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Bandwagon investment equilibrium of a preemption game

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Abstract
In stochastic and competitive environments, investors face an investment dilemma because the environments provide conflicting incentives. Empirical research reports various behaviors exhibited by investors, including voluntary concurrent investments, which are called bandwagon investments. However, the current theoretical understanding is still limited in explaining under which condition the investment bandwagon effect occurs. The authors investigated the closed-loop subgame perfect equilibrium of an investment timing game that describes voluntary simultaneous investments. They showed that investors are on the investment bandwagon when the second mover's additional profit rate exceeds a threshold value. Otherwise, investors sequentially invest. It explains the frequently observed investment herd effect. Moreover, it shows that the investment bandwagon effect does not exist for entering firms.

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Keywords Option exercise games; preemption games; bandwagon investment; closed-loop equilibrium

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1. Introduction

Under stochastic and competitive environments, irreversible investment timing is an essential yet delicate strategic decision. Although the investment timing decision is complicated, it is evident that excessive industry capacity caused by simultaneous investment is undesirable to every investor. However, concurrent investments are frequently observed, and some of the simultaneous investments are even voluntary. Alan Greenspan, former Federal Reserve Board chairman, described these types of investments as irrational exuberance. Is simultaneous investment unreasonable? Under what circumstances are rational investors on the investment bandwagon? This research aims to answer these questions under the stochastic environments. Our research theoretically demonstrates that bandwagon investment is possible even when observing competitor behavior continuously without assuming investment frictions.

We often observe concurrent investments in many industries. In the first half of the year 2015, fifteen China automobile makers invested more than $150 billion into new energy car projects. Due to this massive investment, the Chana automobile industry is expected to produce 21 million vehicles a year by 2020. China's Ministry of Industry and Information Technology is concerned about the overcapacity caused by the large-scale investment (Li 2017). Moreover, concurrent investments happen in various industries (Gilbert and Lieberman 1987; Lieberman 1987a; Henderson and Cool 2003). In particular, recent empirical studies show that the adoption of information technology often occurs simultaneously (Xu et al. 2012; Li et al. 2013).

Concurrent investment was considered a non-timely investment despite the rich empirical evidence. When firms compete for investment opportunities, either sequential or simultaneous investment is possible. If the law of diminishing marginal utility holds, a firm’s investment lowers the unit market price and reduces the profitability of the other firm’s investment opportunity. Hence, when a firm is first to occupy a market, it constructs an entrance barrier against other competitors that enables the first mover to enjoy a temporary monopolistic benefit (Spence 1979; Porter 1998). Conversely, uncertain environments provide the opposite incentives (Mieghem 2003). For example, consumer preference is a major uncertainty for investors, and investors desire to postpone making investments until the revealed preference ensures the investment's profitability. Theoretical research supports the sequential investment equilibrium where the two different incentives are balanced (Thijssen et al. 2012; Boyarchenko and Levendorskiĭ 2014).

This research studies voluntary simultaneous investments. In the case of a large-scale investment in China, it is hard to imagine that 15 companies invested at the same time by mistake. We classify simultaneous investment as either coordination failure or bandwagon investment. Coordination failure refers to a concurrent investment caused by inappropriate investment timing. On the other hand, when firms intentionally invest concurrently, such behavior is called bandwagon investment. Which conditions make the bandwagon invest as the equilibrium investment behavior of rational decision makers? We aim to answer this question.

This research contributes to the investment timing game area by elucidating closed-loop equilibrium where firms invest on a bandwagon. Researchers employ option exercise games, which integrate the real-options approach and game theory, to model the investment timing decisions. Accepting this stream of research, our model is based on the option exercise games framework. The development of information and communication technology provides abundant information that allows investors to monitor competitor's behavior in virtually real time. Regarding game theory, a closed-loop information structure models this situation. We derived the subgame perfect equilibrium under the closed-loop information structure and identified the conditions where bandwagon investment is the equilibrium investment behavior.

2. Related Work

Investment timing decision is important and ubiquitous. For example, Achi et al. (1996) asserted that appropriate investment timing can enhance the return on investment by 3% to 4%. Considering the magnitude and duration of an investment, investment time impacts the investment's performance. In stochastic and competitive environments, researchers describe investment decisions as stochastic games. The investment game has been applied to a wide range of areas, including the energy industry (Chuang et al. 2001; Madlener et al. 2005; Laurikka and Koljonen 2006), supply chain management (Swinney et al. 2011; Vairaktarakis 2012), and real estate development (Grenadier 1996).

Despite the theoretical consensus that supports preemption strategies pursuing first mover's advantages, successful implementation of the preemption strategy is rare. A number of studies have examined competitive advantages from preempting competitors (Spence 1979; Porter 1998; Ghemawat and Cassiman 2007). By increasing industry capacity, a firm can construct an entrance barrier against competitors. Song et al. (1999) conducted an extensive survey, the results of which demonstrated that industry practitioners are aware of the advantages of a preemption strategy. However, Lieberman (1987b) shows that effective preoccupation is seldom observed through an empirical study. Analyzing 20 years' empirical data for the international petrochemical industry, Henderson and Cool (2003) agree with the difficulty of successful preemption.

A stream of research asserts that investment frictions cause poorly timed capital investments. Early studies in stochastic differential games assert that it is difficult to explain concurrent investments without market frictions (Stadler 1992). Consequently, researchers investigated the factors that enable simultaneous investments. Widely discussed frictions include investment time-lag, network effect and information cascading. Grenadier (1996) and Anderson and Yang (2015) state that simultaneous investment occurs when it takes considerable time for investments to take effect. Li et al. (2013) assert that investment externalities such as network effects of information technology promote the occurrence of voluntary concurrent investments. Cartwright (2015) indicates information cascades can cause simultaneous investments.

Our research belongs to another stream of research that investigates simultaneous investment without postulating any frictions. In the deterministic continuous-time preemption game area, Reinganum (1981a, b) and Fudenberg and Tirole (1985) delivered seminal research articles. In an open-loop model, decision-makers do not observe other players' actions. In contrast, players react to competitors' behavior in a closed-loop information structure (Fudenberg and

Levine 1988). Reinganum (1981a, b) provided the open-loop equilibrium of the deterministic investment timing game. Fudenberg and Tirole (1985) extended Reinganum's model into closed-loop information structure. Their result, rent equalization principle, is often cited in subsequent studies. Option exercise games are regarded as an appropriate approach to modeling the investment timing game with continuous decision epochs in stochastic environments (Grenadier 2000; Chevalier-Roignant and Trigeorgis 2011; Chevalier-Roignant et al. 2011; Smit and Trigeorgis 2012). Recently, Azevedo and Paxon (2014) provide a well-organized review paper on option exercise games. This research investigates the closed-loop simultaneous investment equilibrium under option exercise games framework.

A few studies have provided a theoretical explanation of bandwagon investment equilibrium under open-loop information structure. Pawlina and Kort (2006) study an option exercise game between asymmetric players. They find that bandwagon investments occur when the best payoff of simultaneous investments exceeds the payoff with competition. Mason and Weeds (2010) derive a bandwagon investment equilibrium that is fundamentally identical to that of Pawlina and Kort (2006). They investigated the condition of bandwagon investment equilibria with respect to volatility in market profitability. Our research differs from these studies because we model the investment game under closed-loop information structure.

Several research articles address the closed-loop equilibrium where firms invest concurrently. Thijssen et al. (2012) employ a jump-diffusion process to describe the stochastic factor of the profit rate and investigate the subgame perfect equilibrium between two identical investors. Boyarchenko and Levendorskiĭ (2014) extend the model of Thijssen et al. (2012) by considering two asymmetric investors. The studies explain coordination failure that occurs when the investment opportunity becomes available after the market is sufficiently mature and when market profitability suddenly increases. Compared to the studies under open-loop information structure, the major contribution of these studies is extending the strategy space by including an investment intense function. As Fudenberg and Tirole (1985) show, the intensity function is an appropriate device to incorporate coordination failure investment. Therefore, their focuses lie in coordination failure rather than bandwagon investment. Boyer et al. (2012) address tactic collusion and derive Markov perfect equilibrium that is similar to our bandwagon investment equilibrium. However, they do not provide a concrete existence condition of the equilibrium. Hence, there still remain questions for the bandwagon investment under closed-loop information structure.

3. Model

Consider an investment timing game starting at time $t = 0$. Two risk-neutral players, firm A and firm B, compete in a duopoly market. A firm is indexed by $i \in \{A, B\}$, and the competitor is referred by $-i = \{A, B\} \setminus i$. The two firms are identical, meaning their payoff structure, investment opportunity, and information are identical. Each player has an irreversible investment opportunity, which can be made only once and incurs investment cost $I > 0$. The two firms seek the optimal time to invest on continuous decision epoch, $t \geq 0$.

Each firm's profit rate is the product of a stochastic factor $X(t)$ and a decision factor π_i. The investment is risky because an exogenous stochastic factor affects an investment's profitability. A geometric Brownian motion is one of the most common assumption for the shock process (Pawlina and Kort 2006; Boyer et al. 2012; Huisman and Kort 2015), and it is viable for use to model processes such as electricity consumption and airline revenue (Marathe and Ryan 2005). Based on the observations, we assume that the stochastic factor follows a geometric Brownian motion:

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t), \;\; X(0) = x_0 > 0 \tag{1}$$

where $\mu \in \mathbb{R}$ is the drift rate, and $\sigma > 0$ denotes the instantaneous standard deviation. Moreover, we assume that x_0 is sufficiently low, as we will clearly restate later. The filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t_0 \leq t \leq \infty}, \mathbb{P}\right)$ satisfies the usual conditions, where the filtration is generated by the one-dimensional standard Brownian motion, $B(t)$.

The deterministic factor of a firm, π_i, is a function of the firms' investment decisions. If $N_i(t) = 0$, then firm i has not invested up to time t. If firm i already invested by time t, then $N_i(t) = 1$. Since an investment is irreversible, $N_i(t)$ is non-decreasing. Because of the symmetricity assumption, $\pi_i\big(N_i(t), N_{-i}(t)\big) = \pi_{-i}\big(N_{-i}(t), N_i(t)\big)$. Hence, we will omit the player index subscripts as long as doing so will not confuse readers. The first mover's advantage exists, which is expressed as

$$\pi(1,0) > \pi(1,1) > \pi(0,0) > \pi(0,1). \tag{2}$$

When a firm monopolizes the market, it earns the highest payoff rate, $\pi(1,0)$. During the preemption period, the payoff rate is the lowest, $\pi(0,1)$. To focus on the bandwagon investment, we assume that if both firms invest, the investment yields a higher profit rate, $\pi(1,1)$, than in the case where neither firm invests, $\pi(0,0)$. Since simultaneous investments change the players' payoff rate from $\pi(0,0)X(t)$ to $\pi(1,1)X(t)$, bandwagon investment is impossible if $\pi(0,0) > \pi(1,1)$. We postulate the profit stream structure based on the empirical study, Lieberman (1987a), which supports the mobility-deterrence theory.

The players discount the future profit stream at a constant rate $\rho > 0$. We assume that the drift coefficient and discount rate satisfy $\mu < \rho$ to avoid perpetual postponement of investment.

3.1. Subgame of Interest and History of the Subgame

Recall that the initial game starts at time 0 with a sufficiently low value of stochastic factor $X(0) = x_0$. Once one of the firms invests, the decision is irreversible. We call the first firm to invest the leader of the game, and the other firm is the follower. We will use the terms "first mover" and "leader" interchangeably, likewise the terms "second mover" and "follower." The first mover can no longer react to the follower's strategy after its investment time. Therefore, the follower seeks the optimal investment timing without considering further reactions from the leader. Because of this interaction structure, our interest lies in elucidating the players' interactions up to the first mover's investment timing.

Generally, the history of a game contains the players' past actions and state variables. In our model, $h_i(t_0) = \left(\{X(t)\}_{0\le t\le t_0}, \{N_i(t)\}_{0\le t\le t_0}, \{N_{-i}(t)\}_{0\le t\le t_0}\right)$ represents the history of the subgame starting at time t_0. With close-loop information, players update their strategy based on history. Since players have perfect information and the stochastic factor is $\mathcal{F}_t$-adapted, any subset of the initial game is a proper subgame (Riedel and Steg 2014). We are interested in subgames where no firm has invested at the outset of the subgame starting at t_0, namely $N_i(t_0) = 0,\ \forall i \in \{A, B\}$. This implies that the sequences of players' past actions is redundant information for describing the history of the subgame of interest. In addition, information on $\{X(t)\}_{0 \le t \le t_0}$ is equivalent to that of $X(t_0)$, because $X(t)$ is a Markov process. Therefore, the pair of two real numbers, $\left(t_0, X(t_0)\right)$, summarizes the relevant history of the subgame of interest.

3.2. Terminal Payoff

At the moment of the leader's investment, the strategic interaction between the firms ends and the players' expected payoffs are fixed. Terminal payoff refers to the expected payoff that is measured at the investment time.

Suppose that the leader and the follower invest at the same time τ_L given the value of stochastic factor $X(\tau_L) = x$. Because the terminal payoff is measured at the investment time, let $\tau_L = 0$. We can express the expected value of the discounted future profit stream as the follows.

$$M(x) = \mathbb{E}\left[\int_0^\infty e^{-\rho t} X(t)\pi(1,1)dt - I \middle| X(0) = x\right] \tag{3}$$

If player $-i$ preempts player i by investing at time $\tau_{-i,L}$ where $X\left(\tau_{-i,L}\right) = x$, player i faces a single decision maker's optimal investment timing problem because the first mover's investment is irreversible. By setting $\tau_{-i,L} = 0$, we can calculate the terminal payoff of the second mover as follows:

$$F(x) = \max_{\tau_{i,F} \ge 0} \mathbb{E}\left[\begin{matrix} \int_0^{\tau_{i,F}} e^{-\rho t} X(t)\pi(0,1)dt - e^{-\rho \tau_{i,F}} I \\ + \int_{\tau_{i,F}}^\infty e^{-\rho t} X(t)\pi(1,1)dt \end{matrix} \middle| X(0) = x\right]. \tag{4}$$

Let $\tau_{i,F}^*$ denote the optimal $\tau_{i,F}$. Because the follower does not have any incentive to deviate from $\tau_{i,F}^*$, the leader's terminal payoff, $L(x)$, is expressed as follows:

$$L(x) = \mathbb{E}\left[\int_0^{\tau_{i,F}^*} e^{-\rho t} X(t)\pi(1,0)dt - I + \int_{\tau_{i,F}^*}^{\infty} e^{-\rho t} X(t)\pi(1,1)dt \,\middle|\, X(0) = x\right]. \qquad (5)$$

We present the well-known explicit representations of above equations as a proposition.

Proposition 1: Terminal Payoff

The explicit expressions of (3), (4) and (5) are follows.

$$M(x) = \frac{\pi(1,1)}{\rho-\mu} x - I. \qquad (6)$$

$$F(x) = \begin{cases} \frac{I}{r_1 - 1}\left(\frac{x}{x_F^*}\right)^{r_1} + \frac{\pi(0,1)}{\rho-\mu} x & x < x_F^* \\ \frac{\pi(1,1)}{\rho-\mu} x - I & x \geq x_F^* \end{cases} \qquad (7)$$

$$L(x) = \begin{cases} \frac{\pi(1,1)-\pi(1,0)}{\pi(1,1)-\pi(0,1)} \frac{r_1}{r_1 - 1} I \left(\frac{x}{x_F^*}\right)^{r_1} + \frac{\pi(1,0)}{\rho-\mu} x - I & x < x_F^* \\ \frac{\pi(1,1)}{\rho-\mu} x - I & x \geq x_F^* \end{cases}. \qquad (8)$$

where

$$x_F^* = \frac{r_1}{r_1 - 1}\left(\frac{\rho-\mu}{\pi(1,1)-\pi(0,1)}\right) I \qquad (9)$$

and

$$r_1 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1. \qquad (10)$$

The second mover's optimal investment time is

$$\tau_{i,F}^* = inf\{s > \tau_{-i,L} \,|\, X(s) \geq x_F^*\}, \qquad (11)$$

Proof: See Appendix A.

Figure 1 illustrates the terminal payoffs. The first mover's terminal payoff depends on the modularity of deterministic factors. If π is super-modular, i.e., $\pi(1,0) - \pi(0,0) < \pi(1,1) - \pi(0,1)$, then $L(x)$ is increasing concave in $x < x_F^*$ as shown in the left panel of Figure 1. The right panel of Figure 1 illustrates the case that π is sub-modular, i.e., $\pi(1,0) - \pi(0,0) \geq \pi(1,1) - \pi(0,1)$. In this case, $L(x)$ is concave and unimodal in $x < x_F^*$. In both of the cases, $F(x)$ is increasing and convex for $x < x_F^*$. The simultaneous investment payoff, $M(x)$ is linear function in x. For $x \geq x_F^*$, all the payoffs are identical.

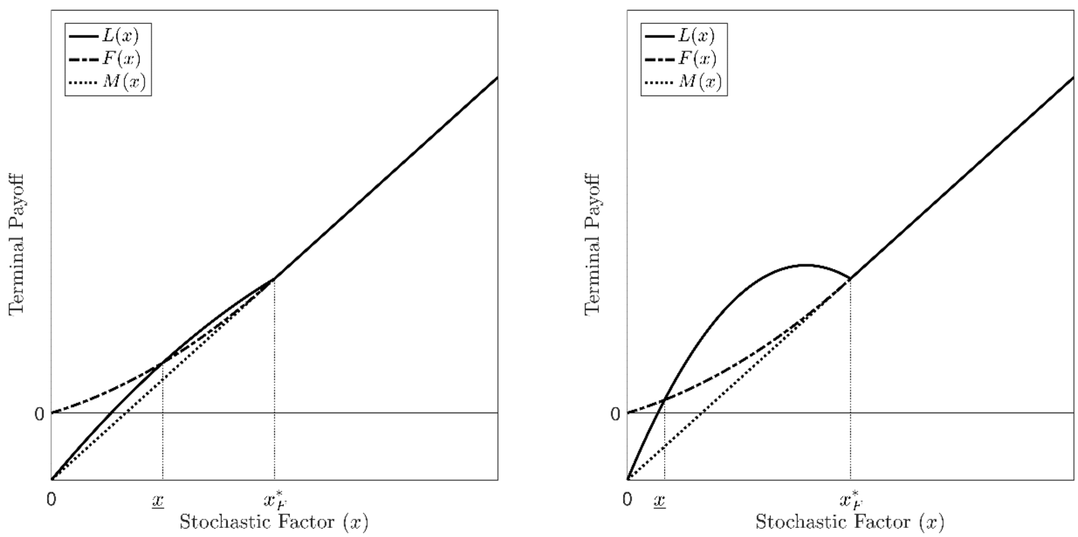

Figure 1 Terminal payoffs

Notice that $L(0) < 0$, $F(0) = 0$, and $L(x_F^*) = F(x_F^*) > 0$. It implies $L(x)$ and $F(x)$ intersect each other only once in $[0, x_F^*)$. Let $\underline{x}$ denote this intersection, i.e.,

$$\underline{x} \in [0, x_F^*) \text{ such that } L(\underline{x}) = F(\underline{x}). \tag{12}$$

The terminal payoff structure provides abundant information about the players' decision. Investors have no incentive to invest when the stochastic factor is lower than $\underline{x}$, because being the follower yields higher payoff than being the leader. In the region of $\left(\underline{x}, x_F^*\right)$, the leader's highest payoff represents the first mover's advantage. While seeking the first mover's advantage, players may invest at the same time. Therefore, simultaneous investment by coordination failure is possible in this region. If the leader invests when the stochastic factor is greater than x_F^*, the follower invests at the same time to the leader. Bandwagon investment equilibrium refers to the equilibrium where both players intentionally invest simultaneously when the stochastic factor lies in the region of $[x_F^*, \infty)$. Recall that we assume an initial value of $X(t)$ is sufficiently low. This means that $x_0 < \underline{x}$. For the subsequent discussion, we define the optimal investment time for the monopolist (τ_L^*) and for simultaneous investment (τ_M^*) as follows.

$$\tau_L^* = \inf\{s \geq t_0 | X(s) \geq x_L^*\}, \text{ where } x_L^* = \frac{r_1}{r_1 - 1}\left(\frac{\rho - \mu}{\pi(1,0) - \pi(0,0)}\right) I. \tag{13}$$

$$\tau_M^* = \inf\{s \geq t_0 | X(s) \geq x_M^*\}, \text{ where } x_M^* = \frac{r_1}{r_1 - 1}\left(\frac{\rho - \mu}{\pi(1,1) - \pi(0,0)}\right) I. \tag{14}$$

We can see that $x_M^* > x_L^*$ and $x_M^* > x_F^*$ because of the relationship in (2).

3.3. Strategy

For the subgame starting with the history, $\left(t_0, X(t_0)\right)$, firm i's intended investment time as the first mover describes the firm's strategy. Players update their strategies according to the information accumulated at the outset of each subgame.

Definition: Simple strategy

For the subgame of interest starting with $\left(t_0, X(t_0)\right)$, the simple strategy of firm i is the intended investment time as the first mover, $\tau_{i,L}\left(t_0, X(t_0)\right) \in [t_0, \infty)$.

Considering the purpose of this study, we consider the reduced form of strategy space that is similar to Boyer et al. (2012). Fudenberg and Tirole (1985) construct a comprehensive strategy space for investment timing games that includes an investment intensity function. Coordination failure equilibrium requires the intensity function, but bandwagon investment equilibrium does not. Hence, we exclude the intensity function from our strategy space.

Note that the actual investment time differs from the intended investment time if player i is preempted by the competitor. The preempted player invests at the optimal investment time as the follower, $\tau_{i,F}^*$.

We are ready to define the performance function and Nash equilibrium of the investment timing game. For the simplicity of notation, define $\tau_L = \min\{\tau_{i,L}, \tau_{-i,L}\}$, $x_\tau = X(\tau_L)$, and $x_{i,\tau} = X(\tau_{i,L})$. Here, we omit the obvious argument $(t_0, X(t_0))$ of τ. The performance function with players' simple strategies $\tau_{i,L}$ and $\tau_{-i,L}$ can be expressed with the terminal payoffs as follows:

$$J_i^{t_0,X(t_0)}\left(\tau_{i,L}\middle|\tau_{-i,L}\right) = \mathbb{E}\left[\int_{t_0}^{\tau_L} e^{-\rho t}\pi(0,0)X(t)dt + e^{-\rho\tau_L}\left\{\begin{matrix} \mathbb{I}_{\{\tau_{i,L}<\tau_{-i,L}\}}L(x_\tau) \\ +\mathbb{I}_{\{\tau_{i,L}>\tau_{-i,L}\}}F(x_\tau) \\ +\mathbb{I}_{\{\tau_{i,L}=\tau_{-i,L}\}}M(x_\tau) \end{matrix}\right\}\middle| X(t_0)\right] \quad (15)$$

3.4. Open-loop and Closed-loop Equilibrium

Consider a game start at $t = 0$ with $X(0) = x_0$. The open-loop equilibrium of is defined as follows.

Definition: Open-loop equilibrium

A pair of simple strategies $\left(\tau_{A,L}^*(0, x_0), \tau_{B,L}^*(0, x_0)\right)$ is the open-loop equilibrium for the investment game starting with $(0, x_0)$ if it satisfies the following conditions for both players:

$$J_i^{0,x_0}\left(\tau_{i,L}^*\middle|\tau_{-i,L}^*\right) \geq J_i^{0,x_0}\left(\tau_{i,L}\middle|\tau_{-i,L}^*\right),\ \forall\tau_{i,L} \in [t_0, \infty), \forall i \in \{A, B\} \quad (16)$$

The definition of the open-loop equilibrium implies that the investors do not observe the competitor's behavior after the beginning of the game.

Definition: Closed-loop strategy and closed-loop equilibrium

For a game starting at time 0, a closed-loop strategy profile for player i is a collection of the simple strategy $\{\tau_{i,L}(t, X(t))\}_{t\geq 0}$. The closed-loop equilibrium is a collection of the open-loop equilibrium, that is $\{\tau^*_{A,L}(t, X(t)), \tau^*_{B,L}(t, X(t))\}_{t\geq 0}$.

Under the closed-loop setting, investors update their investment strategy at each point in time by monitoring the competitor's behavior. Emphasizing on the difference from the open-loop equilibrium, we restate the closed loop equilibrium as subgame perfect equilibrium with an intuitive representation of the investment game. Consider a subgame starting at $t_0 \geq 0$ with $X(t_0)$. At the outset of the subgame of our interest, the investors have two choices, invest or defer. Table 1 illustrates the investment decisions and payoffs at the outset of a subgame.

Table 1 Investment decisions and payoffs at the outset of a subgame

		Player B	
		Invest	Defer
Player A	Invest	$\{M(X(t_0)), M(X(t_0))\}$	$\{L(X(t_0)), F(X(t_0))\}$
	Defer	$\{F(X(t_0)), L(X(t_0))\}$	$\{V(X(t_0)), V(X(t_0))\}$

The first and second element in the braces represents player A's and B's payoff, respectively. If both players decide to invest at time t_0, their payoffs are $M(X(t_0))$. If the player i invests and the other does not, their payoffs are $L(X(t_0))$ and $F(X(t_0))$, respectively. When both of the players defer the investment, then the expected payoff evolves according to the stochastic factor. Ito-Doeblin formula tells that V satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - \rho V(x) + \pi(0,0)x = 0. \quad (17)$$

Suppose that both players defer their investments at the outsets of all the subgames starting $[0, \tau_L^*)$, and at least one investor invests at τ_L^*, where $\tau_L^* = \min\left(\tau_{i,L}^*, \tau_{-i,L}^*\right)$. Then $\left(\tau_{i,L}^*, \tau_{-i,L}^*\right)$ is the subgame perfect equilibrium of the game starting at time 0.

At the bandwagon investment equilibrium, both investors invest at the same time. The firms recognize that if they postpone their investment time until $\tau_{i,L}^*$, their competitor will invest at the same time. Despite the existence of first mover's advantage, firms voluntarily simultaneously invest at the bandwagon invest equilibrium. The primary research question of this research is when the bandwagon investment equilibrium is possible.

4. Results and Discussion

In this section, we investigate the subgame perfect equilibrium of the investment game. Suppose that firm $-i$ does not intend to be the leader, i.e., firm $-i$ yields the first mover position to firm i. Even in this situation, firm i invests within a finite time. Proposition 2 states this property.

Proposition 2: Consider an investment game starts at $t = 0$ with $X(0) = x_0 < \underline{x}$. At least one firm invests no later than τ_M^*.
Proof: See Appendix B.

The latest investment time plays a role of the boundary condition for the subgame perfect equilibrium. Consider a subgame starting at the time, $\tau_M^* - dt$, for infinitesimally small $dt > 0$. The subgame has the initial value of the stochastic factor $X(\tau_M^*) - \varepsilon$, for a small $\varepsilon > 0$. If both players postpone the investment at the outset of this subgame, then τ_M^* is the closed-loop equilibrium of this subgame. If a player invests at the beginning of a subgame, we have a new boundary condition. By applying this procedure iteratively from τ_M^* to 0, we can derive the subgame perfect equilibrium of the investment game starts at time 0. Theorem 1 provides the result.

Theorem 1: Define $\xi_L = \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,0)}$ and $\xi_F = \frac{\pi(0,0)-\pi(1,1)}{\pi(1,1)-\pi(0,0)}$. For an investment game starting at time $t = 0$ with $X(0) = x_0 \leq \underline{x}$, bandwagon investment at $\tau_M^* = \inf\{t \geq 0 | X(t) \geq x_M^*\}$ is the subgame perfect equilibrium, if

$$\xi_F \geq \left(\frac{(\xi_L+1)^{r_1}-1}{r_1 \cdot \xi_L}\right)^{\frac{1}{r_1-1}} - 1. \tag{18}$$

Otherwise, sequential investment is the closed-loop equilibrium, where a firm invests at $\underline{\tau} = \inf\{t \geq 0 \big| X(t) \geq \underline{x}\big\}$ and the other firm invests at $\tau_F^* = \inf\{t \geq 0 | X(t) \geq x_F^*\}$.

Proof: See Appendix C

The originality of this study is condensed in the condition (18). The equilibrium of bandwagon investment and sequential equilibrium is identical to the open-loop equilibrium of Pawlina and Kort (2006) and the Markov perfect equilibrium of Boyer et al. (2012). They state that the simultaneous investment occurs at $\tau_M^* = \inf\{t \geq 0 | X(t) \geq x_M^*\}$. However, no research reports the existence condition of bandwagon investment equilibrium as succinct as ours.

The explicit condition is insightful because it explains why bandwagon investment is frequently observable. Through the investment, the leader acquires additional profit rate of $\pi(1{,}0) - \pi(0{,}0) = \{\pi(1{,}0) - \pi(1{,}1)\} + \{\pi(1{,}1) - \pi(0{,}0)\}$, and the follower does $\{\pi(1{,}1) - \pi(0{,}0)\} + \{\pi(0{,}0) - \pi(0{,}1)\}$. The $\pi(1{,}1) - \pi(0{,}0)$ is common gain for both players. The ξ_L and ξ_F represent the players' pure gain that is normalized by the common gain. Theorem 1 tells that bandwagon investment occurs as long as the second mover's additional profit rate is higher than the threshold value. This can happen when there are investment network effects and second mover's advantages.

Interestingly, bandwagon investment happens not for entering a new industry but for expansion investment. We can interpret $\pi(0{,}0) - \pi(0{,}1)$ as the loss of the second mover due to the leader's investment. Our result also means the follower's fear for the loss from preemption is the driver for bandwagon investment. If a firm has not entered a market, competitor's investment does not diminish the firm's profit. Therefore, $\pi(0,0) - \pi(0,1) = 0$ and $\xi_F = 0$. Only incumbent firms have $\xi_F > 0$. Our result shows that bandwagon investment is possible only when the two firms compete for expansion opportunity that harms competitor's profitability.

Figure 2 shows the regions of each equilibrium according to the deterministic factor π and r_1. The horizontal axis represents ξ_L, and the vertical axis does ξ_F. The solid, dotted, dash-dot, and dashed lines represent the boundaries of bandwagon and sequential investment equilibrium regions with $r_1 = 1.1$, $r_1 = 2$, $r_1 = 5$, and $r_1 = 30$, respectively. As Theorem 1 states, bandwagon investment is the closed-loop equilibrium above the lines, and the sequential investment is below the lines.

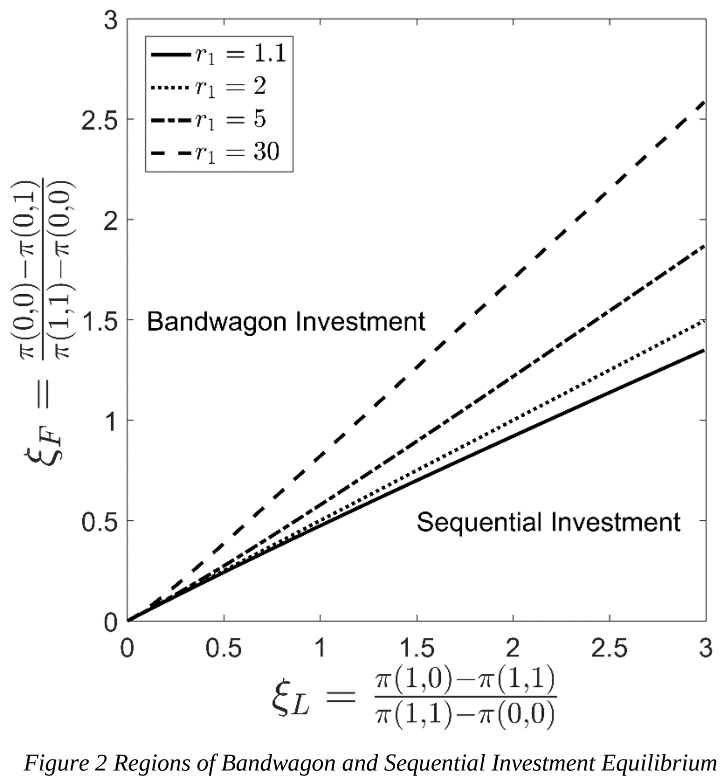

Figure 2 Regions of Bandwagon and Sequential Investment Equilibrium

Our result has potential to deepen our understanding of investment timing games. The threshold value depends on ξ_L and r_1 that is a function of other parameters (μ, σ, ρ).

Figure 2 also illustrates other plausible relationships that we have not proven, yet. First, the leader's gain increases along with ξ_L. Hence, high ξ_L provides more incentive to be the leader, so that sequential investment is more liked happens. However, we could not rigorously prove this plausible relationship which is illustrated in Figure 2. Let $\bar{\bar{\xi}}_F = \left(\frac{(\xi_L+1)^{r_1}-1}{r_1\cdot\xi_L}\right)^{\frac{1}{r_1-1}} - 1$ be the threshold value. Future research should check whether $\frac{\partial\bar{\bar{\xi}}_F}{\partial\xi_L} > 0$ to prove the effect of ξ_L on the equilibrium.

Second, it is well known that r_1, which depends on the parameters μ, σ and ρ, is related to time value of money accounting for risk (Dixit 2002). As r_1 increases, investors prefer the current money over the future money. Since the first mover invests earlier than the bandwagon investment time in the sequential equilibrium, the bandwagon investment region shrinks as r_1 increases. This argument requires to prove $\frac{\partial\bar{\bar{\xi}}_F}{\partial r_1} > 0$. Moreover, the effect of the parameters on the equilibrium is an ongoing research topic (Mason and Weeds 2010). The effect of the parameters on r_1 is well-known (Dixit 2002). Increase in μ, the drift parameter of the stochastic factor, decreases r_1, i.e., $\frac{\partial r_1}{\partial\mu} < 0$. Moreover, increases in discount rate, ρ, increase r_1, i.e., $\frac{\partial r_1}{\partial\rho} > 0$. If $r_1 > \frac{\rho}{\mu}$, then $\frac{\partial r_1}{\partial\sigma} > 0$. Otherwise, $\frac{\partial r_1}{\partial\sigma} \leq 0$. Therefore, identifying the properties of $\frac{\partial\bar{\bar{\xi}}_F}{\partial r_1}$ can deepen our understanding about the effect of the parameters.

5. Concluding Remarks

In stochastic and competitive environments, investors face an investment dilemma because the environments provide conflicting incentives. Empirical research reports various behaviors exhibited by investors, including voluntary concurrent investments, which are called bandwagon investments. However, the current theoretical understanding is still limited in explaining under which condition the investment bandwagon effect occurs. We investigated the closed-loop subgame perfect equilibrium of an investment timing game that describes voluntary simultaneous investments. We showed that investors are on the investment bandwagon when the second mover's additional profit rate exceeds a threshold value. Otherwise, investors sequentially invest. It explains the frequently observed investment herd effect. Moreover, it shows that the investment bandwagon effect does not exist for entering firms.

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Appendices

Appendix A: Proof of Proposition 1

A straightforward stochastic calculus shows equation (3) is equivalent to equation (6).

$$M(x) = \mathbb{E}\left[\int_0^\infty e^{-\rho t} X(t)\pi(1,1)dt - I \middle| X(0) = x\right] = \pi(1,1)\int_0^\infty e^{-\rho t} x e^{\mu t} dt - I = \frac{\pi(1,1)}{\rho - \mu} x - I$$

The second mover's problem (4) yields the following general solution:

$$F(x) = \begin{cases} Ax^{r_1} + \frac{\pi(0,1)}{\rho-\mu} x & x < x_F^* \\ \frac{\pi(1,1)}{\mu-\rho} x - I & x \geq x_F^* \end{cases}. \tag{A.1}$$

where $r_1 > 1$ is the positive root of the quadratic equation $\frac{1}{2}\sigma^2 r_1(r_1 - 1) + (\rho - \mu)r_1 - \rho = 0$, which is provided in (10). It is also known that the first hitting time, $\tau_{i,F}^*$ in equation (11), represents the optimal investment time of the second mover. The threshold value x_F^* and the unknown coefficient A are obtained by solving the following value-matching and smooth-pasting conditions:

$$\begin{cases} A(x_F^*)^{r_1} + \frac{\pi(0,1)}{\rho-\mu} x_F^* = \frac{\pi(1,1)}{\rho-\mu} x_F^* - I \\ Ar_1(x_F^*)^{r_1 - 1} + \frac{\pi(0,1)}{\rho-\mu} = \frac{\pi(1,1)}{\rho-\mu} \end{cases}$$

By plugging the solution of above system of equations into (A.1), we obtain (7) and (9).

The first mover's terminal payoff in equation (5) can be rewritten as

$$L(x) = \mathbb{E}\left[\int_0^{\tau_{i,F}^*} e^{-\rho t} X(t)\{\pi(1,0) - \pi(1,1)\}dt + \int_0^\infty e^{-\rho t} X(t)\pi(1,1)dt - I \middle| X(0) = x\right]$$

$$= \{\pi(1,0) - \pi(1,1)\}\mathbb{E}\left[\int_0^{\tau_{i,F}^*} e^{-\rho t} X(t)dt \middle| X(0) = x\right] + M(x)$$

If $x \geq x_F^*$, $\tau_{i,F}^* = 0$ as shown in equation (11). In this case $L(x) = M(x)$.

If $x < x_F^*$, $\tau_{i,F}^* > 0$. Dixit and Pindyck (1994) provides

$$\mathbb{E}\left[\int_0^{\tau_{i,F}^*} e^{-\rho t} X(t) dt \,\middle|\, X(0) = x\right] = -\frac{x_F^*}{\rho - \mu}\left(\frac{x}{x_F^*}\right)^{r_1} + \frac{x}{\rho - \mu}$$

Therefore, equation (8) is obtained as follows.

$$\begin{aligned} L(x) &= \{\pi(1,0) - \pi(1,1)\}\left\{-\tfrac{1}{\rho-\mu} x_F^* \left(\tfrac{x}{x_F^*}\right)^{r_1} + \tfrac{1}{\rho-\mu} x\right\} + \tfrac{\pi(1,1)}{\mu-\rho} x - I \\ &= -\tfrac{\pi(1,0)-\pi(1,1)}{\rho-\mu} x_F^* \left(\tfrac{x}{x_F^*}\right)^{r_1} + \tfrac{\pi(1,0)-\pi(1,1)}{\rho-\mu} x + \tfrac{\pi(1,1)}{\mu-\rho} x - I \\ &= -\tfrac{\pi(1,0)-\pi(1,1)}{\rho-\mu} \tfrac{r_1}{r_1-1}\left(\tfrac{\rho-\mu}{\pi(1,1)-\pi(0,1)}\right) I \left(\tfrac{x}{x_F^*}\right)^{r_1} + \tfrac{\pi(1,0)}{\rho-\mu} x - I \\ &= -\tfrac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} \tfrac{r_1}{r_1-1} I \left(\tfrac{x}{x_F^*}\right)^{r_1} + \tfrac{\pi(1,0)}{\rho-\mu} x - I \end{aligned}$$

□

Appendix B: Proof of Proposition 2

Consider an investment game starting at $t = 0$. Let $\tau_{-i,L} = \infty$ to represent that firm $-i$ does not intend to be the leader. Then the payoff function in (15) becomes

$$J_i^{0,x_0}(\tau_{i,L}|\infty) = \mathbb{E}\left[\int_0^{\tau_{i,L}} e^{-\rho t}\pi(0,0)X(t)dt + e^{-\rho\tau_{i,L}} L(x_{i,\tau})\middle| x_0\right] \qquad \text{(A.2)}$$

If $x_{i,\tau} \geq x_F^*$,

$$\begin{aligned} J_i^{0,x_0}(\tau_{i,L}|\infty) &= \mathbb{E}\left[\int_0^{\tau_{i,L}} e^{-\rho t}\pi(0,0)X(t)dt + e^{-\rho\tau_{i,L}}\left\{\tfrac{\pi(1,0)}{\rho-\mu} x_{i,\tau} - I\right\}\middle| x_0\right] \\ &= \tfrac{\pi(1,1)-\pi(0,0)}{\rho-\mu}(x_0)^{r_1}(x_{i,\tau})^{1-r_1} - I(x_0)^{r_1}(x_{i,\tau})^{-r_1} + \tfrac{\pi(0,0)}{\rho-\mu} x_0. \end{aligned}$$

Because $\left.\frac{\partial J_i^{0,x_0}}{\partial x_{i,\tau}}\right|_{x_{i,\tau}=x_M^*} = 0$ and $\left.\frac{\partial^2 J_i^{0,x_0}}{\partial x_{i,\tau}^2}\right|_{x_{i,\tau}=x_M^*} < 0$, (A.2) is maximized at $\tau_{i,L} = \tau_M^*$ that is given in (14). Because $x_M^* > x_F^*$, it proves Proposition 2. □

Appendix C: Proof of Theorem 1.

According to Proposition 2, if a player invests at τ_M^*, the other player also invests at the same time. This bandwagon investment is subgame perfect equilibrium if both players defer their investments at the beginning of every subgame starts at $t_0 \in [0, \tau_M^*)$ as shown in Table 1. At the outset of each subgame starting with $X(t_0)$, deferring investment is the dominant strategy if $M\big(X(t_0)\big) \leq F\big(X(t_0)\big)$ and $L\big(X(t_0)\big) \leq V\big(X(t_0)\big)$. Because $X(t_0)$ can be any number in $(0, x_M^*)$ during $[0, \tau_M^*)$, these inequalities should hold for all $x \in (0, x_M^*)$ to have the bandwagon investment subgame perfect equilibrium.

As Figure 1 illustrates, the first condition, $L(x) \leq F(x)\ \forall x \in (0, x_M^*)$, is obvious because of the following properties.

- $F(x)$ is increasing convex in $x \in [0, x_F^*]$ and $F(0) = 0$.
- $M(x)$ is linear and $M(0) = -I < 0$.
- $F(x_F^*) = M(x_F^*)$ and $F'(x_F^*) = M'(x_F^*)$.
- For $x \geq x_F^*$, $F(x) = M(x)$.

Our next task is investigating the second condition, $L(x) \leq V(x)\ \forall x \in (0, x_M^*)$. The general solution of (17) is known as

$$V(x) = A_1 x^{r_1} + A_2 x^{r_2} + \frac{\pi(0,0)x}{\rho-\mu}, \tag{A.3}$$

where r_2 is the negative root of the quadratic equation $\frac{1}{2}\sigma^2 r_2(r_2 - 1) + (\rho - \mu)r_2 - \rho = 0$. We assume that the initial investment game starts with sufficiently low x_0 so that no player has incentive to invest at the beginning of the game. Moreover, the players have not invested at the outset of the subgames of our interest. It implies the investors would not invest when the stochastic factor is lower than x. When the investors simultaneously invest at τ_M^*, their payoffs are $M(x_M^*)$. Therefore, we have two boundary conditions $V(0) = 0$ and $V(x_M^*) = M(x_M^*)$. The particular solution of (17) for the bandwagon investment equilibrium is $V(x) = \left[\frac{\pi(1,1)-\pi(0,0)}{\rho-\mu} x_M^* - I\right] (x_M^*)^{-r_1} x^{r_1} + \frac{\pi(0,0)}{\rho-\mu} x$. By plugging $x_M^* = \frac{r_1}{r_1-1}\left(\frac{\rho-\mu}{\pi(1,1)-\pi(0,0)}\right) I$ into the brackets and rearranging terms,

$$V(x) = \frac{I}{r_1 - 1}(x_M^*)^{-r_1} x^{r_1} + \frac{\pi(0,0)}{\rho - \mu} x. \tag{A.4}$$

For $x \in [x_F^*, x_M^*)$, $L(x) = M(X) = \frac{\pi(1,1)}{\rho-\mu}x - I$. Therefore, $L(x) \le V(x)$ is equivalent to

$$\frac{\pi(1,1) - \pi(0,0)}{\rho - \mu} x - I \le \frac{I}{r_1 - 1}(x_M^*)^{-r_1} x^{r_1} \tag{A.5}$$

Let $g(x) = \frac{\pi(1,1)-\pi(0,0)}{\rho-\mu}x - I$ and $h(x) = \frac{I}{r_1-1}(x_M^*)^{-r_1}x^{r_1}$. We can find the following properties.

- $h(x)$ is increasing convex in $x \ge 0$ because $\frac{I}{r_1-1}(x_M^*)^{-r_1} > 0$ and $r_1 > 1$.
- $g(0) < h(0)$ because $g(0) = -I$ and $h(0) = 0$.
- $g(x_M^*) = h(x_M^*)$
- $h'(0) = 0$ and $h'(x_M^*) = \frac{\pi(1,1)-\pi(0,0)}{\rho-\mu}$

Notice that $h(x)$ is increasing convex in $x \ge 0$. Moreover, $g(0) = -I < h(0) = 0$ and $g(x_M^*) = h(x_M^*)$. Because $h'(0) = 0$ and $h'(x_M^*) = \frac{\pi(1,1)-\pi(0,0)}{\rho-\mu}$, we can see that (A.5) holds for every $x \in [0, x_M^*)$ using Legendre transformation.

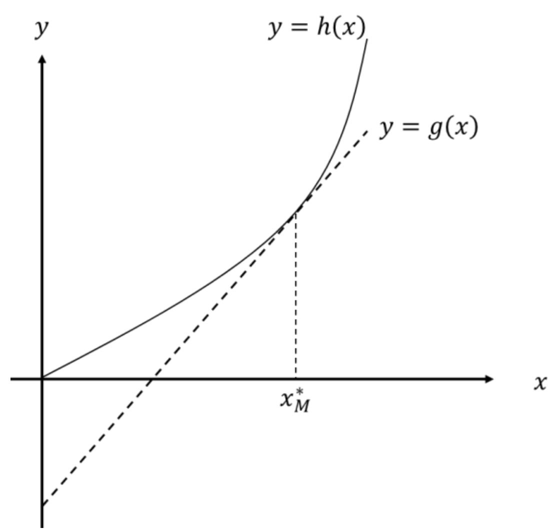

Figure 3 Illustration of (A.5)

Figure 3 depicts this relationship. The dashed line and the solid curve represents $g(x)$ and $h(x)$, respectively. Since the straight line $g(x)$ is a tangent line that meets the increasing convex curve $h(x)$ at $x = x_M^*$, (A.5) holds for all $x \in [0, x_M^*)$. It means that the bandwagon investment at $\tau_M^* = \inf\{s \geq t_0 | X(s) \geq x_M^*\}$ is closed-loop equilibrium for the subgame starting with $X(t_0) \in [x_F^*, x_M^*)$.

For $x < x_F^*$, $L(x) = \frac{\pi(1,1)-\pi(1,0)}{\pi(1,1)-\pi(0,1)} \frac{r_1}{r_1-1} I \left(\frac{x}{x_F^*}\right)^{r_1} + \frac{\pi(1,0)}{\rho-\mu} x - I$. By rearranging the terms, $L(x) \leq V(x)$ is equivalent to

$$\frac{\pi(1,0)-\pi(0,0)}{\rho-\mu} x - I \leq \frac{I}{r_1-1} \left[(x_M^*)^{-r_1} + r_1 \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} (x_F^*)^{-r_1} \right] x^{r_1}. \tag{A.6}$$

Let $G(x) = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu} x - I$ and $H(x) = \frac{I}{r_1-1} \left[(x_M^*)^{-r_1} + r_1 \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} (x_F^*)^{-r_1} \right] x^{r_1}$. To have the bandwagon investment equilibrium, $G(x) \leq H(x)$ should be satisfied for all $x \in (0, x_F^*)$. We can see the following properties

- $H(x)$ is increasing convex in $x \geq 0$ because $\frac{I}{r_1-1} \left[(x_M^*)^{-r_1} + r_1 \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} (x_F^*)^{-r_1} \right] > 0$ and $r_1 > 1$.
- $G(0) < H(0)$ because $G(0) = -I$ and $H(0) = 0$.

Figure 4 shows the two cases that the inequality (A.6) holds $\forall x \in (0, x_F^*)$. The first case is $G(x) \leq H(x)$ for all $x \geq 0$ as illustrated in the left panel. The right panel depicts the other case. Even $G(x) > H(x)$ for some x, (A.6) is satisfied when $G(x) \leq H(x)$ for $x \in (0, x_F^*)$.

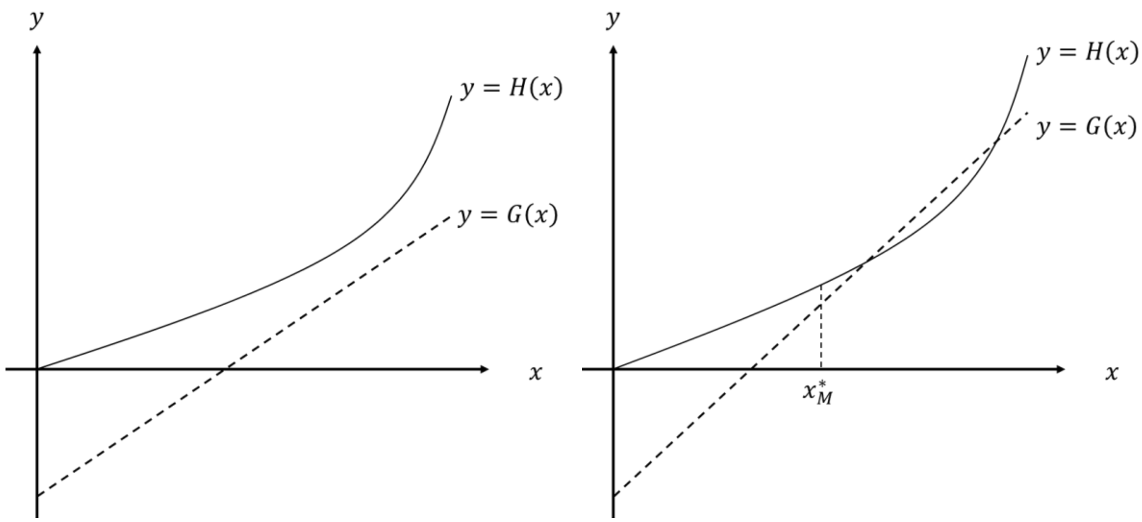

Figure 4 Illustration of (A.6)

Case 1: $G(x) \leq H(x)$ for all $x \geq 0$.

Define a positive constant k such that $G'(k) = H'(k)$. Then $H(x) - G(x)$ is minimized at k. Therefore, if $H(k) - G(k) \geq 0$, then $G(x) \leq H(x)$ for all $x \geq 0$.

Because $H'(x) = \frac{r_1}{r_1-1} I \left[(x_M^*)^{-r_1} + r_1 \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} (x_F^*)^{-r_1}\right] x^{r_1-1}$ and $G'(x) = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu}$, k satisfies $\frac{r_1}{r_1-1} I \left[(x_M^*)^{-r_1} + r_1 \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} (x_F^*)^{-r_1}\right] k^{r_1-1} = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu}$. Note that $H(k) = H'(k) \cdot \frac{k}{r_1}$. Therefore, $H(k) - G(k) = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu} \cdot \left(\frac{1-r_1}{r_1}\right) \cdot k + I$. The condition $H(k) - G(k) \geq 0$ implies

$$k \leq \frac{r_1}{r_1-1} \cdot \frac{\rho-\mu}{\pi(1,0)-\pi(0,0)} I = x_L^* \tag{A.7}$$

Since H is increasing convex function, $k \leq x_L^*$ is equivalent to $H'(k) \leq H'(x_L^*)$. By the construction, $H'(k) = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu}$ and $H'(x_L^*) = \frac{r_1}{r_1-1} I \cdot \frac{1}{x_L^*} \left\{ \left(\frac{x_L^*}{x_M^*}\right)^{r_1} - r_1 \frac{\pi(1,1)-\pi(1,0)}{\pi(1,1)-\pi(0,1)} \left(\frac{x_L^*}{x_F^*}\right)^{r_1} \right\}$.

Plugging (9), (11) and (12) into the equation, $H'(x_L^*) = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu} \left\{ \left(\frac{\pi(1,1)-\pi(0,0)}{\pi(1,0)-\pi(0,0)}\right)^{r_1} - r_1 \frac{\pi(1,1)-\pi(1,0)}{\pi(1,1)-\pi(0,1)} \left(\frac{\pi(1,1)-\pi(0,1)}{\pi(1,0)-\pi(0,0)}\right)^{r_1} \right\}$. Therefore, $H'(k) \leq H'(x_L^*)$ implies

$$1 < \left(\frac{\pi(1,1)-\pi(0,0)}{\pi(1,0)-\pi(0,0)}\right)^{r_1} + r_1 \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} \left(\frac{\pi(1,1)-\pi(0,1)}{\pi(1,0)-\pi(0,0)}\right)^{r_1}. \tag{A.8}$$

Case 2: $G(x) \leq H(x)$ for $x \in (0, x_F^*)$

This condition is equivalent to $G(x_F^*) \leq H(x_F^*)$ and $G'(x_F^*) \geq H'(x_F^*)$. Because $G(x_F^*) = \left[\frac{r_1}{r_1-1}\frac{\pi(1,0)-\pi(0,0)}{\pi(1,1)-\pi(0,1)} - 1\right] I$ and $H(x_F^*) = \frac{I}{r_1-1}\left[\left(\frac{x_F^*}{x_M^*}\right)^{r_1} + r_1\frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)}\right]$, $G(x_F^*) \leq H(x_F^*)$ is equivalent to

$$\frac{r_1}{r_1-1}\frac{\pi(1,1)-\pi(0,0)}{\pi(1,1)-\pi(0,1)} - 1 \leq \frac{1}{r_1-1}\left(\frac{\pi(1,1)-\pi(0,0)}{\pi(1,1)-\pi(0,1)}\right)^{r_1}. \tag{A.9}$$

Let $q_1(x) = \frac{r_1}{r_1-1}x - 1$ and $q_2(x) = \frac{1}{r_1-1}x^{r_1}$. Since $r_1 > 1$, q_2 is increasing convex for $x \geq 0$. Moreover $q_1(0) < q_2(0), q_1(1) = q_2(1)$ and $q_1'(1) = q_2'(1)$. Therefore, $q_1(x) < q_2(x)$ for all $x \in [0,1)$. Because of (2), $\frac{\pi(1,1)-\pi(0,0)}{\pi(1,1)-\pi(0,1)} \in (0,1)$. Therefore, (A.9), equivalently $G(x_F^*) \leq H(x_F^*)$, holds always. Note that $G'(x_F^*) = \frac{\pi(1,0)-\pi(0,0)}{\rho-\mu}$ and $H'(x_F^*) = \frac{\pi(1,1)-\pi(0,1)}{\rho-\mu}\left\{\left(\frac{\pi(1,1)-\pi(0,0)}{\pi(1,1)-\pi(0,1)}\right)^{r_1} + r_1\frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)}\right\}$. Hence, second case implies

$$\frac{\pi(1,0)-\pi(0,0)}{\pi(1,1)-\pi(0,1)} \geq \left(\frac{\pi(1,1)-\pi(0,0)}{\pi(1,1)-\pi(0,1)}\right)^{r_1} + r_1\frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,1)} \tag{A.10}$$

Up to this point, we have shown that bandwagon investment at $\tau_M^* = \inf\{t \geq 0 | X(t) \geq x_M^*\}$ is subgame perfect equilibrium if (A.8) or (A.10) holds. We will prove that if (A.10) holds, then (A.8) also holds. Hence, (A.10) is the only meaningful condition.

Let $a = \pi(1,0) - \pi(1,1)$, $b = \pi(1,1) - \pi(0,0)$, and $c = \pi(0,0) - \pi(0,1)$. We can rewrite (A.8) as $\frac{a+b}{b+c} \geq \left(\frac{b}{b+c}\right)^{r_1} + r_1\frac{a}{b+c}$. With some algebra, it is identical to $1 \leq \left(1 + \frac{c}{b}\right)^{r_1-1}\left(\frac{a}{b} + 1\right) - r_1 \cdot \frac{a}{b}\left(1 + \frac{c}{b}\right)^{r_1-1}$. By defining $\xi_L = \frac{a}{b} = \frac{\pi(1,0)-\pi(1,1)}{\pi(1,1)-\pi(0,0)}$ and $\xi_F = \frac{c}{b} = \frac{\pi(0,0)-\pi(1,1)}{\pi(1,1)-\pi(0,0)}$, (A.8) is equivalent to

$$\xi_F \geq \left(\frac{1}{(1-r_1)\xi_L + 1}\right)^{\frac{1}{r_1-1}} - 1 \text{ for } 0 < \xi_L < \frac{1}{r_1-1}. \tag{A.11}$$

With the similar procedure, (A.10) can be rewritten as

$$\xi_F \geq \left(\frac{(\xi_L+1)^{r_1}-1}{r_1 \cdot \xi_L}\right)^{\frac{1}{r_1-1}} - 1. \tag{A.12}$$

By showing $\left(\frac{(\xi_L+1)^{r_1}-1}{r_1 \cdot \xi_L}\right)^{\frac{1}{r_1-1}} > \left(\frac{1}{(1-r_1)\xi_L+1}\right)^{\frac{1}{r_1-1}}$, we will prove that (A.12) is the only meaningful condition for the bandwagon investment equilibrium. Because $r_1 > 1$, the following inequalities are equivalent to the claim for $0 < \xi_L < \frac{1}{r_1-1}$,

$$\begin{gathered} \frac{(\xi_L+1)^{r_1}-1}{r_1 \cdot \xi_L} > \frac{1}{(1-r_1)\xi_L+1} \\ \frac{1}{r_1 \cdot \xi_L\{(1-r_1)\xi_L+1\}}[(\xi_L+1)^{r_1}\{(1-r_1)\xi_L+1\} - \xi_L - 1] > 0 \\ (1-r_1)\xi_L + 1 > (\xi_L+1)^{1-r_1} \end{gathered} \tag{A.13}$$

Let $f_1(\xi_L) = (1-r_1)\xi_L + 1$ and $f_2(\xi_L) = (\xi_L+1)^{1-r_1}$. We can see that f_1 is linear decreasing function with $f_1(0) = 1$ and $f_1\left(\frac{1}{r_1-1}\right) = 0$, and f_2 is decreasing convex with $f_2(0) = 1$, $f_2'(0) = 1 - r_1$, and $f_2(\xi_L) > 0$ for all ξ_L. Therefore, (A.13) holds always. In sum, (A.12) is the condition for bandwagon investment equilibrium.

Our next task is identifying the sequential investment is the closed-loop equilibrium if the conditions are violated. Because (A.5) holds for every $x \in [x_F^*, x_M^*)$, the players have incentive to invest only when $X(t_0) \in \left[\underline{x}, x_F^*\right)$. Without loss of generality, assume that $L(x_L^*) > V(x_L^*)$ because $L(x)$ is maximized at x_L^*. If player i intend to invest at $\tau_L^* = \inf\{t \geq 0 | X(t) \geq x_L^*\}$, the player $-i$'s best response is $\tau_{L-\varepsilon} = \inf\{t \geq 0 | X(t) \geq x_L^* - \varepsilon\}$ because $F(x_L^*) < L(x_L^* - \varepsilon)$. This procedure is continued until a player intend to invest at $\underline{\tau} = \inf\left\{t \geq 0 \middle| X(t) \geq \underline{x}\right\}$. When $X(t_0) = \underline{x}$, $L\left(\underline{x}\right) = F\left(\underline{x}\right)$. Therefore, a player invests as the leader and the other plyer invests as the follower at $\tau_F^* = \inf\{t \geq 0 | X(t) \geq x_F^*\}$. We refer to Thijssen et al. (2012) and Boyarchenko and Levendorskiĭ (2014) for detailed proof of sequential investment equilibrium.

□