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Normalized CES supply-side system approach: how to replicate Klump, McAdam, and Willman (Review of Economics and Statistics, 2007)

Gerald Eric Daniels Jr. and Venoo Kakar

Abstract

This paper lays out a replication plan for the influential paper by Klump et al. (Factor Substitution and Factor-augmenting Technical Progress in the United States: a Normalized Supply-side System Approach, *Review of Economics and Statistics*, 2007) on using a normalized CES supply-side system approach to estimate the value of the elasticity of substitution between factors of production and identify the growth patterns for biased technical progress. The authors begin with a general discussion of basic principles on carrying out a replication study. Further, they outline key steps to follow to replicate the chosen paper and establish criteria that can be used to determine if the replication confirms or disconfirms the original findings. This paper contributes to the increased interest in improving replications in economics research.

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Authors
Gerald Eric Daniels Jr., ✉ Howard University, Washington, DC, USA, Gerald.Daniels@howard.edu
Venoo Kakar, San Francisco State University, San Francisco, CA, USA

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1 What does replication entail?

For long, the economics profession has placed more emphasis on novelty than on establishing robustness, Coffman *et al.* (2017). The May 2017 issue of the American Economic Association Papers and Proceedings gave visibility to this issue and contained of a series of papers on replication in economics. The profession might be at an inflection point with a growing demand for replication studies arising from the profession as a whole.[1]

An important consideration is related to assessing which papers are of significant importance to be replicated. Coffman *et al.* (2017) suggest that focusing on well-cited papers is ideal direction to take. A comprehensive conceptualization of replication is proposed in Hamermesh (2007) in the following three categories: 1. *pure replications* (using the same sample, model, and estimation methods) 2. *statistical replications* (different sample, but identical model and underlying population); and 3. *scientific replications* (different sample, different population, and perhaps similar but not identical model employing alternative theoretical or conceptual approaches).

These concepts are further delineated in Clemens (2017) where a distinction between replications and robustness tests. There are two types of replications: 1. verification (showing that using the the same statistical analysis produces the same results, either using the original data set or remeasuring using similar methods. Verification can rectify measurement errors, coding errors, or errors in data set construction) and 2. reproduction (resampling the same population by using identical model and estimation techniques. This form of replication can also rectify for sampling error or low power in addition to the ones aforementioned).[2] Chang *et al.* (2017) define a successful replication as "when we could use author-provided files to qualitatively produce their key results." Similarly, Anderson *et al.* (2017) define replicability as "beginning with an authors dataset and applying the mathematical operations

[1] A new section in the Journal of Applied Econometrics (JAE) was devoted exclusively to the issue of replication of empirical results published in the JAE, see Pesaran (2003) and it also contains a data archive of all studies published since 1994. Similarly, the American Economic Review made it mandatory to have an online archive for replication material in 2005.

[2] There are two types of robustness tests: 1. reanalysis test (altering the computer code to include new regression specifications and variable coding from but using the same data set or a new sample representative of the same population, estimating different parameters) and 2. an extension test (using new data on a sample representative of a different population, or gathered on the same sample at a substantially different time, or both. This eliminates the effect of outliers by using the identical computer code on the new data.)

specified by the author, a researcher should obtain the authors numerical results".

While there have been various ideas that guide replication studies, there is a lack of consensus in the scientific literature about its gold standard. According to Duvendack *et al.* (2017) a broad definition of replication is a study that determines the validity of one or more empirical results from an original published study. Berry *et al.* (2017) consider a replication to be "any project that reports results that speak directly to the veracity of the original paper's main hypothesis."

We also believe that replication studies should go a step further in assessing the conditions under which the results from the original study are valid. In other words, the replication study should shed some light on how "generalizable" the results from the original study are. The intent of a replication study should be purely scientific in nature.[3] Sharing data and making code available improves transparency, facilitates comparison of results, and can further the existing knowledge in a field, Höffler (2017).

2 Why Klump, McAdam, and Willman (2007)?

Klump *et al.* (2007) is a seminal paper that contributes significantly to neoclassical growth theory. It is motivated by both previous theoretical and empirical contributions that have challenged the use of the Cobb-Douglas production function in dynamic macroeconomics for which the elasticity of substitution between capital and labor is exactly unity.

Their paper considers a natural alternative: the Consant Elasticity of Substituition (CES) production function with nonunitary elasticity of substitution. Using a normalized CES they estimate a supply-side system of the U.S. economy from 1953 to 1998 by avoiding potential estimation biases and emphasizing data consistency. Previous research was unable to settle the dispute between the use of two production functions due to a lack of high-quality data as well as the difficulties in identifying from the data at the same time an aggregate elasticity of substitution, and growth rates of labor- and capital-augmenting technical change. Hence, the empirical estimates for the elasticity of substitution have not been robust. They

[3]The project Teaching Integrity in Emprical Research (TIER) is taking important steps to facilitate replicability in the classroom. Similarly, Replication WIKI is an excellent database of replication studies in economics including data, codes, methods, and information on softwares.

proposed that the estimation of the CES functions parameters can be much improved by applying the normalization procedure developed by De La Grandville (1989) and Klump & De La Grandville (2000).

They found robust evidence to support that the elasticity of substitution is significantly below unity and that the growth rates of technical progress show an asymmetrical pattern. Technical progress shows an asymmetrical pattern where the growth of labor augmenting technical progress is exponential, while that of capital is hyperbolic or logarithmic. Their results justify more extensive use of CES production functions in dynamic macroeconomics.

3 Replication plan

To replicate Klump *et al.* (2007), we must first begin with defining our technology as the linear homogeneous CES production function with two factors of production, physical capital and labor. In addition, technological change is assumed to augment the efficiency of both factors of production. The elasticity of substitution is defined as the point elasticity. Therefore, the production function can be normalized using fixed baseline values for per capita production, capital intensity, and factor income shares (or marginal rate of substitution).[4] Klump *et al.* allow for the most general case of imperfect competition and include a markup.

The objective of their paper is to support the economic growth literature that relates biased technical change to a non-unitary elasticity of substitution and to demonstrate that technical progress should be purely labor augmenting in the long run while capital-augmenting progress is only transitory. To allow for time varying factor-augmenting technological progress, the functional form for the growth rate of factor efficiency is determined by the Box-Cox transformation. The production function and labor share of income, at the baseline values,

[4]See De La Grandville (1989) and Klump & De La Grandville (2000) for details on normalizing the CES production function.

are given by

$$Y_t = \left[(E_t^N N_t)^{-\rho} + (E_t^K K_t)^{-\rho}\right]^{\frac{-1}{\rho}} \quad \text{and} \tag{1}$$

$$1 - \pi_0 = \frac{w_0 N_0}{w_0 N_0 + q_0 K_0} = (1+\mu)\frac{w_0 N_0}{p_0 Y_0}. \tag{2}$$

Output, labor, capital, and labor share of income are denoted by Y, N, and K, π respectively. Price of output, compensation per employee, and user cost of capital are denoted by p, w, and q, respectively. Efficiency levels for the factor of production is denoted by E_t^i, where $i = N, K$. In addition, the substitution parameter is given by ρ, a function of the elasticity of substitution σ, and is give by the functional form $\rho(\sigma) = \frac{1-\sigma}{\sigma}$. Time is denoted by t and $t = 0$ denotes the baseline values for output, factors of production and their corresponding prices. The parameter μ measures the markup determined by the price elasticity of demand for goods.

When estimating the normalized the CES production function, baseline values should be selected. The authors believe that "appropriate" baseline values can only be detected from the data. Furthermore, the authors advise and use geometric or arithmetic sample averages for output (Y_0), labor (N_0), and capital (K_0) in estimating their model. The sample average may also be used for the baseline value for the labor share of income (π_0) or it can be estimated jointly with the other parameters of the model. The authors choose the latter. The author's also include a scaling parameter, ζ, for output since the non-linearity of the CES functional form does not necessarily coincide with the implied fixed point of the underlying empirical CES function, and this would be the case if the functional form is a log-linear Cobb-Douglas with constant technical growth.

Replicating Klump *et al.* (2007) requires the estimation of the following supply side system:

$$\log\left(\frac{w_t N_t}{p_t Y_t}\right) = \log\left(\frac{1-\bar{\pi}}{1-\mu}\right) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\zeta - g_N(t,\bar{t})\right] \tag{3}$$

$$\log\left(\frac{q_t K_t}{p_t Y_t}\right) = \log\left(\frac{\bar{\pi}}{1-\mu}\right) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) - \log\zeta - g_K(t,\bar{t})\right] \tag{4}$$

$$\log\left(\frac{Y_t}{N_t}\right) = \log\left(\frac{\zeta\cdot\bar{Y}}{\bar{N}}\right) + \frac{\gamma_N \bar{t}}{\lambda_N}\left[\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right] - \frac{\sigma}{1-\sigma}\log\left[(1-\bar{\pi}) + \bar{\pi}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{\sigma-1}{\sigma}} \cdot e^{\frac{1-\sigma}{\sigma}[g_N(t,\bar{t})-g_K(t,\bar{t})]}\right] \tag{5}$$

as well as the Kmenta approximation (Kmenta, 1967) of the production function around the baseline values, fixed points, to allow for the separation of the total factor productivity (TFP) term from the rest of the production function:

$$\log\left(\frac{Y_t}{N_t}\right) = \overbrace{\bar{\pi} g_K(t,\bar{t}) + (1-\bar{\pi}) g_N(t,\bar{t}) - \left(\frac{1-\sigma}{\sigma}\right)\frac{\bar{\pi}(1-\bar{\pi})}{2}[g_N(t,\bar{t}) - g_K(t,\bar{t})]^2}^{\text{TFP}} + \log\left(\frac{\zeta\cdot\bar{Y}}{\bar{N}}\right) - \frac{\sigma}{1-\sigma}\log\left[\bar{\pi}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi})\right]. \tag{6}$$

Geometric sample averages for output, factors of production, labor share of income, and time are denoted by $\bar{Y}$, $\bar{N}$, $\bar{K}$, $\bar{\pi}$, and $\bar{t}$. The growth rate of the efficiency level of is denoted by the function $g_i(t,\bar{t})$ where $i = K, N$. Klump *et al.* use a the Box-Cox transformation, Box & Cox (1964), to determine their general expression for the efficiency levels growth rates:

$g_i(t) = \frac{\gamma_i \bar{t}}{\lambda_i}\left[\left(\frac{t}{\bar{t}}\right)^{\lambda_i} - 1\right], t > 0$, where γ_i is the conventional constant growth rate when $\frac{\partial g_i}{\partial t} = \gamma_i$ and λ_i is the curvature parameter. The equations above are to be estimated as a system of equations. Klump *et al.* (2007) estimate (3)–(5) as a system or (3)–(4) and (6) as a system, replacing logarithm of the per capita production equation with the Kmenta approximation.

3.1 Data

The principle data sources necessary for replicating Klump *et al.* are the National Income and Product Accounts (NIPA) tables for production and income as well as Ho & Jorgenson (1999) for the labor input, Herman (2000) for the current and constant replacement cost of the capital stock, and Auerbach (1983, 2003) for the rental price of capital.[5] The data sample period should be from 1953 to 1998. The output series should be calculated as private nonresidential sector output, and should be determined by total output minus indirect tax revenues, public-sector output, and housing-sector output. Labor income should be given by Compensation to Private-Sector Employees:

$$w_t = \left(1 + \frac{\text{Self-Employed}}{\text{Total Private Employment}}\right) \times \text{Compensation to Private-Sector Employees} \tag{7}$$

When replicating this study, we believe that a verification approach should be used. The data should be remeasured using similiar methods to verify and rectify any potential mearsument errors or coding errors in the original study. Further, the sample period should be extendend to the most recent data available as the result should not be sensitive to a larger sample period.

3.2 Estimation

Before estimation, the data should be checked for internal consistency. The sample average for the markup component, μ, should be non-negative, and the markup component should be stationary. In addition, the data should be checked to see if it is consistent with the

[5]The NIPA series may be found at https://www.bea.gov/itable/.

standard neoclassical growth model. Klump *et al.* (2007) find that their data is not consistent with the standard neoclassical growth model, since capital output ratio is nonstationary.

The following five model specifications should be estimated to replicate the results found in Klump *et al.* (2007) using a nonlinear systems estimator using nonlinear seemingly unrelated regressions (SUR):

1. Estimate (3)–(5) and assume constant factor-augmenting technical progress (i.e. $\lambda_N = \lambda_K = 1$). Set starting value for elasticity of substitution at a small number, suggested value is $\sigma = .3$.

2. Estimate (3)–(5) and assume constant factor-augmenting technical progress. Choose the parameter starting values to ensure a global optimum, use a fine grid search of all parameters (individually and jointly) around broad and plausible ranges.

3. Estimate (3)–(5) and assume constant factor-augmenting technical progress is assumed and impose $\sigma = 1$ and $\gamma = (1 - \bar{\pi})\gamma_N + \bar{\pi}\gamma_K$.

4. Estimate (3)–(5) and estimate (3)–(4) and (6).

5. Estimate (3)–(5) and estimate (3)–(4) and (6), and impose $\lambda_K = -.001$ (approximated close enough to a logarithmic function since $\lambda_K = 0$ renders the equation indeterminate).

4 Interpreting Results

Klump *et al.* (2007) find that most parameters are significant at the 1% level and stable. A fine grid search should be used for the starting values set for parameters to ensure that a global optimum is reached. Also, models should be compared using the log determinate or log likelihood, depending on the statistic provided by the software used. The original study uses the log determinate since this is the statistic provided by RATS software.[6] Lastly, when examining all models, one must examine the stationarity of the residuals for the labor,

[6] A recent study by Stewart (1938) replicates Klump *et al.* (2007) using alternative software, TSP. Using Klump *et al.* (2007) original data, their replication study finds the original study's results to be robust, both numerically and substantively.

capital, and output equations. Thus, a successful replication of this study should possess parameters that are mostly significant for all specifications as well as report the log dermeninate (or log likelihood) for the system and the Augmented Dickey-Fuller (ADF) test for the residuals for each equations.

In Table 1, we present the estimation results of the supply-side system for Klump *et al.* (2007) to be used as a benchmark for a replication of their analysis.

Table 1: Benchmark estimation results of the supply-side system.

Specification	**(1)**	**(2)**	**(3)**	**(4.A)**	**(4.B)**	**(5.A)**	**(5.B)**
ζ	1.000	1.040	1.000	1.029	1.029	1.029	1.029
	(0.012)	(0.007)	(0.012)	(0.006)	(0.006)	(0.006)	(0.007)
$\bar{\pi}$	0.222	0.219	0.222	0.221	0.222	0.222	0.221
	(0.009)	(0.009)	(0.011)	(0.009)	(0.009)	(0.009)	(0.009)
γ_N	0.017	−0.210	0.014	0.015	0.015	0.015	0.016
	(0.001)	(0.121)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
λ_N		1.000		0.439	0.499	0.427	0.562
		(—)		(0.076)	(0.090)	(0.083)	(0.082)
γ_K	0.004	0.813	0.014	0.004	0.004	0.004	0.004
	(0.001)	(0.416)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)
λ_K		1.000		0.118	0.866	−0.001	
		(—)		(0.336)	(0.583)	(—)	
$\lambda_K \approx 0$[g]				0.122	2.210		
				[0.727]	[0.137]		
σ	0.509	0.998	1.000	0.556	0.605	0.557	0.642
	(0.012)	(0.002)	(—)	(0.018)	(0.019)	(0.018)	(0.024)
$1+\mu$	1.038	1.043	1.038	1.042	1.037	1.042	1.038
	(0.012)	(0.012)	(0.013)	(0.011)	(0.012)	(0.012)	(0.012)
TFP	0.014	0.014	0.014	0.013	0.013	0.013	0.013
Log determinant	−18.365	−19.408	−17.430	−19.618	−19.633	−19.614	−19.577
ADF$_N$	−2.500	−3.170	−3.250	−4.310	−4.050	−4.360	−3.880
ADF$_K$	−3.610	−3.540	−3.478	−3.580	−3.570	−3.580	−3.550
ADF$_Y$	−2.150	−3.140	−2.064	−3.960	−3.880	−3.970	−3.840
Global Optimum	No	Yes	Yes	Yes	Yes	Yes	Yes
Time Varying Factor-Augmenting Technical Growth	No	No	No	Yes	Yes	Yes	Yes
Kmenta Approximation	No	No	No	No	Yes	No	Yes

Note: Standard errors in parentheses, p-values in squared brackets.

A successful replication for each specification of the supply-side system should be given by the following:

1. Estimating (3)–(5): a local optimum with constant factor-augmenting technical growth: This model should yield a low and significant elasticity of substitution. The model should perform rather poorly with nonstationary residuals for the labor and output equations based on the ADF test.

2. Estimating (3)–(5): a global optimum with constant factor-augmenting technical growth: This model should be an improvement over model Specification 1 with stationary residuals for the output and labor equations according to the ADF tests. This model specification should have a near unity value for the elasticity of substitution, Klump *et al.* (2007) estimate a value of .998, as well as implausible estimates for the technical progress parameters for both factors. Parameter estimates for the factor-augmented technical progress should be given by a negative growth rate for labor-augmenting technical growth and an unreasonably high value for capital-augmenting technical growth. Although, the TFP based on the Kmenta approximation should be reasonable, around 1.4%.

3. Estimating (3)–(5): a global optimum with constant factor-augmenting technical growth and imposing a Cobb-Douglas technology. This specification should perform worse when comparing the log determinate to the two previously estimated models. In addition, the stationary properties of the residuals for the output equation should be unsatisfactory.

4. **A.** Estimating (3)–(5): a global optimum with technical progress parameters in a time-varying manner (i.e., $\lambda_N, \lambda_K \neq 1$). This specification should yield an improved log determinant when compared to all other specifications. The elasticity of substitution should be significantly below unity, around .56, and have stationary residuals for all equations. All parameter values should be economically reasonable and close to the benchmark values. The parameters for the labor-augmenting technical progress

[g] Wald test of the restriction $\lambda_K = -0.001$.

should display a diminishing and exponential pattern. The curvature parameter for the capital-augmenting technical progress should be negative, and not significant at the standard levels, implying a hyperbolic growth pattern. **B.** Estimating (3)–(4) and (6): the results should be similar to non-approximated specification. Again, the curvature parameter for the capital-augmenting technical progress should not be significant at the standard levels. Further, using a Wald test of $\lambda_K = -.001$ should be strongly nonrejected.

5. **A.** Estimating (3)–(5): a global optimum with technical progress parameters in a time-varying manner and imposing $\lambda_K = -.001$, for a more parsimonious specification. This specification assumes a logarithmic growth pattern for capital-augmenting technical progress. All parameters should be highly significant, have stationary residuals for all equations, and the log determinant should be similar to the unconstrained case, Specification 4.A. **B.** Estimating (3)–(4) and (6), the results should be similar to the unconstrained approximated model, Specification 4.B.

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