+[[\theta(\{x+\Delta\bar{x}\})-\theta(\{x\})]-[[\theta(\{x\})-\theta(\{x\})]]\}$$
$$=(1/2)\{[\theta(\{y\})-\theta(\{y-\Delta\bar{x}\})]+[\theta(\{x+\Delta\bar{x}\})-\theta(\{x\})]\}$$

Similarly we can write that

$$C(\Delta I^{A})=(1/2)\{[[\theta(\{y\})-\theta(\{x\})]-[\theta(\{x+\Delta\bar{x}\})-\theta(\{x\})]] \quad \text{(C-7)}$$
$$+[[\theta(\{y-\Delta\bar{x}\})-\theta(\{x\})]-[\theta(\{x\})-\theta(\{x\})]]\}$$
$$=(1/2)\{[\theta(\{y\})-\theta(\{x+\Delta\bar{x}\})]+[\theta(\{y-\Delta\bar{x}\})-\theta(\{x\})]\}$$

Combining (C-6) and (C-7) we observe again that, as expected,

$$C(\Delta\bar{x})+C(\Delta I^{A})=\theta(\{y\})-\theta(\{x\}) \quad \text{(C-8)}$$

The actual relative change in the poverty index observed between times 0 and 1 will therefore be expressed as

$$(d\theta/\theta) = Gr^A + In^A \tag{C-9}$$

where

$$Gr^A = C(\Delta\bar{x})/\theta \tag{C-10}$$

and

$$In^A = C(\Delta I^A)/\theta \tag{C-11}$$

where θ, as before is the value of the poverty index at the original period 0.

Using (C-9), (C-10), and (C-11) we will now call ∂ the total poverty elasticity of growth, that is, the percentage change in poverty $(d\theta/\theta)$ when the growth in the mean income (consumption) is equal to 1%. Similarly, following Kakwani and Son (2008) call η^A the percentage change in poverty (Gr^A) that is observed when the growth in mean income (consumption) is equal to 1% *and there is no change over time in absolute inequality*.

Kakwani and Son (2008) have then defined the Absolute Pro-Poor Growth Index $(PPGI^A)$ as

$$PPGI^A = \frac{\delta}{\eta^A} \tag{C-12}$$

The measure η^A is also called the neutral absolute growth elasticity of poverty, that is, the elasticity of poverty with respect to growth when the benefits of growth are shared equally, in the absolute sense of equality, by all the members of society. We may therefore conclude (see, Kakwani and Son, 2008) that growth is pro-poor in the absolute sense if $PPGI^A$ is greater than one. Note that if growth is negative, growth will be defined as absolute pro-poor if $PPGI^A$ is less than one (the absolute loss of income resulting from negative growth would be smaller for the poor than for the non-poor.

Finally Kakwani and Son (2008) have also defined a "poverty equivalent growth rate" ($PEGR^A$) in the case where an absolute approach to inequality is adopted. $PEGR^A$ refers to the growth rate that would result in the same level of poverty reduction as the one actually observed, assuming there had been no change in absolute inequality during the growth process.

Let us, as before, call γ the actual growth rate (of the mean income) and let us call γ^A the growth rate that would have been observed had there been no change in absolute inequality. If growth is neutral in the absolute sense (that is, when there is growth without change in absolute inequality) the relative change in poverty will be expressed as $\eta^A\gamma^A$. Here again we would like this hypothetical relative change in poverty to be equal to the one which is actually observed and is equal to $\delta\gamma$. It is then easy to conclude that if $\eta^A\gamma^A = \delta\gamma$, we must have

$$PEGR^A = \gamma^A = (\frac{\delta}{\eta^A})\gamma = (PPGI^A)\times\gamma \qquad \text{(C-13)}$$

Expression (C-13) implies that growth is pro-poor if γ^A is greater than γ.

II) The Cases of Relative and Absolute Pro-Poor Growth When the Poverty Line Varies over Time but Is Exogenously Determined:

A) The Case of Relative Pro-Poor Growth:

Let, as before, $\{x\} = \{x_1,...,x_n\}$ and $\{y\} = \{y_1,...,y_n\}$ represent the vector of incomes at times 0 and 1 and let $\theta(x, z_x)$ and $\theta(y, z_y)$ refer to the poverty index at times 0 and 1, z_x and z_y being the corresponding poverty lines. Finally let $(\Delta\theta / \theta(x, z_x))$ refer to the relative change in the poverty index between times 0 and 1, with $\Delta(\theta) = \theta(y, z_y) - \theta(x, z_x)$.

The relative change $(\Delta\theta / \theta(x, z_x))$ in the poverty index will therefore be expressed as

$$(\Delta\theta / \theta(x, z_x)) = g(\Delta z, (\Delta\bar{x} / \bar{x}), \Delta I^R) \qquad \text{(C-14)}$$

where $\bar{x}$ is the mean income of the distribution given by $\{x\}$, $\Delta\bar{x} = (\bar{y} - \bar{x})$ is the difference between the average income at times 0 and 1, I^R refers to some relative measure of income inequality of the distribution given by $\{x\}$, ΔI^R to the change in relative inequality and Δz to the change in the poverty line.

Using the concept of Shapley decomposition, $(\Delta\theta/\theta(x,z_x))$ may be written as

$$(\Delta\theta/\theta(x,z_x)) = C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R) + C(\Delta z) \tag{C-15}$$

where $C(\Delta\bar{x}/\bar{x})$ refers to the contribution of the relative change over time in the average income, $C(\Delta I^R)$ to the contribution of the change in relative inequality and $C(\Delta z)$ to the contribution of the change in the poverty line between time 0 and time 1.

Let us now express the contribution $C(\Delta\bar{x}/\bar{x})$. It may be written as

$$\begin{aligned} C(\Delta\bar{x}/\bar{x}) = (2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0; \Delta z \neq 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z = 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0; \Delta z = 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z \neq 0)]\} \\ +(2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0; \Delta z = 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z = 0)]\} \end{aligned} \tag{C-16}$$

Similarly the contribution $C(\Delta I^R)$ of the change in relative inequality may be written as

$$
\begin{aligned}
C(\Delta I^R) = (2/6)\{&[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0; \Delta z \neq 0)]\} \\
&+(1/6)\{[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z = 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0; \Delta z = 0)]\} \\
&+(1/6)\{[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z \neq 0)]\} \\
&+(2/6)\{[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0; \Delta z = 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z = 0)]\}
\end{aligned} \tag{C-17}
$$

Finally the contribution of the change in the poverty line will be expressed as

$$
\begin{aligned}
C(\Delta z) = (2/6)\{&[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z = 0)]\} \\
&+(1/6)\{[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0; \Delta z = 0)]\} \\
&+(1/6)\{[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0; \Delta z = 0)]\} \\
&+(2/6)\{[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z = 0)]\}
\end{aligned} \tag{C-18}
$$

Combining (C-16), (C-17) and (C-18) we observe that

$$
\begin{aligned}
&C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R) + C(\Delta z) \\
&= [(\Delta\theta/\theta(x,z_x))\,with\ ((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0; \Delta z \neq 0)] \\
&-[(\Delta\theta/\theta(x,z_x))\,with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0; \Delta z = 0)] \\
&= [(\theta(\{y,z_y\}) - \theta(\{x,z_x\}))/\theta(\{x,z_x\})] - [(\theta(\{x,z_x\}) - \theta(\{x,z_x\}))/\theta(\{x,z_x\})] \\
&= [(\theta(\{y,z_y\}) - \theta(\{x,z_x\}))/\theta(\{x,z_x\})]
\end{aligned} \tag{C-19}
$$

Let us now first define the expression

$[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x})\neq 0;\Delta I^R\neq 0;\Delta z\neq 0)]$. It may clearly be written as $(\theta(y,z_y)-\theta(x,z_x))/\theta(x,z_x)$.

Similarly the expression
$[(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x})=0;\Delta I^R=0;\Delta z=0)]$ may be written as
$(\theta(x,z_x)-\theta(x,z_x))/\theta(x,z_x)=0$

Now let us define the expression
$\Delta\theta/\theta(x,z_x)\,with\,((\Delta\bar{x}/\bar{x})\neq 0;\Delta I^R=0;\Delta z_x=0)$.

It is easy to derive that this expression may be also expressed as
$(\theta(\{x(1+k),z_x\})-\theta(\{x,z_x\})/\theta(\{x,z_x\})$ where
$k=(\Delta\bar{x}/\bar{x})$ and $\theta(\{x(1+k),z_x\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the original incomes multiplied by one plus a factor k equal to the growth rate of the average income between times 0 and 1 and where the poverty line is the one observed at time 0. It should be clear that if all the incomes are multiplied by the same constant k, by definition relative inequality will have remained constant.

Similarly the expression
$(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x})\neq 0;\Delta I^R=0;\Delta z\neq 0)$ may be written as
$(\theta(\{(x(1+k),z_y)\})-\theta(\{x,z_x)\})/\theta(\{x,z_x\})$
where $\theta(\{(x(1+k),z_y)\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the incomes observed at time 1 multiplied by one plus the growth rate of the average income between time 0 and time 1, assuming the poverty line is that observed at time 1.

Let us now define the expression
$(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x})\neq 0;\Delta I^R\neq 0;\Delta z=0)$. It may be written as
$(\theta\{y,z_x\}-\theta\{x,z_x\})/\theta(x,z_x)$.

Similarly we will define the expression
$(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x})=0;\Delta I^R\neq 0;\Delta z\neq 0)$ as
$(\theta\{(y/(1+k)),z_y\}-\theta\{x,z_x\})/\theta\{x,z_x\}$.

It is also easy to see that the expression
$(\Delta\theta/\theta(x,z_x))\,with\,((\Delta\bar{x}/\bar{x})=0;\Delta I^R\neq 0;\Delta z=0)$ will be written as
$(\theta\{(y/(1+k)),z_x\}-\theta\{x,z_x\})/\theta\{x,z_x\}$.

We therefore end up with

$$
\begin{aligned}
&C(\Delta\bar{x}/\bar{x}) \\
&= (2/6)\{[(\theta(y,z_y)-\theta(x,z_x))/\theta(x,z_x)] \\
&\qquad -[(\theta\{(y/(1+k)),z_y\}-\theta\{x,z_x\})/\theta\{x,z_x\}]\} \\
&+(1/6)\{[(\theta\{y,z_x\}-\theta\{x,z_x\})/\theta(x,z_x)] \\
&\qquad -[(\theta\{(y/(1+k)),z_x\}-\theta\{x,z_x\})/\theta\{x,z_x)]\} \\
&+(1/6)\{[(\theta(\{(x(1+k),z_y)\})-\theta(\{x,z_x)\})/\theta(\{x,z_x\})] \\
&\qquad -[(\theta(x,z_y)-\theta(x,z_x))/\theta(x,z_x)]\} \\
&+(2/6)\{[(\theta(\{x(1+k),z_x\})-\theta(\{x,z_x\})/\theta(\{x,z_x\})] \\
&\qquad -[(\theta(x,z_x)-\theta(x,z_x))/\theta(x,z_x)]\}
\end{aligned}
\tag{C-20}
$$

$$
\begin{aligned}
&\leftrightarrow C(\Delta\bar{x}/\bar{x}) \\
&= (2/6)[\theta(y,z_y)-\theta((y/(1+k)),z_y)] \\
&+(2/6)[\theta(x(1+k),z_x)-\theta(x,z_x)] \\
&+(1/6)[\theta(y,z_x)-\theta((y/(1+k)),z_x)] \\
&+(1/6)[\theta(x(1+k),z_y)-\theta(x,z_y)]
\end{aligned}
\tag{C-21}
$$

Similarly we can write that

$$
\begin{aligned}
&C(\Delta I^R) \\
&= (2/6)\{[(\theta(y,z_y)-\theta(x,z_x))/\theta(x,z_x)] \\
&\qquad -[(\theta(\{(x(1+k),z_y)\})-\theta(\{x,z_x)\})/\theta(\{x,z_x\})]\} \\
&+(1/6)\{[(\theta\{(y,z_x\}-\theta\{x,z_x\})/\theta\{x,z_x\}] \\
&\qquad -[(\theta(x(1+k),z_x)-\theta(x,z_x))/\theta(x,z_x)]\} \\
&+(1/6)\{[(\theta\{y/(1+k),z_y\}-\theta\{x,z_x\})/\theta(x,z_x)] \\
&\qquad -[(\theta(\{x,z_y\})-\theta(\{x,z_x\})/\theta(\{x,z_x\})]\} \\
&+(2/6)\{[(\theta\{(y/(1+k)),z_x\}-\theta\{x,z_x\})/\theta\{x,z_x] \\
&\qquad -[(\theta(x,z_x)-\theta(x,z_x))/\theta(x,z_x)]\}
\end{aligned}
\tag{C-22}
$$

so that

$$C(\Delta I^{R}) = (2/6)\{[(\theta(y,z_{y}) - \theta(x(1+k),z_{y}))/\theta(x,z_{x})]$$
$$+(1/6)\{[(\theta\{(y,z_{x}\} - \theta\{x(1+k),z_{x}\})/\theta\{x,z_{x}\}]$$
$$+(1/6)\{[(\theta\{y/(1+k),z_{y}\} - \theta\{x,z_{y}\})/\theta(x,z_{x})]$$
$$+(2/6)\{[(\theta\{(y/(1+k)),z_{x}\} - \theta\{x,z_{x}\})/\theta\{x,z_{x}] \qquad \text{(C-23)}$$

Finally we can also write that

$$\begin{aligned}
&C(\Delta z)\\
&= (2/6)\{[(\theta(y,z_{y}) - \theta(x,z_{x}))/\theta(x,z_{x})]\\
&\qquad -[(\theta\{y,z_{x}\} - \theta\{x,z_{x}\})/\theta(x,z_{x})]\}\\
&+(1/6)\{[(\theta\{(y/(1+k)),z_{y}\} - \theta\{x,z_{x}\})/\theta\{x,z_{x}\}]\\
&\qquad -[(\theta\{(y/(1+k)),z_{x}\} - \theta\{x,z_{x}\})/\theta\{x,z_{x})]\}\\
&+(1/6)\{[(\theta(\{(x(1+k),z_{y})\}) - \theta(\{x,z_{x})\})/\theta(\{x,z_{x}\})]\\
&\qquad -[(\theta(\{x(1+k),z_{x}\}) - \theta(\{x,z_{x}\})/\theta(\{x,z_{x}\})]\}\\
&+(2/6)\{[(\theta(x,z_{y}) - \theta(x,z_{x}))/\theta(x,z_{x})]\\
&\qquad -[(\theta(x,z_{x}) - \theta(x,z_{x}))/\theta(x,z_{x})]\}
\end{aligned} \qquad \text{(C-24)}$$

The latter expression may be also written as

$$\begin{aligned}
C(\Delta z) &= (2/6)\{[(\theta(y,z_{y}) - \theta(y,z_{x}))/\theta(x,z_{x})]\}\\
&+(1/6)\{[(\theta\{(y/(1+k)),z_{y}\}) - (\theta\{(y/(1+k)),z_{x}\})]\}\\
&+(1/6)\{[(\theta(\{(x(1+k),z_{y})\}) - (\theta(\{(x(1+k),z_{x})\})]\}\\
&+(2/6)\{[(\theta(x,z_{y}) - \theta(x,z_{x}))/\theta(x,z_{x})]
\end{aligned} \qquad \text{(C-25)}$$

It is then easy to observe that if we sum the three expressions $C(\Delta\bar{x}/\bar{x}), C(\Delta I^{R}), C(\Delta z)$ given in (C-23), (C-24) and (C-25) we obtain

$$
\begin{aligned}
&C(\Delta\bar{x}/\bar{x}) + C(\Delta I^{R}) + C(\Delta z) \\
&= (2/6)[\theta(y,z_y) - \theta((y/(1+k)), z_y)] \\
&+ (2/6)[\theta(x(1+k), z_x) - \theta(x, z_x)] \\
&+ (1/6)[\theta(y,z_x) - \theta((y/(1+k)), z_x)] \\
&+ (1/6)[\theta(x(1+k), z_y) - \theta(x, z_y)] \\
&+ (2/6)\{[(\theta(y,z_y) - \theta(x(1+k), z_y))/\theta(x,z_x)] \\
&+ (1/6)\{[(\theta\{(y,z_x\} - \theta\{x(1+k), z_x\})/\theta\{x,z_x\}] \\
&+ (1/6)\{[(\theta\{y/(1+k), z_y\} - \theta\{x, z_y\})/\theta(x,z_x)] \\
&+ (2/6)\{[(\theta\{(y/(1+k)), z_x\} - \theta\{x,z_x\})/\theta\{x,z_x] \\
&+ (2/6)\{[(\theta(y,z_y) - \theta(y,z_x))/\theta(x,z_x)]\} \\
&+ (1/6)\{[(\theta\{(y/(1+k)), z_y\}) - (\theta\{(y/(1+k)), z_x\})]\} \\
&+ (1/6)\{[(\theta(\{(x(1+k), z_y)\}) - (\theta(\{(x(1+k), z_x)\})]\} \\
&+ (2/6)\{[(\theta(x,z_y) - \theta(x,z_x))/\theta(x,z_x)]
\end{aligned}
\qquad \text{(C-26)}
$$

which amounts, as expected, to writing

$$
\begin{aligned}
&C(\Delta\bar{x}/\bar{x}) + C(\Delta I^{R}) + C(\Delta z) \\
&\quad = [(\theta(y,z_y) - \theta(x,z_x))/\theta(x,z_x)]
\end{aligned}
\qquad \text{(C-27)}
$$

B) The Case of Absolute Pro-Poor Growth:

Let, as before, $\{x\} = \{x_1,...,x_n\}$ and $\{y\} = \{y_1,...,y_n\}$ represent the vector of incomes at times 0 and 1 and let $\theta(x,z_x)$ and $\theta(y,z_y)$ refer to the poverty index at times 0 and 1, z_x and z_y being the corresponding poverty lines. At this stage we will assume that the poverty lines z_x (at time 0) and z_y (at time 1) are not the same but we will assume that they are exogenously determined. Finally let $(\Delta\theta/\theta(x,z_x))$ refer to the relative change in the poverty index between times 0 and 1, with $\Delta(\theta) = \theta(y,z_y) - \theta(x,z_x)$.

The relative change $(\Delta\theta/\theta(x,z_x))$ in the poverty index will therefore be expressed as

$$(\Delta\theta/\theta(x,z_x)) = g(\Delta z, \Delta\bar{x}, \Delta I^A) \qquad \text{(C-28)}$$

where $\bar{x}$ is the mean income of the distribution given by $\{x\}$, $\Delta\bar{x} = (\bar{y} - \bar{x})$ is the difference between the average incomes at times 0 and 1, I^A refers to some absolute measure of income inequality of the distribution given by $\{x\}$, ΔI^A to the change in absolute inequality and Δz to the change in the poverty line.

Using the concept of Shapley decomposition, $(\Delta\theta/\theta(x,z_x))$ may be written as

$$(\Delta\theta/\theta(x,z_x)) = C(\Delta\bar{x}) + C(\Delta I^A) + C(\Delta z) \qquad \text{(C-29)}$$

where $C(\Delta\bar{x})$ refers to the contribution of the change over time in the average income, $C(\Delta I^A)$ to the contribution of the change in absolute inequality and $C(\Delta z)$ to the contribution of the change in the poverty line between time 0 and time 1.

Let us now express the contribution $C(\Delta\bar{x})$. It may be written as

$$\begin{aligned} C(\Delta\bar{x}) = (2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z \neq 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z = 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z = 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z \neq 0)]\} \\ +(2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z = 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z = 0)]\} \end{aligned} \qquad \text{(C-30)}$$

Similarly the contribution $C(\Delta I^A)$ of the change in absolute inequality may be written as

$$\begin{aligned} C(\Delta I^A) = (2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z \neq 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z = 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z = 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z \neq 0)]\} \\ +(2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z = 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z = 0)]\} \end{aligned} \tag{C-31}$$

Finally the contribution of the change in the poverty line will be expressed as

$$\begin{aligned} C(\Delta z) = (2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z = 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z = 0)]\} \\ +(1/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z = 0)]\} \\ +(2/6)\{&[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z \neq 0)] \\ &-[(\Delta\theta/\theta(x,z_x))\, with\, ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z = 0)]\} \end{aligned} \tag{C-32}$$

Combining (C-30), (C-31) and (C-32) we observe that

$$
\begin{aligned}
& C(\Delta\bar{x}/\bar{x}) + C(\Delta I^{A}) + C(\Delta z) \\
& = [(\Delta\theta/\theta(x,z_x))\, with\ ((\Delta\bar{x}) \neq 0; \Delta I^{A} \neq 0; \Delta z \neq 0)] \\
& -[(\Delta\theta/\theta(x,z_x))\, with\ ((\Delta\bar{x}) = 0; \Delta I^{A} = 0; \Delta z = 0)] \\
& = [(\theta(\{y,z_y\}) - \theta(\{x,z_x\}))/\theta(\{x,z_x\})] \\
& -[(\theta(\{x,z_x\}) - \theta(\{x,z_x\}))/\theta(\{x,z_x\})] \\
& = [(\theta(\{y,z_y\}) - \theta(\{x,z_x\}))/\theta(\{x,z_x\})]
\end{aligned} \qquad \text{(C-33)}
$$

Let us now first define the expression

$(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}) \neq 0; \Delta I^{A} \neq 0; \Delta z \neq 0)]$. It may clearly be written as $(\theta(y,z_y) - \theta(x,z_x))/\theta(x,z_x)$

Similarly the expression
$(\Delta\theta/\theta(x,z_x))\, with((\Delta\bar{x}) = 0; \Delta I^{A} = 0; \Delta z = 0)]$ may be written as
$(\theta(x,z_x) - \theta(x,z_x))/\theta(x,z_x) = 0$

Now let us define the expression

$(\Delta\theta/\theta(x,z_x)\, with\ ((\Delta\bar{x}) \neq 0; \Delta I^{A} = 0; \Delta z_x = 0)$.

It is easy to derive that this expression may be also expressed as

$(\theta(\{x+\Delta\bar{x}, z_x\}) - \theta(\{x,z_x\})/\theta(\{x,z_x\})$

where $\theta(\{(x+\Delta\bar{x}), z_x\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the original incomes to which a sum $\Delta\bar{x}$, equal to the difference between the average incomes at times 1 and 0, has been added to each individual's income at time 0 and where the poverty line is the one observed at time 0. It is should be clear that if the same amount of money $\Delta\bar{x}$ is added to all the incomes, then by definition absolute inequality will have remained constant.

Similarly the expression

$(\Delta\theta / \theta(x, z_x))\, with\ ((\Delta\bar{x}) \neq 0; \Delta I^A = 0; \Delta z \neq 0)$ may be written as
$(\theta(\{(x + \Delta\bar{x}), z_y)\}) - \theta(\{x, z_x)\}) / \theta(\{x, z_x\})$

where $\theta(\{(x + \Delta\bar{x}), z_y)\})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the incomes observed at time 0 to which an equal sum $\Delta\bar{x}$ has been added to all incomes, assuming the poverty line is that observed at time 1.

Let us now define the expression
$(\Delta\theta / \theta(x, z_x))\, with\ ((\Delta\bar{x}) \neq 0; \Delta I^A \neq 0; \Delta z = 0)$. It may be written as
$(\theta\{y, z_x\} - \theta\{x, z_x\}) / \theta(x, z_x)$

Similarly we will define the expression
$(\Delta\theta / \theta(x, z_x))\, with\ ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z \neq 0)$ as
$(\theta\{(y - \Delta\bar{x}), z_y\} - \theta\{x, z_x\}) / \theta\{x, z_x\}$

where $\theta\{(y - \Delta\bar{x})\}$ refers to a hypothetical distribution at time 0 where the individual incomes would be those actually observed at time 1 from which an equal amount $\Delta\bar{x}$ would have been deduced from each individual income.

It is easy to see that the expression
$(\Delta\theta / \theta(x, z_x))\, with\ ((\Delta\bar{x}) = 0; \Delta I^A \neq 0; \Delta z = 0)$ will be written as
$(\theta\{(y - \Delta\bar{x}), z_x\} - \theta\{x, z_x\}) / \theta\{x, z_x\}$

Finally one should observe that the expression

$(\Delta\theta / \theta(x, z_x))\, with\ ((\Delta\bar{x}) = 0; \Delta I^A = 0; \Delta z = 0)$ will be written as
$(\theta(x, z_x) - \theta(x, z_x)) / \theta(x, z_x) = 0$

We therefore end up with

$$\begin{aligned}
&C(\Delta\bar{x}) \\
&= (2/6)\{[(\theta(y,z_y)-\theta(x,z_x))/\theta(x,z_x)] \\
&\qquad -[(\theta\{(y-\Delta\bar{x}),z_y\}-\theta\{x,z_x\})/\theta\{x,z_x\}]\} \\
&+(1/6)\{[(\theta\{y,z_x\}-\theta\{x,z_x\})/\theta(x,z_x)] \\
&\qquad -[(\theta\{(y-\Delta\bar{x}),z_x\}-\theta\{x,z_x\})/\theta\{x,z_x)]\} \\
&+(1/6)\{[(\theta(\{(x+\Delta\bar{x}),z_y)\})-\theta(\{x,z_x)\})/\theta(\{x,z_x\})] \\
&\qquad -[(\theta(x,z_y)-\theta(x,z_x))/\theta(x,z_x)]\} \\
&+(2/6)\{[(\theta(\{(x+\Delta\bar{x}),z_x\})-\theta(\{x,z_x\})/\theta(\{x,z_x\})] \\
&\qquad -[(\theta(x,z_x)-\theta(x,z_x))/\theta(x,z_x)]\}
\end{aligned} \tag{C-34}$$

$$\begin{aligned}
\leftrightarrow C(\Delta\bar{x}) &= (2/6)[\theta(y,z_y)-\theta((y-\Delta\bar{x}),z_y)] \\
&+(2/6)[\theta(x+\Delta\bar{x}),z_x)-\theta(x,z_x)] \\
&+(1/6)[\theta(y,z_x)-\theta((y-\Delta\bar{x}),z_x)] \\
&+(1/6)[\theta(x+\Delta\bar{x}),z_y)-\theta(x,z_y)]
\end{aligned} \tag{C-35}$$

Similarly we can write that

$$\begin{aligned}
&C(\Delta I^A) \\
&= (2/6)\{[(\theta(y,z_y)-\theta(x,z_x))/\theta(x,z_x)] \\
&\qquad -[(\theta(\{(x+\Delta\bar{x}),z_y)\})-\theta(\{x,z_x)\})/\theta(\{x,z_x\})]\} \\
&+(1/6)\{[(\theta\{(y-\Delta\bar{x}),z_y\}-\theta\{x,z_x\})/\theta\{x,z_x\}] \\
&\qquad -[(\theta(x,z_y)-\theta(x,z_x))/\theta(x,z_x)]\} \\
&+(1/6)\{[(\theta\{y,z_x\}-\theta\{x,z_x\})/\theta(x,z_x)] \\
&\qquad -[(\theta(\{(x+\Delta\bar{x}),z_x\})-\theta(\{x,z_x\})/\theta(\{x,z_x\})]\} \\
&+(2/6)\{[(\theta\{(y-\Delta\bar{x}),z_x\}-\theta\{x,z_x\})/\theta\{x,z_x] \\
&\qquad -[(\theta(x,z_x)-\theta(x,z_x))/\theta(x,z_x)]\}
\end{aligned} \tag{C-36}$$

so that

$$\begin{aligned} C(\Delta I^{A}) = (2/6)\{[(\theta(y, z_{y}) - \theta((x+\Delta\bar{x}), z_{y}) / \theta(x, z_{x})] \\ +(1/6)\{[(\theta\{(y-\Delta\bar{x}), z_{y}\} - \theta\{x, z_{y}\}) / \theta\{x, z_{x}\}] \\ +(1/6)\{[(\theta\{y, z_{x}\} - \theta\{(x+\Delta\bar{x}), z_{x}\}) / \theta(x, z_{x})] \\ +(2/6)\{[(\theta\{(y-\Delta\bar{x}), z_{x}\} - \theta\{x, z_{x}\}) / \theta\{x, z_{x}] \end{aligned} \tag{C-37}$$

Finally we can also write that

$$\begin{aligned} & C(\Delta z) \\ & = (2/6)\{[(\theta(y,z_{y}) - \theta(x,z_{x}))/\theta(x,z_{x})] \\ & \quad -[(\theta\{y,z_{x}\} - \theta\{x,z_{x}\})/\theta(x,z_{x})]\} \\ & +(1/6)\{[(\theta\{(y-\Delta\bar{x}),z_{y}\} - \theta\{x,z_{x}\})/\theta\{x,z_{x}\}] \\ & \quad -[(\theta\{(y-\Delta\bar{x}),z_{x}\} - \theta\{x,z_{x}\})/\theta\{x,z_{x})]\} \\ & +(1/6)\{[(\theta(\{(x+\Delta\bar{x}),z_{y})\}) - \theta(\{x,z_{x})\})/\theta(\{x,z_{x}\})] \\ & \quad -[(\theta(\{(x+\Delta\bar{x}),z_{x}\}) - \theta(\{x,z_{x}\})/\theta(\{x,z_{x}\})]\} \\ & +(2/6)\{[(\theta(x,z_{y}) - \theta(x,z_{x}))/\theta(x,z_{x})] \\ & \quad -[(\theta(x,z_{x}) - \theta(x,z_{x}))/\theta(x,z_{x})]\} \end{aligned} \tag{C-38}$$

The latter expression may be also written as

$$\begin{aligned} C(\Delta z) = (2/6)\{[(\theta(y, z_{y}) - \theta(y, z_{x}))/\theta(x, z_{x})]\} \\ +(1/6)\{[(\theta\{(y-\Delta\bar{x}), z_{y}\}) - (\theta\{(y-\Delta\bar{x}), z_{x}\})]\} \\ +(1/6)\{[(\theta(\{(x+\Delta\bar{x}), z_{y})\}) - (\theta(\{(x+\Delta\bar{x}), z_{x}\})]\} \\ +(2/6)\{[(\theta(x, z_{y}) - \theta(x, z_{x}))/\theta(x, z_{x})] \end{aligned} \tag{C-39}$$

It is then easy to observe that if we sum the three expressions $C(\Delta\bar{x}), C(\Delta I^{A}), C(\Delta z)$ given in (C-35), (C-37) and (C-39) we obtain

$$
\begin{aligned}
&C(\Delta\bar{x}) + C(\Delta I^A) + C(\Delta z) \\
&= (2/6)[\theta(y, z_y) - \theta((y - \Delta\bar{x}), z_y)] \\
&+ (2/6)[\theta((x + \Delta\bar{x}), z_x) - \theta(x, z_x)] \\
&+ (1/6)[\theta(y, z_x) - \theta((y - \Delta\bar{x}), z_x)] \\
&+ (1/6)[\theta((x + \Delta\bar{x}), z_y) - \theta(x, z_y)] \\
&+ (2/6)\{[(\theta(y, z_y) - \theta((x + \Delta\bar{x}), z_y)) / \theta(x, z_x)] \\
&+ (1/6)\{[(\theta\{(y - \Delta\bar{x}), z_y\} - \theta\{x, z_y\}) / \theta\{x, z_x\}] \qquad \text{(C-40)} \\
&+ (1/6)\{[(\theta\{y, z_x\} - \theta\{(x + \Delta\bar{x}), z_x\}) / \theta(x, z_x)] \\
&+ (2/6)\{[(\theta\{(y - \Delta\bar{x}), z_x\} - \theta\{x, z_x\}) / \theta\{x, z_x] \\
&+ (2/6)\{[(\theta(y, z_y) - \theta(y, z_x)) / \theta(x, z_x)]\} \\
&+ (1/6)\{[(\theta\{(y - \Delta\bar{x}), z_y\}) - (\theta\{(y - \Delta\bar{x}), z_x\})]\} \\
&+ (1/6)\{[(\theta(\{(x + \Delta\bar{x}), z_y\}) - (\theta(\{(x + \Delta\bar{x}), z_x\})]\} \\
&+ (2/6)\{[(\theta(x, z_y) - \theta(x, z_x)) / \theta(x, z_x)]
\end{aligned}
$$

which amounts, as expected, to writing

$$C(\Delta\bar{x}) + C(\Delta I^A) + C(\Delta z) = [(\theta(y, z_y) - \theta(x, z_x)) / \theta(x, z_x)] \qquad \text{(C-41)}$$

III) The Cases of Relative and Absolute Pro-Poor Growth When the Poverty Line Varies over Time but Is Endogenously Determined

A) Measuring Relative Pro-Poor Growth (in discrete terms) when the poverty line is variable and endogenous:

Let, as before, $\{x\} = \{x_1, ..., x_n\}$ and $\{y\} = \{y_1, ..., y_n\}$ represent the vector of incomes at times 0 and 1 and let $\theta(x)$ and $\theta(y)$ refer to the poverty index at times 0 and 1. Finally let $(\Delta\theta / \theta(x))$ refer to the relative change in the poverty index between times 0 and 1, with $\Delta(\theta) = \theta(y) - \theta(x)$.

The relative change $\Delta(\theta)/\theta$ in the poverty index will now be expressed as

$$\Delta(\theta) = g((\Delta\bar{x}/\bar{x}), \Delta I^R) \qquad \text{(C-42)}$$

where $\bar{x}$ is the mean income of the distribution given by $\{x\}$, $\Delta\bar{x} = (\bar{y} - \bar{x})$ is the difference between the average income at times 0 and 1 and I^R refers to some relative measure of income inequality of the distribution given by (x). The poverty line will always be assumed to be equal to half the median of the corresponding distribution.

Using the concept of Shapley decomposition, $\Delta(\theta)$ may be written as

$$(\Delta\theta/\theta) = C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R) \qquad \text{(C-43)}$$

where $C(\Delta\bar{x}/\bar{x})$ refers to the contribution of the relative change over time in the average income and $C(\Delta I^R)$ to the contribution of the change in relative inequality between time 0 and time 1.

The contribution $C(\Delta\bar{x}/\bar{x})$ may itself be expressed as

$$\begin{aligned} C(\Delta\bar{x}/\bar{x}) = (1/2)\{&[(\Delta\theta/\theta)\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0) - (\Delta\theta/\theta) \\ &with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0)] \\ &+[(\Delta\theta/\theta)\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0) \\ &-(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)]\} \end{aligned} \qquad \text{(C-44)}$$

Similarly the contribution $C(\Delta I^R)$ may be written as

$$\begin{aligned} C(\Delta I^R) = (1/2)\{&[(\Delta\theta/\theta)\, with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0) - (\Delta\theta/\theta) \\ &with((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0)] \\ &+[(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0) - (\Delta\theta/\theta) \\ &with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)]\} \end{aligned} \qquad \text{(C-45)}$$

Combining (C-44) and (C-45) we observe that

$$\begin{aligned} & C(\Delta\bar{x}/\bar{x}) + C(\Delta I^R) \\ & = [(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0)] \\ & - [(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)] \\ & = [(\theta(\{y\}) - \theta(\{x\}))/\theta(\{x\})] \\ & - [(\theta(\{x\}) - \theta(\{x\}))/\theta(\{x\})] \\ & = [(\theta(\{y\}) - \theta(\{x\}))/\theta(\{x\})] \end{aligned} \quad \text{(C-46)}$$

Let us first define the expression $(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R \neq 0)$. It may be written as $(\theta(\{y\}, z_y) - \theta(\{x\}, z_x))/\theta(\{x\}, z_x)$

Let us now define the expression $(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) \neq 0; \Delta I^R = 0)$. It is easy to see that it may be written as

$(\theta(\{x(1+k)\}, z_{x(1+k)}) - \theta(\{x\}, z_x)/\theta(\{x\}, z_x)$

where $k = (\Delta\bar{x}/\bar{x})$ and $\theta(\{x(1+k)\}, z_{x(1+k)})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the original incomes multiplied by a factor k equal to the growth rate of the average income between times 0 and 1 and the poverty line is equal to half the median of the distribution $\{x(1+k)\}$. It should be clear that if all the incomes are multiplied by the same constant k, by definition relative inequality will have remained constant.

Similarly the expression $(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R \neq 0)$ may be written as

$(\theta(\{(y/(1+k))\}, z_{y/(1+k)}) - \theta(\{x\}, z_x)/\theta(\{x\}, z_x)$

where $\theta(\{(y/(1+k))\}, z_{y/(1+k)})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the incomes observed at time 1 divided by one plus the growth rate of the average income between time 0 and time 1, $z_{y/(1+k)}$ being the poverty line (half the median) corresponding to this distribution $\{y/(1+k)\}$.

Finally $(\Delta\theta/\theta)\, with\ ((\Delta\bar{x}/\bar{x}) = 0; \Delta I^R = 0)$ will evidently be expressed as $(\theta(\{x\}, z_x) - \theta(\{x\}, z_x)/\theta(\{x\}, z_x) = 0$

We therefore end up with

$$\begin{aligned}
&C(\Delta\bar{x}/\bar{x}) \\
&= (1/2)\{[(\theta(\{y\},z_y) - \theta(\{x\},z_x))/\theta(\{x\},z_x)] \\
&-[(\theta(\{(y/(1+k))\},z_{y/(1+k)}) - \theta(\{x\},z_x))/\theta(\{x\},z_x)]]\} \\
&+(1/2)\{[(\theta(\{x(1+k)\},z_{x(1+k)}) - \theta(\{x\},z_x))/\theta(\{x\},z_x)] \\
&-[(\theta(\{x\},z_x) - \theta(\{x\},z_x))/\theta(\{x\},z_x)]\} \\
&= (1/2)\{[(\theta(\{y\},z_y) - \theta(\{(y/(1+k))\},z_{y/(1+k)})/\theta(\{x\},z_x)] \\
&+[(\theta(\{x(1+k)\},z_{x(1+k)}) - \theta(\{x\},z_x))/\theta(\{x\},z_x)]\}
\end{aligned} \quad \text{(C-47)}$$

Similarly we can write that

$$\begin{aligned}
&C(\Delta I^R) = (1/2)\{[(\theta(\{y\},z_y) - \theta(\{x\},z_x))/\theta(\{x\},z_x)] \\
&-[(\theta(\{x((1+k)\},z_{x(1+k)}) - \theta(\{x\},z_x))/\theta(\{x\},z_x)]]\} \\
&+(1/2)\{[(\theta(\{y/(1+k)\},z_{y/(1+k)}) - \theta(\{x\},z_x))/\theta(\{x\},z_x)] \\
&-[(\theta(\{x\},z_x) - \theta(\{x\},z_x))/\theta(\{x\},z_x)]\} \\
&= (1/2)\{[(\theta(\{y\},z_y) - \theta(\{x(1+k)\},z_{x(1+k)})/\theta(\{x\},z_x)] \\
&+[(\theta(\{y/(1+k)\},z_{y/(1+k)}) - \theta(\{x\},z_x))/\theta(\{x\},z_x)]\}
\end{aligned} \quad \text{(C-48)}$$

Combining (C-47) and (C-48) we observe again that, as expected,

$$C(\Delta\bar{x}) + C(\Delta I^A) = \theta(\{y\}) - \theta(\{x\}) \quad \text{(C-49)}$$

The actual relative change in the poverty index observed between times 0 and 1 will therefore be expressed as

$$(d\theta/\theta) = Gr^A + In^A \quad \text{(C-50)}$$

where

$$Gr^A = C(\Delta\bar{x})/\theta \quad \text{(C-51)}$$

and

$$In^{A} = C(\Delta I^{A}) / \theta \tag{C-52}$$

where θ, as before is the value of the poverty index at the original period 0.

B) Measuring Absolute Pro-Poor Growth (in discrete terms) when the poverty line is variable and endogenous:

Let, as before, $\{x\} = \{x_1, ..., x_n\}$ and $\{y\} = \{y_1, ..., y_n\}$ represent the vector of incomes at times 0 and 1 and let $\theta(\{x\}, z_x)$ and $\theta(\{y\}, z_y)$ refer to the poverty index at times 0 and 1. Throughout this section it is assumed that the poverty line varies over time and is endogenous because it is always defined as being equal to half the median of the corresponding distribution. Finally let $(\Delta\theta / \theta(\{x\}, z_x))$ refer to the relative change in the poverty index between times 0 and 1, with $\Delta(\theta) = \theta(\{y\}, z_y) - \theta(\{x\}, z_x)$.
The absolute change $\Delta(\theta)$ in the poverty index will now be expressed as

$$\Delta(\theta) = f(\Delta\bar{x}, \Delta I^{A}) \tag{C-53}$$

where $\bar{x}$ is the mean income of the distribution given by $\{x\}$, $\Delta\bar{x}$ is the change over time in the average income, so that $\Delta\bar{x} = \bar{y} - \bar{x}$ and I^{A} refers to some absolute measure of income inequality of the distribution given by (x).

Using the concept of Shapley decomposition, $\Delta(\theta)$ may be written as

$$\Delta(\theta) = C(\Delta\bar{x}) + C(\Delta I^{A}) \tag{C-54}$$

where $C(\Delta\bar{x})$ refers to the contribution of the change over time in the average income and $C(\Delta I^{A})$ to the contribution of the change in absolute inequality between time 0 and time 1.

The contribution $C(\Delta\bar{x})$ may itself be expressed as

$$\begin{aligned} C(\Delta\bar{x}) = (1/2)\{&[\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^{A} \neq 0) - \Delta\theta(\Delta\bar{x} = 0; \Delta I^{A} \neq 0)] \\ &+ [\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^{A} = 0) - \Delta\theta(\Delta\bar{x} = 0; \Delta I^{A} = 0)]\} \end{aligned} \tag{C-55}$$

Similarly the contribution $C(\Delta I^A)$ may be written as

$$\begin{aligned} C(\Delta I^A) = (1/2)\{&[\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^A \neq 0) - \Delta\theta(\Delta\bar{x} \neq 0; \Delta I^A = 0)] \\ &+ [\Delta\theta(\Delta\bar{x} = 0; \Delta I^A \neq 0) - \Delta\theta(\Delta\bar{x} = 0; \Delta I^A = 0)]\} \end{aligned} \quad \text{(C-56)}$$

Combining (C-55) and (C-56) we observe that

$$\begin{aligned} &C(\Delta\bar{x}) + C(\Delta I^A) \\ &= [\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^A \neq 0)] - [\Delta\theta(\Delta\bar{x} = 0; \Delta I^A = 0)] \\ &= [\theta(\{y\}) - \theta(\{x\})] - [\theta(\{x\}) - \theta(\{x\})] = [\theta(\{y\}) - \theta(\{x\})] \end{aligned} \quad \text{(C-57)}$$

Let us first define the expression $\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^A \neq 0)$. It may be written as $\theta(\{y\}, z_y) - \theta(\{x\}, z_x)$.

Let us now define the expression $\Delta\theta(\Delta\bar{x} \neq 0; \Delta I^A = 0)$.

It is easy to derive that this expression may be also expressed as $\theta(\{x + \Delta\bar{x}\}, z_{x+\Delta\bar{x}}) - \theta(\{x\}, z_x)$ where $\theta(\{x + \Delta\bar{x}\}, z_{x+\Delta\bar{x}})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the original incomes at time 0 to which an amount equal to $\Delta\bar{x} = \bar{y} - \bar{x}$ has been assumed to have been added to every individual and where the poverty line $z_{x+\Delta\bar{x}}$ is equal to half the median of the distribution $\{x + \Delta\bar{x}\}$.

Similarly the expression $\Delta\theta(\Delta\bar{x} = 0; \Delta I^A \neq 0)$ may be also written as $\theta(\{y - \Delta\bar{x}\}, z_{y-\Delta\bar{x}}) - \theta(\{x\}, z_x)$ where $\theta(\{y - \Delta\bar{x}\}, z_{y-\Delta\bar{x}})$ refers to the poverty rate which is observed in a distribution where the incomes are equal to the incomes at time 1 minus the difference $\Delta\bar{x}$ between the average income at time 1 and at time 0 ($\Delta\bar{x} = \bar{y} - \bar{x}$) and where the poverty line is equal to half the median of the distribution $\{y - \Delta\bar{x}\}$.

We therefore end up with

$$
\begin{aligned}
&C(\Delta\bar{x})\\
&=(1/2)\{[\theta(\{y\},z_y)-\theta(\{x\},z_x)]\\
&\qquad -[\theta(\{y-\Delta\bar{x}\},z_{y-\Delta\bar{x}})-\theta(\{x\},z_x)]\}\\
&+(1/2)\{[\theta(\{x+\Delta\bar{x}\},z_{x+\Delta\bar{x}})-\theta(\{x\},z_x)]\\
&\qquad -[\theta(\{x\},z_x)-\theta(\{x\},z_x)]\}\\
&=(1/2)\{[\theta(\{y\},z_y)-\theta(\{y-\Delta\bar{x}\},z_{y-\Delta\bar{x}})]\\
&\qquad +[\theta(\{x+\Delta\bar{x}\},z_{x+\Delta\bar{x}})-\theta(\{x\},z_x)]\}
\end{aligned}
\tag{C-58}
$$

Similarly we can write that

$$
\begin{aligned}
&C(\Delta I^A)\\
&=(1/2)\{[\theta(\{y\},z_y)-\theta(\{x\},z_x)]\\
&\qquad -[\theta(\{x+\Delta\bar{x}\},z_{x+\Delta\bar{x}})-\theta(\{x\},z_x)]\}\\
&+(1/2)\{[\theta(\{y-\Delta\bar{x}\},z_{y-\Delta\bar{x}})-\theta(\{x\},z_x)]\\
&\qquad -[\theta(\{x\},z_x)-\theta(\{x\},z_x)]\}\\
&=(1/2)\{[\theta(\{y\},z_y)-\theta(\{x+\Delta\bar{x}\},z_{x+\Delta\bar{x}})]\\
&\qquad +[\theta(\{y-\Delta\bar{x}\},z_{y-\Delta\bar{x}})-\theta(\{x\},z_x)]\}
\end{aligned}
\tag{C-59}
$$

Combining (C-58) and (C-59) we observe again that, as expected,

$$C(\Delta\bar{x})+C(\Delta I^A)=\theta(\{y\},z_y)-\theta(\{x\},z_x) \tag{C-60}$$

The actual relative change in the poverty index observed between times 0 and 1 will therefore be expressed as

$$(d\theta/\theta)=Gr^A+In^A \tag{C-61}$$

where

$$Gr^{A} = C(\Delta\bar{x}) / \theta \qquad \text{(C-62)}$$

and

$$In^{A} = C(\Delta I^{A}) / \theta \qquad \text{(C-63)}$$

where θ, as before is the value of the poverty index at the original period 0.