Discussion Paper
No. 2018-34 | April 19, 2018 | http://www.economics-ejournal.org/economics/discussionpapers/2018-34

Exchange rates expectations and chaotic dynamics: a replication study

Jorge Belaire-Franch

Abstract

In this paper the author analyzes the behavior of exchange rates expectations for four currencies, by considering a re-calculation and an extension of Resende and Zeidan (Expectations and chaotic dynamics: empirical evidence on exchange rates, *Economics Letters*, 2008). Considering Lyapunov exponent-based tests results, they are not supportive of chaos in exchange rates expectations, although the so-called 0−1 test strongly supports the chaos hypothesis.

(Published in Special Issue Replication Study)

JEL C12 C15
Keywords Deterministic chaos; exchange rates; expectations; Lyapunov exponents; 0−1 test

Authors
Jorge Belaire-Franch, University of Valencia, Spain, jorge.belaire@uv.es

The author declares that he has no relevant or material financial interests that relate to the research described in this paper.

Citation Jorge Belaire-Franch (2018). Exchange rates expectations and chaotic dynamics: a replication study. Economics Discussion Papers, No 2018-34, Kiel Institute for the World Economy.
http://www.economics-ejournal.org/economics/discussionpapers/2018-34

Received December 20, 2016 **Accepted as Economics Discussion Paper** April 18, 2018 **Published** April 19, 2018

1 Introduction

Resende and Zeidan (2008) test for deterministic chaos in exchange rates expectations for four currencies, by means of Fernández-Rodríguez et al. (2005) test, FSA hereafer. Their work, however, presents some formal drawbacks. In the next sections, We re-analyze the problem at hand more accurately and we apply alternative statistical techniques.

Methods are described on sections 2 through 4. Section 5 briefly reviews the original paper and section 6 presents the results with a discussion. Section 7 concludes.

2 FSA test

The null hypothesis of the test is deterministic chaos. Given a time series of length T, $\{y_t\}_{t=1}^{T}$, FSA suggest the estimation of the dominant Lyapunov exponent, λ, to be the bootstrap aggregation, or bagging, of 100 block bootstrap samples of the largest Lyapunov exponents.

Since the largest Lyapunov exponent does not increase when increasing the sample size in a deterministic process, but increases otherwise, FSA propose using $\langle\lambda_{max}(T_i)\rangle$ to test for the stability of the largest Lyapunov exponent, where T_i is the size of block i. This non-increasing property of the largest Lyapunov exponent with the sample size for chaotic processes may be tested by the regression:

$$\langle\lambda_{max}(T_i)\rangle = \alpha_0 + \alpha_1 T_i + \varepsilon_{T_i} \tag{1}$$

for $T_i = T_1, \ldots, T_r = T$, where r is the number of blocks. Now our interest is on testing the null hypothesis $H_0 : \alpha \leq 0$. The statistic is $\hat{\alpha}_1/s_1$ where s_1 is the standard error of $\hat{\alpha}_1$, computed through the Newey-West HAC covariance matrix estimator.[1]

In order to estimate the Lyapunov exponents, we will compute Rosenstein's et al. (1995) algorithm with the parameters selected as suggested in FSA (2005).

[1]We are grateful to Professor Andrada-Félix for providing the computer code.

3 Shintani and Linton (2004) test

Let $\{y_t\}_{t=1}^T$, be a random scalar sequence generated by the nonlinear autoregressive model:

$$y_t = \theta_0(y_{t-1}, \ldots, y_{t-d}) + u_t$$

where $\theta_0 : \mathbb{R}^d \to \mathbb{R}$ is a nonlinear dynamic map and $\{u_t\}$ is a sequence of random variables. The model can be expressed in terms of a map with an error vector $U_t = (u_t, 0, \ldots, 0)$ and the map function $F : \mathbb{R}^d \to \mathbb{R}^d$ such that

$$Z_t = F(Z_{t-1}) + U_t$$

where $Z_t = (y_t, \ldots, y_{t-d+1})' \in \mathbb{R}^d$. Let J_t be the Jacobian of the map F evaluated at Z_t. We can obtain the neural network estimator of the largest Lyapunov exponent. Now we distinguish between the sample size T used for estimating the Jacobian $\hat{J}_t$ and the block length M, which is the number of evaluation points used for estimating the Lyapunov exponent.

Our interest is to test the null hypothesis $H_0 : \lambda \geq 0$ against the alternative $H_1 : \lambda < 0$, therefore the test is one-sided. Given a consistent estimate tandard deviation of the estimator $\hat{\lambda}$, $\hat{\Phi}$, the test statistic is:

$$\hat{t} = \frac{\hat{\lambda}_M}{\sqrt{\hat{\Phi}/M}}$$

Under the null hypothesis, $\hat{t}$ is asymptotically distributed as a $N(0, 1)$. We compute the heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator.[2]

Three alternative selection procedures can be applied: Full, Block and ES (equally spaced subsamples, see Shintani and Linton, 2004, for the details). In all cases, given the number of hidden units (r), the lag length (d) has been selected based on the Bayesian information criterion. The quadratic spectral kernel with optimal bandwidth has been used for the heteroskedasticity and autocorrelation consistent covariance estimation.

4 The $0 - 1$ test

Gottwald and Melbourne (2004) designed the so-called $0 - 1$ test for chaos. Given an observation $\phi(j)$ for $j = 1, 2, \ldots, N$, the $0 - 1$ test algorithm com-

[2]We are grateful to Professor Shitani for providing the computer code.

prises the following steps:

1. For $c \in (0, \pi)$, compute the translation variables

$$p_c(n) = \sum_{j=1}^{n} \phi(j) \cos jc, \quad q_c(n) = \sum_{j=1}^{n} \phi(j) \sin jc,$$

   for $n = 1, 2, ..., N$.

2. Compute the mean square displacement of the transition variables for several values of $c \in (0, \pi)$ and for $n = 1, 2, \ldots, n_{\text{cut}}$, where $n_{\text{cut}} \ll N$:

$$D_c(n) = M_n(c) - V_{osc}(c, n)$$

   where

$$M_c(n) = \frac{1}{N} \sum_{j=1}^{N} [p_c(j+n) - p_c(j)]^2 + [q_c(j+n) - q_c(j)]^2$$

   and

$$V_{osc} = \left(\frac{1}{N} \sum_{j=1}^{N} \phi(j) \right)^2 \frac{1 - \cos nc}{1 - \cos c}$$

3. Form the vectors $\xi = (1, 2, \ldots n_{\text{cut}})$ and $\Delta = (D_c(1), D_c(2), \ldots, D_c(n_{\text{cut}}))$ and estimate the asymptotic growth rate as the sample correlation coefficient:

$$K_c = \text{corr}(\xi, \Delta) = \frac{\text{cov}(\xi, \Delta)}{\sqrt{\text{var}(\xi)\text{var}(\Delta)}}.$$

Webel (2012) computed the test on DAX asset returns, assuming that asset returns time series $\{r_t\}_{t=1}^{N}$ are defined by:

$$r_t = s_t + \varepsilon_t$$

where $\{s_t\}$ is the one-dimensional signal, an observable of an underlying $k-$dimensional deterministic dynamical system, and ε_t is the random noise. Webel suggests filtering out noise by wavelet denoising techniques, and then applying the 0-1 test to the denoised series. More specifically, Webel (2012) computed Maximal Overlap Discrete Wavelet Transforms (MODWT) for each returns series of 1655 observations, with four Daubechies filters. In this work we follow his guidelines, and compute the denoised original RZ residuals for two filters: Coiflet and Haar and a coarsest resolution chosen to be $[\log_2(167)] = 7$ and a soft thresholding rule with resolution dependent universal thresholds. For each filter, 100 random frequencies are drawn from a uniform distribution on [/8, 7/8]. Tests statistics are computed for each frequency, hence the cut-off value is set to $n_{\text{cut}} = 17$, as suggested by Gottwald and Melbourne (2009). The final statistic for each filter is the median of these 100 single frequency test statistics.

5 Resende and Zeidan (2008) results

On page 34 Resende and Zeidan's (2008) Table 1 displays the coefficient and the correspondig p-value. We assume that "coefficient" means $\hat{\alpha}_1$ for each currency. However, p-value computation deserves some concern.[3] As FSA argue, the null hypothesis (chaos) includes a wide range of deterministic chaotic processes. Thus, they compute some critical values for just two sample sizes: 380 and 2000 observations, by combining the empirical distribution of the statistics from 250 replications of three of those processes: Logistic, Henon and Lorenz. In Resende and Zeidan (2008), the sample size is 167 observations, hence they should compute the new empirical distribution. Nevertheless, there is no mention to this issue neither to the mentioned procedure to compute the p-values.[4]

6 Our results

The analyzed data, as in Resende and Zeidan (2008), correspond to New York Money Market Survey (NYMMS), regarding expected future exchange rates from 30 traders, at one week horizon, for the British pound (BP), the deutschemark (DM), the yen (JY) and the Swiss franc (SF). The data set comprises 168 observations for each currency. Provided the evidence about unit roots, we compute first differences to render the data stationary. Next, Resende and Zeidan (2008) compute the residuals from fitted ARMA models

[3]We will assume that the negative p-value for Japanese Yen and $m = 3$ is a typo.

[4]We asked the authors for the computer code. In a mail tracing back to 2008, they argued they hired someone to make the calculations. In a recent reply, they could not provide the software either, arguing it is a ten years old paper.

for each first-differenced currency time series, to calculate FSA test on such residuals. [5] We compute the statistics directly on the residuals series for embedding dimension ranging from 2 to 6.[6]

Table 1: FSA test.

		DIM2	DIM3	DIM4	DIM5	DIM6
JY	Coefficient (RZ residuals)	0.0017	0.0002	-0.0000	0.0002	0.0002
	Coefficient (RZ 2008)	0.0010	0.0008	0.0012	0.0005	-0.0001
	p-value (RZ residuals)	0.0000	0.0240	0.0070	0.0000	0.3390*
	p-value (RZ 2008)	0.0002	-0.0003	0.0005	0.0002	0.8865*
SF	Coefficient (RZ residuals)	0.0018	0.0013	0.0012	0.0012	0.0006
	Coefficient (RZ 2008)	0.0007	0.0008	0.0012	0.0005	0.0009
	p-value (RZ residuals)	0.0000	0.0000	0.0000	0.0000	0.0019
	p-value (RZ 2008)	0.0069	0.0003	0.0006	0.0012	0.0003
DM	Coefficient (RZ residuals)	0.0017	0.0018	0.0013	0.0013	0.0010
	Coefficient (RZ 2008)	0.0033	0.0017	0.0015	0.0017	0.0010
	p-value (RZ residuals)	0.0000	0.0000	0.0000	0.0000	0.0000
	p-value (RZ 2008)	0.0009	0.0000	0.0009	0.0007	0.0007
BP	Coefficient (RZ residuals)	0.0011	0.0011	0.0008	0.0011	0.0010
	Coefficient (RZ 2008)	0.0000	0.0014	0.0007	0.0009	0.0002
	p-value (RZ residuals)	0.0000	0.0000	0.0000	0.0000	0.0003
	p-value (RZ 2008)	0.0001	0.0006	0.0010	0.0626*	0.0016

DIM stands for embedding dimension. (RZ) are Resende and Zeidan (2008) results. * indicates chaos evidence.

In Resende and Zeidan (2008), there is an inconsistency between their Table 1 results and their conclusions. The computed p-values display evidence of chaos for BP at embedding dimension 5, and statistical evidence of chaos for JY at embedding dimension 6. However, they conclude that there is evidence of chaos for DM and JY. Our results do not confirm their results displayed in the text, either in their Table 1. We just find weak evidence of chaos for JY, for just one embedding dimension as well, but with a rather different p-value: 0.339 vs. 0.886.[7]

[5] The original residuals have been kindly provided by Rodrigo Zeidan.

[6] When fitting an ARMA(p,q) model, the sample size of the residuals is reduced depending on the number of AR lags of the fitted model. This reduction is not accounted for in Resende and Zeidan (2008).

[7] We contacted the original authors to reconcile the results, but were unable to deter-

Regarding Shintani and Linton (2004) test, the lag (d) and the number of hidden units (r) are jointly selected based on BIC. QS kernel with optimal bandwidth is used for the heteroskedasticity and autocorrelation consistent covariance estimation. Based on the Full estimation version of this test, Table 2, there is no evidence of chaos, just marginally for BP, confirming FSA test results. [8]

In addition, we compute the 0-1 test, Table 3. THe conclusions are sharply differente. Hence, with just one exception (JY with C(12)), the test statistics are close to one as they range from 0.9891 to 0.9980. They strongly confirm the hypothesis that fluctuations of exchange rates expectations are, at least, partially chaotic.

7 Conclusions

In this paper, we have revised and extended Resende and Zeidan (2008) results. We have stressed some important details of the testing process using FSA test. In addition we have applied Shintani and Linton test and the 0-1 test.

Results of FSA and Shintani and Linton test confirm Resende and Zeidan (2008) conclusions, hence exchange rates expectations are not chaotic. However, results with 0-1 test point towards chaotic motion (at least partially, after filtering out noise) in all exchange rates expectations.

8 References

Fernández-Rodríguez, F., Sosvilla-Rivero, S. and Andrada-Félix, J. 2005. Testing chaotic dynamics via Lyapunov exponents. *Journal of Applied Econometrics*, Vol 20, no. 7, 911-930.

Gottwald, G. A., Melbourne, I., 2004. A new test for chaos in deterministic systems. *Proc. R. Soc.* A 460. 603611.

mine the source of the discrepancies. It should be due to differences in any of the relevant parameters to compute the statistics, since the residuals series used in their work are the same that we have used in this paper.

[8]For the estimation based on blocks (Block) and ES, conclusions are identical and are available upon request.

Gottwald, G.A., Melbourne, I., 2009. On the implementation of the 01 test for chaos. *SIAM Journal on Applied Dynamical Systems* 8, 129145.

Resende, M., Zeidan, R. 2008. Expectations and chaotic dynamics: Empirical evidence on exchange rates. *Economics Letters*, Vol 99, no. 1, 33–35.

Rosenstein, M.T, Collins, J.J., De Luca, C.J. 1995. A practical method for calculating largest Lyapunov exponents from small data sets. *Physica D*, 65, 117–134.

Shintani, M. and Linton, O. 2004. Nonparametric neural network estimation of Lyapunov exponents and a direct test for chaos. *Journal of Econometrics*, Vol 120, no. 1, 1–33.

Webel, K., 2012.Chaos in German stock returns New evidence from the 01 test. *Economics Letters*, Vol. 115(3), 487-489.

Table 2: Shintani and Linton test. Full.

	(d, r)	(1,1)	(2,1)	(3,1)	(4,2)
BP	t value	-19.776	-18.883	-11.947	-0.842
	p-value	0.0000	0.000	0.000	0.200*
	(d, r)	(1,1)	(2,1)	(3,1)	(4,1)
DM	t value	-362.317	-22.984	-27.220	-9.598
	p-value	0.0000	0.000	0.000	0.0000
	(d, r)	(1,1)	(2,1)	(3,1)	(4,1)
JY	t value	-63.579	-17.383	-14.719	-17.407
	p-value	0.0000	0.000	0.000	0.0000
	(d, r)	(1,1)	(2,1)	(3,1)	(4,1)
SF	t value	-35.281	-15.795	-15.524	-9.351
	p-value	0.0000	0.000	0.000	0.0000

The t-statistics are presented with corresponding one-sided p-values for $H_0 : \lambda \geq 0$. * indicates chaos evidence.

Table 3: $0-1$ test.

	Haar	C(12)
BP	0.9974	0.9965
DM	0.9980	0.9891
JY	0.9877	0.7496
SF	0.9978	0.9958

The entries are the $0-1$ statistics. Haar is the Haar filter and C(12) is the Coiflet filter with filter length equal to 12.

Economics
The Open-Access, Open-Assessment E-Journal

Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:

http://www.economics-ejournal.org/economics/discussionpapers/2018-34

The Editor