No. 2007-3
May 21, 2007

Economics
The Open Access, Open Assessment E-Journal

Long Run Macroeconomic Relations in the Global Economy

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Abstract:

This paper presents tests of long run macroeconomic relations involving interest rates, equity, prices and exchange rates suggested by arbitrage in financial and goods markets. It uses the global vector autoregressive (GVAR) model to test for long run restrictions in each country/region conditioning on the rest of the world. Bootstrapping is used to compute both the empirical distribution of the impulse responses and the log-likelihood ratio statistic for over-identifying restrictions. The paper also examines the speed with which adjustments to the long run relations take place via the persistence profiles. It finds strong evidence in favour of a long run version of uncovered interest parity and to a lesser extent the Fisher equation across a number of countries, but the test results for the purchasing power parity relation are much weaker. Also the transmission of shocks and subsequent adjustments in financial markets are much faster than those in goods markets.

JEL: C32, E17, F47, R11
Keywords: Global VAR, Fisher relationship, Uncovered Interest Rate Parity, Purchasing Power Parity, persistence profile

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* A preliminary version of this paper was presented at the Joint ANU (CAMA)–ECB-Lowy Institute workshop on "Globalisation and Regionalism", December 7-8, 2005, Sydney as well as at the workshop on "Factor Models in Theory and Practice", 11-12 November 2006, European University Institute. Comments by the associate editor and two anonymous referees as well as Anindya Banerjee, Luisa Corrado, Alex Al-Haschimi, Timo Henckel, Filippo di Mauro, Warwick McKibbin, Adrian Pagan, Luca Sala, and particularly by Alexander Chudik, are gratefully acknowledged. For Stephane Dees; any views expressed represent those of the authors and not necessarily those of the European Central Bank. Sean Holly, M. Hashem Pesaran and L. Vanessa Smith gratefully acknowledge financial support from the ECB.

1 Introduction

This paper presents tests of long run macroeconomic relations in a structural vector autoregressive model of the global economy with a particular focus on interest rates, real output, inflation and exchange rates. The long run relations considered admit both within as well as cross-country parametric restrictions. For example, although the Fisher equation involves only domestic variables - given the other long run channels such as uncovered interest parity and possibly purchasing power parity (PPP) - it could be misleading to focus only on the Fisher equation on a country by country basis. However, to our knowledge all empirical studies of the long run relations in the literature relate to single countries or when a multi-country framework is adopted, the countries are treated in isolation, and cross country dependencies are generally ignored. Similar considerations are also relevant to the analysis of output convergence, the term premium and purchasing power parity.

In this paper we adopt a global perspective and use the Global VAR (GVAR) model developed in Dees, di Mauro, Pesaran and Smith (2007, DdPS) to test for a number of long run relations in the world economy. We also address a number of issues concerning how and at what speed adjustments towards the long run take place in financial and goods markets. The GVAR approach consists of a comprehensive modelling framework that allows us to consider the responses to various types of global and country specific shocks through a number of transmission channels. These channels include both trade flows and financial linkages - notably, through capital, equity and currency markets. The use of the GVAR for testing long run relationships has the added advantage that the tests do not depend on the choice of the reference country.

Using this approach, we find that while the Fisher hypothesis and uncovered interest parity cannot be rejected for a number of countries, strict PPP can only be detected for two countries (Australia and Switzerland) and a weaker form with relative productivity differences also playing a role (Norway and the UK). Bootstrapping is used to compute error bands for the impulse responses, and the critical values for the likelihood ratio (LR) statistic used to test the long run over-identifying restrictions. In particular, the testing of long run restrictions for each country is carried out while taking into account the inter-relations that prevail in the global economy.

The imposition of long run relations on the GVAR tends to make the impulse responses more in line with our theoretical priors as compared to the ones based on the unrestricted GVAR reported in DdPS. This is particularly the case when one considers the effects of US monetary policy shocks (rises in the US policy rate) on inflation and interest rates in the US and elsewhere. As in DdPS there is a price puzzle with the inflation in the US rising on impact, but this effect dies out quite rapidly after a couple of quarters. The impacts of the US monetary shock on the rest of the world is much more muted, with the exception of Canada where short term interest rates rise by half of the US interest rate response.

Finally, perhaps not surprisingly, we find that the transmission of shocks across markets and economies and the subsequent adjustments are much faster in financial markets as compared to the ones in goods markets.

In Section 2 we set out a number of theory-based long run relations that can be tested in the context of a global model. In Section 3 we turn to an analysis of the GVAR. The use of persistence profiles, impulse response functions, and generalized error variance decomposition for the GVAR are discussed in Section 4. Section 5 reports the empirical results, and Section 6 provides some concluding remarks. The proof of pair-wise validity of the PPP in the GVAR, and the mathematical details of the derivation of the generalized error variance decomposition and the sieve bootstrap procedure applied to the GVAR are provided in the Appendix.

2 Long Run Equilibrium Conditions

The GVAR model developed in Dees, di Mauro, Pesaran and Smith (2007) - hereafter DdPS - comprises country-specific VARX* models that relate the core variables of each economy, $\mathbf{x}_{it}$, to their foreign counterparts, $\mathbf{x}_{it}^*$. The country specific models are thereafter combined to form a GVAR in which all the variables are endogenous. The high dimensional nature of the model is circumvented at the estimation stage by constructing the country specific foreign variables, $\mathbf{x}_{it}^*$, using predetermined coefficients such as trade weights, and by noting that for relatively small open economies $\mathbf{x}_{it}$ can be treated as weakly exogenous (or forcing) for the long run relations. The model for the US economy is treated differently due to the dominant role that the US plays in the world economy.

The core variables considered are the log of real per capita output (y_{it}), the log of the general price level (p_{it}), the rate of price inflation ($\Delta p_{it} = p_{it} - p_{i,t-1}$), the short term interest rate (ρ_{it}^S), the long term interest (bond) rate (ρ_{it}^L), the log exchange rate in terms of the US dollar (e_{it}), the log of real equity prices (q_{it}), and the log of nominal oil prices (p_t^o). The associated country-specific foreign variables are

$$y_{it}^* = \Sigma_{j=0}^N w_{ij} y_{jt},\ \rho_{it}^{S*} = \Sigma_{j=0}^N w_{ij} \rho_{jt}^S,\ \rho_{it}^{L*} = \Sigma_{j=0}^N w_{ij} \rho_{jt}^L, \tag{2.1}$$

$$p_{it}^* = \Sigma_{j=0}^N w_{ij} p_{jt},\ e_{it}^* = \Sigma_{j=0}^N w_{ij} e_{jt},\ q_{it}^* = \Sigma_{j=0}^N w_{ij} q_{jt}, \tag{2.2}$$

where w_{ij} is the share of country j in the trade (exports plus imports) of country i, such that $w_{ii} = 0$ and $\Sigma_{j=0}^N w_{ij} = 1$.[1] The focus of the present paper is on the long run relations that might exist amongst the domestic variables $y_{it}, p_{it}, e_{it}, \rho_{it}^S, \rho_{it}^L, q_{it}$, and their foreign counterparts, $y_{it}^*, p_{it}^*, e_{it}^*, \rho_{it}^{S*}, \rho_{it}^{L*}, q_{it}^*$. To separate the long run relations from the short run dynamics it is necessary that the variables under consideration are nonstationary (typically unit root processes, or integrated of order 1 or more), so that the errors from the long run relations could be stationary. In the case where the core variables are $I(1)$ or higher, the long run relations will also form a set of cointegrating relations.

In the context of the global economy, arbitrage is at work in both financial and goods markets. In financial markets arbitrage equates risk adjusted expected rates of return on all financial assets.[2] For individual economies theory-based long run relations can be derived either from inter-temporal optimization conditions as in a Dynamic Stochastic General Equilibrium (DSGE) model, e.g Gali and Monacelli (2005) in the context of a small open economy, or from arbitrage and solvency conditions. Garratt, Lee, Pesaran and Shin (2003, 2006, Ch. 4) discuss these alternative approaches and derive the long run conditions in the case of a core model of the UK economy with real money balances but without bond and real equity returns. Long run implications of a small open economy New Keynesian macroeconomic model are also discussed in Pesaran and Smith (2006). In

[1] As noted in DdPS time varying weights or weights based on other measures of connectivity of countries such as capital flows or physical proximity can also be considered. However, for empirical purposes, trade weights are likely to be more reliable as well as being readily available historically.

[2] Of course, in the short run risk premia can be moving around in ways that is very difficult to model.

this paper we shall consider the following six relationships as possible long run conditions linking the core variables of the i^{th} economy to those in the rest of the world economy

$$ee_{it}+p^*_{it}-p_{it}-b_i(y_{it}-y^*_{it})=a_{i1}+\zeta_{i1,t}\sim I(0),\ b_i>0, \tag{2.3}$$

$$y_{it}-y^*_{it}=a_{i2}+\zeta_{i2,t}\sim I(0), \tag{2.4}$$

$$\rho^S_{it}-\Delta p_{it}=a_{i3}+\zeta_{i3,t}\sim I(0), \tag{2.5}$$

$$\rho^S_{it}-\rho^L_{it}=a_{i4}+\zeta_{i4,t}\sim I(0), \tag{2.6}$$

$$q_{it}-c_iy_{it}-d_i(\rho^L_{it}-\Delta p_{it})=a_{i5}+\zeta_{i5,t}\sim I(0),\ c_i>0,\ d_i>0, \tag{2.7}$$

$$\rho^S_{it}-\rho^{S*}_{it}-E_t(\Delta e^*_{i,t+1})=a_{i6}+\zeta_{i6,t}\sim I(0). \tag{2.8}$$

The first relationship represents the PPP modified to allow for the possibility of different rates of growth of productivity (the Ballassa-Samuelson effect). It relates the (log) effective exchange rate, $ee_{it}=\Sigma^N_{j=0}w_{ij}e_{ijt}$, where $e_{ijt}=e_{it}-e_{jt}$ is the logarithm of the bilateral exchange rate of country i with country j, to the log price ratio, $p^*_{it}-p_{it}$, and the per capita output gap, $y_{it}-y^*_{it}$.[3] The modified PPP relationship can also be derived from foreign account solvency conditions (see Garratt, Lee, Pesaran and Shin (2006, Section 4.4)).

The second relationship, (2.4), postulates that domestic and foreign outputs (in logs) are convergent in the long run. Although, the neoclassical growth model does not explicitly address the issue of cross-country output convergence, it is often argued that in an interrelated global economy technological progress (taken to be an unobserved $I(1)$ process) is likely to be shared eventually by all countries, either directly or through trade in investment goods. The output convergence condition then follows under complete cross country technological diffusion combined with the Solow-Swan growth process applied to each country separately (see, for example, Pesaran (2007)). In the case where the output convergence condition holds the modified PPP condition (2.3) reduces to the standard PPP condition given by $ee_{it}+p^*_{it}-p_{it}\sim I(0)$. If the condition for relative output convergence is not met, the weights used in the construction of p^*_{it} and y^*_{it} need not be the same for the validity of the modified PPP (see Chudik (2006)).

The third relationship, the Fisher equation, suggests that the real interest rate is stationary. The fourth relationship between the short and the long rate is the term structure condition that the vertical spread in the yield curve is stationary. The fifth relationship relates to equity markets and has real equity prices varying in line with real output but also dependent on the real long-term interest rate, where the real long-term interest rate is inversely proportional to the subjective rate of time preference. In the event where the real long-term interest rate is stationary, (2.7) predicts a long run relationship between real equity prices and real output. The long run real equity price equation can be derived from a log-linear approximation of the first order Euler equation

[3] Note that ee_{it} differs from $e^*_{it}=\Sigma^N_{j=0}w_{ij}e_{jt}$ defined in (2.2). The latter is defined in terms of the US dollar exchange rates, whilst the former is measured in terms of the bilateral exchange rates.

in consumption-based asset pricing models, or can be obtained more directly from present value relations.[4]

The final relationship is the uncovered interest parity (UIP) condition, where $E_t(\Delta e^*_{i,t+1})$ is the expected rate of depreciation of country i^{th} currency as defined above. In most cases where exchange rates follow random walks, or more generally, can be approximated by $I(1)$ processes we have $E_t(\Delta e^*_{i,t+1}) \sim I(0)$ and the UIP condition reduces to

$$\rho^S_{it} - \rho^{S*}_{it} \sim I(0).$$

Also, because of the Fisher relationship and the term structure conditions, the UIP can be considered equally in terms of long-term interest rates.

3 Long Run Analysis within the GVAR Framework

3.1 Modelling of Real Exchange Rate in the GVAR

So far the country specific models in the version of the GVAR model developed in DdPS are formulated in terms of $\tilde{e}_{it} = e_{it} - p_{it}$, $\Delta p_{it}, y_{it}, q_{it}, \rho^S_{it}, \rho^L_{it}$, and p^o. These VARX* models allow specification and testing of a number of long run relations described in Section 2 such as uncovered interest parity, the term structure, the Fisher's inflation parity relation, the output gap relation, $y_{it} - y^*_{it}$, as well as relations that link bond and equity markets. However, they do not permit the specification and testing of PPP. This is because in a multi-country set up PPP is best formulated in terms of effective exchange rates, ee_{it}, rather than the US dollar rate, e_{it}.

To incorporate the PPP relationship in the GVAR model, we first note that since $e_{ijt} = e_{it} - e_{jt}$, then the (log) real effective exchange rate, $\overline{re}_{it} = ee_{it} + p^*_{it} - p_{it}$, can be written equivalently as (recall that $\Sigma^N_{j=0} w_{ij} = 1$)

$$\begin{aligned}\overline{re}_{it} &= \sum_{j=0}^{N} w_{ij}(e_{it} - e_{jt}) + p^*_{it} - p_{it}, \\ &= e_{it} - e^*_{it} + p^*_{it} - p_{it}, \\ &= \tilde{e}_{it} - \tilde{e}^*_{it},\end{aligned}$$

where $\tilde{e}^*_{it} = e^*_{it} - p^*_{it}$, and $e^*_{it} = \Sigma^N_{j=0} w_{ij} e_{it}$ is as defined above. This suggests a modification to the DdPS version of the GVAR so that the real effective exchange rate $\overline{re}_{it}$ is included amongst the endogenous variables in place of $\tilde{e}_{it} = e_{it} - p_{it}$. Accordingly, in what follows we consider the following set of endogenous variables

$$\mathbf{x}_{it} = (\overline{re}_{it}, \Delta p_{it}, y_{it}, q_{it}, \rho^S_{it}, \rho^L_{it})', \; i = 1, 2, ..., N,$$

and

$$\mathbf{x}_{0t} = (\Delta p_{0t}, y_{0t}, q_{0t}, \rho^S_{0t}, \rho^L_{0t}, p^o_t)',$$

with the following corresponding country specific foreign variables

$$\mathbf{x}^*_{it} = (\Delta p^*_{it}, y^*_{it}, q^*_{it}, \rho^{*S}_{it}, \rho^{*L}_{it}, p^o_t)', \; i = 1, 2, .., N$$

[4] See, for example, Campbell, Lo and MacKinlay (1997, Ch. 7).

and

$$\mathbf{x}_{0t}^{*} = (\tilde{e}_{0t}^{*}, \Delta p_{0t}^{*}, y_{0t}^{*})'.$$

PPP can then be specified in terms of $\tilde{e}_{it}$ and $\tilde{e}_{it}^{*}$, with PPP holding if $\overline{re}_{it} = \tilde{e}_{it} - \tilde{e}_{it}^{*} \sim I(0)$.

The above formulation of the PPP in the GVAR model has two main advantages:

Remark 1 *Tests of the PPP hypothesis do not depend on the choice of the reference country. The asymmetric treatment of the US model in the GVAR reflects the dominant role that US plays in the global economy (namely that not all foreign variables can be treated as weakly exogenous in the US model), rather than the choice of the US dollar as the reference currency.*

Remark 2 *If PPP holds in terms of effective exchange rates for all countries, namely if $\overline{re}_{it} \sim I(0)$ for $i = 0, 1, 2, ..., N$, then it also follows that*

$$e_{ijt} + p_{jt} - p_{it} \sim I(0),$$

for all $i, j = 0, 1, 2, ..., N$, namely that PPP holds for all country pairs. A proof is provided in the Appendix.

Similarly, the long run relations $y_{it} - y_{it}^{*} \sim I(0)$, and $\rho_{it}^{S} - \rho_{it}^{S*} \sim I(0)$, imply $y_{it} - y_{jt} \sim I(0)$, and $\rho_{it}^{S} - \rho_{jt}^{S} \sim I(0)$ for all i and j. This result is particularly pertinent when N is relatively large and a full system approach to the analysis of cointegration might not be possible. By focussing on possible cointegration of y_{it} and y_{it}^{*} for each i we are also able to shed light on the possibility of pair-wise cointegration (Pesaran, 2007).

3.2 Individual Country Model Specifications

We consider the same VARX*(2,1) specification across all countries[5]

$$\mathbf{x}_{it} = \mathbf{h}_{i0} + \mathbf{h}_{i1} t + \mathbf{\Phi}_{i1} \mathbf{x}_{i,t-1} + \mathbf{\Phi}_{i2} \mathbf{x}_{i,t-2} + \mathbf{\Psi}_{i0} \mathbf{x}_{it}^{*} + \mathbf{\Psi}_{i1} \mathbf{x}_{i,t-1}^{*} + \mathbf{u}_{it} \tag{3.1}$$

The corresponding error correction model is given by[6]

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} - \boldsymbol{\alpha}_i \boldsymbol{\beta}_i' [\mathbf{z}_{i,t-1} - \boldsymbol{\gamma}_i (t-1)] + \mathbf{\Psi}_{i0} \Delta \mathbf{x}_{it}^{*} + \mathbf{\Gamma}_i \Delta \mathbf{z}_{i,t-1} + \mathbf{u}_{it}, \tag{3.2}$$

where $\mathbf{z}_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime})'$, $\boldsymbol{\alpha}_i$ is a $k_i \times r_i$ matrix of rank r_i and $\boldsymbol{\beta}_i$ is a $(k_i + k_i^{*}) \times r_i$ matrix of rank r_i. By partitioning $\boldsymbol{\beta}_i$ as $\boldsymbol{\beta}_i = (\boldsymbol{\beta}_{ix}', \boldsymbol{\beta}_{ix*}')'$ conformable to $\mathbf{z}_{it}$, the r_i error correction terms defined by (3.2) can be written as

$$\boldsymbol{\beta}_i' (\mathbf{z}_{it} - \boldsymbol{\gamma}_i t) = \boldsymbol{\beta}_{ix}' \mathbf{x}_{it} + \boldsymbol{\beta}_{ix*}' \mathbf{x}_{it}^{*} + (\boldsymbol{\beta}_i' \boldsymbol{\gamma}_i) t, \tag{3.3}$$

which clearly allows for the possibility of cointegration both within $\mathbf{x}_{it}$ and between $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^{*}$ and consequently across $\mathbf{x}_{it}$ and $\mathbf{x}_{jt}$ for $i \neq j$. Conditional on r_i cointegrating relations, the co-trending restrictions, $\boldsymbol{\beta}_i' \boldsymbol{\gamma}_i = \mathbf{0}$, can then be tested.

Using $\mathbf{z}_{it}$, (3.1) can be rewritten as

$$\mathbf{A}_{i0} \mathbf{z}_{it} = \mathbf{h}_{i0} + \mathbf{h}_{i1} t + \mathbf{A}_{i1} \mathbf{z}_{i,t-1} + \mathbf{A}_{i2} \mathbf{z}_{i,t-2} + \mathbf{u}_{it}, \tag{3.4}$$

[5] In the empirical implementation of the GVAR we also considered VARX*(2,2) models but the VARX*(2,1) was generally preferred using lag order selection procedures such as the Bayesian information criterion. Owing to data limitations, we did not consider lag orders for the domestic and foreign variables to be greater than two.

[6] Here we consider the trend restricted version, case IV, discussed in Pesaran, Shin and Smith (2000) which ensures that the deterministic trend property of the country-specific models remains invariant to the cointegrating rank assumptions.

where

$$\mathbf{A}_{i0} = (\mathbf{I}_{k_i}, -\mathbf{\Psi}_{i0}), \quad \mathbf{A}_{i1} = (\mathbf{\Phi}_{i1}, \mathbf{\Psi}_{i1}), \quad \mathbf{A}_{i2} = (\mathbf{\Phi}_{i2}, \mathbf{\Psi}_{i2}),$$

and $\mathbf{\Psi}_{i2} = \mathbf{0}_{k_i \times k_i^*}$. The dimensions of $\mathbf{A}_{i0}$, $\mathbf{A}_{i1}$ and $\mathbf{A}_{i2}$ are $k_i \times (k_i + k_i^*)$ and $\mathbf{A}_{i0}$ has full row rank, namely $Rank(\mathbf{A}_{i0}) = k_i$, for $i = 0, 1, ..., N$.

3.3 Combining the Country-Specific Models into the GVAR

The main difference between the US model and the model for the rest of the countries is that $\overline{re}_{0t}$ is not included in the US model, and the oil price variable, p_t^o, is included as an endogenous variable in the US model, whilst $\overline{re}_{it}$ is included as endogenous and p_t^o as weakly exogenous in the rest of the country models for $i = 1, 2, ..., N$. The inclusion of $\tilde{e}_{0t}^*$ as a weakly exogenous variable in the US model and the presence of $\overline{re}_{it}$, $i = 1, 2, ..., N$ as endogenous variables in the model for the remaining countries leads to the $k \times 1$ vector of the global variables defined by $\mathring{\mathbf{x}}_t = (\tilde{\mathbf{x}}_{0t}', \tilde{\mathbf{x}}_{1t}', ..., \tilde{\mathbf{x}}_{Nt}')'$, where $\tilde{\mathbf{x}}_{0t} = \left(\tilde{e}_{0t}, \Delta p_{0t}, y_{0t}, q_{0t}, \rho_{0t}^S, \rho_{0t}^L, p_t^o\right)'$ for $i = 0$, and $\tilde{\mathbf{x}}_{it} = \left(\tilde{e}_{it}, \Delta p_{it}, y_{it}, q_{it}, \rho_{it}^S, \rho_{it}^L\right)'$ for $i > 1$, as a first step in solving the GVAR model. It is easy to see that the variables $\mathbf{z}_{it}$ are linked to the global variables, $\mathring{\mathbf{x}}_t$, through the identity

$$\mathbf{z}_{it} = \mathbf{W}_i \mathring{\mathbf{x}}_t, \tag{3.5}$$

where $\mathbf{W}_i$, $i = 0, 1, ..., N$, are $(k_i + k_i^*) \times k$ 'link' matrices defined in terms of the trade weights such that the above identity is satisfied.

As an illustration consider a simple case where $N = 2$, $\mathbf{x}_{0t} = (\Delta p_{0t}, y_{0t}, p_t^o)'$, $\mathbf{x}_{0t}^* = (\tilde{e}_{0t}^*, \Delta p_{0t}^*, y_{0t}^*)'$, $\mathbf{x}_{it} = (\overline{re}_{it}, \Delta p_{it}, y_{it})'$, for $i = 1, 2$, and $\mathbf{x}_{it}^* = (\Delta p_{it}^*, y_{it}^*, p_t^o)'$, then we have (recall that $w_{ii} = 0$)

$$\begin{aligned}
\mathbf{z}_{0t} &= \begin{pmatrix} \Delta p_{0t} \\ y_{0t} \\ p_t^o \\ \tilde{e}_{0t}^* \\ \Delta p_{0t}^* \\ y_{0t}^* \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ w_{00} & 0 & 0 & 0 & w_{01} & 0 & 0 & w_{02} & 0 & 0 \\ 0 & w_{00} & 0 & 0 & 0 & w_{01} & 0 & 0 & w_{02} & 0 \\ 0 & 0 & w_{00} & 0 & 0 & 0 & w_{01} & 0 & 0 & w_{02} \end{pmatrix} \begin{pmatrix} \tilde{e}_{0t} \\ \Delta p_{0t} \\ y_{0t} \\ p_t^o \\ \tilde{e}_{1t} \\ \Delta p_{1t} \\ y_{1t} \\ \tilde{e}_{2t} \\ \Delta p_{2t} \\ y_{2t} \end{pmatrix} \\
&= \mathbf{W}_0 \mathring{\mathbf{x}}_t,
\end{aligned}$$

$$\begin{aligned}
\mathbf{z}_{1t} &= \begin{pmatrix} \overline{re}_{1t} \\ \Delta p_{1t} \\ y_{1t} \\ \Delta p_{1t}^* \\ y_{1t}^* \\ p_t^o \end{pmatrix} = \begin{pmatrix} -w_{10} & 0 & 0 & 0 & 1 - w_{11} & 0 & 0 & -w_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & w_{10} & 0 & 0 & 0 & w_{11} & 0 & 0 & w_{12} & 0 \\ 0 & 0 & w_{10} & 0 & 0 & 0 & w_{11} & 0 & 0 & w_{12} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{e}_{0t} \\ \Delta p_{0t} \\ y_{0t} \\ p_t^o \\ \tilde{e}_{1t} \\ \Delta p_{1t} \\ y_{1t} \\ \tilde{e}_{2t} \\ \Delta p_{2t} \\ y_{2t} \end{pmatrix} \\
&= \mathbf{W}_1 \mathring{\mathbf{x}}_t,
\end{aligned}$$

$$
\begin{aligned}
\mathbf{z}_{2t} &= \begin{pmatrix} \overline{re}_{2t} \\ \Delta p_{2t} \\ y_{2t} \\ \Delta p^{*}_{2t} \\ y^{*}_{2t} \\ p^{o}_{t} \end{pmatrix} = \begin{pmatrix} -w_{20} & 0 & 0 & 0 & -w_{21} & 0 & 0 & 1-w_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & w_{20} & 0 & 0 & 0 & w_{21} & 0 & 0 & w_{22} & 0 \\ 0 & 0 & w_{20} & 0 & 0 & 0 & w_{21} & 0 & 0 & w_{22} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{e}_{0t} \\ \Delta p_{0t} \\ y_{0t} \\ p^{o}_{t} \\ \tilde{e}_{1t} \\ \Delta p_{1t} \\ y_{1t} \\ \tilde{e}_{2t} \\ \Delta p_{2t} \\ y_{2t} \end{pmatrix} \\
&= \mathbf{W}_2 \mathring{\mathbf{x}}_t .
\end{aligned}
$$

One could easily re-order the variables in $\mathring{\mathbf{x}}_t$ so that oil prices are included as the last rather than the third variable in the US model and the fourth variable in the rest of the model. The re-ordering of the variables/countries does not impact the analysis of the long run relations in the global economy.

As set out above, due to the fact that $\tilde{e}_{0t}$ is not included in the US model, but is included in $\mathring{\mathbf{x}}_t$, the total number of equations in the country specific models will be one less than the number of unknown elements in $\mathring{\mathbf{x}}_t$, and without a further restriction (or equation) $\mathring{\mathbf{x}}_t$ cannot be solved uniquely from the knowledge of the country-specific models. The final equation is provided by noting that $e_{0t} = 0$, and hence $\tilde{e}_{0t} = e_{0t} - p_{0t} = -p_{0t}$. For example, in the case of the above illustration $\mathring{\mathbf{x}}_t$ is a 10×1 vector, whilst there are 9 endogenous variables in the global model.

To deal with the problem of exchange rate modelling in a closed system first using (3.5), equation (3.4) can be written as

$$
\mathbf{A}_{i0}\mathbf{W}_i\mathring{\mathbf{x}}_t = \mathbf{h}_{i0} + \mathbf{h}_{i1}t + \mathbf{A}_{i1}\mathbf{W}_i\mathring{\mathbf{x}}_{t-1} + \mathbf{A}_{i2}\mathbf{W}_i\mathring{\mathbf{x}}_{t-2} + \mathbf{u}_{it} \tag{3.6}
$$

for $i = 0, 1, ..., N$, and then stacked to yield the model for $\mathring{\mathbf{x}}_t$ as

$$
\mathring{\mathbf{H}}_0\mathring{\mathbf{x}}_t = \mathbf{h}_0 + \mathbf{h}_1\mathbf{t} + \mathring{\mathbf{H}}_1\mathring{\mathbf{x}}_{t-1} + \mathring{\mathbf{H}}_2\mathring{\mathbf{x}}_{t-2} + \mathbf{u}_t
$$

where

$$
\mathring{\mathbf{H}}_j = \begin{pmatrix} \mathbf{A}_{0j}\mathbf{W}_0 \\ \mathbf{A}_{1j}\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_{Nj}\mathbf{W}_N \end{pmatrix}, \ \mathbf{h}_j = \begin{pmatrix} \mathbf{h}_{0j} \\ \mathbf{h}_{1j} \\ \vdots \\ \mathbf{h}_{Nj} \end{pmatrix}, \ \mathbf{u}_t = \begin{pmatrix} \mathbf{u}_{0t} \\ \mathbf{u}_{1t} \\ \vdots \\ \mathbf{u}_{Nt} \end{pmatrix}
$$

for $j = 0, 1, 2$, and $\mathring{\mathbf{H}}_0$ is a $k \times (k+1)$ matrix. To solve for the endogenous variables of the global economy, we set $\mathbf{x}_t = (\mathbf{x}'_{0t}, \tilde{\mathbf{x}}'_{1t}, ..., \tilde{\mathbf{x}}'_{Nt})'$, with $\mathbf{x}_{0t} = \left(p_{0t}, y_{0t}, q_{0t}, \rho^{S}_{0t}, \rho^{L}_{0t}, p^{o}_{t}\right)'$, and $\tilde{\mathbf{x}}_{it} = \left(\tilde{e}_{it}, \Delta p_{it}, y_{it}, q_{it}, \rho^{S}_{it}, \rho^{L}_{it}\right)'$, for $i = 1, 2, ..., N$. Note that we are now solving for the US price level and not the US inflation rate, although it is inflation that is being solved for in the case of the other countries. It is then easily seen that

$$
\mathring{\mathbf{x}}_t = \mathbf{S}_0\mathbf{x}_t - \mathbf{S}_1\mathbf{x}_{t-1},
$$

where $\mathbf{S}_i$, for $i = 0, 1$ are $(k+1) \times k$ matrices defined by

$$
\mathbf{S}_0 = \begin{pmatrix} \begin{matrix} -1 \\ 1 \end{matrix} & \mathbf{0}_{2\times(k-1)} \\ \mathbf{0}_{k-1\times 1} & \mathbf{I}_{k-1} \end{pmatrix}, \text{ and } \mathbf{S}_1 = \begin{pmatrix} \begin{matrix} 0 \\ 1 \end{matrix} & \mathbf{0}_{2\times(k-1)} \\ \mathbf{0}_{k-1\times 1} & \mathbf{0}_{(k-1)\times(k-1)} \end{pmatrix}.
$$

Hence

$$\mathring{\mathbf{H}}_0\left(\mathbf{S}_0\mathbf{x}_t-\mathbf{S}_1\mathbf{x}_{t-1}\right)=\mathbf{h}_0+\mathbf{h}_1\mathbf{t}+\mathring{\mathbf{H}}_1\left(\mathbf{S}_0\mathbf{x}_{t-1}-\mathbf{S}_1\mathbf{x}_{t-2}\right)+\mathring{\mathbf{H}}_2\left(\mathbf{S}_0\mathbf{x}_{t-2}-\mathbf{S}_1\mathbf{x}_{t-3}\right)+\mathbf{u}_t$$

or

$$\mathbf{H}_0\mathbf{x}_t=\mathbf{h}_0+\mathbf{h}_1\mathbf{t}+\mathbf{H}_1\mathbf{x}_{t-1}+\mathbf{H}_2\mathbf{x}_{t-2}+\mathbf{H}_3\mathbf{x}_{t-3}+\mathbf{u}_t \tag{3.7}$$

where

$$\begin{aligned}\mathbf{H}_0&=\mathring{\mathbf{H}}_0\mathbf{S}_0\\ \mathbf{H}_1&=\mathring{\mathbf{H}}_1\mathbf{S}_0+\mathring{\mathbf{H}}_0\mathbf{S}_1\\ \mathbf{H}_2&=\mathring{\mathbf{H}}_2\mathbf{S}_0-\mathring{\mathbf{H}}_1\mathbf{S}_1\\ \mathbf{H}_3&=-\mathring{\mathbf{H}}_2\mathbf{S}_1.\end{aligned}$$

The GVAR is then obtained as

$$\mathbf{x}_t=\mathbf{a}_0+\mathbf{a}_1t+\mathbf{F}_1\mathbf{x}_{t-1}+\mathbf{F}_2\mathbf{x}_{t-2}+\mathbf{F}_3\mathbf{x}_{t-3}+\boldsymbol{\varepsilon}_t \tag{3.8}$$

where $\mathbf{F}_j=\mathbf{H}_0^{-1}\mathbf{H}_j$, $\mathbf{a}_j=\mathbf{H}_0^{-1}\mathbf{h}_j$, for $j=0,1,2,3$, and $\boldsymbol{\varepsilon}_t=\mathbf{H}_0^{-1}\mathbf{u}_t$. Once the GVAR is solved for $\mathbf{x}_t$, one can then compute p_{it} and e_{it} for all i, noting that $e_{0t}=0$.

4 Persistence Profiles, Impulse Responses and Forecast Error Variance Decomposition

4.1 Persistence Profiles

The persistence profiles (PP) refer to the time profiles of the effects of system or variable-specific shocks on the cointegrating relations in the GVAR model, whilst the impulse responses refer to the time profile of the effects of variable-specific shocks or identified shocks (such as monetary policy or technology shocks identified using a suitable economic theory) on all the variables in the model. The impulse responses of shocks to specific variables are known as the generalized impulse response functions (GIRF).[7] Derivation of PP's and GIRF's are based on the following moving average representation of the GVAR model given by (3.8), which we write as

$$\mathbf{x}_t=\mathbf{d}_t+\sum_{j=0}^{\infty}\mathbf{A}_j\boldsymbol{\varepsilon}_{t-j}, \tag{4.1}$$

where $\mathbf{d}_t$ represents the deterministic (perfectly forecastable) component of $\mathbf{x}_t$, and $\mathbf{A}_j$ can be derived recursively as

$$\mathbf{A}_j=\mathbf{F}_1\mathbf{A}_{j-1}+\mathbf{F}_2\mathbf{A}_{j-2}+\mathbf{F}_3\mathbf{A}_{j-3},\ j=1,2,... \tag{4.2}$$
$$\text{with } \mathbf{A}_0=\mathbf{I}_k,\ \mathbf{A}_j=\mathbf{0},\text{ for } j<0.$$

In the context of the GVAR the cointegrating relations are given in terms of the country-specific variables, namely $\boldsymbol{\beta}_i'\mathbf{z}_{it}$, whilst the variables in the GVAR are given by

[7] Persistence profiles applied to cointegrating models are discussed in Pesaran and Shin (1996). Generalized impulse response functions were introduced in Koop, Pesaran and Potter (1996) and adapted to VAR models in Pesaran and Shin (1998).

$\mathbf{x}_t$, and appropriate mappings between $\mathbf{z}_{it}$ and $\mathbf{x}_t$ should be used. Note that from the preceding discussions $\mathbf{z}_{it} = \mathbf{W}_i \mathring{\mathbf{x}}_t = \mathbf{W}_i (\mathbf{S}_0\mathbf{x}_t - \mathbf{S}_1\mathbf{x}_{t-1})$, and

$$\mathbf{z}_{it} = \mathbf{W}_i (\mathbf{S}_0\mathbf{d}_t - \mathbf{S}_1\mathbf{d}_{t-1}) + \mathbf{W}_i\mathbf{S}_0\mathbf{A}_0\boldsymbol{\varepsilon}_t + \sum_{j=1}^{\infty} \mathbf{W}_i (\mathbf{S}_0\mathbf{A}_j - \mathbf{S}_1\mathbf{A}_{j-1}) \boldsymbol{\varepsilon}_{t-j}.$$

Therefore, the PP of $\boldsymbol{\beta}'_{ji}\mathbf{z}_{it}$, with respect to a system-wide shock to $\boldsymbol{\varepsilon}_t$ is given by

$$\mathcal{PP}(\boldsymbol{\beta}'_{ji}\mathbf{z}_{it}; \boldsymbol{\varepsilon}_t, n) = \frac{\boldsymbol{\beta}'_{ji}\mathbf{W}_i\mathbf{B}_n\boldsymbol{\Sigma}_\varepsilon\mathbf{B}'_n\mathbf{W}'_i\boldsymbol{\beta}_{ji}}{\boldsymbol{\beta}'_{ji}\mathbf{W}_i\mathbf{B}_0\boldsymbol{\Sigma}_\varepsilon\mathbf{B}'_0\mathbf{W}'_i\boldsymbol{\beta}_{ji}}, \; n = 0, 1, 2, ... \tag{4.3}$$

where $\boldsymbol{\beta}'_{ji}$ is the j^{th} cointegrating relation in the i^{th} country ($j = 1, 2, ..., r_i$), n is the horizon, $\boldsymbol{\Sigma}_\varepsilon$ is the covariance matrix of $\boldsymbol{\varepsilon}_t$ and

$$\mathbf{B}_0 = \mathbf{S}_0\mathbf{A}_0, \text{ and } \mathbf{B}_n = \mathbf{S}_0\mathbf{A}_n - \mathbf{S}_1\mathbf{A}_{n-1}.$$

Similarly, the PP of $\boldsymbol{\beta}'_{ji}\mathbf{z}_{it}$ with respect to a variable specific shock, say the ℓ^{th} element of $\mathbf{x}_t$ is given by

$$\mathcal{PP}(\boldsymbol{\beta}'_{ji}\mathbf{z}_{it}; \varepsilon_{\ell t}, n) = \frac{\boldsymbol{\beta}'_{ji}\mathbf{W}_i\mathbf{B}_n\boldsymbol{\Sigma}_\varepsilon\mathfrak{e}_\ell}{\sqrt{\sigma_{\ell\ell}}}, \; n = 0, 1, 2, ...$$

where $\sigma_{\ell\ell}$ is the ℓ^{th} diagonal element of $\boldsymbol{\Sigma}_\varepsilon$ and $\mathfrak{e}_\ell$ is a $k \times 1$ selection vector with its element corresponding to the ℓ^{th} variable in $\mathbf{x}_t$ is unity and zeros elsewhere.

4.2 Impulse Responses

The GIRF's of a unit (one standard error) shock to the ℓ^{th} element of $\mathbf{x}_t$ on its j^{th} element is given by

$$\mathcal{GIRF}(\mathbf{x}_t; u_{\ell t}, n) = \frac{\mathfrak{e}'_j\mathbf{A}_n\mathbf{H}_0^{-1}\boldsymbol{\Sigma}_u\mathfrak{e}_\ell}{\sqrt{\mathfrak{e}'_\ell\boldsymbol{\Sigma}_u\mathfrak{e}_\ell}}, \; n = 0, 1, 2, ...; \ell, j = 1, 2, ..., k.$$

For a structurally identified shock, $\mathrm{v}_{\ell t}$, such as a US monetary policy shock the GIRF is given by

$$\mathcal{SGIRF}(\mathbf{x}_t; \mathrm{v}_{\ell t}, n) = \frac{\mathfrak{e}'_j\mathbf{A}_n(\mathbf{P}^0_{H_0}\mathbf{H}_0)^{-1}\boldsymbol{\Sigma}_\mathrm{v}\mathfrak{e}_\ell}{\sqrt{\mathfrak{e}'_\ell\boldsymbol{\Sigma}_\mathrm{v}\mathfrak{e}_\ell}}, \; n = 0, 1, 2, ...; \ell, j = 1, 2, ..., k, \tag{4.4}$$

where $\boldsymbol{\Sigma}_\mathrm{v}$ is the covariance matrix of the structural shocks and $\mathbf{P}^0_{H_0}\mathbf{H}_0$ is defined by the identification scheme used to identify the shocks. For example, for identification of the US monetary policy shock using the triangular approach of Sims (1980), starting with the US model

$$\mathbf{x}_{0t} = \mathbf{h}_{00} + \mathbf{h}_{01}t + \boldsymbol{\Phi}_{01}\mathbf{x}_{0,t-1} + \boldsymbol{\Phi}_{02}\mathbf{x}_{0,t-2} + \boldsymbol{\Psi}_{00}\mathbf{x}^*_{0t} + \boldsymbol{\Psi}_{01}\mathbf{x}^*_{0,t-1} + \mathbf{u}_{0t}, \tag{4.5}$$

the structural shocks are identified by

$$\mathbf{v}_{0t} = \mathbf{P}_0\mathbf{u}_{0t}$$

where $\mathbf{P}_0$ is a lower triangular matrix obtained as the $k_0 \times k_0$ Cholesky factor of the variance covariance matrix $\mathbf{\Sigma}_{u_0}$, such that $\mathbf{\Sigma}_{u_0} = \mathbf{P}_0\mathbf{P}_0'$. Premultiplying the GVAR model by

$$\mathbf{P}_{H_0}^0 = \begin{pmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{k_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{k_N} \end{pmatrix}, \tag{4.6}$$

it follows that

$$\mathbf{P}_{H_0}^0\mathbf{H}_0\mathbf{x}_t = \mathbf{P}_{H_0}^0\mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{P}_{H_0}^0\mathbf{H}_2\mathbf{x}_{t-2} + \mathbf{P}_{H_0}^0\mathbf{H}_3\mathbf{x}_{t-3} + \mathbf{v}_t,$$

with $\mathbf{v}_t = (\mathbf{v}_{0t}', \mathbf{u}_{1t}', ..., \mathbf{u}_{Nt}')'$ and

$$\mathbf{\Sigma}_{\mathrm{v}} = Cov\left(\mathbf{v}_t\right) = \begin{pmatrix} V(\mathbf{v}_{0t}) & Cov(\mathbf{v}_{0t}, \mathbf{u}_{1t}) & \cdots & Cov(\mathbf{v}_{0t}, \mathbf{u}_{Nt}) \\ Cov(\mathbf{u}_{1t}, \mathbf{v}_{0t}) & V(\mathbf{u}_{1t}) & \cdots & Cov(\mathbf{u}_{1t}, \mathbf{u}_{Nt}) \\ \vdots & \vdots & & \vdots \\ Cov(\mathbf{u}_{Nt}, \mathbf{v}_{0t}) & Cov(\mathbf{u}_{Nt}, \mathbf{u}_{1t}) & \cdots & V(\mathbf{u}_{Nt}) \end{pmatrix}.$$

By using the definition of the generalized impulse responses with respect to the structural shocks

$$\mathcal{SGIRF}(\mathbf{x}_t; \mathrm{v}_{\ell t}, n) = E(\mathbf{x}_{t+n}|\ \Omega_{t-1}, \mathfrak{e}_\ell'\mathbf{v}_t = \sqrt{\mathfrak{e}_\ell'\mathbf{\Sigma}_{\mathrm{v}}\mathfrak{e}_\ell}) - E(\mathbf{x}_{t+n}|\Omega_{t-1})$$

formula (4.4) readily follows. See DdPS for further details.

4.3 Forecast Error Variance Decomposition

Traditionally the forecast error variance decomposition (FEVD) of a VAR model is performed on a set of orthogonalized shocks whereby the contribution of the j^{th} orthogonalized innovation to the mean square error of the n-step ahead forecast of the model is calculated. In the case of the GVAR, the shocks across countries, that is u_{it} and u_{st} for $i \neq s$, are not orthogonal. In fact there is evidence that on average the shocks across countries are positively correlated. This invalidates the standard application of the orthogonalized FEVD to the GVAR model. An alternative approach, which is invariant to the ordering of the variables, would be to consider the proportion of the variance of the n-step forecast errors of $\mathbf{x}_t$ which is explained by conditioning on the non-orthogonalized shocks u_{jt}, $u_{j,t+1}$,..., $u_{j,t+n}$, for $j = 1, ..., k$, while explicitly allowing for the contemporaneous correlations between these shocks and the shocks to the other equations in the system. Analogously to the generalized impulse response functions, the generalized forecast error variance decomposition of shocks to specific variables can be derived as

$$\mathcal{GFEVD}(\mathbf{x}_{(\ell)t}; u_{(j)t}, n) = \frac{\sigma_{jj}^{-1}\sum_{l=0}^{n}\left(\mathfrak{e}_\ell'\mathbf{A}_l\mathbf{H}_0^{-1}\mathbf{\Sigma}_u\mathfrak{e}_j\right)^2}{\sum_{l=0}^{n}\mathfrak{e}_\ell'\mathbf{A}_l\mathbf{H}_0^{-1}\mathbf{\Sigma}_u\mathbf{H}_0^{-1\prime}\mathbf{A}_l'\mathfrak{e}_\ell}, \text{ for } n = 0, 1, 2, ... \tag{4.7}$$

and $\ell = 1, ..., k$, which gives the proportion of the n-step ahead forecast error variance of the ℓ^{th} element of $\mathbf{x}_t$ accounted for by the innovations in the j^{th} element of $\mathbf{x}_t$.[8] Notice that due to the non-diagonal form of $\mathbf{\Sigma}_u$, the elements of $\mathcal{GFEVD}(\mathbf{x}_{(\ell)t}; u_{(j)t}, n)$ across j need not sum to unity. For the derivation of the generalized forecast error variance decomposition see the Appendix. Similarly to the GIRF case under structural identification of the shocks we have

$$\mathcal{SGFEVD}(\mathbf{x}_{(\ell)t}; \mathrm{v}_{(j)t}, n) = \frac{\sigma_{jj}^{-1} \sum_{l=0}^{n} \{\mathfrak{e}'_\ell \mathbf{A}_l (\mathbf{P}^0_{H_0} \mathbf{H}_0)^{-1} \mathbf{\Sigma}_{\mathrm{v}} \mathfrak{e}_j\}^2}{\sum_{l=0}^{n} \mathfrak{e}'_\ell \mathbf{A}_l (\mathbf{P}^0_{H_0} \mathbf{H}_0)^{-1} \mathbf{\Sigma}_{\mathrm{v}} (\mathbf{P}^0_{H_0} \mathbf{H}_0)^{-1\prime} \mathbf{A}'_l \mathfrak{e}_\ell}, \text{ for } n = 0, 1, 2, ...$$

The above expressions can be used to compute the effects of shocking (displacing) a given endogenous variable in country i on all the variables in the global economy at different horizons. In choosing the variables of interest recall that $\mathbf{x}_t = (\mathbf{x}'_{0t}, \tilde{\mathbf{x}}'_{1t}, ..., \tilde{\mathbf{x}}'_{Nt})'$, with $\mathbf{x}_{0t} = \left(p_{0t}, y_{0t}, q_{0t}, \rho^S_{0t}, \rho^L_{0t}, p^o_t\right)'$, and $\tilde{\mathbf{x}}_{it} = \left(\tilde{e}_{it}, \Delta p_{it}, y_{0t}, q_{0t}, \rho^S_{0t}, \rho^L_{0t}\right)'$, for $i = 1, 2, ..., N$. Also note that the PP or GIRF of a unit shock to the US price level are the same as the PP or GIRF of a unit shock to the US inflation.

5 Empirical Results

Using the methodology described above, this section presents the results of the transmission process of shocks in the global economy. After a brief review of the GVAR model used, we present the results of the tests for the long run restrictions imposed, before looking at the persistence profiles of the implied GVAR model. Based on this model, we analyse the transmission of shocks to oil and equity prices as well as monetary policy shocks in the global economy through impulse response analysis and forecast error variance decomposition. A similar set of shocks are considered in the GIRF analysis by DdPS. This allows us to empirically evaluate the effects of imposing the theory-based long run restrictions on the short run as well as the long run properties of the model. Finally, we show how the results change when the alternative definition of the exchange rate *viz a viz* the US dollar is used. All tables and figures can be found at the end of the paper.

5.1 The GVAR Model

The version of the GVAR model developed by DdPS and used in this paper covers 33 countries: 8 of the 11 countries that originally joined the euro area on January 1, 1999 are grouped together, while the remaining 25 countries are modeled individually (see Table 1 for the list of countries included in the GVAR model and composition of regional groups). Therefore, the present GVAR model contains 26 countries/regions estimated over the sample period 1979(2)-2003(4).

As noted earlier the endogenous variables included in the country specific models are the logarithm of real output (y_{it}); the quarterly rate of inflation, Δp_{it}, the real effective

[8] Note that formula (4.7) is associated with performing GFEVD for the errors, $\mathbf{u}_{it}$, in the country-specific models. GFEVD can also be performed for the errors of the global model, $\boldsymbol{\varepsilon}_t$.

exchange rate, $\overline{re}_{it}$, the short-term interest rate, ρ_{it}^S, and if relevant real equity prices, q_{it}, and the long-term interest rate, ρ_{it}^L. The time series data for the euro area were constructed as cross section weighted averages of $y_{it}, \Delta p_{it}, q_{it}, \rho_{it}^S, \rho_{it}^L$ over Germany, France, Italy, Spain, Netherlands, Belgium, Austria and Finland, using average Purchasing Power Parity GDP weights over the 1999-2001 period.

The trade shares used to construct the country-specific foreign variables (the "starred" variables) are given in the 26×26 trade-share matrix provided in a Supplement to DdPS available on request. Table 2 presents the trade shares for the eleven focus economies (ten countries plus the euro area), with the "Rest" category showing the trade shares for the remaining countries.

With the exception of the US model, all individual models include the country-specific foreign variables, $y_{it}^*, \Delta p_{it}^*, q_{it}^*, \rho_{it}^{*S}, \rho_{it}^{*L}$ and oil prices (p_t^o). The country-specific foreign variables are obtained from the aggregation of data on the foreign economies using as weights the trade shares in Table 2. Because the set of weights for each country reflects its specific geographical trade composition, foreign variables vary across countries. We use fixed trade weights based on the average trade flows computed over the three years 1999-2001. It is clearly possible to use different types of weights for aggregation of different types of variables. The problem is one of data availability and empirical feasibility. However, we do not think that the choice of the weights is critical for the results. We have addressed this issue in DdPS partly by considering time-varying trade weights. Also in the case of equity and bond prices that tend to move very closely across different economies it is unlikely that using other weights could matter much.

Subject to appropriate testing, the country-specific foreign variables are treated as weakly exogenous when estimating the individual country models. The concept of weak exogeneity in the context of the GVAR is discussed in DdPS and relates to the standard assumption in the small-open-economy macroeconomic literature. Whether such exogeneity assumptions hold in practice depends on the relative sizes of the countries/regions in the global economy. Following Johansen (1992) and Granger and Lin (1995) this assumption implies no long run feedbacks from the domestic/endogenous variables to the foreign variables, without necessarily ruling out lagged short run feedbacks between the two sets of variables. In this case the star variables are said to be 'long run forcing' for the domestic variables, and implies that the error correction terms of the individual country VECMs do not enter in the marginal model of the foreign variables. A formal test of this assumption for the country-specific foreign variables (the "starred" variables) and the oil price variable is provided in DdPS, where it is shown that apart from three cases the weak exogeneity assumption holds for all foreign variables and oil prices in non-US models.

Recall that the specification of the US model differs from that of the other countries in that oil prices are included as an endogenous variable, while only $\overline{re}_{US,t}^*$, $y_{US,t}^*$, and $\Delta p_{US,t}^*$ are included included in the US model as weakly exogenous. The endogeneity of oil prices reflects the large size of the US economy. The omission of $q_{US,t}^*$, $\rho_{US,t}^{*S}$ and $\rho_{US,t}^{*L}$ from the vector of US-specific foreign financial variables reflects the results of tests showing that these variables are not weakly exogenous with respect to the US domestic financial variables, in turn reflecting the importance of the US financial markets within the global financial system. However, the assumption that $\overline{re}_{US,t}^*$, $y_{US,t}^*$, and $\Delta p_{US,t}^*$ are weakly exogenous in the US model can not be rejected.

Having defined the variables to be included in the individual country models, VARX* specifications are determined for all countries. The VARX* models are estimated separately for each country conditional on the star variables, $\mathbf{x}_{it}^*$, taking into account the possibility of cointegration both within the domestic variables, $\mathbf{x}_{it}$, and across $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$. The estimation is based on reduced rank regressions containing weakly exogenous I(1) regressors following the methodology developed by Harbo, Johansen, Nielsen and Rahbek (1998) and Pesaran, Shin and Smith (2000). The individual country models are then linked in a consistent manner as described in Section 3 to generate impulse reponse functions for all the variables in the world economy simultaneously, while persistence profiles are used to examine the effect of system-wide shocks on the long-run relations.

The issue of parameter instability is dealt with in DdPS, where a number of structural stability tests are conducted along the lines of Stock and Watson (1996), and it is found that although there is evidence of structural instability, this is mainly confined to error variances and does not seem to adversely affect the coefficient estimates. In view of changing error variances we use robust standard errors when investigating the impact effects of the foreign variables, and base our analysis of impulse responses on the bootstrap means and confidence bounds rather than the point estimates.

5.2 Testing and Interpreting Long-Term Restrictions

The modelling strategy chosen begins with the determination of the number of cointegrating vectors for each country-specific model all of which have a VARX*(2,1) specification. The number of cointegration relationships is derived from cointegration tests (see DdPS).

The tests yield a number of 3 cointegration vectors for most of the eleven focus countries, except China (only one vector) and the US (2 cointegrating vectors). In the case of the UK and Norway, while the tests indicate that 4 cointegrating vectors could not be rejected (borderline), we decided to impose only 3 cointegrating relations. The choice of 3 cointegrating relations for these countries was motivated by the empirical results. In particular, the persistence profiles, which allow us to check whether a restriction corresponding to a long run relation is valid by converging to zero and whether it produces reasonable speed of convergence, were more satisfactory with 3 cointegrating relations. Impulse responses were also more reasonable in such cases.

Once the number of cointegrating relationships is determined, we proceed to incorporate the long-run structural relations, suggested by economic theory as outlined in Section 2, in our otherwise unrestricted country-specific models. We consider over-identifying restrictions for 11 of the 26 countries namely, the US, the euro area, China, Japan, UK, Sweden, Switzerland, Norway, Australia, Canada and New Zealand. These countries represent the major economies in the world, and with the exception of China have developed bond and capital markets. The over-identifying restrictions are imposed simultaneously on the 11 countries, while the remaining 15 individual country VECM models are estimated subject to just-identifying restrictions. We also experimented by imposing over-identifying restrictions on each of the 11 countries separately, imposing just-identifying restrictions on the remaining countries. The results obtained were very similar.

The choice of the possible long-term relations was arrived at on the basis of a satisfactory performance of the GVAR model in terms of stability (eigenvalues), persistence profiles and impulse response functions. In particular, various combinations of the cointegrating relations outlined in Section 2 were imposed on the individual country models. Long run relations with persistence profiles not converging to zero were not considered.

In the case where there were two sets of cointegrating relations, both sets producing convergent persistence profiles, the choice was made based on the speed with which the persistence profiles approached zero, and the stability of the resultant GVAR model by checking that none of its eigenvalues exceeded unity.

Table 3 reports the long-run restrictions that correspond to each country, for the case where the inflation coefficient in the Fisher equation is restricted to unity in all the focus countries.

In a second step, the inflation coefficient in the Fisher equation is left unrestricted as in Table 4. According to the value of the t-statistic on the inflation coefficient, we then determine the country models for which the Fisher equation can be left unrestricted. It is worth noting that the value of the inflation coefficient can in this case be interpreted as the importance of the inflation term in the Central Bank policy feedback rule. In accordance with the Taylor principle, the coefficient on inflation should be greater than one if the Central Bank wants to ensure that the real interest rates move in the right direction to stabilize output and inflation. This is in fact the case in the US, Japan and the UK. In the euro area, Canada and Australia, the coefficient on inflation is not significantly different from one. For the remaining countries, China, Sweden, Switzerland, Norway and New Zealand, the inflation coefficient was estimated to be less than one. This is a difficult result to interpret and requires further investigation, at least in the case of the latter four economies. However, as recently argued by Nelson (2005) the low estimate of the inflation coefficient in the case of some of these countries - New Zealand in particular - could be explained by the extensive use of price and wage controls during the 1980's and early 1990's.

Building on the initial results reported in Tables 3 and 4, the final set of over-identified long-run restrictions for the 11 focus countries are summarized in Table 5. These restrictions are tested using the log-likelihood ratio (LR) statistic at the 1% significance level. The critical values reported are computed by bootstrapping from the solution of the GVAR model (see Appendix for the computational details). The results in Table 5 show that only in the case of Norway and the UK (and to a lesser extent Japan) are the LR statistics greater than their bootstrapped critical values.[9] In all other cases the long run relations are not rejected by the data. Furthermore, all the long run relations have well behaved persistence profiles (see Figure 1) indicating that the effects of shocks on the long run relations are transitory and die out eventually.

Overall, the test results support the term premium condition (i.e. $\rho_{it}^{L} - \rho_{it}^{S} \sim I(0)$) in nine out of the ten focus countries where the condition is relevant (there are no long run rates in China). The UIP condition can also be maintained in the case of six countries (euro area, Japan, UK, Australia, Sweden, and New Zealand). A strict Fisher equation is supported in the case of euro area, Canada and Australia, with the less strict version of the hypothesis holding for all the remaining economies. But, strict PPP can only be detected for three countries (UK, Australia, Norway) and a weaker form with relative productivity differences cannot be rejected in the case of Switzerland. However, the combination of the Fisher relationship and UIP together imply relative PPP. Hence, among the eleven focus countries, we reject both absolute and relative PPP only in the case of the US and China. For the US, this result can be explained by the role of the US dollar as a reserve currency. As suggested by Juselius and MacDonald (2003), the peculiar role of the US dollar has

[9] Alternative restrictions and specifications chosen did not appear to alter this result.

facilitated relatively cheap financing of the large US current account deficits explaining why an adequate adjustment toward PPP between the USA and the rest of world has not taken place. In the case of China, as the country has remained in transition towards the market economy over the period, it is therefore not surprising that such "market failures" can be found.

5.3 Contemporaneous Effects and Cross-Section Correlations

The country specific models are estimated with the set of over-identifying long-run restrictions imposed, as presented in Table 5. Regarding the contemporaneous effects of the foreign variables on their domestic counterparts (Table 6), as in the unrestricted case of DdPS we continue to find only weak linkages across the short-term interest rates, ρ^s and ρ^{*s}, with Sweden no longer constituting an exception. The contemporaneous elasticity of real equity prices remains significant and slightly above one in most case, while we also continue to observe significant linkages across the long-term rates with the exception of New Zealand. In terms of real output the elasticity of UK real output with respect to $y^*_{uk,t}$ is now more in line with other countries, increasing to 0.67 from 0.33 reported in DdPS. In contrast, the real output elasticity of Australia decreases from 0.52 (reported in DdPS) to 0.36. Finally, inflation elasticities show the greatest variability when compared to the estimates in DdPS that do not impose any long run restrictions on the relationship between inflation and interest rates. In particular, inflation elasticity in Japan (with respect to foreign inflation) is now significant, dropping from -0.04 to -0.34. In the UK it drops from -0.15 to -0.52 though remaining insignificant; in Canada it remains significant, dropping from 0.73 to 0.38; in Australia it is not significant, dropping from 0.51 to 0.21; in Sweden it falls from 1.23 to 1.10 retaining its significance; the same applies to Norway with the inflation elasticity falling from 1.11 to 0.68, while for New Zealand the estimate increases from 0.23 to 0.38, but is not statistically significant. The inflation elasticity in the euro area remains significant and at 0.22 is almost the same as before, while in the US it increases slightly from 0.06 to 0.13 although still remaining insignificant. Thus, in most cases it appears that the imposition of the Fisher equation tends to reduce inflation elasticities, showing a higher degree of independence of domestic inflation from their foreign counterparts.

Turning to the effectiveness of the country specific foreign variables in reducing the cross-section correlation of the variables, we deal with this as in DdPS, by computing average pair-wise cross-section correlations of the country specific residuals over the estimation period. What is worth noting, is that now with the use of the real effective exchange rate we no longer observe high correlations for the exchange rate variable after conditioning on the foreign variables. In fact first differencing the exchange rate reduces the cross section correlations from an average value of 20% for the real exchange rate to 5% for the real effective exchange rate.

The above results indicate the importance of the Fisher restriction for the global economy, while the UIP restriction appears to be robust to the finding of strong significant relations between bond markets. Such relations are even stronger in the case of equity markets, while overall they remain limited in the case of the short term interest rate set by monetary authorities.

5.4 Persistence Profiles

As detailed above, we use persistence profiles (Pesaran and Shin, 1996) to examine the effect of system-wide shocks on the dynamics of the long run relations. As can be seen from equation (4.3), the value of these profiles is unity on impact, while it should tend to zero as the horizon, $n \rightarrow \infty$, if the vector under investigation is indeed a cointegrating vector. It is important, once a system is shocked, that the analysis of long run (cointegration) relationships is accompanied by some estimates of the speed with which the relationships under consideration return to their equilibrium states. Figure 1 shows the persistence profiles corresponding to the model including the long-term restrictions displayed in Table 5. The chart labelled "All" displays the profiles corresponding to all long run relations. This chart shows that after a shock, all variables return to their long run equilibrium within a ten year period, although in many cases the convergence is much more rapid, often taking less than five years. The other charts decompose the profiles according to the type of restrictions imposed. We can see therefore that the Fisher relationships are not very persistent, any departure from these relationships being corrected within 2 years. The term-premium relationships also display similar profiles, albeit somewhat more persistent. By contrast, the UIP and PPP relationships are very persistent. Therefore, departures of inflation and interest rates from their long run equilibrium values could take a long time before they are corrected. This result seems perfectly in line with both economic intuition and previous findings that show that UIP and PPP relations, when they are valid, hold only in the very long run. For instance, Lothian and Wu (2005) find that uncovered interest-rate parity can deviate from equilibrium for a long period of time because of slow adjustment of expectations to actual regime changes or to anticipations of extended periods of regime changes or other big events that never materialize. Similarly, PPP might only be valid in the long run owing to factors like transaction costs, various trade restrictions, the existence of non-traded goods, imperfect competition, foreign exchange market intervention or statistical issues related to the measure of price indices (Obstfeld and Rogoff, 2000). Finally, the last chart shows the persistence profiles corresponding to the remaining 15 countries, for which no restriction is imposed. In all cases, the persistence profiles of these remaining cointegration relationships converge to zero quite rapidly.

Figure 2 presents bootstrap mean estimates of the persistence profiles for the euro area together with 90% bootstrap error bands. For the Fisher and term premium restrictions the bands approach zero at a much faster rate as compared to the UIP relation, the bands of which are wider reflecting the slower convergence of this cointegrating relation to equilibrium. Bootstrap error bands for the rest of the countries are available on request.

5.5 Impulse Response Analysis

We show the consequences of imposing the long run restrictions for the impulse response functions, where we focus on the propagation of a shock to oil prices, US real equity prices, and a US monetary policy shock. The long-run restricted generalized impulse response functions (GIRFs) are generally more in line with our theoretical priors as compared to the unrestricted ones in DdPS, and tend to have narrower confidence bands. This is true for the final set of restrictions shown in Table 5, as well as for the comparison of the impulse responses based on Tables 3 and 4. These results are available upon request.

All GIRF figures are based on the set of restrictions given in Table 5. In particular, Figures 3-5 show how the effect of oil, real equity prices as well as monetary policy shocks differ across the main industrial economies by plotting the various impulse response functions across the different markets. In order to evaluate the significance of the responses, the figures display bootstrap mean estimates of the GIRFs together with 90% bootstrap bounds computed from the long-run restricted GVAR model. For the monetary policy shock in the US, we entertain the ordering $\mathbf{x}_{0t}$ = (oil, long-term interest rate, equity prices, inflation, output, short-term interest rate) in the US model, which corresponds to ordering B in DdPS.

In our sample period a positive one standard error shock to oil prices is equivalent to an increase of around 10% in nominal oil prices. This shock has a significant positive effect on inflation in the short term in most countries, increasing inflation by around 0.1 percentage points. On real GDP, the oil price shock has generally a negative impact; this is however significant only in a couple of cases. The impact is much stronger and significant on real equity prices, which decrease by more than 2% in the US, Canada and the UK after one year. The impact is even larger for real equity prices in the euro area, Sweden and Switzerland (between 4 and 6% after one year). The effect of the oil price shock on other financial variables (interest and exchange rates) remain limited and statistically insignificant in most cases.

A negative one-standard error shock to real US equity prices, which amounts to a decrease by 4.5% on impact and by around 5% in the long-run, has a significant negative effect on US real output over quarters 1 to 6, with a maximum impact of -0.3%. The effect on the other US variables is also significant in the short-run. Such a negative real equity price shock tends to lower inflation, results in a slight depreciation of the US dollar real effective exchange rate and a slight decrease in nominal interest rates (both short and long-term). The transmission of the shock to the rest of the world economy seems to take place through equity markets. Indeed, real equity prices fall in most economies by a significant amount. Moreover, the magnitude of the impact is very close to that of the US in most countries and even larger in the case of Sweden and Norway. Apart from the transmission through equity markets, the US equity shock does not affect macroeconomic activity in the rest of the world. Real GDP is significantly affected only in the case of Canada and Switzerland. The impact on inflation seems more significant, though remaining relatively limited. As regards exchange rates, the slight depreciation of the US dollar in real effective terms seems to find a significant counterpart in a real appreciation of the Canadian dollar, the effects on the other real exchange rates remaining largely non-significant. The impact on the short- and long-term interest rates follows the US responses with some significant decline in most countries.

Finally, there is a significant response to a one-standard error US monetary policy shock of real output in the US. Compared with the results reported in DdPS, the impacts are stronger and remain permanent. On inflation, as in DdPS, there is a price puzzle in the short-term. This effect fades away rapidly and becomes insignificant after a couple of quarters. The impact on the rest of the world remains limited and in most cases non-significant. Real output falls significantly only in Canada and Norway. Financial variables are barely affected by the US monetary policy shock. Finally as regards the transmission of the increase in US short term rates, the other central banks tend to increase their interest rates slightly. However, these increases are not significant in most cases. The short-term interest rate of Canada is the most affected by the US monetary policy shock, increasing by half the US interest rate response.

5.6 Forecast Error Variance Decomposition

Tables 7 and 8 show the forecast error variance decomposition of the euro area and US real output and inflation in terms of their top ten determinants from the eleven focus countries. In particular, each table shows the proportion of the forecast error variances of the euro area and US real output and inflation explained by conditioning on contemporaneous and expected future values of the top ten variables (which are identified in terms of their relative contributions at the eighth quarterly horizon). These tables also show the sums across the top ten and the total number of determinants, the latter being equal to the number of endogenous variables, k, in the GVAR. Note that the sum across the total number of determinants is greater than 100% because of the positive correlation that exists across the shocks from the various countries in the global economy.

The greatest proportion of euro area real output forecast error variance is explained by domestic variables (real output, inflation and short-term interest rates). Among the foreign real outputs, China and the US contribute the most to the euro area real output forecast error variance. Among the financial variables, the main determinants are the US and Swiss long-term interest rates. The contribution of the Chinese real effective exchange rate is also relatively large (between 2 and 3%). Overall, after two years half of the euro area real output forecast error variance is explained by domestic variables, around 20% by foreign financial variables, and 15% by foreign outputs (China and the US).

For euro area inflation, apart from euro area real output and the real effective exchange rate, almost all US variables contribute to the variance of inflation forecast errors in the euro area. Oil price is the third most important factor in explaining the forecast error variance of euro area inflation. It is also worth noting the relatively large contribution from real effective exchange rates (in the euro area, but also in Canada and Japan). Overall, after two years, domestic variables contribute to around 40% of euro area inflation forecast error variance, while oil prices contribute almost 15%, the rest of the determinants being foreign variables.

Similarly to the euro area case, the US real output variable explains the greatest proportion of US real output forecast error variance. US real equity prices and short-term interest rates are also among the three main factors that help explain forecast error variance of US real output. Among the foreign variables, Chinese real output and the real effective exchange rates of Japan, the euro area and Canada contribute the most. The contribution of oil prices is also relatively significant. On the whole, after two years, domestic variables explain 60% of US real output forecast error variance, oil prices around 5%, the rest of the determinants being foreign variables.

Finally, all US variables help in determining the forecast error variance of US inflation, along with the real effective exchange rate in the euro area, Canada, Japan and Australia, and also oil prices.[10]

Overall, the results in Tables 7 and 8 show that the contribution of other determinants to the forecast error variance of inflation is larger compared to real output, and in terms of magnitude more so for the US than the euro area. It is perhaps not surprising that the main determinants are typically the variables in countries that are fairly significant trade partners of the country under consideration or are significant trade partners of the

[10] Note that US inflation does not appear in the bottom panel of Table 8 as it, on itself, explains a high proportion, 96.27%, on impact, reducing abruptly to 34.8% and 17.12% in the first and second quarters respectively, reaching 1.96% by the eighth quarter.

largest foreign contributor.

5.7 Does the Definition of the Real Exchange Rate Affect the Results?

As a robustness check, we have also performed alternative simulations in which the real exchange rate is computed vis-à-vis the US dollar, that is the real exchange rate variable is defined as $\overline{re}_{i,us} = (e_i - p_i) - (e_{us} - p_{us})$ rather than $\overline{re}_i = (e_i - p_i) - (e_i^* - p_i^*)$. The results in terms of tests of long-run restrictions, persistence profiles and impulse responses proved to be very similar to those presented above (these are available upon request). However, we prefer defining the exchange rate in effective terms owing to the formulation of PPP in a multi-country set-up and due to the need for consistency in the definition of UIP (defined in terms of differences between domestic and weighted averages of foreign interest rates, the latter being computed in the same way as the effective exchange rate).

6 Concluding Remarks

We considered applying long-run structural relations to a global model, with the aim of developing a model with a transparent and coherent foundation. We then tested for the number of cointegrating relationships in each country/block of countries using persistence profiles among others to examine the validity and reasonableness of theory-based long run restrictions on the cointegrating relations.

The critical values for testing the long run relations are obtained via bootstrapping from the solution of the GVAR model. We report generalized impulse responses together with bootstrapped standard errors. We are able to impose a number of restrictions on the long run relations of the model that are consistent with the data. For example, we are not able to reject the uncovered interest parity, and to a lesser extent the Fisher condition. However, we have more difficulty with absolute purchasing power parity. We can only successfully impose this for a subset of countries. We are only able to find some evidence in favour of relative purchasing power parity, a result also found in the literature (Sarno and Taylor, 2002).

The imposition of long-run theory restrictions also has the added advantage that the estimated impulse responses tend to have narrower confidence bounds as compared to the estimates reported in DdPS that do not impose such restrictions. These new results also confirm, with a greater degree of confidence, that financial shocks (to equity and bond prices) tend to be transmitted much faster than shocks to real output and/or inflation.

The next challenge is to link these long run restrictions to recent developments in the theoretical modelling of open economies and to use restrictions from dynamic stochastic general equilibrium models in order to generate a set of short run restrictions that can be used to refine further the GVAR approach to modelling the global economy.

Appendix

Proof of Remark 2 . Let $\overline{\mathbf{re}}_t = (\overline{re}_{0t}, \overline{re}_{1t}, ..., \overline{re}_{Nt})$, $\mathbf{e}_t = (e_{0t}, e_{1t}, ..., e_{Nt})'$, $\mathbf{p}_t = (p_{0t}, p_{1t}, ..., p_{Nt})'$ and denote the $(N+1) \times (N+1)$ matrix with elements w_{ij} by $\mathbf{W}$, and write

$$\overline{\mathbf{re}}_t = (\mathbf{I}_{N+1} - \mathbf{W})(\mathbf{e}_t - \mathbf{p}_t) = \mathbf{A}\tilde{\mathbf{e}}_t,$$

where $\mathbf{A} = (\mathbf{I}_{N+1} - \mathbf{W})$, and $\tilde{\mathbf{e}}_t = \mathbf{e}_t - \mathbf{p}_t$. Since $\Sigma_{j=0}^N w_{ij} = 1$ for all i, it readily follows that $\mathbf{A}\boldsymbol{\tau}_{N+1} = 0$, where $\boldsymbol{\tau}_{N+1}$ is an $(N+1) \times 1$ vector of ones. Hence $\mathbf{A}$ is rank deficient and only N out of the $N+1$ elements of $\tilde{\mathbf{e}}_t$ can be solved uniquely. Here, without loss of generality, we provide a solution in terms of $\tilde{e}_{0t} = -p_{0t}$. To this end consider the following partitioned form of $\overline{\mathbf{re}}_t = \mathbf{A}\tilde{\mathbf{e}}_t$, and recall that $\mathbf{A}\boldsymbol{\tau}_{N+1} = 0$ to obtain:

$$\begin{aligned}
\begin{pmatrix} \overline{re}_{0t} \\ \overline{\mathbf{re}}_{t,-0} \end{pmatrix} &= \begin{pmatrix} a_{00} & \mathbf{a}'_{01} \\ \mathbf{a}_{10} & \mathbf{A}_{11} \end{pmatrix} \begin{pmatrix} \tilde{e}_{0t} \\ \tilde{\mathbf{e}}_{t,-0} \end{pmatrix} \\
&= \begin{pmatrix} a_{00} & \mathbf{a}'_{01} \\ \mathbf{a}_{10} & \mathbf{A}_{11} \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{\mathbf{e}}_{t,-0} - \tilde{e}_{0t}\boldsymbol{\tau}_N \end{pmatrix} + \tilde{e}_{0t} \begin{pmatrix} a_{00} & \mathbf{a}'_{01} \\ \mathbf{a}_{10} & \mathbf{A}_{11} \end{pmatrix} \boldsymbol{\tau}_{N+1}, \\
&= \begin{pmatrix} a_{00} & \mathbf{a}'_{01} \\ \mathbf{a}_{10} & \mathbf{A}_{11} \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{\mathbf{e}}_{t,-0} - \tilde{e}_{0t}\boldsymbol{\tau}_N \end{pmatrix},
\end{aligned}$$

where $\overline{\mathbf{re}}_{t,-0} = (r\bar{e}_{1t}, r\bar{e}_{2t}, ..., r\bar{e}_{Nt})'$, $\tilde{\mathbf{e}}_{t,-0} = (\tilde{e}_{1t}, \tilde{e}_{2t}, ..., \tilde{e}_{Nt})'$,

$$\begin{aligned}
a_{00} &= 1 - w_{00} = 1, \ \mathbf{a}_{01} = (-w_{01}, -w_{02}, ..., -w_{0N})', \\
\mathbf{a}_{10} &= (-w_{10}, -w_{20}, ..., w_{N0})',
\end{aligned}$$

$\mathbf{A}_{11}$ is an $N \times N$ matrix with unit diagonal elements and $-w_{ij}$ on its off-diagonals. Hence

$$\overline{re}_{0t} = \mathbf{a}'_{01}(\tilde{\mathbf{e}}_{t,-0} - \tilde{e}_{0t}\boldsymbol{\tau}_N),$$

and

$$\overline{\mathbf{re}}_{t,-0} = \mathbf{A}_{11}(\tilde{\mathbf{e}}_{t,-0} - \tilde{e}_{0t}\boldsymbol{\tau}_N).$$

Assuming $\mathbf{A}_{11}$ is full rank now yields

$$\tilde{\mathbf{e}}_{t,-0} - \tilde{e}_{0t}\boldsymbol{\tau}_N = \mathbf{A}_{11}^{-1}\overline{\mathbf{re}}_{t,-0} \sim I(0),$$

or focusing on the i^{th} element

$$\tilde{e}_{it} - \tilde{e}_{0t} \sim I(0), \text{ for all } i = 1, 2, ..., N,$$

which can be written as

$$e_{it} - p_{it} - (e_{0t} - p_{0t}) = e_{it} + p_{0t} - p_{it} \sim I(0).$$

Hence also

$$(e_{it} - p_{it}) - (e_{jt} - p_{jt}) \sim I(0), \text{ for all } i \text{ and } j.$$

It is, therefore, established that if $\mathbf{A}_{11}$ is non-singular and PPP holds for all real effective exchange rates then it must also hold in terms of the US, and more generally for any country pairs. It is also worth noting that nonsingularity of A_{11} means that trade weights are such that no country or group of countries is isolated from the rest of the world economies.[11] ■

[11] We are grateful to Alexander Chudik for this last point.

A.1 Derivation of the Generalized Forecast Error Variance Decomposition

Consider the MA representation (4.1) of the GVAR model. The forecast error of predicting $\mathbf{x}_{t+n}$ conditional on the information at time $t-1$ is given by

$$\boldsymbol{\xi}_t(n) = \sum_{l=0}^{n} \mathbf{A}_l \boldsymbol{\varepsilon}_{t+n-l}, \text{ for } n = 0, 1, 2, ...,$$

with the $\mathbf{A}_l$ matrices computed using (4.2) and the total forecast error covariance matrix is given by

$$\boldsymbol{\Omega}_n = \sum_{l=0}^{n} \mathbf{A}_l \boldsymbol{\Sigma}_\varepsilon \mathbf{A}_l'.$$

In what follows we will consider the forecast error covariance matrix of predicting $\mathbf{x}_{t+n}$ conditional on the information at time $t-1$, and the contemporaneous and expected future shocks to the j^{th} equation, $\varepsilon_{jt}, \varepsilon_{j,t+1}, ..., \varepsilon_{j,t+n}$. The forecast error of predicting $\mathbf{x}_{t+n}$ in this case is given by[12]

$$\boldsymbol{\xi}_t^j(n) = \sum_{l=0}^{n} \mathbf{A}_l \left[\boldsymbol{\varepsilon}_{t+n-l} - E(\boldsymbol{\varepsilon}_{t+n-l}|\varepsilon_{j,t+n-l})\right]. \tag{A.1}$$

Assuming that $\boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Sigma}_\varepsilon)$ we obtain that

$$E(\boldsymbol{\varepsilon}_{t+n-l}|\varepsilon_{j,t+n-l}) = (\sigma_{jj}^{-1}\boldsymbol{\Sigma}_\varepsilon \mathfrak{e}_j)\varepsilon_{j,t+n-l} \tag{A.2}$$

for $j = 1, .., k$ and $l = 0, 1, .., n$, where $\mathfrak{e}_j$ is a $k \times 1$ selection vector with its element corresponding to the j^{th} variable in $\mathbf{x}_{t+n}$ is unity and zeros elsewhere. Substituting (A.2) in (A.1) yields

$$\boldsymbol{\xi}_t^j(n) = \sum_{l=0}^{n} \mathbf{A}_l(\boldsymbol{\varepsilon}_{t+n-l} - \sigma_{jj}^{-1}\boldsymbol{\Sigma}_\varepsilon \mathfrak{e}_j \varepsilon_{j,t+n-l}),$$

and the forecast error covariance matrix in this case becomes

$$\boldsymbol{\Omega}_n^j = \sum_{l=0}^{n} \mathbf{A}_l \boldsymbol{\Sigma}_\varepsilon \mathbf{A}_l' - \sigma_{jj}^{-1} \sum_{l=0}^{n} \mathbf{A}_l \boldsymbol{\Sigma}_\varepsilon \mathfrak{e}_j \mathfrak{e}_j' \boldsymbol{\Sigma}_\varepsilon \mathbf{A}_l'.$$

It follows that the decline in the n-step ahead forecast error variance of $\mathbf{x}_t$ as a result of conditioning on the expected future shocks to the j^{th} equation is given by

$$\Delta_{jn} = \boldsymbol{\Omega}_n - \boldsymbol{\Omega}_n^j = \sigma_{jj}^{-1} \sum_{l=0}^{n} \mathbf{A}_l \boldsymbol{\Sigma}_\varepsilon \mathfrak{e}_j \mathfrak{e}_j' \boldsymbol{\Sigma}_\varepsilon \mathbf{A}_l'.$$

Obtaining the change Δ_{jn} of the n-step ahead forecast error variance of $\mathbf{x}_t$ with respect to the ℓ^{th} variable as

$$\Delta_{(\ell)jn} = \mathfrak{e}_\ell' \left(\boldsymbol{\Omega}_n - \boldsymbol{\Omega}_n^j\right) \mathfrak{e}_\ell = \sigma_{jj}^{-1} \sum_{l=0}^{n} \left(\mathfrak{e}_\ell' \mathbf{A}_l \boldsymbol{\Sigma}_\varepsilon \mathfrak{e}_j\right)^2 \ , \ell, j = 1, .., k$$

and scaling it by the n-step ahead forecast error variance of the ℓ^{th} variable of $\mathbf{x}_t$ yields the generalized forecast error variance decomposition (GFEVD) formula for the errors in the global

[12] Note that as the $\boldsymbol{\varepsilon}_t s$ are serially uncorrelated, $E(\boldsymbol{\varepsilon}_{t+n-l}|\varepsilon_{jt}, \varepsilon_{j,t+1}, ..., \varepsilon_{j,t+n}) = E(\boldsymbol{\varepsilon}_{t+n-l}|\varepsilon_{j,t+n-l})$, $l = 0, 1, ..., n$.

model. Note that the above derivation is based on the errors ε_t of the global model. However, as in the case of impulse response analysis, we perform GFEVD for the errors in the country-specific models, $\mathbf{u}_{it}$, in which case the above can be easily adjusted using (3.7) to yield (4.7). The formula for the case of structural shocks follows similarly.

A.2 Bootstrapping the GVAR

To derive the empirical distribution of the impulse response functions we employ the sieve bootstrap. The sieve bootstrap has been studied by Kreiss (1992), Bühlmann (1997) and Bickel and Bühlmann (1999) among others and has now become a standard tool when bootstrapping time series models.[13] The method rests on the assumption that the precise form of the parametric model generating the data is not known and that the true model belongs to the class of linear processes having an autoregressive representation of infinite order. Taking the estimated finite order vector autoregressive process that describes in our case the GVAR model to be an approximation to the underlying infinite order vector autoregressive process, we can use the sieve bootstrap for the basis of deriving critical values for the likelihood ratio statistic used to test the long run overidentifying restrictions and for constructing bootstrap confidence regions.

In the case of stationary multivariate models, the sieve bootstrap has been used successfully to handle parameter estimation (Paparoditis, 1996). In the context of non-stationary time series, Park (2002) established an invariance principle applicable for the asymptotic analysis of the sieve bootstrap, which led Chang and Park (2003) to establish its asymptotic validity in the case of ADF unit root tests. Subsequently, Chang, Park and Song (2006) established the consistency of the sieve bootstrap for the OLS estimates of the cointegrating parameters assuming there exists one cointegrating relation amongst the variables under consideration.

When bootstrapping unit root tests based on first order autoregressions, Basawa et al. (1991) show that the bootstrap samples need to be generated with the unit root imposed in order to achieve consistency for the bootstrap unit root tests. While our focus is not on bootstrapping unit root or cointegration tests, it seems natural to impose the unit root and cointegrating properties of the model when bootstrapping the statistics of interest. See also Li and Maddala (1997) who study the bootstrap cointegrating regression by means of simulation.

We begin by estimating the individual country VARX*$(\hat{p}_i, \hat{q}_i)$ models in their error correction form subject to reduced rank restrictions having imposed the long run over-identifying restrictions. In general the estimates of these country-specific models can be written as

$$\begin{aligned}\mathbf{x}_{it} &= \hat{\mathbf{h}}_{i0} + \hat{\mathbf{h}}_{i1}t + \hat{\mathbf{\Phi}}_{i1}\mathbf{x}_{i,t-1} + ... + \hat{\mathbf{\Phi}}_{i\hat{p}_i}\mathbf{x}_{i,t-\hat{p}_i} + \hat{\mathbf{\Psi}}_{i0}\mathbf{x}^*_{it} + \hat{\mathbf{\Psi}}_{i1}\mathbf{x}^*_{i,t-1} \\ &\quad + ... + \hat{\mathbf{\Psi}}_{i\hat{q}_i}\mathbf{x}^*_{i,t-\hat{q}_i} + \hat{\mathbf{u}}_{it}\end{aligned} \tag{A.1}$$

for $i = 0, 1, 2, ..., N$ and $t = 1, 2, ..., T$, where $\hat{p}_i$ and $\hat{q}_i$ are the lag orders of the endogenous and foreign variables, respectively, which can be typically chosen by some information criterion such as the Bayesian information criterion or the AIC. We denote by $\hat{r}_i$ the estimated number of cointegrating relations for country i. In estimating the cointegrating rank we entertain the case of an unrestricted intercept and restricted trend, the latter restricted to lie in the cointegrating space so as to avoid giving rise to quadratic trends in the level of the process.

Having estimated the country specific models given by (A.1), these are then consistently combined using the link matrices $\mathbf{W}_i$ to form the GVAR$(\hat{p})$ model expressed in terms of the

[13] Another popular method is the block bootstrap by Künsch (1989). Choi and Hall (2000) discuss the substantial advantages of the sieve bootstrap over the block bootstrap for linear time series.

global variables vector $\mathbf{x}_t$ as

$$\hat{\mathbf{H}}_0\mathbf{x}_t = \hat{\mathbf{h}}_0 + \hat{\mathbf{h}}_1\mathbf{t} + \hat{\mathbf{H}}_1\mathbf{x}_{t-1} + \hat{\mathbf{H}}_2\mathbf{x}_{t-2} + ... + \hat{\mathbf{H}}_{\hat{p}}\mathbf{x}_{t-\hat{p}} + \hat{\mathbf{u}}_t, \tag{A.2}$$

with $\hat{p} = max(\hat{p}_i, \hat{q}_i)$, or alternatively,

$$\mathbf{x}_t = \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1\mathbf{t} + \hat{\mathbf{F}}_1\mathbf{x}_{t-1} + \hat{\mathbf{F}}_2\mathbf{x}_{t-2} + ... + \hat{\mathbf{F}}_{\hat{p}}\mathbf{x}_{t-\hat{p}} + \hat{\varepsilon}_t, \tag{A.3}$$

where $\hat{\mathbf{F}}_j = \hat{\mathbf{H}}_0^{-1}\hat{\mathbf{H}}_j$, $\hat{\mathbf{a}}_j = \hat{\mathbf{H}}_0^{-1}\hat{\mathbf{h}}_j$, for $j = 0, 1, ..., \hat{p}$, $\hat{\varepsilon}_t = \hat{\mathbf{H}}_0^{-1}\hat{\mathbf{u}}_t$ and $\hat{\mathbf{\Sigma}}_\varepsilon = \sum_{t=1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_t'/T$. The total number of variables in the GVAR model is given by $k = \Sigma_{i=0}^{N} k_i$, where k_i is the number of endogenous regressors in country i, $i = 0, 1, ..., N$.

Note that while in the empirical analysis above a VARX*$(2, 1)$ specification is chosen for the individual country models, the resulting GVAR model is of order $\hat{p} = 3$ as it is solved in terms of the US price level in order to accommodate the inclusion of the real effective exchange rate included amongst the endogenous country specific variables. Using the estimates from the fitted model (A.3) obtained from the observed data for $\hat{p} = 3$, we generate B bootstrap samples denoted by $\mathbf{x}_t^{(b)}$, $b = 1, 2, ..., B$ from the process

$$\mathbf{x}_t^{(b)} = \hat{\mathbf{b}}_0 + \hat{\mathbf{b}}_1 t + \hat{\mathbf{F}}_1\mathbf{x}_{t-1}^{(b)} + \hat{\mathbf{F}}_2\mathbf{x}_{t-2}^{(b)} + \hat{\mathbf{F}}_3\mathbf{x}_{t-3}^{(b)} + \boldsymbol{\varepsilon}_t^{(b)},\ t = 1, 2, ..., T, \tag{A.4}$$

by resampling the residuals $\hat{\boldsymbol{\varepsilon}}_t$ of the fitted model, with $\mathbf{x}_0^{(b)} = \mathbf{x}_0$, $\mathbf{x}_{-1}^{(b)} = \mathbf{x}_{-1}$ and $\mathbf{x}_{-2}^{(b)} = \mathbf{x}_{-2}$, where $\mathbf{x}_0$ and $\mathbf{x}_{-1}$ are the observed initial data vectors and $\mathbf{x}_{-2}$ is the vector containing the observed price level of the US with the rest of the variables set to zero. Prior to any resampling the residuals $\hat{\boldsymbol{\varepsilon}}_t$ are recentered to ensure that their bootstrap population mean is zero. The sieve bootstrap effectively reinterprets the familiar parametric AR model as a device for nonparametric estimation. The errors $\boldsymbol{\varepsilon}_t^{(b)}$ could also be drawn by parametric methods. Both these methods will be described in what follows. Simulating the GVAR model is clearly preferable to simulating the country specific models separately. The latter requires that the country specific foreign variables, $\mathbf{x}_{it}^*$, and their lagged values are treated as strictly exogenous which might not be appropriate and could lead to unstable outcomes for $\mathbf{x}_t$.

Once a set of $\mathbf{x}_t^{(b)}$, $b = 1, 2, ..., B$ are generated, as the GVAR model given in (A.4) is solved for $e_{it} - p_{it}$ and for inflation for all countries (except the US), the price level and the nominal exchange rate for all countries are then recovered. The corresponding foreign variables, $\mathbf{x}_{it}^{*(b)}$, are then constructed using the trade weights, and the inflation and real effective exchange rate variable, $\overline{re}_{it}$, are recreated using the observed initial data vectors $\mathbf{x}_0$, $\mathbf{x}_{-1}$ and $\mathbf{x}_{-2}$ referred to above.

For each replication b, the individual country models are then estimated in their error correction form which for the trend restricted version under a VARX*$(2, 1)$ specification, is given by

$$\Delta\hat{\mathbf{x}}_{it}^{(b)} = \hat{\mathbf{c}}_{i0}^{(b)} - \hat{\boldsymbol{\alpha}}_i^{(b)}\hat{\boldsymbol{\beta}}_i'[\mathbf{z}_{i,t-1}^{(b)} - \hat{\gamma}_i^{(b)}(t-1)] + \hat{\mathbf{\Psi}}_{i0}^{(b)}\Delta\mathbf{x}_{it}^{(b)} + \hat{\mathbf{\Gamma}}_i^{(b)}\Delta\mathbf{x}_{i,t-1}^{(b)} + \hat{\mathbf{u}}_{it}^{(b)}, \tag{A.5}$$

where $\mathbf{z}_{it}^{(b)} = (\mathbf{x}_{it}^{(b)\prime}, \mathbf{x}_{it}^{*(b)\prime})'$, $\hat{\boldsymbol{\alpha}}_i^{(b)}$ is a $k_i \times r_i$ matrix of rank r_i and $\hat{\boldsymbol{\beta}}_i$ is a $(k_i + k_i^*) \times r_i$ matrix of rank r_i that contains the long run vectors. The country-specific lag orders p_i and q_i, and the number of cointegrating relations, r_i, are fixed over all replications at their estimated values $\hat{p}_i$, $\hat{q}_i$ and $\hat{r}_i$ based on the historical observations, with $\boldsymbol{\beta}_i's$ fixed at the their maximum likelihood estimates subject to the long run economic theory restrictions. The VARX* form of (A.5) is then derived as

$$\mathbf{x}_{it}^{(b)} = \hat{\mathbf{a}}_{i0}^{(b)} + \hat{\mathbf{a}}_{i1}^{(b)} t + \hat{\mathbf{\Phi}}_{i1}^{(b)} \mathbf{x}_{i,t-1}^{(b)} + ... + \hat{\mathbf{\Phi}}_{i\hat{p}_i}^{(b)} \mathbf{x}_{i,t-\hat{p}_i}^{(b)} \\ + \hat{\mathbf{\Psi}}_{i0}^{(b)} \mathbf{x}_{it}^{*(b)} + \hat{\mathbf{\Psi}}_{i1}^{(b)} \mathbf{x}_{i,t-1}^{*(b)} + ... + \hat{\mathbf{\Psi}}_{i\hat{q}_i}^{(b)} \mathbf{x}_{i,t-\hat{q}_i}^{*(b)} + \hat{\mathbf{u}}_{it}^{(b)}. \tag{A.6}$$

We denote by $E\hat{C}M_{ij,t-1}^{(r)}$ the estimated error correction terms that correspond to the $\hat{r}_i$ cointegrating relationships for country i, where $i = 0, 1, ..., N$ and $j = 1, 2, ..., \hat{r}_i$.

A.2.1 The Empirical Distribution of the Log-likelihood Ratio Statistic for Testing Over-identifying Restrictions on the Cointegrating Relations

The estimation of the individual country VECM models, subject to deficient rank restrictions on the long-run multiplier matrix, does not lead to a unique choice for the cointegrating relations. The exact identification of $\boldsymbol{\beta}_i$ requires r_i restrictions per each of the r_i cointegrating vectors where r_i is the number of cointegrating relations for country i. We further consider over-identifying restrictions for 11 of the 26 countries namely, US, euro area, China, Japan, UK, Sweden, Switzerland, Norway and the other developing economies Australia, Canada and New Zealand. Let $\boldsymbol{\theta}_i = vec(\boldsymbol{\beta}_i)$ where $\boldsymbol{\beta}_i = (\boldsymbol{\beta}_{i1}', \boldsymbol{\beta}_{i2}', ..., \boldsymbol{\beta}_{ir_i}')'$, and let $\hat{\boldsymbol{\theta}}_i$ be the maximum likelihood (ML) estimator of $\boldsymbol{\theta}_i$ obtained subject to the r_i^2 exactly-identifying restrictions and $\tilde{\boldsymbol{\theta}}_i$ be the ML estimator of $\boldsymbol{\theta}_i$ obtained under the total number of restrictions m_i. Then, the log-likelihood ratio statistic for testing the over-identifying restrictions is given by

$$\mathcal{LR} = 2\{\ell_n(\hat{\boldsymbol{\theta}}_i; r_i) - \ell_n(\tilde{\boldsymbol{\theta}}_i; r_i)\} \tag{A.7}$$

where $\ell_n(\hat{\boldsymbol{\theta}}_i; r_i)$ represents the maximized value of the log-likelihood function under the just-identifying restrictions, and $\ell_n(\tilde{\boldsymbol{\theta}}_i; r_i)$ is the maximized value of the log-likelihood function under the over-identifying restrictions.

Under the null hypothesis that the over-identifying restrictions hold the log-likelihood ratio statistic $\mathcal{LR}$ defined by (A.7) is asymptotically distributed as a χ^2 variate with degrees of freedom equal to the number of over-identifying restrictions, namely $m_i - r_i^2 > 0$. But in small samples and to take account of the global interactions, the critical values for the $\mathcal{LR}$ statistics are computed by bootstrapping the GVAR using 2000 replications. For each bootstrap replication b, the vector error-correction model given by (A.5) is estimated for each country i, $i = 0, 1, ..., N$. For the b^{th} replication the LR statistic is then computed as

$$\mathcal{LR}^{(b)} = 2\{\ell_n^{(b)}(\hat{\boldsymbol{\theta}}_i; r_i) - \ell_n^{(b)}(\tilde{\boldsymbol{\theta}}_i; r_i)\}, \text{ for } b = 1, 2, ..., 2000. \tag{A.8}$$

These statistics are sorted in an ascending order and the value that exceeds 99% of the bootstrapped statistics yields the appropriate 99% critical value for testing the over-identifying restrictions.

A.2.2 The Empirical Distribution of the Impulse Response Functions

On the assumption that the error term $\mathbf{u}_t$ associated with equation (A.2) has a multivariate normal distribution, recall from Section 4 that the $k \times 1$ vector of the generalized impulse response functions for a one standard error shock to the j^{th} equation corresponding to a particular shock in a particular country on $\mathbf{x}_{t+n}$ is given by

$$\mathcal{GIRF}(\mathbf{x}_t; u_{\ell t}, n) = \frac{\mathfrak{e}_j' \mathbf{A}_n \mathbf{\Sigma}_u \mathfrak{e}_\ell}{\sqrt{\mathfrak{e}_\ell' \mathbf{\Sigma}_u \mathfrak{e}_\ell}}, \; n = 0, 1, 2, ...; \ell, j = 1, 2, ..., k \tag{A.9}$$

where $\mathfrak{e}_j$ is a $k \times 1$ selection vector with its element corresponding to the j^{th} variable in country i being unity and zeros elsewhere. This result also holds in non-Gaussian but linear settings where the conditional expectations can be assumed to be linear. The corresponding generalized impulse response function for the case of a structural shock to the US is given by

$$\mathcal{SGIRF}(\mathbf{x}_t; \mathrm{v}_{\ell t}, n) = \frac{\mathfrak{e}_j' \mathbf{A}_n (\mathbf{P}_{H_0}^0 \mathbf{H}_0)^{-1} \mathbf{\Sigma}_{\mathrm{v}} \mathfrak{e}_\ell}{\sqrt{\mathfrak{e}_\ell' \mathbf{\Sigma}_{\mathrm{v}} \mathfrak{e}_\ell}}, \; n = 0, 1, 2, ...; \ell, j = 1, 2, ..., k. \tag{A.10}$$

For each bootstrap replication $b = 1, 2, ..., B$, having estimated the individual country models using the simulated data $\mathbf{x}_t^{(b)}$, the GVAR is reconstructed as described above and the impulse responses are calculated based on the formulas (A.9) and (A.10) as $\mathcal{GIRF}_{j,n}^{(b)}$, $\mathcal{SGIRF}_{j,n}^{(b)}$ $\forall n$. These statistics are then sorted into ascending order $\forall n$ and the $(1-\gamma)100\%$ confidence interval is calculated by using the $\gamma/2$ and $(1-\gamma/2)$ quantiles, say $s_{\gamma/2}$ and $s_{(1-\gamma/2)}$ respectively, of the bootstrap distribution of $\mathcal{GIRF}_{j,n}$ and $\mathcal{SGIRF}_{j,n}$.[14] The empirical distributions of the persistence profiles and forecast error variance decomposition are derived similarly based on the formulae in section (4).

A.2.3 Generating the Simulated Errors

A.2.3.1 Parametric Approach

Under the parametric approach the errors are generated from a multivariate distribution with zero means and covariance matrix $\hat{\mathbf{\Sigma}}_\varepsilon$ given by $\hat{\mathbf{\Sigma}}_\varepsilon = \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t'$. To obtain the simulated errors for the k variables in the GVAR model we first generate kT draws from an i.i.d. distribution which we denote by $\mathbf{v}_t^{(b)}$, $t = 1, 2, ..., T$. In our application we generate $\mathbf{v}_t^{(b)}$ as $IIN(\mathbf{0}, \mathbf{I}_k)$ although other parametric distributions could also be entertained. Invoking the spectral decomposition, the variance-covariance matrix of the estimated GVAR residuals are decomposed as $\hat{\mathbf{\Sigma}}_\varepsilon = \hat{\mathbf{P}} \hat{\mathbf{\Lambda}} \hat{\mathbf{P}}'$, where $\hat{\mathbf{\Lambda}}$ is a diagonal matrix containing the eigenvalues of $\hat{\mathbf{\Sigma}}_\varepsilon$ on its diagonal and $\hat{\mathbf{P}}$ is an orthogonal matrix consisting of its eigenvectors. Note that the Choleski decomposition of $\hat{\mathbf{\Sigma}}_\varepsilon$ is not applicable in this case due to the semi-positive definite nature of this matrix that follows from the underlying common factor structure of the GVAR. The errors $\boldsymbol{\varepsilon}_t^{(b)}$, $t = 1, 2, ..., T$, are then computed as $\boldsymbol{\varepsilon}_t^{(b)} = \hat{\mathbf{A}} \mathbf{v}_t^{(b)}$, where $\hat{\mathbf{A}} = \hat{\mathbf{P}} \hat{\mathbf{\Lambda}}^{1/2}$.

A.2.3.2 Non-Parametric Approach

To obtain a bootstrap sample for the k variables in the GVAR model, we initially pre-whiten the residuals $\hat{\boldsymbol{\eta}}_t$ by using the generalized inverse of $\hat{\mathbf{A}}$ as given above, denoted $\hat{\mathbf{A}}_g^-$, so that $\hat{\boldsymbol{\eta}}_t = \hat{\mathbf{A}}_g^- \, \hat{\varepsilon}_t$. The generalized inverse of $\hat{\mathbf{A}}$ is required due to the semi-positive definite nature of this matrix as was pointed out earlier. We then resample with replacement from the kT elements of the matrix obtained from stacking of the vectors $\hat{\boldsymbol{\eta}}_t$, for $t = 1, 2, ..., T$. This is done in order to reduce the repetition of the bootstrap samples. The bootstrap error vector is then obtained as $\boldsymbol{\varepsilon}_t^{(b)} = \hat{\mathbf{A}} \hat{\boldsymbol{\eta}}_t^{(b)}$, where $\hat{\mathbf{A}}$ is given as above, and $\hat{\boldsymbol{\eta}}_t^{(b)}$ is the $k \times 1$ vector of re-sampled values from $(\hat{\boldsymbol{\eta}}_1, \hat{\boldsymbol{\eta}}_2, ..., \hat{\boldsymbol{\eta}}_T)$.

[14] Note that the GVAR is solved for the US price level and $e_{it} - p_{it}$. Impulse responses for US inflation and the real effective exchange rates, $\overline{re_{it}}$, can be readily obtained by using appropriate linear transformations of the impulse responses for e_{it} and p_{it}.

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Tables and Figures

Table 1. Countries and Regions in the GVAR Model

Unites States	Euro Area	Latin America
China	Germany	Brazil
Japan	France	Mexico
United Kingdom	Italy	Argentina
	Spain	Chile
Other Developed Economies	Netherlands	Peru
Canada	Belgium	
Australia	Austria	
New Zealand	Finland	
Rest of Asia	Rest of W. Europe	Rest of the World
Korea	Sweden	India
Indonesia	Switzerland	South Africa
Thailand	Norway	Turkey
Philippines		Saudi Arabia
Malaysia		
Singapore		

Note: This Table has been reproduced from Table 1 in DdPS (2007).

Table 2. Trade Weights Based on Direction of Trade Statistics

	USA	EA	China	Japan	UK	Canada	Australia	Sweden	Switz.	Norway	NZ	Rest*
USA	0.000	0.155	0.073	0.124	0.052	0.241	0.011	0.008	0.012	0.004	0.003	0.215
EA	0.227	0.000	0.056	0.072	0.238	0.019	0.012	0.057	0.090	0.028	0.002	0.199
China	0.229	0.164	0.000	0.250	0.029	0.020	0.025	0.010	0.007	0.003	0.003	0.260
Japan	0.319	0.132	0.123	0.000	0.032	0.024	0.035	0.007	0.009	0.003	0.005	0.311
UK	0.180	0.537	0.020	0.042	0.000	0.021	0.013	0.027	0.028	0.023	0.003	0.106
Canada	0.803	0.046	0.021	0.035	0.023	0.000	0.004	0.003	0.003	0.006	0.001	0.055
Australia	0.182	0.119	0.080	0.193	0.057	0.018	0.000	0.010	0.009	0.002	0.061	0.269
Sweden	0.104	0.514	0.024	0.035	0.115	0.010	0.008	0.000	0.018	0.099	0.001	0.072
Switz.	0.113	0.670	0.015	0.039	0.066	0.008	0.005	0.015	0.000	0.004	0.001	0.064
Norway	0.090	0.449	0.020	0.030	0.181	0.047	0.003	0.132	0.008	0.000	0.000	0.040
NZ	0.181	0.119	0.055	0.141	0.054	0.018	0.248	0.008	0.006	0.002	0.000	0.168

Note: This Table has been reproduced from Table 2 in DdPS (2007). Trade weights are computed as shares of exports and imports displayed in rows by region such that a row, but not a column, sums to one. *"Rest" gathers the remaining countries. The complete trade matrix used in the GVAR model is given in a Supplement that can be obtained from the authors on request. Source: Direction of Trade Statistics, 1999-2001, IMF.

Table 3. Inflation Coefficient in the Fisher Equation Restricted (1974Q4-2003Q4)

Country	r	Fisher	Term Premium	UIP	PPP	$LR(df)$
US	2	$\rho^S - \Delta p$	$\rho^L - \rho^S$			86.03(16)
EA	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		120.11(30)
China	1	$\rho^S - \Delta p$				23.12(10)
Japan	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		132.95(30)
UK	3	$\rho^S - \Delta p$		$\rho^L - \rho^{L*}$	$\underset{(0.77)}{-0.20}y + \overline{re} + \underset{(0.77)}{0.20}y^*$	168.34(29)
Canada	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^S - \rho^{S*}$		106.32(30)
Australia	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$		$\overline{re}$	116.75(30)
Sweden	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		105.14(30)
Switzerland	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$		$\underset{(0.16)}{-0.14}y + \overline{re} + \underset{(0.16)}{0.14}y^*$	114.25(29)
Norway	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$		$\underset{(0.14)}{-0.20}y + \overline{re} + \underset{(0.14)}{0.20}y^*$	131.95(29)
New Zealand	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		104.38(30)

Note: The country specific models for all countries including those not presented above have a VARX*(2,1) specification. r denotes the number of cointegrating vectors. The cointegrating rank for the rest of the countries is selected based on MacKinnon et al (1998) upper five percent critical values. The exchange rate variable is defined as $\overline{re} = (e - p) - (e^* - p^*)$. LR is the log-likelihood ratio statistic for testing the long run relations, with the number of over-identifying restrictions provided in brackets.

Table 4. Inflation Coefficient in the Fisher Equation Unrestricted (1974Q4-2003Q4)

Country	r	Fisher	Term Premium	UIP	PPP	$LR(df)$
US	2	$\rho^S - \underset{(0.32)}{2.06}\Delta p$	$\rho^L - \rho^S$			61.26(15)
EA	3	$\rho^S - \underset{(0.22)}{1.13}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		119.71(29)
China	1	$\rho^S - \underset{(0.09)}{0.56}\Delta p$				16.80(9)
Japan	3	$\rho^S - \underset{(0.42)}{2.11}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		117.46(29)
UK	3	$\rho^S - \underset{(0.24)}{1.62}\Delta p$		$\rho^L - \rho^{L*}$	$-\underset{(0.73)}{0.16}y + \overline{re} + \underset{(0.73)}{0.16}y^*$	153.20(28)
Canada	3	$\rho^S - \underset{(0.21)}{1.26}\Delta p$	$\rho^L - \rho^S$	$\rho^S - \rho^{S*}$		104.37(29)
Australia	3	$\rho^S - \underset{(0.25)}{1.19}\Delta p$	$\rho^L - \rho^S$		$\overline{re}$	115.95(29)
Sweden	3	$\rho^S - \underset{(0.08)}{0.75}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		100.22(29)
Switzerland	3	$\rho^S - \underset{(0.08)}{0.56}\Delta p$	$\rho^L - \rho^S$		$-\underset{(0.12)}{0.41}y + \overline{re} + \underset{(0.12)}{0.41}y^*$	104.84(28)
Norway	3	$\rho^S - \underset{(0.15)}{0.72}\Delta p$	$\rho^L - \rho^S$		$\underset{(0.24)}{0.08}y + \overline{re} - \underset{(0.24)}{0.08}y^*$	129.91(28)
New Zealand	3	$\rho^S - \underset{(0.08)}{0.65}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		97.28(29)

See the notes to Table 3.

Table 5. Over-identified Long Run Relations for the Eleven Focus Countries (1974Q4-2003Q4)

Country	r	Fisher	Term Premium	UIP	PPP	$LR(df)$	$99\%CV$
US	2	$\rho^S - \underset{(0.32)}{2.06}\Delta p$	$\rho^L - \rho^S$			61.26(15)	63.55
EA	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		120.11(30)	127.87
China	1	$\rho^S - \underset{(0.09)}{0.56}\Delta p$				16.80(9)	40.27
Japan	3	$\rho^S - \underset{(0.42)}{2.11}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		117.46(29)	106.23
UK	3	$\rho^S - \underset{(0.24)}{1.62}\Delta p$		$\rho^L - \rho^{L*}$	$\overline{re}$	153.24(29)	111.92
Canada	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$	$\rho^S - \rho^{S*}$		106.32(30)	111.48
Australia	3	$\rho^S - \Delta p$	$\rho^L - \rho^S$		$\overline{re}$	116.75(30)	126.04
Sweden	3	$\rho^S - \underset{(0.08)}{0.75}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		100.22(29)	112.33
Switzerland	3	$\rho^S - \underset{(0.08)}{0.56}\Delta p$	$\rho^L - \rho^S$		$-\underset{(0.12)}{0.41}y + \overline{re} + \underset{(0.12)}{0.41}y^*$	104.84(28)	114.31
Norway	3	$\rho^S - \underset{(0.10)}{0.77}\Delta p$	$\rho^L - \rho^S$		$\overline{re}$	130.03(29)	117.20
New Zealand	3	$\rho^S - \underset{(0.08)}{0.65}\Delta p$	$\rho^L - \rho^S$	$\rho^L - \rho^{L*}$		97.28(29)	106.00

Note: The country specific models for all countries including those not presented above have a VARX*(2,1) specification. r denotes the number of cointegrating vectors. The cointegrating rank for the rest of the countries is selected based on MacKinnon et al (1998) upper five percent critical values. The exchange rate variable is defined as $\overline{re} = (e - p) - (e^* - p^*)$. LR is the log-likelihood ratio statistic for testing the long run relations, with the number of over-identifying restrictions provided in brackets. The bootstrapped upper one percent critical value of the LR statisticis provided in the last column. See Appendix for the computational details.

Table 6. Contemporaneous Effects of Foreign Variables on their Domestic Counterparts Based on the Over-Identified Models in Table 5

	Domestic Variables				
Country	y	Δp	q	ρ^S	ρ^L
US	0.59	0.13	-	-	-
	[3.55]	[1.60]	-	-	-
EuroArea	0.42	0.22	1.06	0.06	0.63
	[3.19]	[2.87]	[9.27]	[2.74]	[7.44]
China	-0.03	0.52	-	0.14	-
	[-0.22]	[2.00]	-	[2.46]	-
Japan	0.50	-0.34	0.63	-0.04	0.44
	[2.22]	[-2.53]	[4.40]	[-0.75]	[5.03]
UK	0.67	-0.52	0.78	0.27	0.81
	[3.09]	[-1.62]	[12.26]	[1.33]	[5.88]
Canada	0.46	0.38	1.07	0.54	0.95
	[4.31]	[2.87]	[13.13]	[2.93]	[15.75]
Australia	0.36	0.21	0.96	0.37	0.79
	[1.94]	[1.06]	[3.75]	[2.11]	[3.58]
Sweden	1.27	1.10	1.14	0.77	0.89
	[3.28]	[4.09]	[12.06]	[1.77]	[5.00]
Switzerland	0.51	0.48	0.75	0.08	0.27
	[3.64]	[3.09]	[2.17]	[1.09]	[3.51]
Norway	0.85	0.68	1.06	0.03	0.58
	[1.93]	[3.45]	[7.76]	[0.11]	[3.41]
NewZealand	0.57	0.38	1.13	0.37	0.22
	[1.98]	[1.76]	[6.58]	[0.98]	[0.88]

Note: White's heteroskedastic robust t-ratios are given in square brackets, [ ].

Table 7. Forecast Error Variance Decomposition of EA Real Output and Inflation in Terms of their Top Ten Determinants from the Eleven Focus Countries Together with the Sum Across Both the Top Ten and Total (134) Determinants

	Quarters						
	0	2	4	6	8	10	12
	EA Real Output (%)						
EA GDP	83.2	65.4	51.8	42.3	35.5	30.5	26.7
China GDP	1.0	3.2	6.7	10.2	13.2	15.4	17.1
EA INFL	1.2	3.8	7.4	10.0	11.7	13.0	13.9
US LIR	0.6	5.0	7.8	9.2	9.9	10.2	10.5
Canada IR	5.3	3.8	3.7	4.0	4.4	4.7	4.9
EA IR	0.7	0.3	0.9	2.1	3.3	4.5	5.6
Canada EQ	1.9	2.2	2.6	2.7	2.8	2.8	2.7
US GDP	0.4	1.2	1.8	2.2	2.4	2.5	2.6
China REER	0.5	1.4	1.8	2.2	2.4	2.6	2.7
Switzerland LIR	1.1	2.1	2.3	2.2	2.2	2.1	2.0
Sum of Top 10	95.9	88.3	86.7	87.1	87.7	88.3	88.7
Sum of Total	186.7	171.7	174.2	182.6	191.6	199.8	206.7
	EA Inflation (%)						
EA INFL	69.7	53.7	43.5	35.5	29.3	24.5	20.8
US LIR	7.9	11.3	15.4	18.9	21.6	23.8	25.4
US POIL	8.7	13.8	14.8	14.9	14.5	13.8	13.0
Canada REER	5.7	8.0	9.8	11.3	12.5	13.3	13.9
US EQ	0.1	2.1	4.4	6.8	9.1	11.1	12.6
US IR	1.7	2.4	4.3	6.6	8.9	10.9	12.3
EA REER	1.0	4.8	7.0	8.1	8.4	8.4	8.1
US INFL	18.2	12.7	9.4	7.4	6.0	5.0	4.2
Japan REER	3.9	4.6	5.0	4.9	4.8	4.7	4.5
EA GDP	2.1	3.9	4.2	4.0	3.7	3.3	2.9
Sum of Top 10	118.8	117.3	117.8	118.4	118.8	118.6	117.7
Sum of Total	266.4	242.2	233.2	228.6	226.0	224.5	223.3

Note: The results show the proportion of forecast error variances of US real output and inflation explained by conditioning on contemporaneous and expected future values of 10 focus variables identified in terms of their relative contributions at the eighth quarterly horizon. REER stands for real effective exchange rate.

Table 8. Forecast Error Variance Decomposition of US Real Output and Inflation in Terms of their Top Ten Determinants from the Eleven Focus Countries Together with the Sum Across Both the Top Ten and Total (134) Determinants

	Quarters						
	0	2	4	6	8	10	12
	US Real Output (%)						
US GDP	86.4	63.8	51.4	42.7	36.3	31.6	28.2
US EQ	0.9	18.8	21.5	19.4	16.7	14.2	12.2
US IR	8.9	6.4	3.7	4.7	7.1	9.5	11.7
China GDP	0.1	0.6	1.8	3.4	4.8	5.9	6.7
Japan REER	0.7	2.0	2.8	3.7	4.6	5.4	6.2
EA IR	0.2	0.4	1.6	3.0	4.4	5.6	6.6
US POIL	0.1	2.0	2.8	3.4	4.0	4.5	5.0
UK INFL	0.6	2.0	2.7	3.5	4.0	4.4	4.8
EA REER	0.0	0.6	1.5	2.7	3.9	4.8	5.6
Canada REER	0.2	1.4	2.0	2.8	3.7	4.4	5.1
Sum of Top 10	98.1	97.9	91.5	89.4	89.5	90.6	91.9
Sum of Total	241.8	239.7	237.3	242.0	247.9	253.3	257.7
	US Inflation (%)						
US LIR	18.6	37.5	40.8	42.0	42.5	42.9	43.2
Canada REER	58.3	52.5	47.4	43.1	39.7	36.9	34.6
US IR	10.6	29.3	33.0	34.3	34.3	33.9	33.1
Japan REER	21.7	19.7	17.8	16.1	14.6	13.3	12.2
US POIL	19.2	16.4	15.0	13.8	12.8	12.0	11.3
EA REER	10.9	10.2	9.5	8.6	7.8	7.1	6.5
Australia REER	7.2	6.0	5.7	5.6	5.6	5.6	5.6
US EQ	2.3	0.6	1.9	3.6	5.1	6.3	7.2
EA IR	2.6	4.1	4.2	4.0	3.7	3.4	3.1
US GDP	0.0	0.8	1.8	2.7	3.3	3.9	4.3
Sum of Top 10	151.5	177.1	177.1	173.7	169.5	165.2	161.1
Sum of Total	440.1	355.8	328.3	310.9	298.5	288.9	281.4

Note: The results show the proportion of forecast error variances of EA real output and inflation explained by conditioning on contemporaneous and expected future values of 10 focus variables identified in terms of their relative contributions at the eighth quarterly horizon. REER stands for real effective exchange rate.

Figure 1. Peristence Profiles of the Long Run Relations for the Over-Identifying Models in Table 5

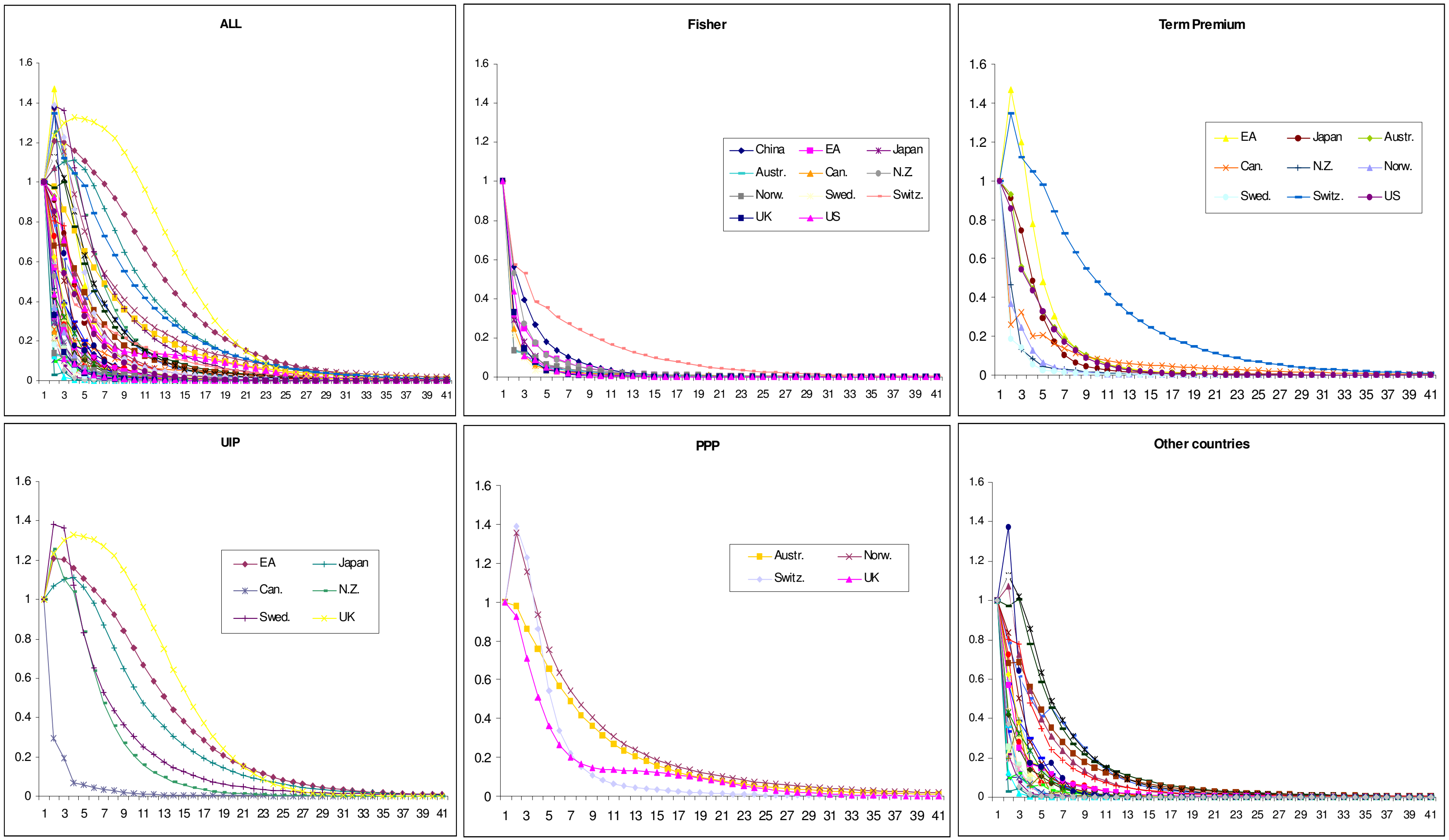

Figure 2: Persistence Profiles for the Euro Area Cointegrating Relations and Speed of Convergence to Equilibrium (Bootstrap Mean Estimates together with 90% Bootstrap Bounds) Based on the Over-Identified Models in Table 5

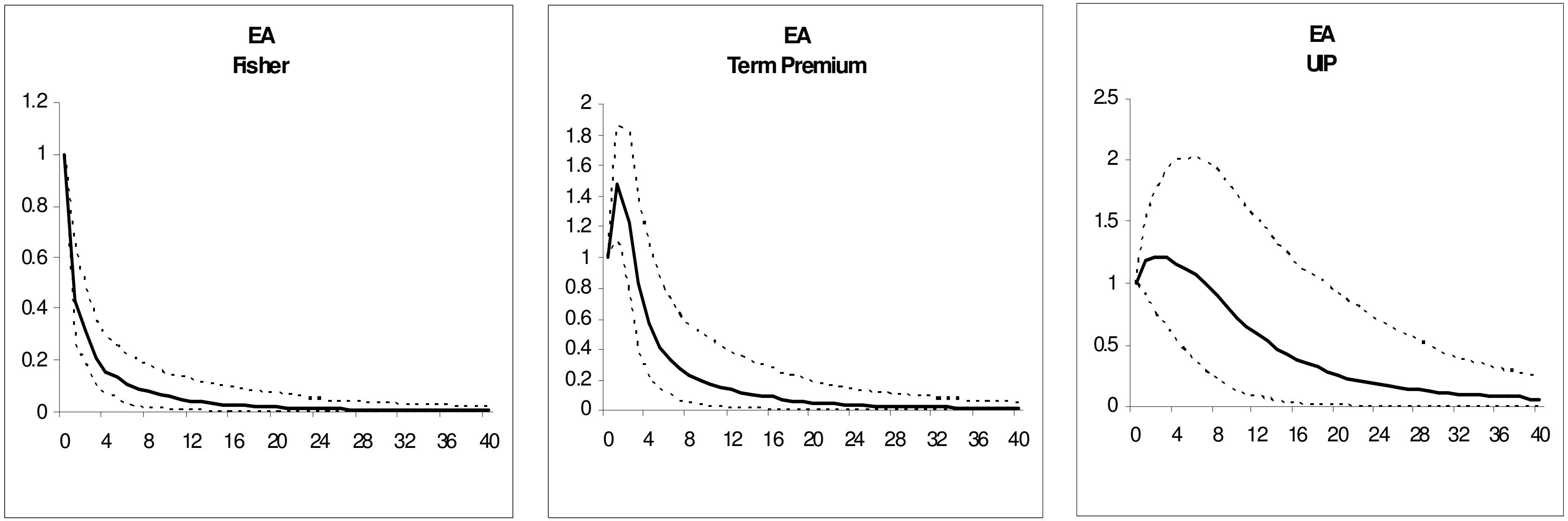

Figure 3(i). Generalized Impulse Responses of a Positive Unit (1 s.e.) Shock to Nominal Oil Prices on Real Output Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

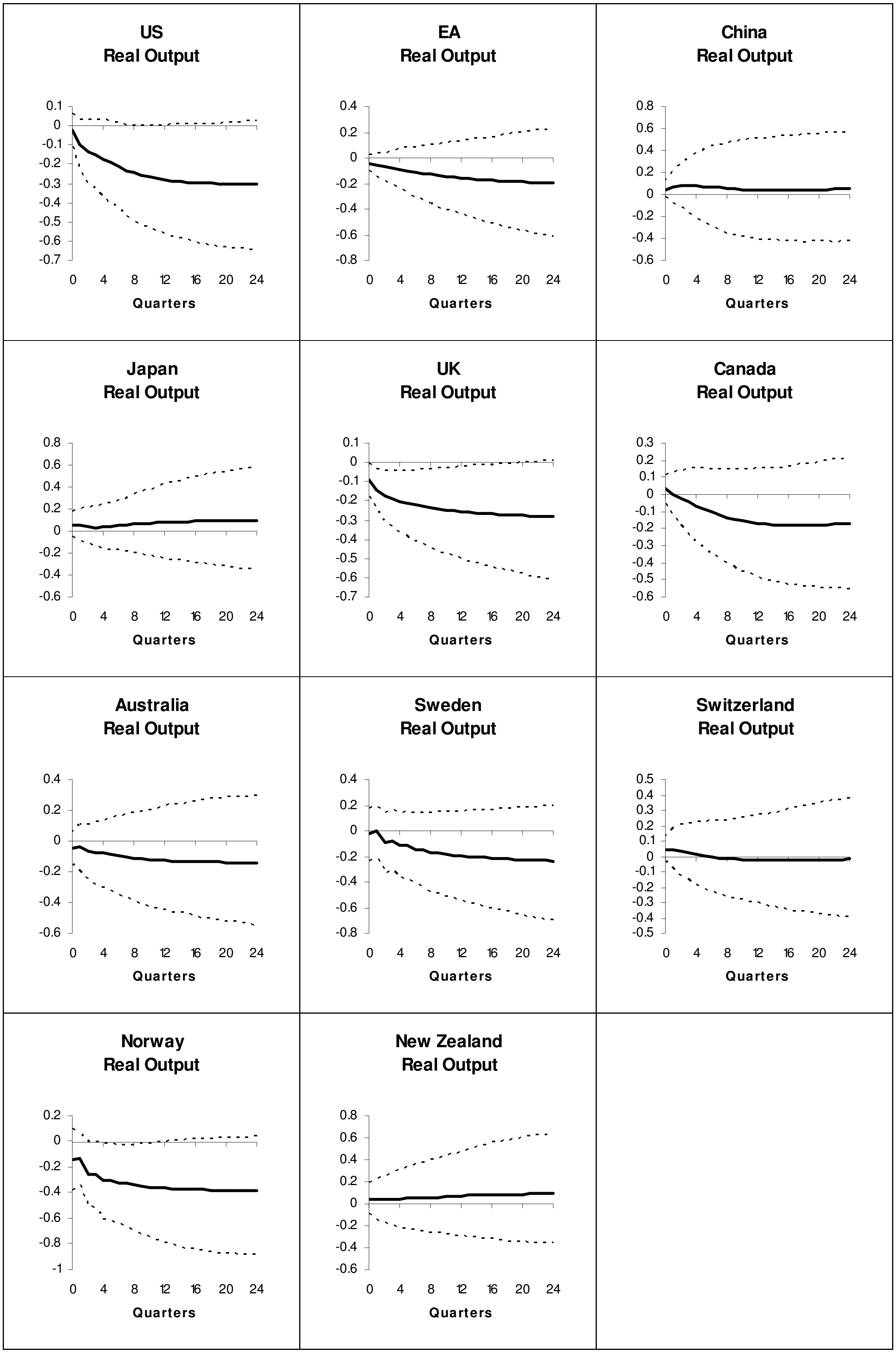

Figure 3(ii). Generalized Impulse Responses of a Positive Unit (1 s.e.) Shock to Oil Prices on Inflation Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

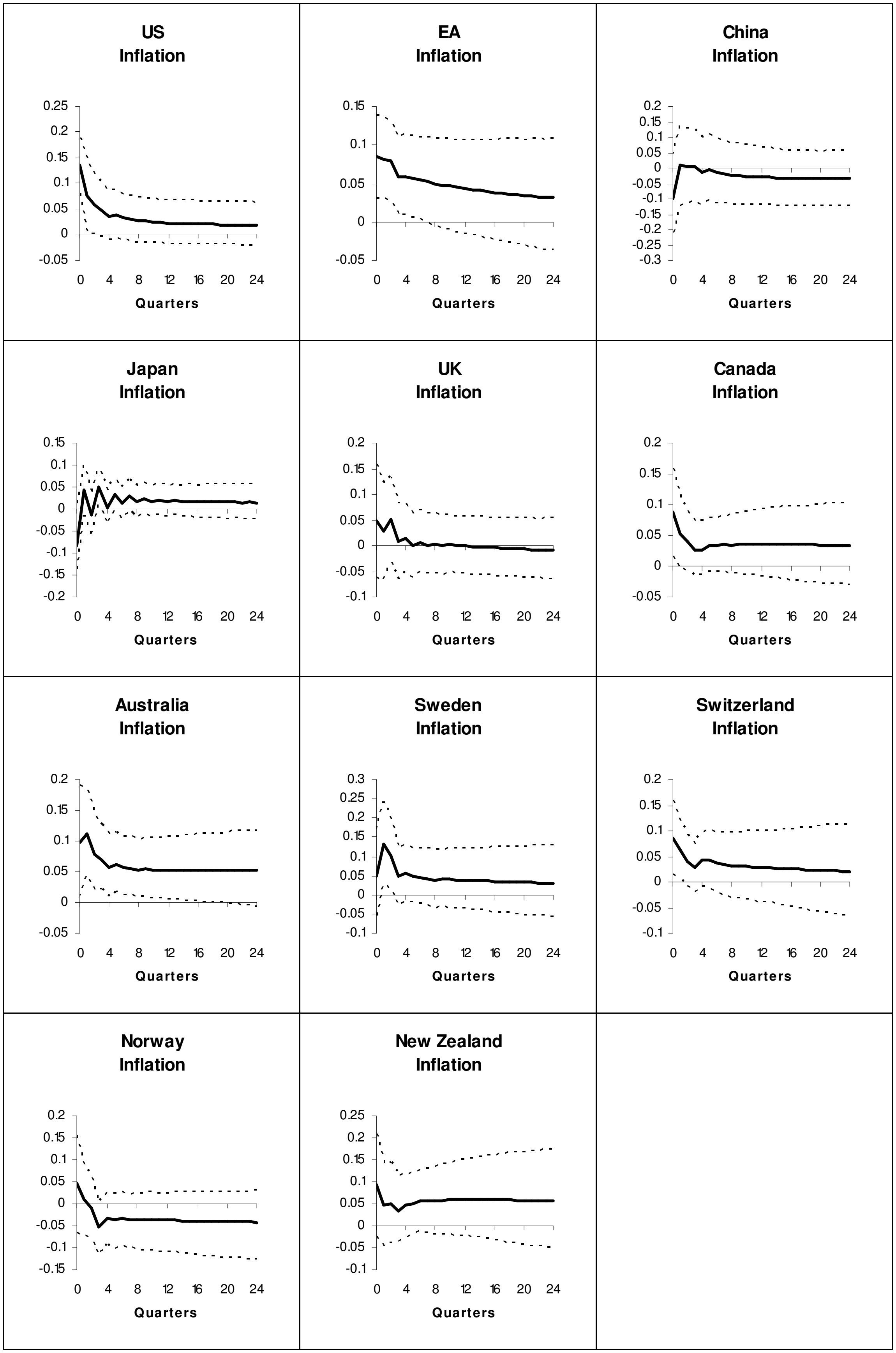

Figure 3(iii). Generalized Impulse Responses of a Positive Unit (1 s.e.) Shock to Oil Prices on Real Equity Prices Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

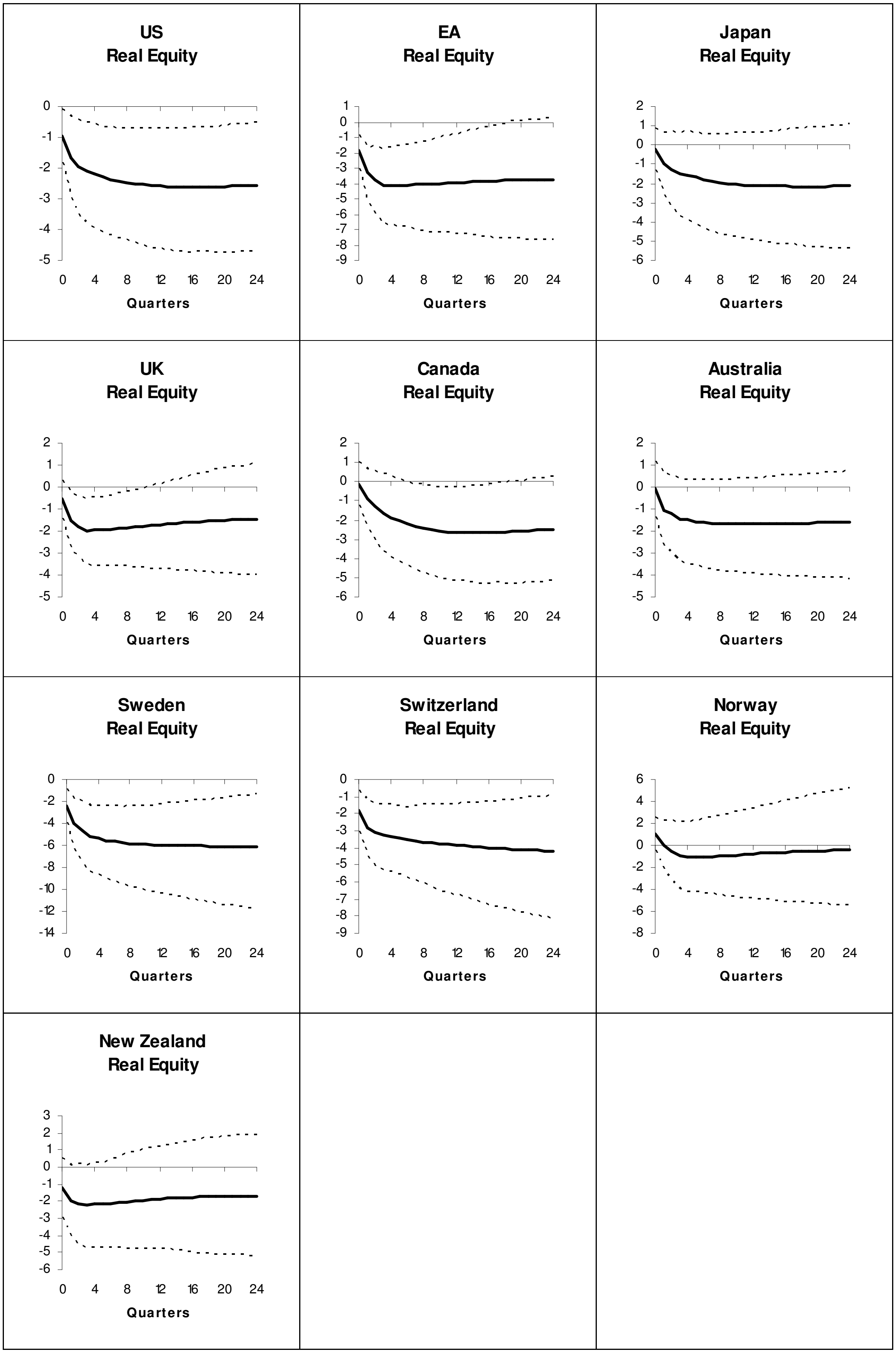

Figure 3(iv). Generalized Impulse Responses of a Positive Unit (1 s.e.) Shock to Oil Prices on Real Effective Exchange Rates Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

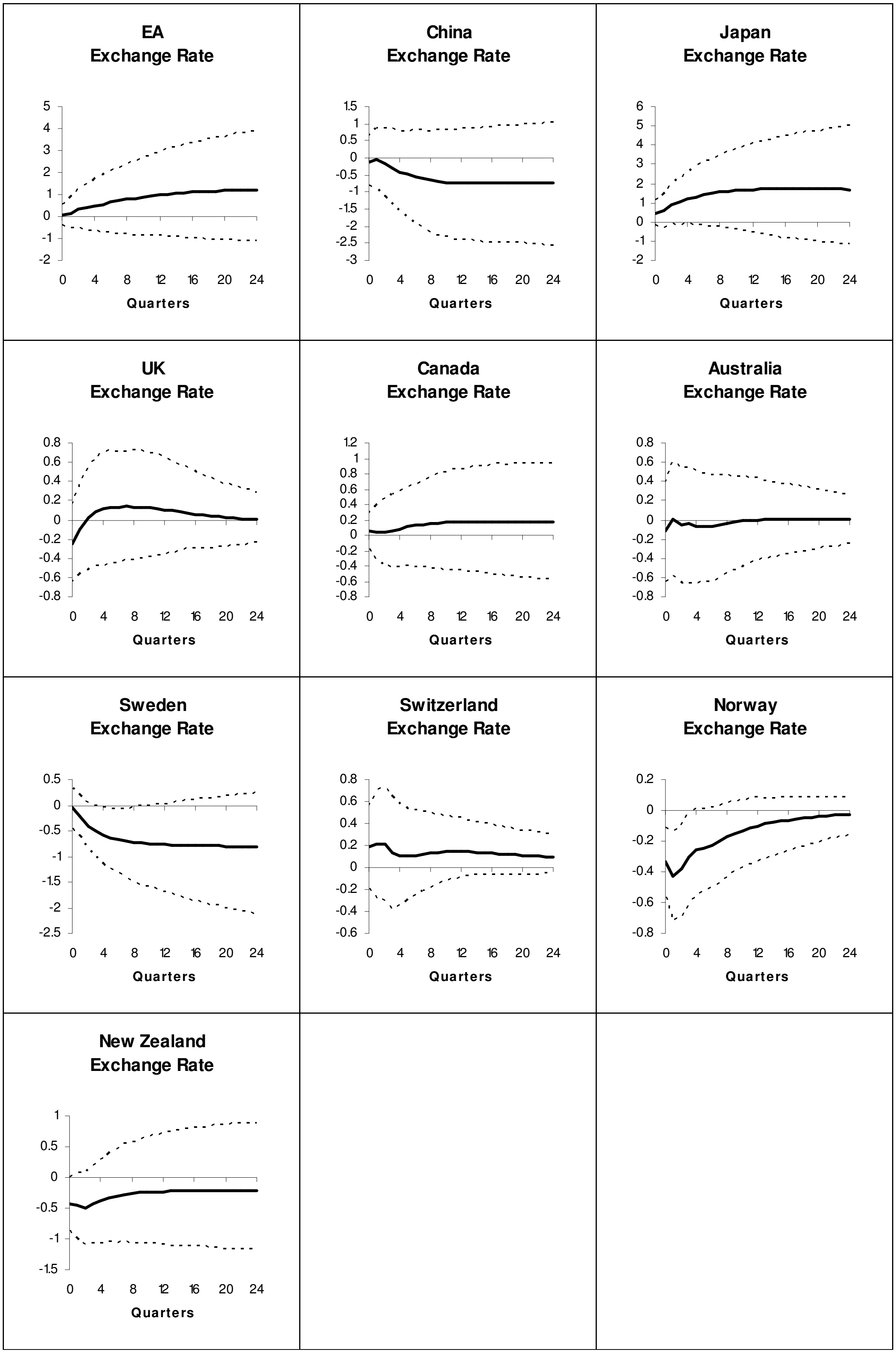

Figure 3(v). Generalized Impulse Responses of a Positive Unit (1 s.e.) Shock to Oil Prices on Nominal Short-Term Interest Rates Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

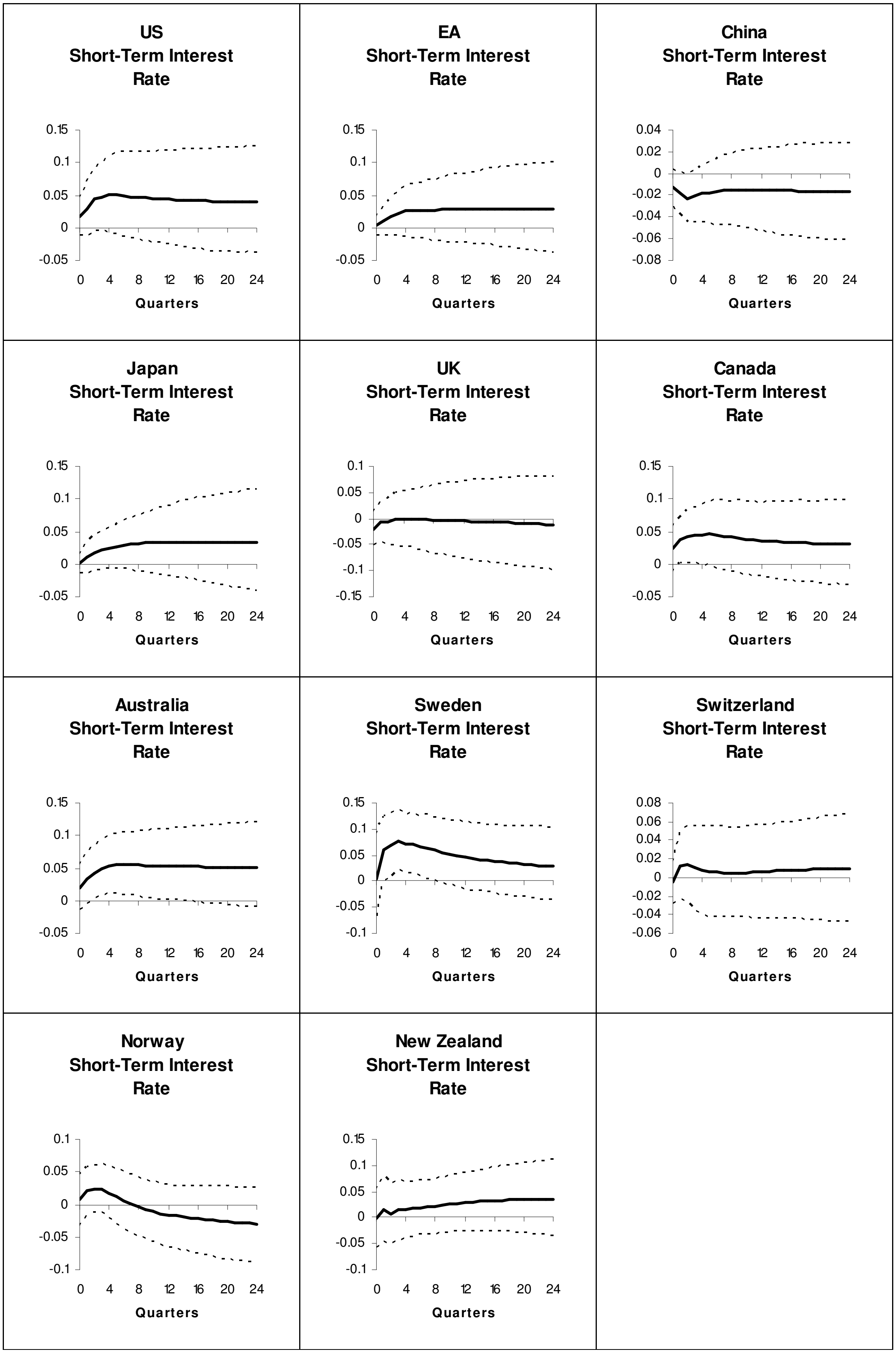

Figure 3(vi). Generalized Impulse Responses of a Positive Unit (1 s.e.) Shock to Oil Prices on Nominal Long-Term Interest Rates Across Countries and the Nominal Oil Price (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

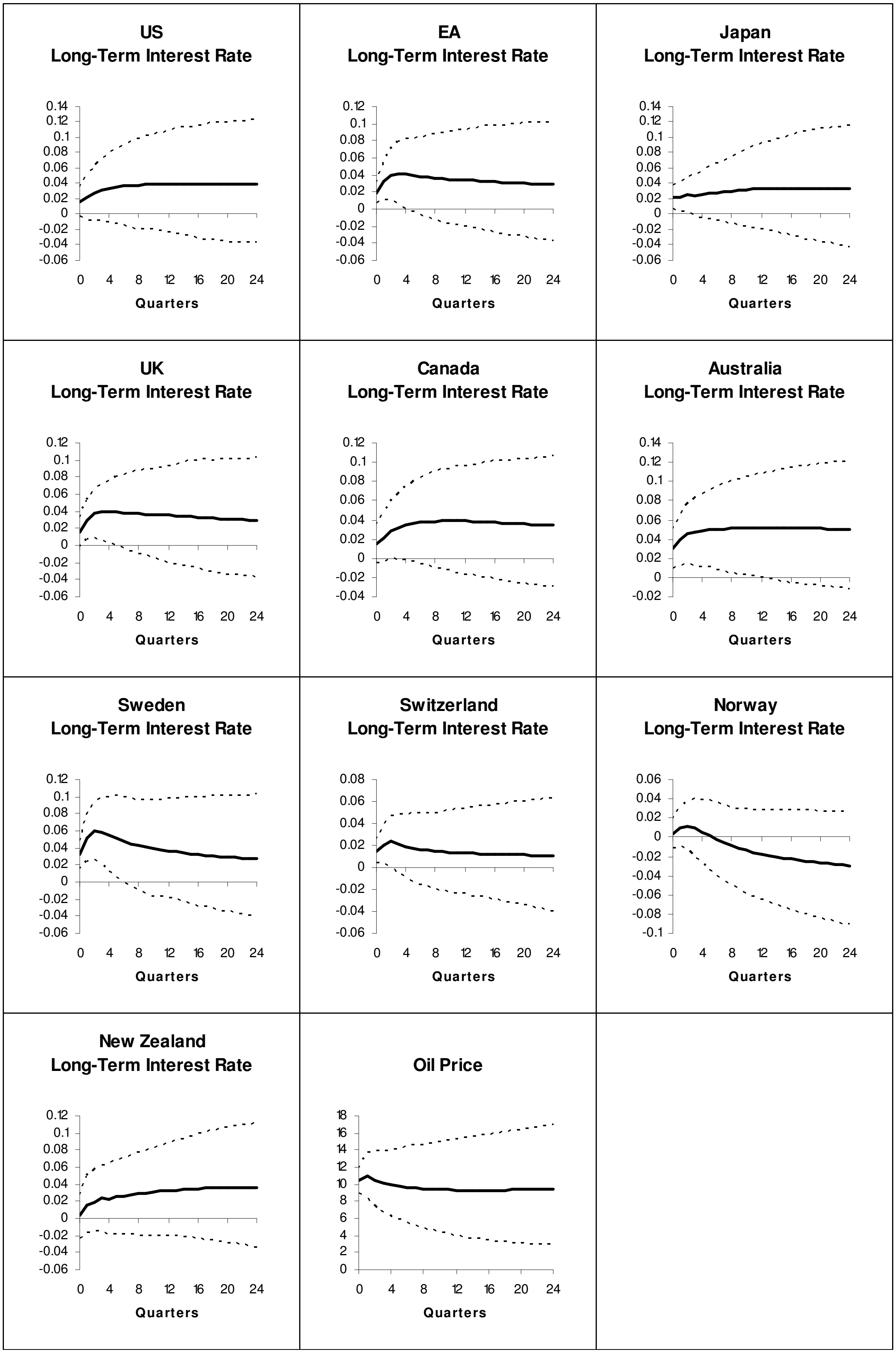

Figure 4(i). Generalized Impulse Responses of a Negative Unit (1 s.e.) Shock to US Real Equity Prices on Real Output Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

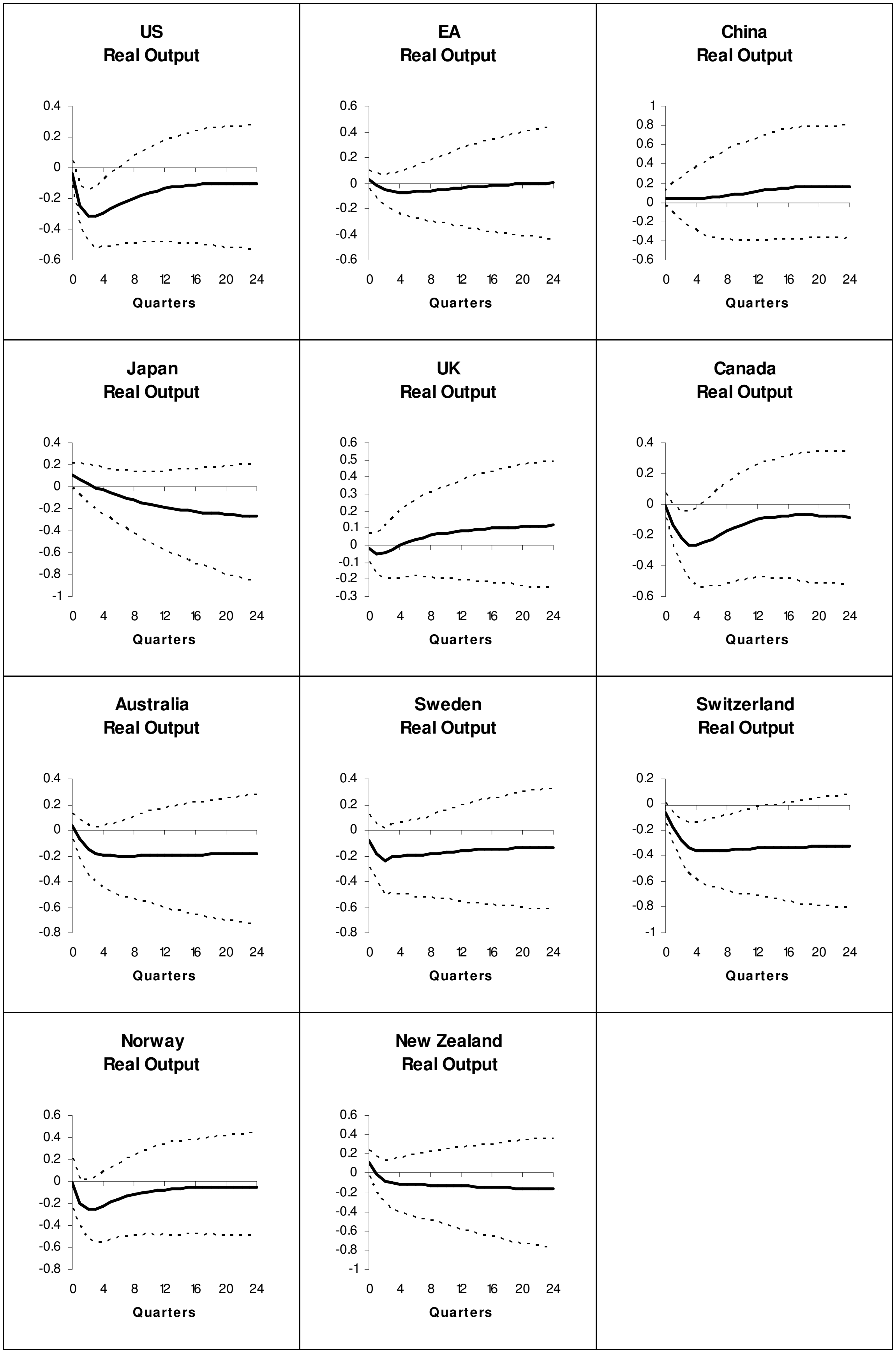

Figure 4(ii). Generalized Impulse Responses of a Negative Unit (1 s.e.) Shock to US Real Equity Prices on Inflation Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

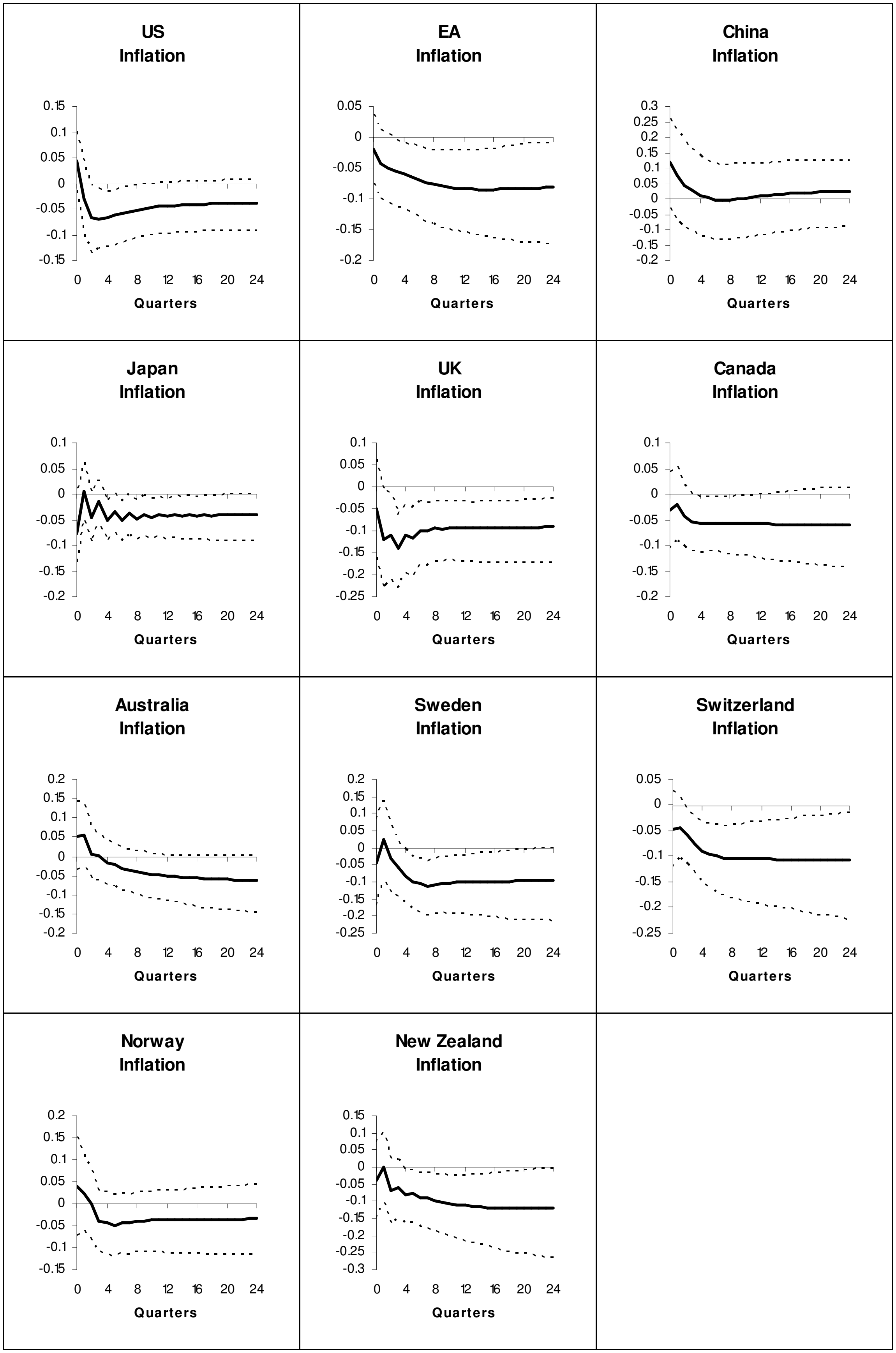

Figure 4(iii). Generalized Impulse Responses of a Negative Unit (1 s.e.) Shock to US Real Equity Prices on Real Equity Prices Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

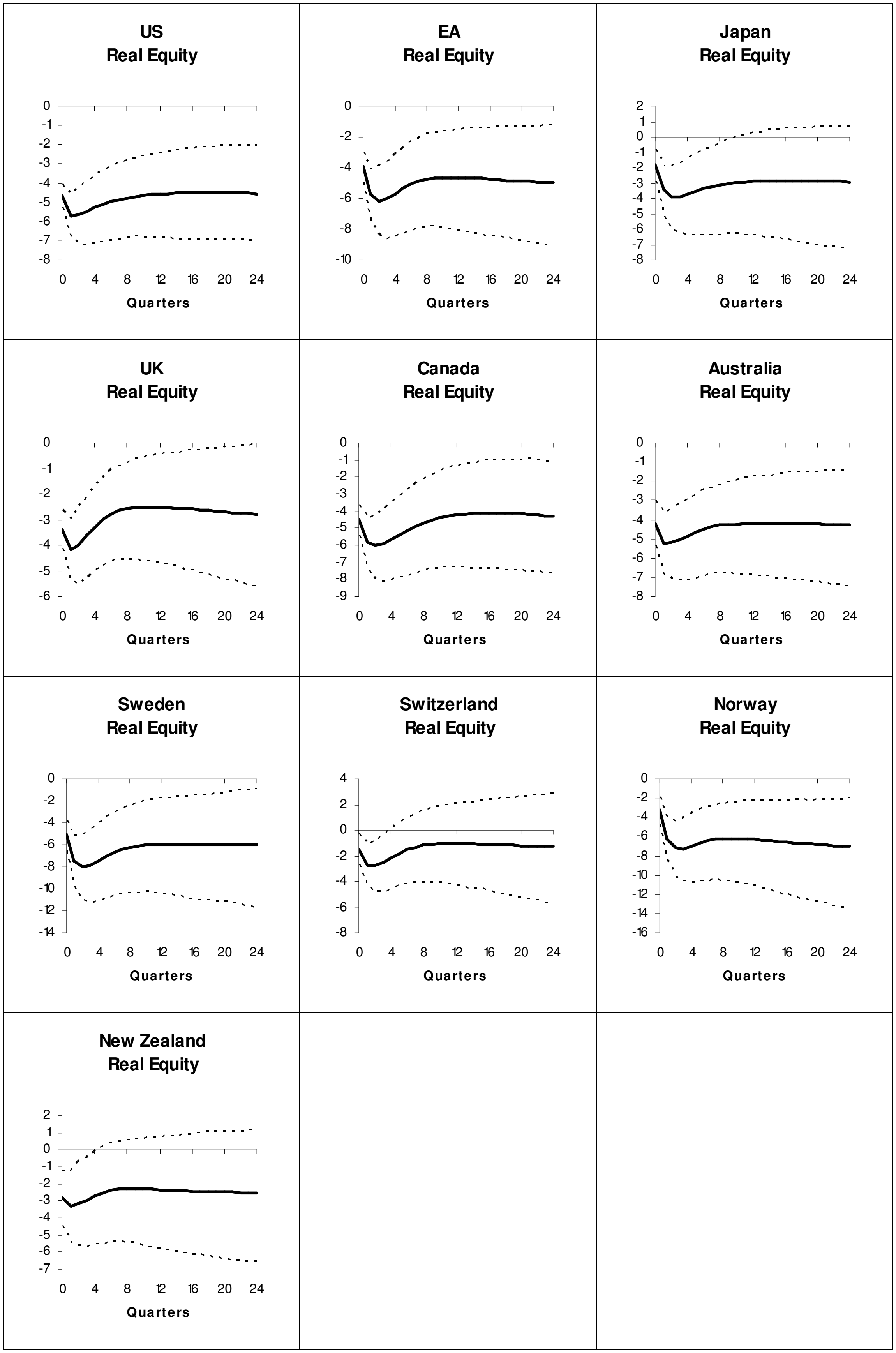

Figure 4(iv). Generalized Impulse Responses of a Negative Unit (1 s.e.) Shock to US Real Equity Prices on Real Effective Exchange Rates Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

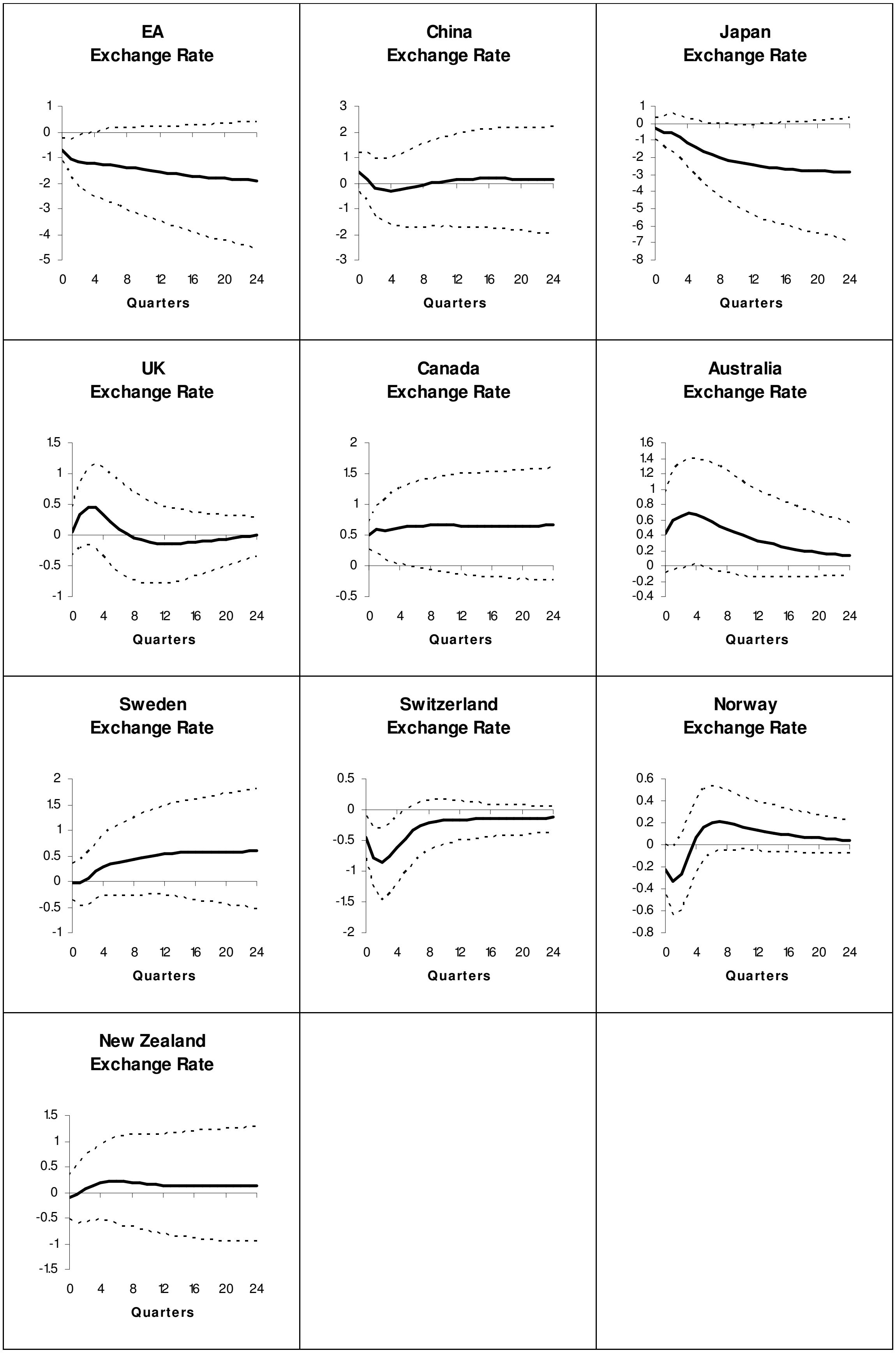

Figure 4(v). Generalized Impulse Responses of a Negative Unit (1 s.e.) Shock to US Real Equity Prices on Nominal Short-Term Interest Rates Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

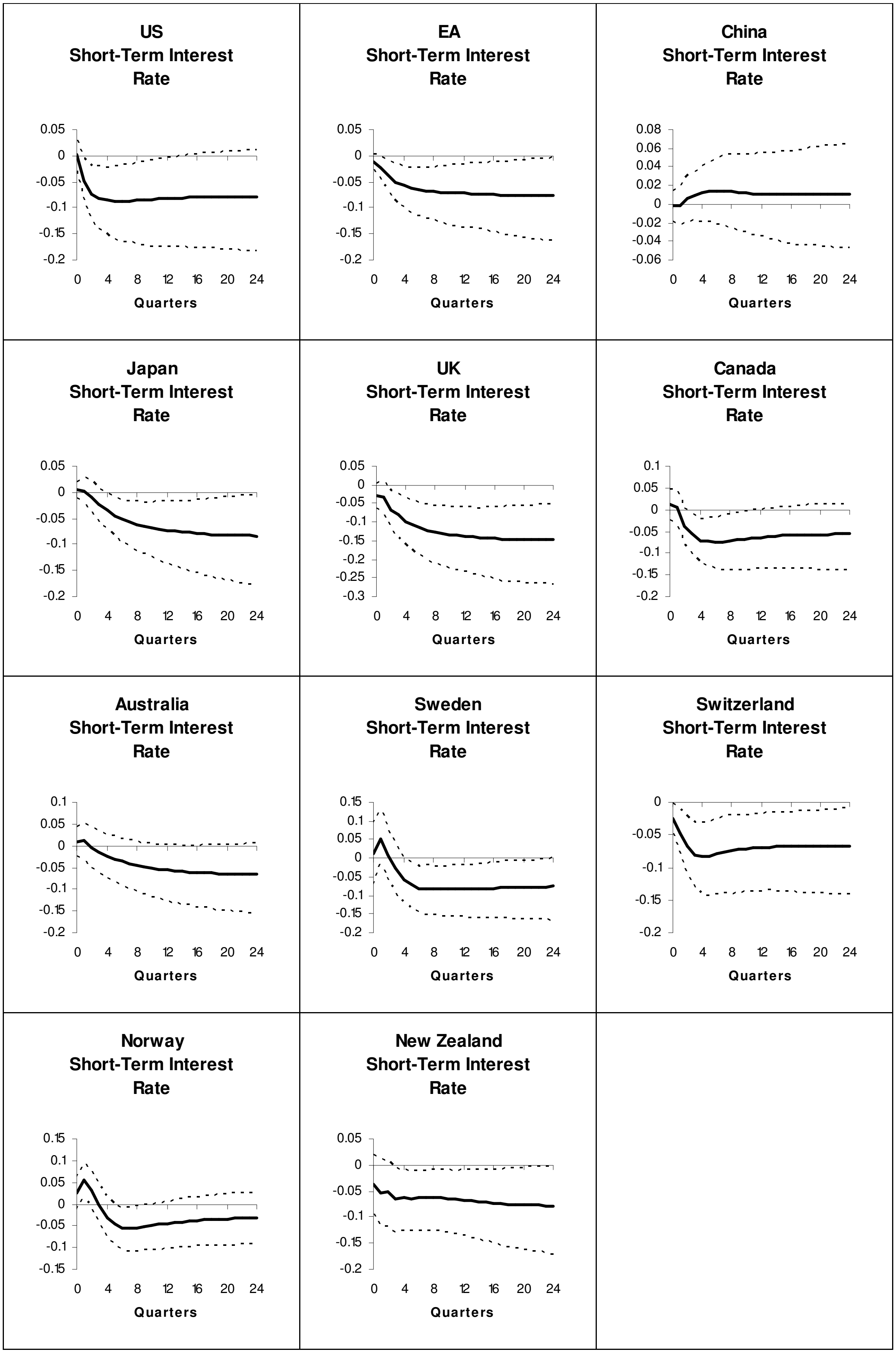

Figure 4(vi). Generalized Impulse Responses of a Negative Unit (1 s.e.) Shock to US Real Equity Prices on Nominal Long-Term Interest Rates Across Countries and the Nominal Oil Price (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

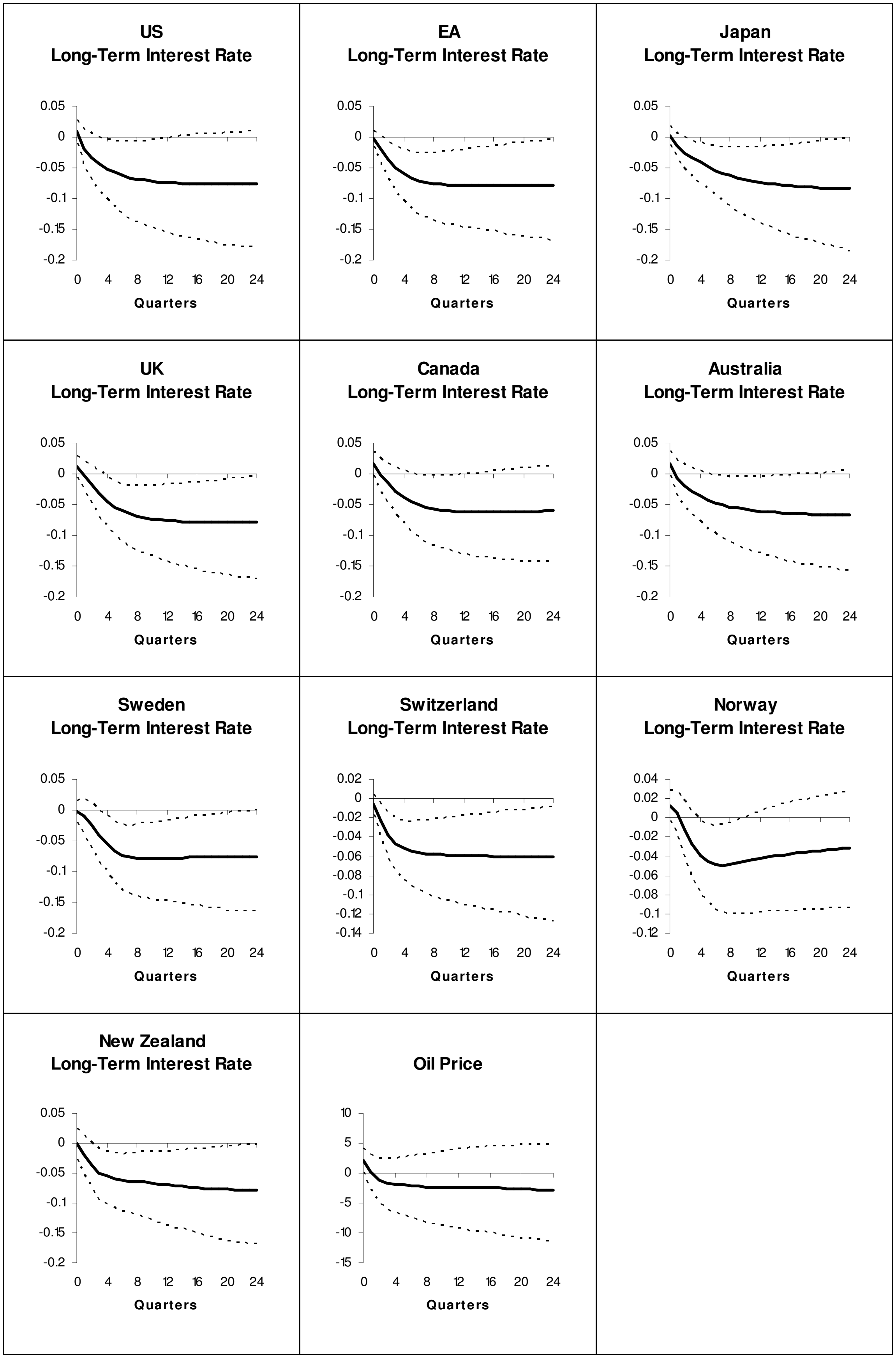

Figure 5(i). Impulse Responses of a Positive Unit (1 s.e.) Shock to US Monetary Policy on Real Output Across Countries Under Ordering {OIL, LIR, EQ, INFL, GDP, IR} (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

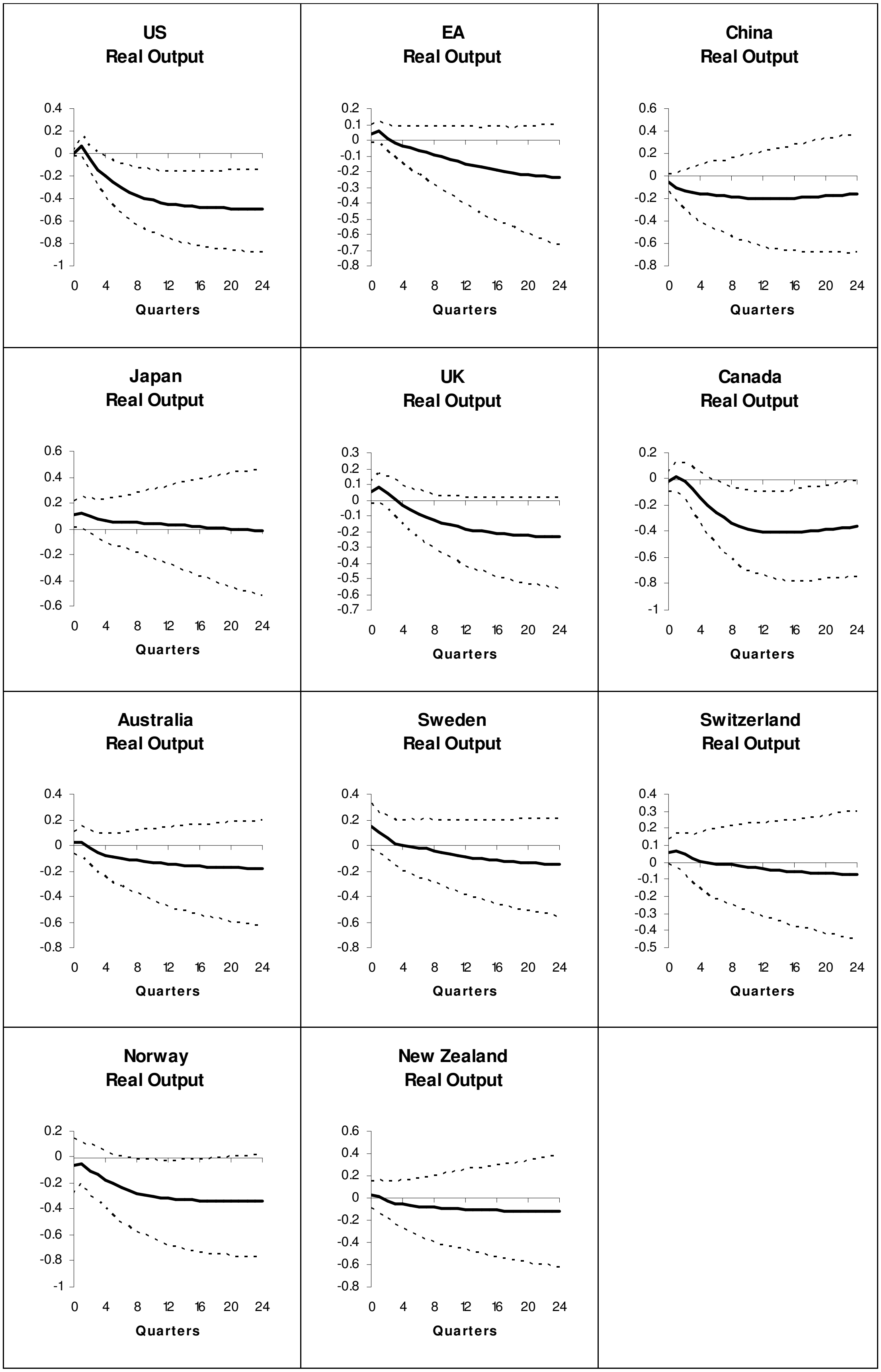

Figure 5(ii). Impulse Responses of a Positive Unit (1 s.e.) Shock to US Monetary Policy on Inflation Across Countries (Bootstrap Mean Estimates together with 90 percent Bootstrap Error Bounds)

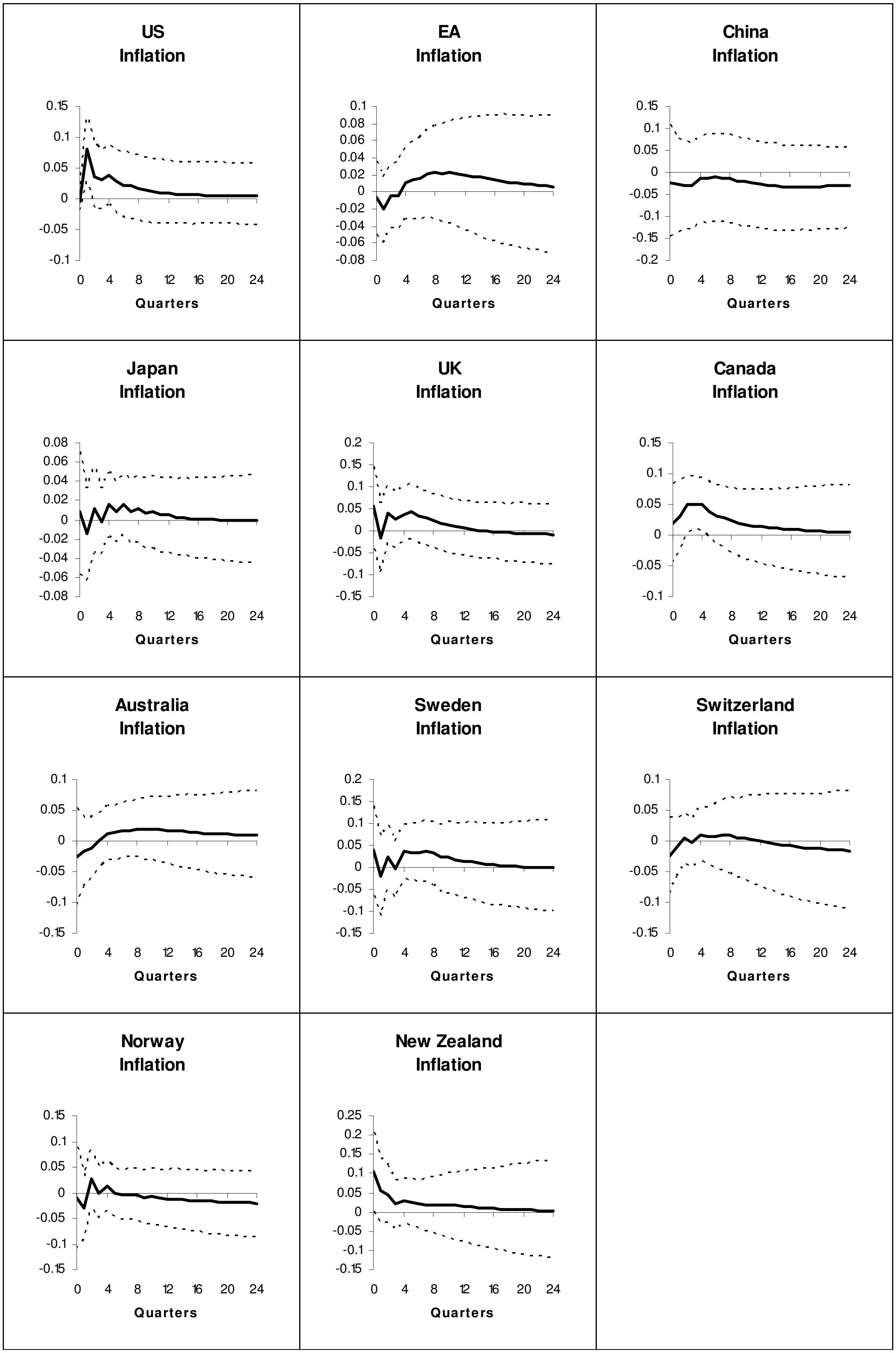

Figure 5(iii). Impulse Responses of a Positive Unit (1 s.e.) Shock to US Monetary Policy on Real Equity Prices Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

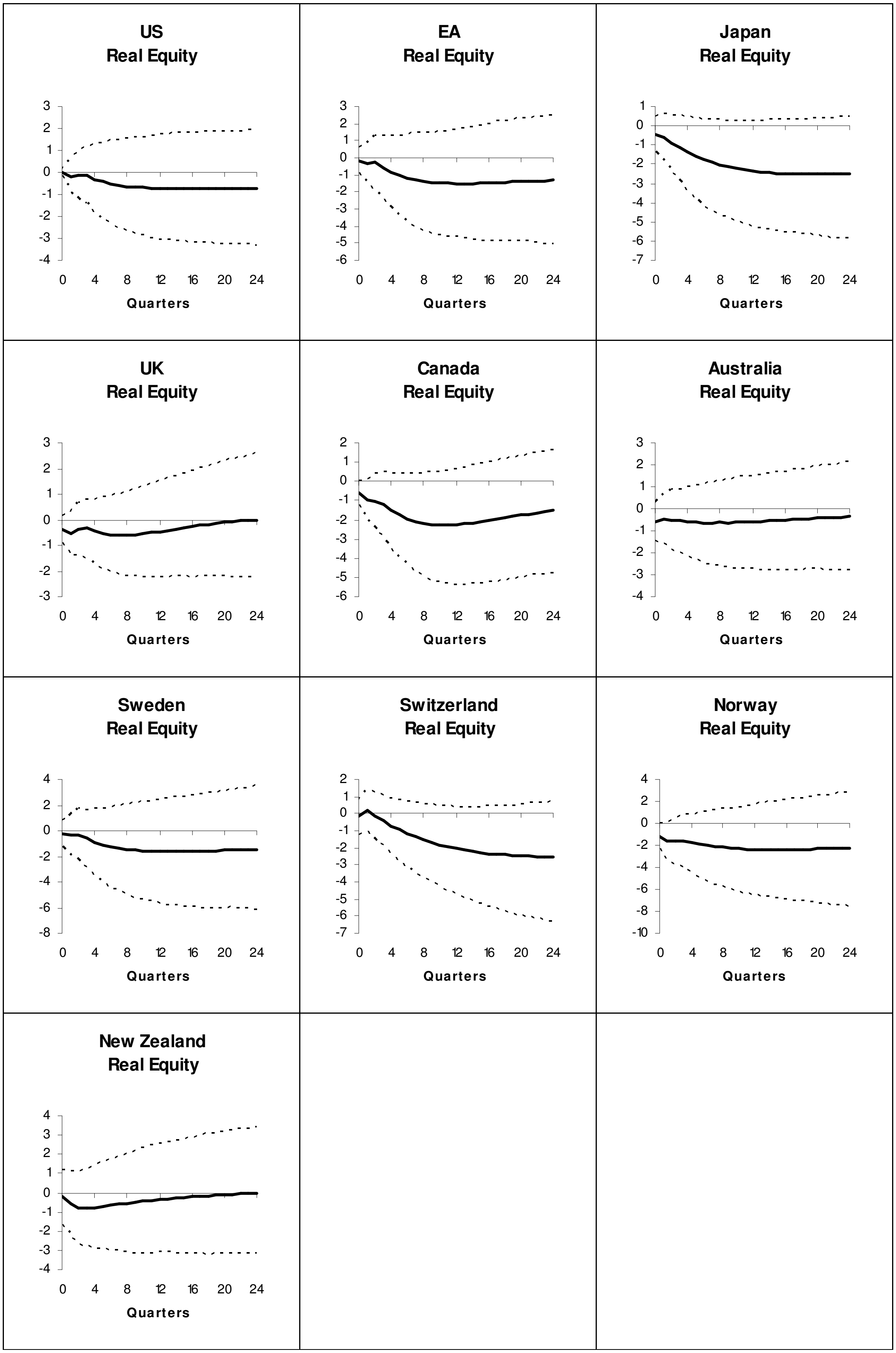

Figure 5(iv). Impulse Responses of a Positive Unit (1 s.e.) Shock to US Monetary Policy on Real Effective Exchange Rates Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

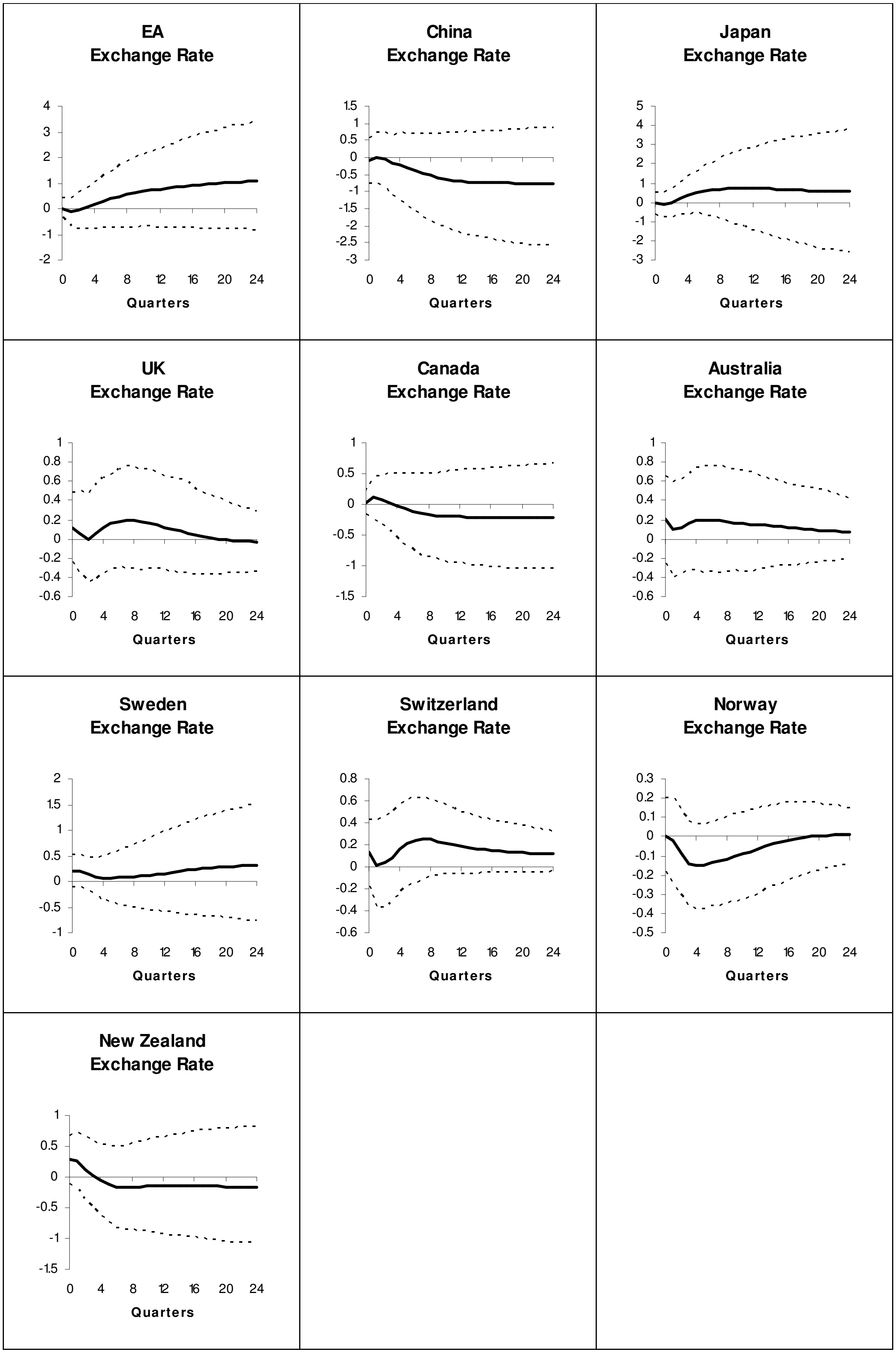

Figure 5(v). Impulse Responses of a Positive Unit (1 s.e.) Shock to US Monetary Policy on Nominal Short-Term Interest Rates Across Countries (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)

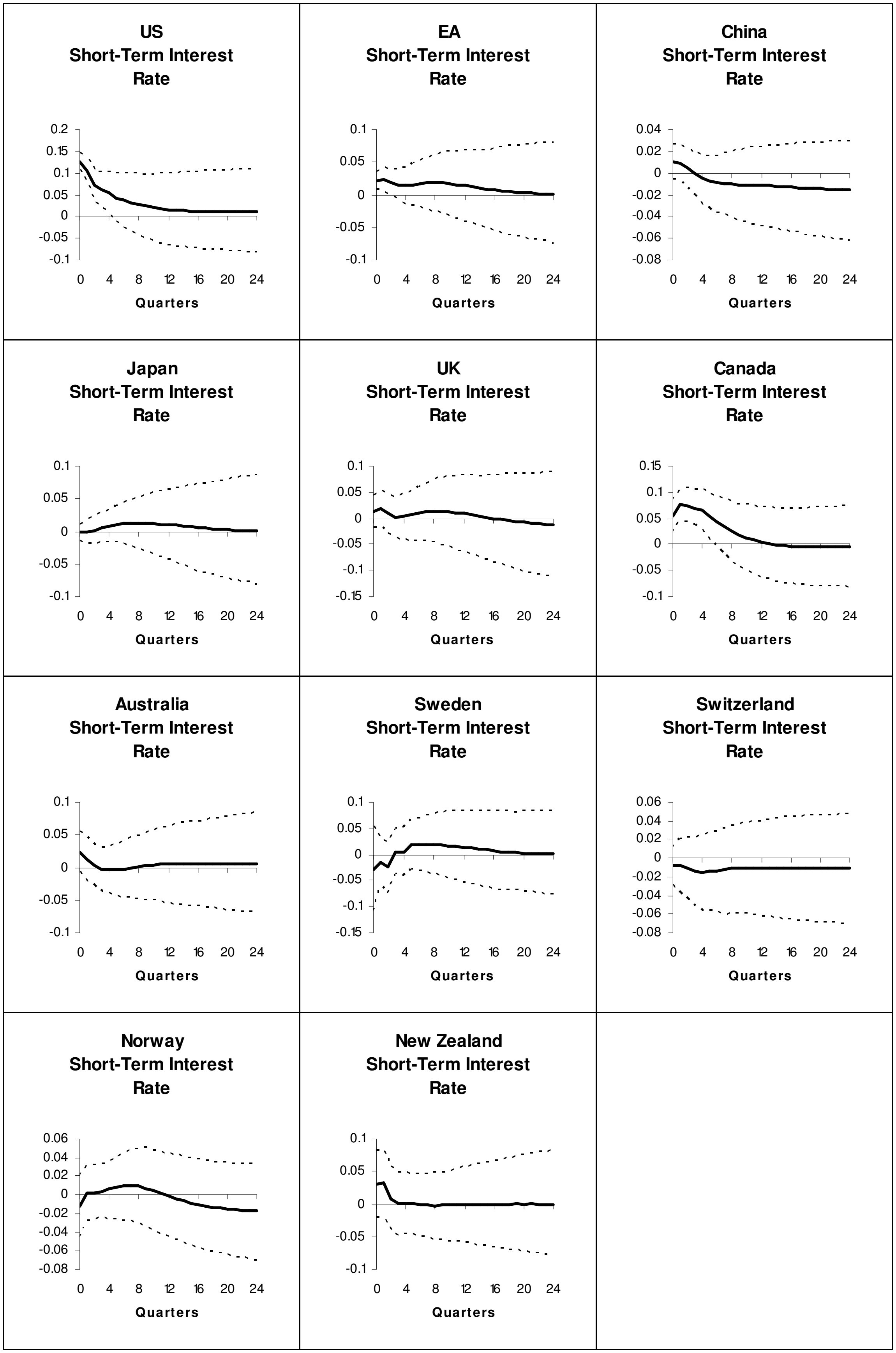

Figure 5(vi). Impulse Responses of a Positive Unit (1 s.e.) Shock to U.S. Monetary Policy on Nominal Long-Term Interest Rates Across Countries and the Nominal Oil Price (Bootstrap Mean Estimates with 90 percent Bootstrap Error Bounds)