Can a cusp catastrophe model describe the effect of sanctions on exchange rates?

Meysam Bolgorian

Abstract
Fluctuations of exchange rates, like any other economic variables, are very common in financial markets. However, sometimes because of political and economic tensions, exchange rates exhibit abrupt crashes that lead to structural break. In this paper, the author answers the question whether a catastrophe model can be used for modeling the collapse of exchange rates caused by economic sanctions. For this goal, he uses a cusp catastrophe model for fitting the dynamics of fluctuations of the Iranian Rial against the US Dollar. Using two sentiment variables, i.e. trading volume and ratio of institutional to individual trades of gold futures contracts, the author has shown that the collapse of Iranian currency can be best explained by cusp catastrophe theory.

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1. Introduction

Collapse of exchange rates has great economic and social consequences for economies. As a result, studying the causes and effects of such large movements is one of the most appealing interest of researchers and policymakers. Economic researches suggest that a variety of factors contributing to fluctuations of exchange rates. e.g., the openness of an economy, the domestic and foreign money supplies, the exchange rate regime, interest rates, central bank independence, levels of output, income, inflation, and unpredictable circumstances (Stancik (2007)). Changes in these factors would create macroeconomic shocks which explain a large proportion of uncommon fluctuations of exchange rates (Balassa (1964), Samuelson (1964) and Steinsson (2008)).

However, recent political tensions in the international area emphasize the importance of outburst of unpredictable circumstance in creation of shocks in economies. Some of the main recent international political tensions include: 1) Russian military intervention in Ukraine, which began at the beginning of 2014. Sanctions imposed by the United States and other countries contributed to the collapse of Russian Ruble. 2) Detention of an American pastor by Turkey in 2018 which led to the imposition of sanctions by the United States against Turkey. These sanctions had a significant negative impact on Turkey Lira. 3) Exit of United States from 2015 nuclear agreement between Iran and six world powers in early 2018 and re- imposition of economic and financial sanctions on Iran. Announcement of US intention for exiting from nuclear accord led to collapse of Iranian currency.

Although often financial and economic sanctions are used for reducing the tensions, in many cases the sanction itself causes large volatility in exchange rates. Recent economic sanctions imposed on countries like Russia, Turkey, Iran, Venezuela, etc. had a negative impact on their currencies, which in many cases led to devaluation as well as significant fluctuations of these countries' currencies. As Frankel (1981) mention, exchange rates are more sensitive to expectations concerning future events. As a result, in periods which are dominated by sanction-related news, people’s expectations are altered and exchange rates are likely to be more volatile. Therefore, we believe that, the large part of these fluctuations are due to pessimistic sentiments
of households in countries with sanction exposure, which lead to unprecedented high demand for foreign currencies which, besides the supply shocks of economic sanctions, eventually causes the collapse of under-sanction currencies.

Above-mentioned mechanism of currency collapses provides a justification of using catastrophe theory. Catastrophe theory tries to reveal part of information one might need to understand crashes in many phenomena including economic variables. It describes how slight, continuous changes in control parameters, or independent variables affecting the state of the system, can have sudden, discontinuous effects on the dependent variables. Based on this, one can reasonably expect that catastrophe theory might be a good candidate model for describing the unpredictable behavior of exchange rates.

The number of papers which used catastrophe theory in economics and finance is small. Zeeman (1977) used catastrophe theory to model stock market crashes. Varian (1979) combined Kaldor’s model of the business cycle by catastrophic events. He found that the nonlinearities in investment introduced by catastrophe theory lead to cycles. Scapens et al. (1981) applied catastrophe theory to study corporate failure and found that crashes occurs due to a sudden drop in corporate credit worthiness. Fischer and Jammernegg (1986) estimated the cusp catastrophe-augmented Phillips curve model by Woodcock and Davis (1979). They found that the augmented model explain USA data behavior better than traditional models. More recently, Barunik and Vosvrda (2009) and Barunik and Kukacka (2015) used the cusp catastrophe model to explain stock market crashes. Both studies provide results which show that the catastrophe model explains the behavior of the stock market better than other traditional models. Diks and Wang (2016) estimated a cusp catastrophe model for the housing market in six different countries. They found that the model fits the data well emphasizing that the interest rate plays an important role in explaining multiple equilibria. Wesselbaum (2017) employ catastrophe theory to explain discontinuous jumps in state variables of dynamic systems. He start by estimating the bank failure model by Ho and Saunders (1980). His findings suggest that the model does not generate catastrophic events during the financial crisis.
For empirical testing, we used data from Iranian currency markets. Iran foreign exchange market is very interesting. Since the Islamic revolution in 1979, Iran economy has been the target of unilateral and international sanctions. Nevertheless, many harsh financial sanctions were imposed on Iran economy in recent decade after the commence of Iran nuclear activities. At the beginning of 2012, all Iranian banks announced as institutions in breach of EU sanctions were disconnected from the SWIFT, the world’s platform of electronic financial transactions. Due to 2012 SWIFT sanctions, Iranian Rial lost 66 percent of its value against the US Dollar in a very short period of time. More recently, when US intent for exiting from 2015 nuclear deal was announced at the beginning of the year 2018, Iranian currency experienced another crash and lost 75 percent of its value against the US Dollar.

In this paper, following the above-mentioned strand of literature, we fit a cusp catastrophe model to exchange rate data of Iranian financial markets. Up to our knowledge, this is the first paper in the literature which uses catastrophe theory for modeling the large fluctuations in exchange rates. Our paper provides findings, which can have very important policy implications. First, our results suggest that currency collapse can be better explained by catastrophe model. This result shows that besides the external shocks like economic and financial sanctions which primarily produce supply shocks into economies, households’ expectations play important role in creation of the collapse of currencies.

More importantly, regarding the independent variable for controlling the dynamics of exchange rates, our results show that trading volume as well as the ratio of institutional to individual trades can in (gold) future markets can be a better measure of sentiment which eventually describe the abrupt behavior of exchange rates. Furthermore, our results are well consistent with finding in literature which show that future markets are efficient in a way that could provide a reliable approximation of future exchange rates. (See e.g. Frankel (1981)).

The rest of paper is organized as follows. In next section, the catastrophe theory and the most commonly used form of it .i.e. cusp model is introduced. In section 3, we describe data and empirical results. Finally, some concluding remarks are provided in section 4.
2. Catastrophe models

2.1. Theory

Catastrophe theory has been first advanced by Thom (1972). This proposition was very important because previous models, mostly described phenomena with smooth and continuous behavior. Nevertheless, many economic and even natural phenomena are involved with abrupt and sudden regularities. But it was the Zeeman, who first used the catastrophe theory in the field of economics and other social science (Zeeman, 1974). Zeeman proposed the application of cusp catastrophe model to study stock markets and qualitatively explained the bull and bear markets as a result of interaction between two main types of investors: fundamentalist and chartists (Diks, Wang, 2016).

Catastrophe theory provides a mathematical tool for describing dynamics of system involving abrupt and discontinuous behavior. Particularly, it provides basis for modeling dynamic system where slightly changing forces lead to abrupt changes in behavior (Diks, Wang, 2016). In its simple form, catastrophe models are based on single variable nonlinear dynamical system as follows:

$$\frac{dy_t}{dt} = -\frac{\partial V(y_t; \theta)}{dy_t}$$ (1)

Where $y_t$ represents the state of system. The equation shows that the system changes in response to a change in $V(y_t; \theta)$, where $V(y_t; \theta)$ is a potential function. Control parameter $\theta$ determines the specific structure of the system. When the spatial derivative of the potential function equals 0, i.e. $\frac{\partial V(y_t; \theta)}{dy_t} = 0$, it is expected that system is in equilibrium. This equilibrium can be a maximum or a minimum of the potential function. When $V(y_t^*; \theta)$ is a (local) minimum, the equilibrium point of state variable $y_t^*$ is stable; this means that system will return to it after a small change. Similarly, an equilibrium point $y_t^*$ is unstable if the potential function is a (local) maximum. In this case even a small change will cause the system divert from the unstable equilibrium state. When control variables impacting $\theta$ change, the local stability properties of these equilibria can change which is why these systems can give rise to unexpected bifurcations.
when the control variables change. Therefore, catastrophe theory can be employed in systems in which equilibrium states can be driven towards instability, such as gradient dynamical systems with critical points (Diks, Wang, 2016).

The cusp catastrophe model is the most commonly used catastrophe model. Its dynamics is given by the biquadratic potential function:

\[ V(y_t, \alpha, \beta) = -\frac{1}{4}y_t^4 + \frac{1}{2}\beta y_t^2 + \alpha y_t. \]  

(2)

Which has an equilibrium surface as:

\[ \frac{dV(y_t, \alpha, \beta)}{dt} = -y_t^3 + \beta y_t + \alpha = 0. \]  

(3)

Where \( \alpha \) and \( \beta \) are control variables (Barunik and Vosvrda (2009)). Assuming two independent variables, \( x_1 \) and \( x_2 \), we can write two control variables as:

\[ \alpha = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 \]  

(4)

\[ \beta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \]  

(5)

It is clear from above discussion that surface in equation (3) is the surface of equilibrium point of dynamic system \( y_t \). For real-world applications, we must add non-deterministic behavior into the system, as the current state of the system usually does not determine its next states completely. We may obtain a stochastic form by superimposing an additive Gaussian white noise term. The system is then described by a stochastic differential equation of the form:

\[ dy = a(y, x_1, x_2)dt + b(y)dW \]  

(6)

In which \( a(.,.) \) is part of SDE which depend on the state and two control variables. Here \( W \) is a standard Wiener process. Following Hartelman (1997) and Wagenmakers et al. (2005), the limiting PDF of \( y \) is

\[ f(y|x) = c \cdot \left[ \text{Exp} \left( -\frac{1}{4} \left( \frac{y-\mu}{\sigma} \right)^4 + \frac{1}{2}\beta \left( \frac{y-\mu}{\sigma} \right)^2 + \alpha \left( \frac{y-\mu}{\sigma} \right) \right) \right] \]  

(7)

The constant \( c \) is normalizing factor that guarantees the integral of a normalized PDF i.e. \( f \) over its entire range equals one (see Barunik and Vosvrda (2009)). \( \mu \) and \( \sigma \) are location and

2.2 Cusp catastrophe behavior of exchange rates

Zeeman (1974) was the first who tried to explain the unstable behavior of a stock market using the catastrophe model. The exchange rate markets (currency markets) have many common properties with the stock markets. Particularly, their dynamics are connected in some ways. Fluctuations of exchange rates have also been found that to be mainly driven by heterogeneous expectations of agents especially reflected in future markets (See e.g. Frankel (1981)). Finally, since Zeeman has argued that the catastrophe model could be used to model financial markets, it might also be useful in the currency market.

A common cusp catastrophe model is shown in Fig 1. The vertical axis of 3-dimentional space .i.e. y is state variable. In our analysis this variable is exchange rate of Iranian Rial per US Dollar. The folded surface with three levels of sheets shows the three equilibrium prices of the system. The control parameters .i.e. $\alpha$ and $\beta$ are shown a 2-dimensional plane.

Fig 1. Cusp catastrophe surface (Figure taken from Diks and Wang (2016))
When the control parameters drive the state to move across the multiple equilibrium area of the cusp equilibrium surface, critical transitions occur. Each point on the top and bottom sheets of this surface is correspondent to equilibrium of the system. Path A shows a moves on the control surface, from left on the top sheet to fold curve. On this path, state variable suddenly jump to the bottom sheet. In this way, a small perturbation in control parameters can lead to a sudden large crash in the state of the system. Alternatively, path B on the control surface outside of the cusp bifurcation shows a smoother move. This path moves to the bottom sheet slowly and smoothly, without any sudden crash (Diks and Wang, 2016).

One can understand the mechanism of currency collapse in terms of the cusp catastrophe model as follows. Assume that the control parameters are such that the currency market has multiple equilibria, and the market is on the top sheet of the equilibria surface. This means that currency market is in the high price equilibrium, however there is a lower and stable equilibrium on the bottom sheet of the equilibrium surface. A collapse can then be produced by any event that changes the control parameters such that can push the state variable over the fold curve so that it jumps to the bottom sheet. A similar mechanism has recently been described by Diks and Wang (2016) for housing markets.

3. **Empirical results**

It is clear from above discussion that catastrophe theory is an extension of traditional especially linear models. For this reason, in order to test the presence of bifurcation, the fit of cusp model is compared to fit of both a linear and a logistic model. In order to judge which model provides better fit for data, we use two criteria for comparing the results of three different models as follows:

The cusp catastrophe model better fit the empirical data if:

1. Akaike and Bayesian information criteria of cusp model is lower than two other models:
2. Likelihood of the cusp is significantly higher than that of the linear regression models.
3.1. Data description

We use daily rate of Iranian Rial against US Dollar since the November 2011 to September 2018. Figure 2 shows the time series of this data.

The studied time window contains two collapses of Iranian currency in early 2012 and of mid-2018. In order to use cusp catastrophe model, we must first investigate exchange rate dynamics in Iranian financial markets. As a first step in this analysis, one must study the local stability and bifurcation of the state variable. A bifurcation is a qualitative change in the dynamics of a nonlinear model, for example a change of stability of a steady state, when the control variables change (Bolt et al. (2014)).

In order to better understand the dynamics of Iranian Rial against US Dollar, Fig. 3 represents the probability distribution of data.
Kernel density estimate of the Iranian Rial per US Dollar in Fig. 3 indicates bimodality which is the candidate of the test for bifurcations. The two modes of density are corresponding to the first and second round of economics sanctions in 2012 and 2018 respectively.

As independent variables, we have chosen daily trading volume and ratio of institutional to individual trades of gold futures contracts. We believe that trading volume is a good measure of investors' expectations about the future of exchange rate. This is because the price of future markets provides a good approximation about the future spot exchange rate. Furthermore, investors use these contracts in order for hedging themselves against the collapse of home currency because gold future prices are strongly correlated with appreciation of the Dollar against Iranian Rial. Furthermore, the ratio of institutional to individual trades of gold futures contracts might be the best measure of household expectation of future inflations which represents the fear or concerns of ordinary people about their future livelihood. Table 1 shows the descriptive statistics of state and independent variables.

Furthermore, since we are comparing the cusp model with two other regression models, we use augmented Dickey- Fuller method for testing the stationarity of state variable i.e. exchange rate. Results show that the data rates are stationary because augmented Dickey- Fuller statistics exceed the critical value at 1% significance level.
Table 1. Descriptive statistics for state and independent variables.

<table>
<thead>
<tr>
<th></th>
<th>Rial per US Dollar</th>
<th>Gold Future Trading Volume</th>
<th>Ratio of institutional to individual Trades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>36,405</td>
<td>9,614</td>
<td>0.019</td>
</tr>
<tr>
<td>Median</td>
<td>33,930</td>
<td>6,814</td>
<td>0.013</td>
</tr>
<tr>
<td>Maximum</td>
<td>138,160</td>
<td>85,871</td>
<td>0.32</td>
</tr>
<tr>
<td>Minimum</td>
<td>13,350</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>15,451</td>
<td>9,163</td>
<td>0.023</td>
</tr>
<tr>
<td>Skewness</td>
<td>3.3</td>
<td>2.43</td>
<td>3.51</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>16.1</td>
<td>11.6</td>
<td>27.04</td>
</tr>
</tbody>
</table>

3.2. Model results

Table 2 presents the estimated parameters of the cusp model, i.e., the estimates of the model equation (3)–(5). Furthermore, the model compares the cusp model with a simple linear model and a logistic model. Our Results show that the cusp model generates a better fit. Higher $R^2$ value of cusp model imply that it fits the data much better. The cusp model clearly outperforms the two other models based on much lower values of AIC and BIC.

Table 2. Estimation results for Iranian Rial per US Dollar. The $R^2$ for cusp model is Cobb’s pseudo-$R^2$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>-5</td>
<td>$\beta_0$</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-4.2</td>
<td>$\beta_1$</td>
<td>-1.6</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>16.9</td>
<td>$\beta_2$</td>
<td>-0.36</td>
</tr>
<tr>
<td>$R^2_{cusp}$</td>
<td>0.82</td>
<td>$AIC_{cusp}$ ($BIC_{cusp}$)</td>
<td>872(917)</td>
</tr>
<tr>
<td>$R^2_{linear}$</td>
<td>0.53</td>
<td>$AIC_{linear}$ ($BIC_{linear}$)</td>
<td>7010(7032)</td>
</tr>
<tr>
<td>$R^2_{logistic}$</td>
<td>0.67</td>
<td>$AIC_{logistic}$ ($BIC_{logistic}$)</td>
<td>3375(3403)</td>
</tr>
</tbody>
</table>
4. Concluding remarks

The abrupt behavior of the exchange rate has always been a great concern for both policymakers and academics. For this, in this paper, we managed to use cusp catastrophe model for exchange data of Iranian financial market which has been under economic and financial sanctions. Our findings provide insights which may help better understanding of currencies collapses. We thus arrived at results which show that the stochastic cusp catastrophe model may better explain the currencies collapses than other traditional regression models.

Finally it must be mentioned that our results are based on the data from a single financial market. Thus, it is needless to say that further works must be done using other financial markets’ data.
References


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