A New Keynesian model with unemployment: the effect of on-the-job search

Zeynep Kantur and Kerim Keskin

Abstract
Although New Keynesian models with labor market frictions found an increase in unemployment and a decrease in labor market tightness in response to a positive technology shock (which appears to be in line with recent empirical findings), the volatilities of unemployment and labor market tightness are not as high as their empirical counterparts. This calls for the introduction of new tools that will amplify the volatilities of these variables. This paper contributes to the theoretical literature by studying the effect of employment-to-employment flows in a New Keynesian model with labor market frictions. In that regard, the authors assume two types of firms which offer different wage levels, thereby incentivizing low-paid agents to search on-the-job. Differently from the literature, the main source of wage dispersion is the assumption of different bargaining powers of firms motivated by the strength of labor unions. The authors show that the proposed model generates a higher volatility of unemployment and labor market tightness in response to a positive technology shock compared to the model without on-the-job search without causing a change in the responses of the other variables.

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Keywords New Keynesian model; employment-to-employment flow; unemployment fluctuations; the Shimer puzzle; search and matching

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1 Motivation

As highlighted by many scholars over the years, employer-to-employer flow is an important transition mechanism in the labor market and should not be disregarded in theoretical models. Fallick and Fleischman (2001), Nagypal (2008), and Bjelland et al. (2011) empirically showed that a significant part of the transitions in the labor market is employer-to-employer transitions.\footnote{Fallick and Fleischman (2001) found that on average 2.6\% of the employed agents change their jobs each month. This number corresponds to the double of employment-to-unemployment flow. Nagypal (2008) reported that almost half of the separations is job-to-job transitions. In a recent paper, two ratios are explicitly calculated by Bjelland et al. (2011) using the recently developed longitudinal linked employer and employee data of U.S. from the Census Bureau’s LEHD program. It is discovered that the ratio of the number of people experiencing an employer-to-employer transition to the total number of employees is 4.1\% and the ratio of the same to the total number of people separating from their jobs is 27.3\%. In earlier related studies, it is estimated by Blanchard and Diamond (1990) that hirings from other jobs is about 50\% of hirings from unemployment, and it is pointed out by Pissarides (1994) that the fraction of new hires coming from other jobs is approximately 20\% in the U.S. and 40\% in the U.K..} Fujita (2010) reported some basic statistics of on-the-job search activity in U.K. using Labour Force Survey (LFS) providing a valuable source to get stylized facts about on-the-job search activity. Comparing the unemployment rate with the ratio of on-the-job searchers to the employed agents, the paper shows that on average 5.5\% of the employed workers participate in on-the-job search activity in U.K. during the period 2002Q1 2009Q2, which is higher than the 5\% unemployment rate for the aforementioned period.

Before the influential and spurring study by Shimer (2005), the bulk of relevant literature ignored employer-to-employer flow and mainly focused on the transitions from unemployment to employment. Shimer (2005) argued that the search and matching (SM) model of Mortensen-Pissarides is incompetent to generate observed fluctuations in the unemployment rate and labor market tightness in response to a positive productivity shock. Stemming from this idea, some scholars concentrated on finding a way to amplify the impact of productivity shock on the unemployment rate. For example, in order to match the

In SM-based models, a permanent positive productivity shock leads to a decrease in unemployment. Recently, this finding is challenged by several papers relying on an argument that the identification of productivity shock is problematic in SM-based models. Among these papers, Barnichon (2010) estimated the impact of a positive technology shock on labor market variables using structural VAR with long-run restrictions as in Galí (1999), and found that a permanent increase in labor productivity (i.e., output per hour)\(^2\) leads to an increase in unemployment and to a decrease in labor market tightness. In two other studies, Canova et al. (2009) utilized different labor market variables, whereas Galí (2010) employed structural VAR in a five-variable model. The findings in both of these papers are similar to those of Barnichon (2010). Motivated by these results, we study a New Keynesian (NK) model in this paper formulating an extension to the model of Galí (2010) by introducing on-the-job search.\(^3\)

There is a bulk of theoretical studies focusing on the effect of job-to-job transitions on the real economy. For instance, Pissarides (1994) introduced on-the-job search which takes place only at short job tenures because of the accumulation of job-specific human capital, and found an increased jumpiness in vacancies and a dampened response of unemployment to changes in aggregate economic condition. Shimer (2006) focused on subgame perfect equilibria for the bargaining stage, noting that the standard strategic bargaining

\(^2\)Since labor is the sole input of the production function, the terms labor productivity and output per hour can be interchangeably used.

\(^3\)This is a highly tractable model allowing for a relatively simple and transparent analysis given that the related studies in the literature have richer models which are commonly solved through simulations. It is worth noting that a very similar model is used by Blanchard and Galí (2010). And, the reader is referred to Section 6 of Blanchard and Galí (2010) for a detailed literature review on studies that combine certain key elements of NK and SM models.
solutions are inapplicable due to the non-convexity of the set of feasible payoffs; and he characterized market equilibria in which more productive firms pay higher wages and analyzed the quantitative predictions of his model. Cahuc et al. (2006) formulated a model with strategic wage bargaining, on-the-job search, and counteroffers; and they estimated the influence of productivity, bargaining power, and between-firm competition on wages. Finally, Krause and Lubik (2007b) utilized a model with two types of firms and showed that on-the-job search is crucial for explaining the observed cyclical upgrading of workers to better employment opportunities in booms.

In a similar line of research, Van Zandweghe (2010) and Krause and Lubik (2007a) integrated on-the-job search into a business cycle model. Van Zandweghe (2010) introduced price stickiness into the SM model of Krause and Lubik (2006), mainly concentrated on the monetary policy implications, and found that on-the-job search dampens the responses of real marginal cost and inflation after a tightening monetary policy shock. Krause and Lubik (2007a) used a similar model, but assumed that matches become productive with a lag. They found that on-the-job search amplifies the number of posted vacancies which leads to a decrease in unemployment in response to a positive technology shock. The findings in these papers contradict with the aforementioned results of Barnichon (2010) and Galí (2010).

To the best of our knowledge, we present the first business cycle model with on-the-job search which builds on the empirical finding that the levels of unemployment and productivity are positively correlated. We differ from the existing on-the-job search models not only in our motivation, but also in our way of model construction. The majority of earlier studies, including Van Zandweghe (2010) and Krause and Lubik (2007a), created wage dispersion by introducing different cost levels for different type of firms. Although we preserve this assumption in our model, there is an additional source of wage dispersion:
the difference in bargaining powers of firms. As a matter of fact, we state this as the main source of wage dispersion. For this assumption, labor unions constitute a good motivation and support. It is well-known that individuals and firms are not the only actors in the labor market; but there are also intermediate associations whose aim is to protect their members’ rights and to attain higher wage and lower unemployment levels for their members. In our model, we initially assume that there are two types of unions: weak union and strong union. Given this assumption, a firm operating in a sector associated with the weak union would have relatively higher bargaining power in comparison to that operating in a sector associated with the strong union. This one-to-one relation becomes helpful in simplifying the framework. In particular, it helps us to remove unions from the model considering the fact that unions would only have an indirect influence for which we do not have a particular interest. Accordingly, we argue that there are two types of firms; the ones facing a weak union and the ones facing a strong union. For the sake of simplicity, these types are referred to as aggressive and passive, respectively. As it is turns out, this simplified model is enough to capture the fluctuations in the labor market.4

As shown by Barnichon (2010), a NK model with search frictions is capable of fulfilling two arguments of the Shimer puzzle: A positive technology shock leads (i) to an increase in the unemployment rate and (ii) to a decrease in labor market tightness. Be that as it may, it is also discussed by the author that the Shimer puzzle is still visible because the magnitude of responses is significantly below its empirical counterparts. The theoretical model we propose in this paper can be thought of as a step forward in addressing this shortcoming since the responses of the unemployment rate and labor market tightness are amplified in our model. The interpretation is: In the standard NK model, firms are demand constrained so that an increase in productivity leads to a sluggish adjustment in aggregate

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4It is worth noting that although the model does not include unions per se, such an interpretation always exists due to the aforementioned one-to-one relation between the types of unions and the types of firms.
demand to the new productivity level due to nominal rigidities. Accordingly, firms employ less labor during this process. Hence, a positive change in technology leads to an increase in unemployment. When on-the-job search is introduced, a positive technology shock leads not only to a decrease in the flow of unemployment-to-employment, but also to an increase in the flow of employment-to-employment. The job finding ratio of on-the-job searchers is procyclical and the posted vacancies are mostly filled by on-the-job searchers rather than unemployed. As a result, on-the-job search fundamentally amplifies the responses of the unemployment rate and labor market tightness. The proposed model achieves this without leading to significant differences between the responses in the models with and without on-the-job search. This outcome fulfills our aim of increasing the volatilities of unemployment and labor market tightness without leading to another change in the model dynamics.

The structure of the paper is as follows. In Section 2, we formulate the model specifying the differences between our model and that of Galí (2010). Moreover, we solve the model and present the calibration values which will be used in the following section. We report our results in Section 3 and conclude in Section 4.

2 Model with On-the-job Search

In this section, we extend the model of Galí (2010) by introducing on-the-job search for a particular group of households. In that regard, we assume two types of firms offering different wage levels. Workers who earn relatively less would be willing to search for better-paid jobs. In this paper, unlike the existing literature, we capture wage dispersion through heterogeneity in bargaining powers of firms. We assume two types of firms: aggressive (A) and passive (P). The bargaining power of aggressive firms over workers is assumed to be greater than that of passive firms. As a result, passive firms offer higher wage levels compared to aggressive firms.
There are five assumptions about job search in this paper:

(i) Search is *indirect*: Job-seekers do not know the type of firms during job search.

(ii) The outside option of individuals is unemployment regardless of their prior-to-search state in the labor market. Put differently, if an on-the-job searcher is matched with a new firm, bargaining process does not start unless the individual resigns from his/her job.\(^5\)

(iii) Wages are *flexible*: A worker’s wage is updated every period as if he/she is newly matched. Hence, a worker at a passive firm has no incentive to do on-the-job search. It then follows as a fact that an on-the-job searcher was a worker at an aggressive firm at the end of the previous period.

(iv) It is obvious that if an on-the-job searcher matches with a passive firm, then he/she prefers to resign from his/her existing job. If the new match is an aggressive firm, however, he/she would be indifferent. Here we assume that a matched on-the-job searcher resigns for sure to negotiate with the new match.\(^6\)

(v) The job finding rates are different for on-the-job searchers (denoted by \(p_t\)) and for unemployed individuals (denoted by \(q_t\)).

The sequence of events is as follows: At the beginning of a period, firms announce vacancies. Unemployed agents and workers at an aggressive firm search for jobs. Both types of job search take place at the same time. Afterwards, matched parties start bargaining wages.\(^7\) If both parties agree on a wage level, then the individual starts working at that

\(^5\)This assumption is, in fact, quite standard in the literature. A technical reason behind this assumption is that if the outside option of an on-the-job searcher is his/her current job, then there would be a continuum of wage levels which harms the simplicity of the model.

\(^6\)This assumption does not cause a qualitative difference in our results.

\(^7\)We determine wages via a simple surplus splitting rule.
particular period. Individuals work until the end of the period at which they may get separated by an exogenous rate of \( \delta \in [0,1] \). Those who are separated are unemployed at the beginning of the next period. If an individual does not get separated, he/she works at the same firm in the next period; unless he/she searches on-the-job and is matched with a new firm.

Finally, we assume that only a fraction \( \varphi \in [0,1] \) of workers can search on-the-job. Notice that, left to themselves, all individuals working at an aggressive firm would prefer to search on-the-job for the prospect of a wage increase. However, in real life, on-the-job search has additional costs and frictions in comparison to job search by an unemployed individual. Such additional frictions are not explicitly modeled in this paper. Instead we refer to this on-the-job search intensity parameter to capture those frictions. A low \( \varphi \) implies that workers face too many frictions, so that only a small portion of them can search on-the-job even if all of them want to.

2.1 Households

We assume that the economy is populated by a continuum of households of measure one and that the representative household is a member of a large family.\(^8\) The large family assumption enables us to assume full risk sharing within the family and helps us avoid distributional issues which may arise due to heterogeneity of firms. The representative family maximizes the objective function

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \log(C_t) - \chi \frac{N_t^{1+\phi}}{1+\phi} \right) \right\}
\]

where \( \beta \in [0,1] \) is the discount factor and \( \phi \) indicates the inverse of the Frisch labor supply elasticity. The standard utility function shows that agents get utility from consumption

\(^8\)See Merz (1995).
and disutility from supplying labor, and that there is no burden of unemployed agents in terms of utility. The aggregate consumption level for different types of consumption goods are denoted by

\[ C_t = \left( \int_0^1 C_t(i) \frac{1}{\epsilon} di \right)^\frac{1}{1-\epsilon}, \]

and the fraction of employed agents is shown by \( N_t \). In this model, we assume full participation of households meaning that all agents are either employed or unemployed (but willing to work). This implies that the fraction of unemployed agents is \( u_t = 1 - N_t \).

The budget constraint of the family is given by

\[ \int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t^A(j) N_t^A(j) dj + \int_0^1 W_t^P(j) N_t^P(j) dj + \Pi_t \]

where \( P_t(i) \) is the price of good \( i \) and \( B_t \) denotes one-period riskless nominal bond holdings of a family member at the price of \( Q_t \). The nominal wage level paid by aggressive and passive firms are denoted by \( W_t^A \) and \( W_t^P \), respectively. \( N_t^A \) and \( N_t^P \) are the fraction of employed agents working at aggressive and passive firms, respectively. The transfers and profit of final good firms are embedded in \( \Pi_t \).

The optimal condition for individual goods is given by

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad (1) \]

where

\[ P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^\frac{1}{1-\epsilon} \]

is the price index for consumption goods. Consequently, the Euler equation is given by

\[ Q_t = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\}. \]
It is worth noting that in this model, differently from the standard business cycle models, wages are not adjusted according to the labor supply decision of the household. As an attribute of the SM model, wages are demand-determined and set according to a bilateral bargaining between workers and firms.

2.2 Firms

There are intermediate goods and final goods sectors. Unlike the conventional NK models, price stickiness is introduced at the final good production stage.\(^9\) Intermediate goods sector is perfectly competitive and the sole factor of production is labor.

2.2.1 Final Goods Producers

The final goods sector is monopolistically competitive, and firms are indexed by \(i \in [0, 1]\). They produce differentiated goods by utilizing an identical technology and use intermediate goods produced by aggressive and passive firms as inputs. The production function of a final good producer is

\[
Y_t(i) = (Z_{tA}(i))^\gamma(Z_{tP}(i))^{1-\gamma}
\]

where \(Z_{tA}(i)\) and \(Z_{tP}(i)\) denote the quantity of intermediate goods produced by aggressive and passive firms, respectively. The weight parameter \(\gamma \in (0, 1)\) is the share of intermediate goods produced by aggressive firms.

Under the assumption of flexible prices, final good producers would set the price of their good optimally subject to the demand equation (1) at each period. Therefore, the profit maximization condition suggests that \(P_t(i) = \mathcal{M} \cdot MC_t\) where \(MC_t\) denotes the real marginal cost and \(\mathcal{M} = \frac{\epsilon - 1}{\epsilon}\) is the desired markup. The quantity demanded for

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\(^9\)The assumption is first proposed by Walsh (2005) and used by Blanchard and Galí (2010), Galí (2010), and other papers with labor market frictions in DSGE models.
intermediate goods $Z_t^A$ and $Z_t^P$ are given by

$$Z_t^A(i) = \gamma \frac{P_t(i)}{P_t^A} Y_t(i)$$

$$Z_t^P(i) = (1 - \gamma) \frac{P_t(i)}{P_t^P} Y_t(i)$$

where $P_t^A$ and $P_t^P$ are the price levels of intermediate goods produced by aggressive and passive firms, respectively. Accordingly, the real marginal cost of production is the weighted average of input prices:

$$MC_t = \left( \frac{P_t^A}{\gamma P_t} \right)^\gamma \left( \frac{P_t^P}{(1-\gamma)P_t} \right)^{1-\gamma}.$$  \hspace{1cm} (2)

Final good producers set the price of their goods to maximize the expected discounted profits due to Calvo (1983) type price-setting. At each period, a firm is able to reset its price with probability $1 - \theta$. This implies that the price levels of the $\theta$ fraction of final good producers remain constant in any given period. The re-optimizing firms’ price level is

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_0^{\infty} \theta^k \Lambda_{t,t+k} P_{t+k}^* Y_{t+k} MC_{t+k}}{\sum_0^{\infty} \theta^k \Lambda_{t,t+k} P_{t+k}^{* - 1} Y_{t+k}}$$  \hspace{1cm} (3)

where $\Lambda_{t,t+k} = \beta^k (C_t/C_{t+k})$ denotes the stochastic discount factor.

2.2.2 Intermediate Goods Producers

There are two types of intermediate goods firms in a perfectly competitive environment: aggressive and passive firms. These firms differ in their bargaining powers under the assumption that the (relative) bargaining power of aggressive firms over workers is higher compared to that of passive firms. The technology $A_t$ is exogenous and common to all types of firms.
The production functions of aggressive and passive firms are, respectively, given by

\[ Z_t^A = A_t N_t^A \quad \text{and} \quad Z_t^P = A_t N_t^P. \]

As in the model of Barnichon (2010), since labor is the sole input of the production functions, the terms ‘labor productivity’ and ‘output per hour’ can be interchangeably used in our model.

Employment in aggressive and passive firms evolve according to

\[ N_t^A = (1 - \delta) N_{t-1}^A + H_t^A \quad \text{and} \quad N_t^P = (1 - \delta) N_{t-1}^P + H_t^P \]

where \( H_t^A \) and \( H_t^P \) are the newly hired agents in aggressive and passive firms at time \( t \), respectively. Finally, as in Blanchard and Galí (2010) and Galí (2010), we assume that new hires start working in the period they are hired.

2.2.3 Labor Market

As mentioned earlier, the main difference between the two types of intermediate good producers is their bargaining powers over workers. The bargaining power of aggressive firms (\( \xi^A \)) is greater than that of passive firms (\( \xi^P \)). Given the above-defined production functions, the wage level in aggressive firms turns out to be less than the wage level in passive firms. As a consequence, agents working at aggressive firms prefer searching for new jobs hoping to match with a passive firm; but only a fraction, \( \varphi \), of these workers is allowed.

At the beginning of period \( t \), there is a pool of on-the-job searchers and unemployed
agents which are respectively denoted by $OJS_t$ and $U^0_t$. Thus, we have

\[ OJS_t = \varphi (1 - \delta) N^A_{t-1} \quad \text{and} \quad U^0_t = 1 - (1 - \delta) N_{t-1}. \]

Accordingly, we can define job searchers at period $t$ as $Pool_t = U^0_t + OJS_t$. Total hiring at period $t$ is denoted by $H_t$. Differently from Blanchard and Galí (2010) and Galí (2010), the number of posted vacancies is not equal to the number of newly hired workers in aggressive firms. The reason is that there is a reallocation of workers in aggressive firms. In particular, we can define the posted vacancies in aggressive firms as

\[ V_t^A = H_t^A + H_t^o \]

where $H_t^A$ indicates the change in the number of workers in aggressive firms and $H_t^o$ is defined as $q_t OJS_t$. On the passive firm side, however, the number of posted vacancies is equal to the number of newly hired workers since there is no reallocation of workers. Therefore, we have $V_t^P = H_t^P$.

In our model, labor market tightness and the “average” job finding rate of job searchers can be interchangeably used. They are defined as

\[ x_t = \frac{H_t}{Pool_t} = \frac{H_t^o + H_t^u}{U^0_t + OJS_t} \]

where $H_t^u = q_t U^0_t$ is the fraction of newly hired workers from the unemployment pool. Moreover, we define the end-of-period unemployment rate as $U_t = 1 - N_t$.

Following Blanchard and Galí (2010) and Galí (2010), the cost for posting a vacancy is defined in terms of the CES bundle of final goods.\(^{10}\) The cost per vacancy is an increasing

\(^{10}\)In Blanchard and Galí (2010) and Galí (2010), the number of vacancies is equal to the number of newly hired workers. However, in our model, they are different due to the reallocation of newly hired on-the-job searchers. Therefore, we use “cost for posting a vacancy” rather than the concept of cost for hiring.
function of the technology level and the corresponding ratio of the posted vacancies to the pool of job searchers. In particular, we assume

\[ G^A_t = A_t B \left( \frac{V^A_t}{P_{\text{pool}_t}} \right)^\alpha \quad \text{and} \quad G^P_t = A_t B \left( \frac{H^P_t}{P_{\text{pool}_t}} \right)^\alpha \]

where \( \alpha \geq 0 \) and \( B \) is a positive constant.\(^{11}\)

### 2.2.4 Price Setting

Let \( P^A_t \) and \( P^P_t \) denote the price levels of the intermediate goods produced by aggressive and passive firms, respectively. These prices are taken as given. Moreover, let \( W^A_t \) and \( W^P_t \) represent the nominal wage levels of aggressive and passive firms, respectively. Profit maximization requires the following conditions to be satisfied for all \( t \):

\[
\left( \frac{P^A_t}{P_t} \right) A_t = \frac{W^A_t}{P_t} + G^A_t - \beta (1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} (1 - \varphi_{p_{t+1}}) G^A_{t+1} \right\} \quad (4)
\]

\[
\left( \frac{P^P_t}{P_t} \right) A_t = \frac{W^P_t}{P_t} + G^P_t - \beta (1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} G^P_{t+1} \right\} \quad (5)
\]

Note that the LHS of the equations (4) and (5) represent the real marginal product of labor, and their RHS denote the real marginal cost including vacancy posting costs. It is worth noting that the equation (5) resembles the findings of Galí (2010). However, our introduction of on-the-job search leads an additional term to appear in the equation (4).

We can define the net vacancy posting costs of aggressive and passive firms as

\[
B^A_t = G^A_t - (1 - \delta) E_t \left\{ \Lambda_{t,t+1} (1 - \varphi_{p_{t+1}}) G^A_{t+1} \right\}
\]

\[
B^P_t = G^P_t - (1 - \delta) E_t \left\{ \Lambda_{t,t+1} G^P_{t+1} \right\}
\]

\(^{11}\)The reader is referred to Blanchard and Galí (2010) and Galí (2010) for a discussion on this issue.
As a result, ceteris paribus, an increase in $\varphi$ leads to an increase in the net vacancy posting cost of aggressive firms. We then rewrite the equations (4) and (5) as follows:

$$\left( \frac{P_A}{P_t} \right) A_t = \frac{W_A^t}{P_t} + B_t^A \tag{6}$$

$$\left( \frac{P_P}{P_t} \right) A_t = \frac{W_P^t}{P_t} + B_t^P \tag{7}$$

Finally, we describe the price dynamics. Plugging the log-linearized version of the equation (3) into the (log-linearized) law of motion for the aggregate price level,\textsuperscript{12} we have

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^*$$

where $p_t$ denotes the log-linearized aggregate price level.\textsuperscript{13} The dynamic Phillips equation is derived as

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \kappa \hat{\mu}_t$$

where $\kappa = (1 - \theta)(1 - \theta\beta)/\theta$ and $\hat{\mu}_t$ denotes the deviation of the average price markup from its steady state value. We write the marginal cost by log-linearizing the equation (2) and plugging it into $\hat{\mu}_t = p_t - mc_t - \mu$, which is derived from the log-linearization of $P_t(i) = M \cdot MC_t$. It then follows that

$$\hat{\mu}_t = \mu_t - \mu = \gamma \mu_t^A + (1 - \gamma)\mu_t^P - \mu.$$

\textsuperscript{12}See Chapter 3 of Galí (2008) for log-linearization and detailed derivations.

\textsuperscript{13}In this paper, we use lower case letters for the log transformations of the corresponding variables. However, since $p_t$ denotes the job finding rate of on-the-job searchers, we make an exception for the log-linearized aggregate price level.
\textbf{2.2.5 Wage Determination}

When on-the-job search is introduced, the set of feasible payoffs is typically non-convex. Hence, the axiomatic Nash bargaining solution and the standard strategic bargaining solutions may be inapplicable (see Shimer, 2006, pg. 815). Keeping this in mind, we impose a surplus splitting rule.

We assume that wages are \textit{flexible}. That is, each type of firm negotiates the wage level with its workers every period. For a representative family member, the expected value resulting from being a worker at an aggressive firm is given by

\begin{equation}
V_{t+1}^{NA} = \frac{W_{t+1}^A}{P_t} - \chi C_t N_t^\phi + E_t \left\{ \Lambda_{t,t+1} \left[ \delta(1-q_{t+1})V_{t+1}^U + \right.ight.
\left. + \left( \delta q_{t+1}\tau_{t+1} + (1-\delta)(\varphi(p_{t+1}\tau_{t+1} + (1-p_{t+1})) + (1-\varphi)) \right) V_{t+1}^{NA} \right.
\left. + \left( \delta q_{t+1}(1-\tau_{t+1}) + (1-\delta)\varphi p_{t+1}(1-\tau_{t+1}) \right) V_{t+1}^{NP} \right\}.
\end{equation}

where \( \tau_t \) is the probability of a job searcher matching with an aggressive firm. For a representative family member, the expected value resulting from being a worker at a passive firm is given by

\begin{equation}
V_{t+1}^{NP} = \frac{W_{t+1}^P}{P_t} - \chi C_t N_t^\phi + E_t \left\{ \Lambda_{t,t+1} \left[ \delta(1-q_{t+1})V_{t+1}^U + \right.ight.
\left. + \left( \delta q_{t+1}\tau_{t+1} \right) V_{t+1}^{NA} \right.
\left. + \left( \delta q_{t+1}(1-\tau_{t+1}) + (1-\delta) \right) V_{t+1}^{NP} \right\}.
\end{equation}

Moreover, the expected value of an unemployed agent is

\begin{equation}
V_{t+1}^U = E_t \left\{ \Lambda_{t,t+1} \left[ (1-q_{t+1})V_{t+1}^U + q_{t+1}(\tau_{t+1}V_{t+1}^{NA} + (1-\tau_{t+1})V_{t+1}^{NP}) \right] \right\}.
\end{equation}

Accordingly, we define the surplus of a representative family member resulting from an
employment relationship with an aggressive firm as $S_{t}^{HA} = V_{t}^{NA} - V_{t}^{U}$ and that with a passive firm as $S_{t}^{HP} = V_{t}^{NP} - V_{t}^{U}$, which in turn implies the following equations:

\[ S_{t}^{HA} = \frac{W_{t}^{A}}{P_{t}} - \chi C_{t} N_{t}^{\phi} + (1 - \delta) E_{t} \left\{ \Lambda_{t,t+1} \left[ \left( \varphi p_{t+1}(\tau_{t+1} - 1) + 1 - q_{t+1}\tau_{t+1} \right) S_{t+1}^{HA} + \left( \varphi p_{t+1}(1 - \tau_{t+1}) - q_{t+1}(1 - \tau_{t+1}) \right) S_{t+1}^{HP} \right] \right\}. \]

\[ S_{t}^{HP} = \frac{W_{t}^{P}}{P_{t}} - \chi C_{t} N_{t}^{\phi} + (1 - \delta) E_{t} \left\{ \Lambda_{t,t+1} \left[ \left( 1 - q_{t+1}(1 - \tau_{t+1}) \right) S_{t+1}^{HP} - \left( q_{t+1}\tau_{t+1} \right) S_{t+1}^{HA} \right] \right\}. \]

The surpluses of aggressive and passive firms from the profit maximization conditions, which are respectively denoted by $S_{t}^{FA}$ and $S_{t}^{FP}$, are given by

\[ S_{t}^{FA} = MRPN_{t}^{A} - \frac{W_{t}^{A}}{P_{t}} + (1 - \delta) E_{t} \left\{ \Lambda_{t,t+1}(1 - \varphi p_{t+1}) S_{t+1}^{FA} \right\} \]  (8)

\[ S_{t}^{FP} = MRPN_{t}^{P} - \frac{W_{t}^{P}}{P_{t}} + (1 - \delta) E_{t} \left\{ \Lambda_{t,t+1} S_{t+1}^{FP} \right\} \]  (9)

where $MRPN_{t}^{A} = (P_{t}^{A}/P_{t})A_{t}$ and $MRPN_{t}^{P} = (P_{t}^{P}/P_{t})A_{t}$ stand for the marginal productivity of labor in aggressive and passive firms, respectively. It follows from the equations (4) and (8) that $S_{t}^{FA} = G_{t}^{A}$ and from the equations (5) and (9) that $S_{t}^{FP} = G_{t}^{P}$.

Recalling that $\xi^{A}$ and $\xi^{P}$ respectively denote the bargaining powers of aggressive and passive firms, the surplus splitting rule stipulates that firms and workers determine the wage levels according to the following maximization problems:

\[ \max_{W_{t}^{A}}(S_{t}^{HA}(j))^{1-\xi^{A}}(S_{t}^{F}(j))^{\xi^{A}} \]

\[ \max_{W_{t}^{P}}(S_{t}^{HP}(j))^{1-\xi^{P}}(S_{t}^{F}(j))^{\xi^{P}} \]
subject to the corresponding value functions. The solutions to these maximization problems are as follows:

\[(1 - \xi^A)S_t^F(j) = \xi^A S_t^{HA}(j) \quad \text{ and } \quad (1 - \xi^P)S_t^F(j) = \xi^P S_t^{HP}(j).\]

The real wage levels in aggressive and passive firms are

\[
\frac{W_t^A}{P_t} = MRS_t + \eta^A \left( G_t^A - \beta(1 - \delta)E_t \left\{ \frac{C_t}{C_{t+1}}(\varphi p_{t+1}(\tau_{t+1} - 1) + 1 - q_{t+1}\tau_{t+1})G_{t+1}^A \right\} \right) \\
- \eta^P \left( \beta(1 - \delta)E_t \left\{ \frac{C_t}{C_{t+1}}(\varphi p_{t+1}(1 - \tau_{t+1}) - q_{t+1}(1 - \tau_{t+1}))G_{t+1}^P \right\} \right) \tag{10}
\]

and

\[
\frac{W_t^P}{P_t} = MRS_t + \eta^P \left( G_t^P - \beta(1 - \delta)E_t \left\{ \frac{C_t}{C_{t+1}}(1 - q_{t+1}(1 - \tau_{t+1}))G_{t+1}^P \right\} \right) \\
+ \eta^A \left( \beta(1 - \delta)E_t \left\{ \frac{C_t}{C_{t+1}}(q_{t+1}\tau_{t+1})G_{t+1}^A \right\} \right) \tag{11}
\]

where \(\eta^A = (1 - \xi^A)/\xi^A\) and \(\eta^P = (1 - \xi^P)/\xi^P\).

In SM models the surplus splitting rule implies that the wage equation is a convex combination of marginal rate of substitution and marginal productivity of labor. However, due to firm heterogeneity and the introduction of on-the-job search, we have additional terms in the wage equations. In both equations the probability of a job searcher matching with an aggressive firm, \(\tau_t\), appears owing to the indirect search assumption. In addition to that, since only the workers in aggressive firms can search on-the-job, the on-the-job search intensity, \(\varphi\), exists in equation (10). In particular, an increase in the ratio of on-the-job searchers leads to a decrease in the wage level in aggressive firms. The reason is that high ratio of on-the-job searchers causes an increase in the future cost of hiring in aggressive firms, thereby leading aggressive firms to decrease their current period’s labor
cost considering higher future hiring costs.

2.3 Technological Process and Monetary Policy

The monetary policy is assumed to follow Taylor type interest rule represented by

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \]

where \( i_t \) is the nominal interest rate and \( \rho = -\log \beta \). We assume that \( a_t = \log A_t \) follows an \( AR(1) \) process

\[ a_t = \rho_a a_{t-1} + \epsilon^a_t \]

where \( \epsilon^a_t \) denotes the technology shock and \( \rho_a \) is the autoregressive coefficient of the technological process.

2.4 Market Clearing Conditions and Solving the Model

In this section, we emphasize certain characteristics of the model. First, the steady state is independent of the monetary policy rule and the degree of price stickiness. Second, we assume that the level of technology \( A = 1 \) at the steady state. Finally, following Blanchard and Galí (2010) and Galí (2010), we assume that hiring costs are paid in terms of final goods. This implies that the goods market clearing condition is

\[ Y_t = C_t + G^A_t V^A_t + G^P_t V^P_t \]

at the equilibrium. To solve the model, we log-linearize the system of equations around zero inflation steady state. See the Appendix for the log-linearized equations.
2.5 Calibration and the Steady State

In this section, the calibration of model parameters is illustrated. Here we use the conventional parameter values if they do not contradict with the model. All parameters are determined according to quarterly values. We set the discount factor $\beta$ to 0.99 and the inverse of the Frisch labor supply elasticity $\phi$ to 5. The parameter for price stickiness is set to be its average duration in one year: $\theta = 0.75$. The gross markup of prices over marginal cost value $M$ is taken to be 1. Following Blanchard and Galí (2010), we set the parameter $\alpha$ in the hiring cost function to 1. Moreover, having no evidence on the share of intermediate goods produced by aggressive firms in the production of final goods, we set $\gamma$ to 0.5.

To calibrate the labor market parameters, we first pin down the steady state values of unemployment and the job finding rate of unemployed agents using the average values in the U.K. economy. Accordingly, the average value for $u$ is 0.05 and the approximate value for $q$ is 0.25. Since we assume full participation of agents, we have $N = 1 - u = 0.95$. Furthermore, since there is no hard evidence on the number of aggressive firms, we assume that $N^A = N^P = 0.475$. The separation rate $\delta$ is calculated using $\delta = qu/(1-q)N \approx 0.02$. As for the coefficients in the Taylor rule, we take the calibration values from the literature. In particular, we set $\phi_\pi = 1.5$ and $\phi_y = 0.125$.

We now turn to the calibration of the parameters related to on-the-job search. To do so, we use the values reported by Fujita (2010). We set the ratio of on-the-job searchers to the employed agents to 5.5%, which implies that 10% of the aggressive workers search on-the-job. Therefore we set $\varphi$ to 0.1. The probability of finding a better-paid job, $p(1-\tau)$, is reported to be approximately 0.1. As $\tau$ can be calculated as approximately 0.8 with the given information, the job finding rate of on-the-job searchers is approximately 0.47. This means that the probability of filling an empty vacancy is higher for an on-the-job searcher
in comparison to an unemployed agent. Following Blanchard and Galí (2010), we set the ratio of total hiring cost to the output level to 0.01 and calculate the value of $B$ which determines the level of hiring costs. Both for an economy with and without on-the-job search, we use the same value $B = 2.28$.\footnote{This value is calculated using the parameters and calibration values of the model without on-the-job search.}

Finally, for the case without on-the-job search, we assume that $\varphi = 0$ and $\xi^A = \xi^P = 0.5$. As for the case with on-the-job search, we assume $\varphi = 0.1$ and $\xi^A = 1$, and calculate the corresponding values for $\xi^P$ and $\chi$. Accordingly, we set the relative bargaining power of passive firm, $\xi^P$, to 0.12 and the disutility of labor, $\chi$, to 1.15.

The above calibration values are summarized in Table 1.

\section{Results}

As discussed earlier, the aim of this paper is to examine the effect of on-the-job search on labor market dynamics, especially on unemployment and labor market tightness. To do so, we compare the responses of these variables to a one percentage point increase in technology in the models with and without on-the-job search. This positive technology shock dies out gradually according to $AR(1)$ process with an autoregressive coefficient $\rho_a$ of 0.9.

Figure 1 shows the dynamic responses of unemployment and labor market tightness. The solid line with star signs and the dashed line with dots demonstrate the responses of the corresponding variable in the models with and without on-the-job search, respectively. In particular, when we set the level of on-the-job search intensity to 0%, the level deviation of the unemployment rate is 0.0026 in the first period, and when we set its level to 10% (based on the aforementioned calibration), then the same response increases to 0.0045. These
correspond to 5.2% and 9% increases in the unemployment rate, respectively. Furthermore, when we set the level of on-the-job search intensity to 0%, the percentage deviation of labor market tightness is $-0.1535$ in the first period, and when we set its level to 10%, then the same response decreases to $-0.2697$.

![Figure 1: The Impulse Responses to a Positive Technology Shock](image)

In the standard NK model, after a positive technology shock, we observe an increase in the unemployment rate. The mechanism behind this is as follows: After a positive technology shock, aggregate demand cannot adjust immediately due to nominal rigidities in the short run. Since firms become more productive, they decrease their demand for labor and post less vacancies. Consequently, the unemployment rate increases. Since employment-to-employment transitions is another channel affecting the flow of employment, the introduc-
tion of on-the-job search amplifies the increase in unemployment. More precisely, not only there is a decrease in vacancies posted by firms, but also a significant fraction of vacancies are filled by on-the-job searchers. Some of on-the-job searchers rematch with aggressive firms, and some of them fill the positions posted by passive firms. The remaining vacancies are filled by unemployed agents. However, the number of vacancies filled by unemployed agents is less than the number of filled vacancies in the model without on-the-job search, which is even less than the number of separated agents. Moreover, in addition to the exogenous separation, the jobs left by the matched on-the-job searchers are destructed. Therefore, the unemployment pool expands, so that we observe a higher increase in the unemployment rate. Because of the same reason we also observe a higher decrease in labor market tightness compared to the model without on-the-job search.

Since we concentrate on the directions and the volatilities of unemployment and labor market tightness, we just report the responses of these variables. Impulse response graphs of the other variables are reported in C. For those variables, there are no significant differences between the responses in the models with and without on-the-job search. This outcome fulfills our aim of increasing the volatilities of unemployment and labor market tightness without leading to another change in the model dynamics.

4 Conclusion

It is well-observed that a permanent increase in the productivity level leads to an increase in the unemployment rate and to a decrease in labor market tightness. Noting that SM-based models imply the opposite of these observations, Barnichon (2010) and Galí (2010) used a business cycle model with nominal rigidities and labor market frictions; and through their models, they were able to qualitatively replicate these empirical findings. On top of these, Barnichon (2014) recently replicated the magnitude of these responses.
In this paper, we introduce on-the-job search into the NK model suggested by Galí (2010). In that regard, we assume two-tier sector including firms with different bargaining powers, and we let a fraction of workers to search on-the-job. This assumption amplifies the flow of employment and increases the volatilities of unemployment and labor market tightness without causing change in the responses of the other variables.
References


Appendix

A Calibration

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.99 discount factor</td>
</tr>
<tr>
<td>φ</td>
<td>5 inverse of Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>θ</td>
<td>0.75 degree of price stickiness</td>
</tr>
<tr>
<td>M</td>
<td>1 desired markup</td>
</tr>
<tr>
<td>γ</td>
<td>0.5 share of aggressive good in final output</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ</td>
<td>0.02 separation rate</td>
</tr>
<tr>
<td>u</td>
<td>0.05 unemployment rate</td>
</tr>
<tr>
<td>q</td>
<td>0.25 the job finding rate of unemployed agents</td>
</tr>
<tr>
<td>α</td>
<td>1 weight of labor market tightness in cost function</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>φπ</td>
<td>1.5 feedback parameter of inflation</td>
</tr>
<tr>
<td>φπy</td>
<td>0.125 feedback parameter of output</td>
</tr>
<tr>
<td>ρa</td>
<td>0.9 persistence of the technology shock</td>
</tr>
<tr>
<td>εa</td>
<td>1 technology shock</td>
</tr>
</tbody>
</table>

Table 1: Baseline calibration and steady state values
B  Log-linearized equations

1. Goods market clearing condition
\[ \hat{y}_t = (1 - \theta_1 - \theta_2) \hat{c}_t + \theta_1 (\hat{g}_A^t + \hat{v}_A^t) + \theta_2 (\hat{g}_P^t + \hat{v}_P^t) \quad \text{where} \quad \theta_1 = \frac{G^AV^A}{Y}, \quad \theta_2 = \frac{G^PV^P}{Y}. \]

2. Aggregate production function
\[ \hat{y}_t = \hat{a}_t + \gamma \hat{n}_A^t + (1 - \gamma) \hat{n}_P^t \]

3. Production function of intermediate goods firms
\[ \hat{z}_A^t = \hat{a}_t + \hat{n}_A^t \]
\[ \hat{z}_P^t = \hat{a}_t + \hat{n}_P^t \]

4. Net hiring in intermediate goods firms
\[ \delta \hat{h}_A^t = \hat{n}_A^t - (1 - \delta) \hat{n}_{A,t-1} \]
\[ \delta \hat{h}_P^t = \hat{n}_P^t - (1 - \delta) \hat{n}_{P,t-1} \]

5. Gross hiring in an aggressive firm
\[ \hat{v}^A_t = (1 - \theta_3) \hat{h}_t^A + \theta_3 \hat{h}_t^O \quad \text{where} \quad \theta_3 = \frac{H^O}{V^A}. \]

6. Hiring cost of intermediate goods firms
\[ \hat{g}_A^t = \hat{a}_t + \alpha (\hat{v}_A^t - \hat{\text{pool}}_t) \]
\[ \hat{g}_P^t = \hat{a}_t + \alpha (\hat{v}_P^t - \hat{\text{pool}}_t) \]

7. Labor market tightness
\[ \hat{x}_t = \hat{h}_t - \hat{\text{pool}}_t \]

8. The pool of job searchers
\[ \hat{\text{pool}}_t = (1 - \theta_4) \hat{o}_j s_t^0 + \theta_4 \hat{u}_t^0 \quad \text{where} \quad \theta_4 = \frac{U^0}{\text{Pool}}. \]
9. The unemployment pool (Beginning-of-period unemployment)
\[ \hat{U}_t^0 = -\frac{(1 - \delta)N}{U^0} \hat{n}_{t-1} \]

10. End-of-period unemployment
\[ \hat{u}_t = U^0 \hat{U}_t^0 - qU^0 \hat{q}_t = -N \hat{n}_t \]

11. Employment
\[ \hat{n}_t = (1 - \theta_5)\hat{n}_t^A + \theta_5 \hat{n}_t^P \quad \text{where} \quad \theta_5 = \frac{N^P}{N}. \]

12. The job finding rate of unemployed agents
\[ \hat{q}_t = \hat{h}_t^U - \hat{u}_t^0 \]

13. The job finding rate of on-the-job searchers
\[ \hat{p}_t = \hat{h}_t^O - \hat{ojs}_t^O \]

14. The evolution of the job finding rate of on-the-job searchers
\[ \hat{p}_t(1 - \hat{\tau}_t) = 0 \]

15. Hiring from the unemployment pool
\[ \hat{h}_t^U = (1 - \theta_6)\hat{h}_t^A + \theta_6 \hat{h}_t^P \quad \text{where} \quad \theta_6 = \frac{H^P}{H^U}. \]

16. The pool of on-the-job searchers
\[ \hat{ojs}_t^0 = \hat{ojs}_{t-1}^0 \]

17. Total hiring
\[ \hat{h}_t = (1 - \theta_7)\hat{h}_t^O + \theta_7 \hat{h}_t^U \quad \text{where} \quad \theta_7 = \frac{H^U}{H}. \]

18. The Euler equation
\[ \hat{c}_t = \hat{c}_{t+1} - \hat{r}_t \]
19. The Fisher equation
\[ \hat{r}_t = \hat{i}_t - \pi_{t+1} \]

20. The forward-looking Phillips equation
\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \hat{m}_t \]

21. Optimal hiring condition of an aggressive firm
\[ \hat{\mu}^A_t = \hat{a}_t - (1 - \phi_a) \hat{w}^A_t - \phi_a \hat{b}^A_t \]

22. Optimal hiring condition of a passive firm
\[ \hat{\mu}^P_t = \hat{a}_t - (1 - \phi_p) \hat{w}^P_t - \phi_p \hat{b}^P_t \]

23. Net hiring condition of an aggressive firm
\[ \hat{b}^A_t = \frac{\hat{g}^A_t}{1 - (1 - \delta) \beta (1 - \varphi_p)} - \frac{(1 - \delta) \beta (1 - \varphi_p)}{1 - (1 - \delta) \beta (1 - \varphi_p)} (\hat{g}^A_{t+1} - \hat{r}_t) + \frac{(1 - \delta) \beta \varphi_p}{1 - (1 - \delta) \beta (1 - \varphi_p)} \hat{p}_{t+1} \]

24. Net hiring condition of a passive firm
\[ \hat{b}^P_t = \frac{\hat{g}^P_t}{1 - (1 - \delta) \beta} - \frac{(1 - \delta) \beta}{1 - (1 - \delta) \beta} (\hat{g}^P_{t+1} - \hat{r}_t) \]

25. Wage equation for an aggressive firm
\[ W^A \hat{w}^A_t = MRS(\hat{c}_t + \phi \hat{n}_t) + \eta^A G^A \hat{q}^A_t - \beta (1 - \delta) \left( \eta^A G^A (\varphi_p (\hat{q}_{t+1} + \hat{g}^A_{t+1} - \hat{r}_t) + \varphi_p (\hat{q}_{t+1} + \hat{r}_{t+1} + \hat{g}^A_{t+1} - \hat{r}_t) + \varphi_p (\hat{q}_{t+1} + \hat{n}_{t+1} + \hat{g}^A_{t+1} - \hat{r}_t) + \varphi_p (\hat{q}_{t+1} + \hat{r}_{t+1} + \hat{g}^A_{t+1} - \hat{r}_t)) \right) \]

26. Wage equation for a passive firm
\[ W^P \hat{w}^P_t = MRS(\hat{c}_t + \phi \hat{n}_t) + \eta^P G^P \hat{g}^P_t - \beta (1 - \delta) \left( \eta^P G^P ((1 - q) (\hat{g}^P_{t+1} - \hat{r}_t) + q \hat{q}_{t+1} - q \tau (\hat{q}_{t+1} + \hat{r}_{t+1} + \hat{g}^P_{t+1} - \hat{r}_t)) + \eta^A G^A q \tau (\hat{q}_{t+1} + \hat{r}_{t+1} + \hat{g}^A_{t+1} - \hat{r}_t)) \right) \]
27. Marginal cost
\[
\hat{mc}_t = \gamma(-\hat{\mu}_t^A) + (1 - \gamma)(-\hat{\mu}_t^P)
\]

28. Markup of an aggressive firm
\[
\hat{\mu}_t^A = \hat{p}_t - \hat{p}_t^A
\]

29. Markup of a passive firm
\[
\hat{\mu}_t^P = \hat{p}_t - \hat{p}_t^P
\]

30. Rate of inflation
\[
\pi = \hat{p}_t - \hat{p}_{t-1}
\]

31. Weighted average wage level
\[
\hat{w}_t = \gamma \hat{w}_t^A + (1 - \gamma)\hat{w}_t^P
\]

32. Interest rate rule (Taylor rule)
\[
\hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t
\]

33. Technological progress
\[
a_t = \rho_a a_{t-1} + \epsilon_t^a
\]

34. Ratio of hirings in an aggressive firm
\[
\tau_t = \hat{\nu}_t^A - \hat{h}_t
\]
C  Impulse Responses

Figure 2: Impulse Responses to a Positive Technology Shock
Please note:

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