Optimal inflation target: insights from an agent-based model

Jean-Philippe Bouchaud, Stanislao Gualdi, Marco Tarzia, and Francesco Zamponi

Abstract
Which level of inflation should Central Banks be targeting? The authors investigate this issue in the context of a simplified Agent Based Model of the economy. Depending on the value of the parameters that describe the micro-behaviour of agents (in particular inflation anticipations), they find a surprisingly rich variety of behaviour at the macro-level. Without any monetary policy, our ABM economy can be in a high inflation/high output state, or in a low inflation/low output state. Hyper-inflation, stagflation, deflation and business cycles are also possible. The authors then introduce a Central Bank with a Taylor-rule-based inflation target, and study the resulting aggregate variables. The main result is that too low inflation targets are in general detrimental to a CB-controlled economy. One symptom is a persistent under-realisation of inflation, perhaps similar to the current macroeconomic situation. This predicament is alleviated by higher inflation targets that are found to improve both unemployment and negative interest rate episodes, up to the point where erosion of savings becomes unacceptable. The results are contrasted with the predictions of the standard DSGE model. (Published in Special Issue Agent-based modelling and complexity economics)

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Most Central Banks around the world nowadays adjust their monetary policy to reach a 2%/year inflation target. The rationale for choosing 2% rather than 1% or 3% is however not clear, as with many other “magic numbers” religiously used in economic policy. The recent crisis has put to the fore the problem of negative nominal interest rates, which can be seen as a consequence of low inflation targets and thus low baseline rates. As emphasized by O. Blanchard in 2010 [1], “As a matter of logic, higher average inflation and thus higher average nominal interest rates before the crisis would have given more room for monetary policy to be eased during the crisis.” This view is however disputed by many economists, who strongly argue against a raise of the inflation target (see e.g. [2, 3] for a recent overview). A major argument to that effect is the credibility of Central Banks, who have succeeded in anchoring low-inflation expectations in the minds of economic agents. If the inflation target is changed in the face of new circumstances, these expectations may un-moor, and the very efficiency of monetary policy may suffer as a consequence. Clearly, the fear of a lurking run-away inflation is weighing heavily on the debate.

Yet, the question of an “optimal” inflation target is well worth considering, and policy makers are eager to receive inputs from academic research. As Federal Reserve Chairwoman J. Yellen recently declared [4]: “We very much look forward to seeing research by economists that will help inform our future decisions on this”. Of course, optimality needs to be defined and different criteria (i.e. welfare functions) may lead to different results. More important still is the modelling framework used to describe the economy. A clear puzzle is that standard monetary theories imply zero or negative optimal inflation rates, at variance with Central Banks’ inflation targets [5]. The standard DSGE machinery – the “workhorse” of monetary economists [6] – has recently been extended to cope with non-zero inflation rates, and generally concludes that the optimal inflation rate should be smaller than 2% [7]. However, DSGE models are based on a series of highly debatable assumptions, and have been under intense fire after the 2007 crisis [8–11].

Another route is provided by Agent Based Models (ABM) in which “reasonable” behavioural rules replace the representative DSGE agent with a fully rational long-term plan. ABMs can include a number of economically relevant features which would be very difficult to accommodate within the DSGE straight-jacket [12–15]. Many simplifying assumptions are of course necessary, but a considerable advantage of ABMs is that interaction-induced, collective effects are present, when DSGE models reduce the whole economy to a small number of representative agents. As a consequence, the global “equilibrium” state of the economy is an emergent property in the former case, while it is a deus ex machina in the latter case.

In particular, crises (i.e. large swings in the output) can occur endogenously within ABMs [16, 17]. DSGE models, on the other hand, only describe small, mean-reverting fluctuations around the postulated equilibrium and crises can only result from exogenous, unpredictable shocks.1 As a case in point, we found in [19] that the aggregate behavior of the economy is not a smooth function of the baseline interest rate: the fact that firms are risk averse and fear going into debt leads to more unemployment that can spiral into a destabilizing feedback loop. This is one of Blanchard’s “dark corners” [20] that ABMs can help uncovering.

To our knowledge, the optimal inflation target question has not been investigated using ABMs (although see [21] where the efficiency of inflation targeting policies is discussed within the framework of an ABM). In the present paper, we take on this issue using a simplified, bare-bone ABM dubbed “Mark-0”, studied in great details in [17, 19], following previous work by the group of Delli Gatti et al. [16] (see also [21, 22]). As discussed in [17], the Mark-0 economy can be in different states or “phases”, good or bad, depending on various parameters. These parameters describe in a phenomenological way the behaviour of agents (firms, households and banks), and their response to different economic stimuli. Interestingly, small changes in the value of these parameters can indeed induce sharp variations in aggregate output, unemployment or inflation [17, 19]. This allows us to consider different “baseline” economies (high inflation/low unemployment, or low inflation/high unemployment) and study the influence of the chosen inflation target on the total output, on the real interest rate and on the probability of negative nominal interest rates.

Our main conclusion is that in general, increasing the inflation target reduces unemployment and reduces the probability of negative rates. Unsurprisingly, it also reduces real interest rates on savings. Actually, trying to impose low inflation on an economy that would naturally run at full steam with high inflation can lead to a complete collapse (high unemployment and deflation). However, high inflation policies can be dangerous and may generate hyper-inflation if agents lose faith in the ability of Central Banks to fulfill their mandate.

Our results are based on a stylized model that is arguably unrealistic on several counts – but the very same can be said about DSGE models. Still, the Mark-0 model, although highly simplified, contains plausible ingredients that are most probably present in reality. For example, our model encodes in a schematic manner the consumption

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1 R. Lucas famously argued that the 2008 crisis was not predicted because economic theory predicts that such events cannot be predicted [18].
behavior of households facing inflation, that is in fact similar to the standard Euler equation for consumption in general equilibrium models [6]. On the other hand, the effect of inflation on the behaviour of indebted firms appears to be absent in DSGE models. The fact that our results strongly contrast with those of standard DSGE models is in our opinion enough to warrant in-depth investigations of more realistic ABMs, and more empirical work on the micro/behavioural assumptions that underpin these models.

Disclaimer: the parameters chosen in the following are not the result of a precise calibration. We only made reasonable guesses, in particular to obtain reasonable values of yearly inflation. All the numbers quoted below are not intended to be taken literally (although we believe they should be taken seriously!).

II. A SHORT RECAP ON MARK-0

The Mark-0 model with a Central Bank (CB) and interest rates has been described in full details in [17, 19], where pseudo-codes are also provided. We will not repeat here the full logic of the model, but only focus on the elements that are relevant for determining inflation in the three sectors: households, firms and the CB. The pseudo-code of the model explored in this work is provided in Appendix B.

First, we need some basic notions. The model is defined in discrete time, where the unit time between \( t \) and \( t+1 \) is plausibly of the order of months. For definiteness, we will choose in the following the unit time scale to be 6 months. Each firm \( i \) at time \( t \) produces a quantity \( Y_i(t) \) of perishable goods that it attempts to sell at price \( p_i(t) \), and pays a wage \( W_i(t) \) to its employees. The demand \( D_i(t) \) for good \( i \) depends on the global consumption budget of households \( C_B(t) \), itself determined as an inflation rate-dependent fraction of the household savings. \( D_i \) is a decreasing function of the firm price \( p_i \), with a price sensitivity parameter that can be tuned. To update their production, price and wage policy, firms use reasonable “rules of thumb” [17] that also depend on the inflation rate through their level of debt (see below). For example, production is decreased and employees are made redundant whenever \( Y_i > D_i \), and vice-versa.\(^2\)

The model is fully “stock-flow consistent” (i.e. all the stocks and flows within the toy economy are properly accounted for).

The instantaneous inflation rate \( \pi(t) \) is defined as:

\[
\pi(t) = \frac{\bar{p}(t) - \bar{p}(t-1)}{\bar{p}(t-1)}; \quad \bar{p}(t) = \frac{\sum_i p_i(t) Y_i(t)}{\sum_i Y_i(t)},
\]

where \( \bar{p}(t) \) is the production-weighted average price. We will assume that firms, households and the CB do not react to the instantaneous value of \( \pi(t) \), but rather to a smoothed, exponential moving average \( \pi_{ema}(t) \), computed as

\[
\pi_{ema}(t) := \omega \pi(t) + (1 - \omega) \pi_{ema}(t-1),
\]

where we fix \( \omega = 0.2 \), which corresponds to an averaging time of \( \approx 4.5 \) time steps, i.e. roughly 2 years in our setting.

In Mark-0 we assume a linear production function with a constant unit productivity, which means that output and employment coincide. The unemployment rate \( u \) is defined as:

\[
u(t) := 1 - \frac{\sum_i Y_i(t)}{N},
\]

where \( N \) is the number of firms, which also coincides with the total workforce [17].

A. The Central Bank policy

In this work, we consider a single-mandate CB that attempts to steer the economy towards a target inflation level \( \pi^* \) (in [19], we in fact considered a double-mandate CB also targeting a certain employment level \( \varepsilon^* \)). The monetary policy\(^3\) followed by the CB for fixing the base interest rate is described by a standard Taylor-rule of the form [6, 23]:

\[
\rho_0(t) = \rho^* + \phi_\pi [\pi_{ema}(t) - \pi^*]
\]

\(^2\) As a consequence of these adaptive adjustments, the economy is on average always ‘close’ to the global market clearing condition one would posit in a fully representative agent framework. However, small fluctuations persists in the limit of large system sizes giving rise to a rich phenomenology [17], including business cycles.

\(^3\) Note that this is in our model the only action taken by the CB to achieve the target; in particular, no actions on the quantity of circulating money, such as quantitative easing or printing money can be taken by the CB.
where \( \rho^* \) is the “natural” interest rate and \( \phi_{s*} > 0 \) quantifies the intensity of the policy\(^4\). We assume that the banking sector sets the interest rates on deposits and loans (\( \rho^d(t) \) and \( \rho^l(t) \) respectively) uniformly for all lenders and borrowers\(^5\). The rate \( \rho^d \) increases and \( \rho^l \) decreases when firm defaults increase, in such a way that the banking sector – which fully absorbs these defaults – makes zero profit at each time step (see [19] for more details).

### B. Households

The effect of inflation on households is the standard trade-off between investment (at rate \( \rho^i \)) and consumption. We therefore assume that the total consumption budget of households \( C_B(t) \) is given by:

\[
C_B(t) = c(t) \left[ S(t) + W(t) + \rho^d(t)S(t) \right] \quad \text{with} \quad c(t) = \left[ c_0 \left[ 1 + \gamma_c \left( \pi(t) - \rho^d_{ema}(t) \right) \right] \right]^+_0 ,
\]
where \( S(t) \) is the savings, \( W(t) \) the total wages, \( \pi(t) \) is the expected inflation in the next period – see Eq. (6) below – and \( c(t) \) is the consumption propensity, which is clipped to the interval \([0,1]\). This is expressed by the symbol \([x]_0^+\) which means that the quantity \( x \) is boxed between 0 and 1, i.e. \([x]_0^+ = 1 \) if \( x > 1 \), \([x]_0^+ = 0 \) if \( x < 0 \), and \([x]_0^+ = x \) otherwise. This propensity is equal to \( c_0 \) when the difference between expected inflation and the interest paid on their savings is zero, and increases (decreases) when this difference is positive (negative). The parameter \( \gamma_c > 0 \) determines the sensitivity of households to the real interest rate. In spirit, Eq. (5) is similar to the standard Euler equation of DSGE models (see e.g. [6, 23]).

We furthermore posit that the expected inflation \( \pi(t) \) is given by a linear combination of the realised inflation \( \pi^{ema}(t) \) and the CB target inflation \( \pi^* \) (see also [21]):

\[
\pi(t) = \tau^{ema} \pi^{ema}(t) + \tau^* \pi^* .
\]

The parameters \( \tau^{ema} \) and \( \tau^* \) can be interpreted as capturing the trust of economic agents in the ability of the CB to enforce its inflation target. They are therefore expected to depend on the commitment of the CB, measured by the Taylor-rule parameter \( \phi_{s*} \).

When \( \tau^{ema} = 0 \) and \( \tau^* = 1 \), agents fully trust that the target inflation will be realised. When \( \tau^{ema} > 0 \), they are also influenced by the past realised inflation when they form their expectations. When \( \tau^{ema} > 1 \), they expect more inflation to be realised in the next period. As we will see below, this can give rise to hyper-inflation episodes, which is the scenario that prevents (in the mind of many policy makers and of the public opinion) higher inflation targets. In principle, \( \tau^{ema} \) and \( \tau^* \) could themselves be time dependent, as economic agents compare the realised inflation to the target inflation and “learn” about the credibility of the CB – see below and [21] for a discussion of this point. In the present paper, we will treat \( \tau^{ema} \) and \( \tau^* \) as time independent.

### C. Firms

#### 1. Financial fragility

Each firm is characterized by its production \( Y_i \) (equal to its workforce), demand for its goods \( D_i \), price \( p_i \), wage \( W_i \) and its cash balance \( \xi_i \), which, when negative, is the debt of the firm. We characterize the financial fragility of the firm through the debt-to-payroll ratio

\[
\Phi_i = -\frac{\xi_i}{W_i Y_i}.
\]

If \( \Phi_i(t) < \Theta \), i.e. when the flux of credit needed from the bank is not too high compared to the size of the company (measured as the total payroll), the firm is allowed to continue its activity. If on the other hand \( \Phi_i(t) \geq \Theta \), the firm defaults and the corresponding default cost is absorbed by the banking sector. The parameter \( \Theta \) controls the maximum leverage in the economy, and models the risk-control policy of the banking sector.

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\(^4\) Note that in our previous paper [19], we had \( \phi_{s*} \to 10\phi_{s*} \) in Eq. (4). The factor 10, that was useful in the context of [19], has been eliminated here to conform with the standard definition.

\(^5\) This is, in our model, the only role played by the banking sector: a transmission belt of the CB policy. In reality, the banking sector has much more freedom, and can sometimes make the CB policy ineffective, e.g. by restricting credit even in presence of a strong incentive from the CB.
2. Production update

If the firm is allowed to continue its business, it adapts its price, wages and production according to reasonable “rules of thumb” – see [17, 19]. In particular, the production update is chosen as:

\[
\begin{align*}
\text{If } Y_i(t) < D_i(t) & \implies Y_i(t+1) = Y_i(t) + \min\{\eta_i^-(D_i(t) - Y_i(t)), u^*_i(t)\} \\
\text{If } Y_i(t) > D_i(t) & \implies Y_i(t+1) = Y_i(t) - \eta_i^-(Y_i(t) - D_i(t))
\end{align*}
\]

(8)

where \( u^*_i(t) \) is the maximum number of unemployed workers available to the firm \( i \) at time \( t \) (see [19, Appendix A]). The coefficients \( \eta^\pm \in [0, 1] \) express the sensitivity of the firm’s target production to excess demand/supply. We postulate that the production adjustment depends on the financial fragility \( \Phi_i \) of the firm: firms that are close to bankruptcy are arguably faster to fire and slower to hire, and vice-versa for healthy firms. In order to model this tendency, we posit that the coefficients \( \eta^\pm \) for firm \( i \) (belonging to \([0, 1]\)) are given by:

\[
\begin{align*}
\eta_i^- &= \eta_0^- (1 + \Gamma \Phi_i(t))_0^1 \\
\eta_i^+ &= \eta_0^+ (1 - \Gamma \Phi_i(t))_0^1,
\end{align*}
\]

where \( \eta^\pm_0 \) are fixed coefficients, identical for all firms\(^6\). The factor \( \Gamma > 0 \) measures how the financial fragility of firms influences their hiring/firing policy, since a larger value of \( \Phi_i \) then leads to a faster downward adjustment of the workforce when the firm is over-producing, and a slower (more cautious) upward adjustment when the firm is under-producing.

In [19] we argue that \( \Gamma \) should in fact depend on the difference between the interest rate and the inflation: high cost of credit makes firms particularly wary of going into debt and their sensitivity to their financial fragility is increased. Therefore, we postulate that interest rates influence the firm’s policy through the financial fragility sensitivity \( \Gamma \), as:

\[
\Gamma = \max \{ \alpha_r (\rho^{ema}(t) - \hat{\pi}(t)), 0 \},
\]

(10)

where \( \alpha_r \) (similarly to \( \alpha_c \) above) captures the influence of the real interest rate on loans on the hiring/firing policy of the firms.

3. Price update

Prices are updated through a random multiplicative process which takes into account the production-demand gap experienced in the previous time step and if the price offered is competitive (with respect to the average price). The update rule for prices reads:

\[
\begin{align*}
\text{If } Y_i(t) < D_i(t) & \implies \begin{cases} 
\text{If } p_i(t) < \bar{p}(t) & \implies p_i(t+1) = p_i(t)(1 + \gamma \xi_i(t))(1 + \hat{\pi}(t)) \\
\text{If } p_i(t) \geq \bar{p}(t) & \implies p_i(t+1) = p_i(t)(1 + \hat{\pi}(t))
\end{cases} \\
\text{If } Y_i(t) > D_i(t) & \implies \begin{cases} 
\text{If } p_i(t) > \bar{p}(t) & \implies p_i(t+1) = p_i(t)(1 - \gamma \xi_i(t))(1 + \hat{\pi}(t)) \\
\text{If } p_i(t) \leq \bar{p}(t) & \implies p_i(t+1) = p_i(t)(1 + \hat{\pi}(t))
\end{cases}
\end{align*}
\]

(11)

where \( \xi_i(t) \) are independent uniform \( U[0, 1] \) random variables and \( \gamma \) is a parameter setting the relative magnitude of the price adjustment, chosen to be 0.1 throughout this work. The \((1 + \hat{\pi}(t))\) factor implies that firms also anticipate inflation when they set their prices. This is precisely the dreaded self-reflexive mechanism that may lead to hyper-inflation when expected future inflation is dominated by past realised inflation (the parameter \( \tau_{ema} \)), rather than by the CB inflation target (the parameter \( \tau^* \)).

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\(^6\) Note that in our previous work, the clipping was such that \( \eta^\pm_0 \in [0, 2\eta^\pm_0] \). This modification is irrelevant.
4. Wage update

The wage update rule follows (in spirit) the choices made for price and production. At each time step firm $i$ updates the wage paid to its employees as:

$$ W_i^{T}(t+1) = W_i(t)[1 + \gamma(1 - \Gamma \Phi_i)(1 - u(t))\xi_i(t)(1 + g\tilde{\pi}(t)) \] $$(12)

$$ W_i(t+1) = W_i(t)[1 - \gamma(1 + \Gamma \Phi_i)u(t)\xi_i(t)(1 + g\tilde{\pi}(t)) \] $$(12)

where $P_i(t)$ is the profit of the firm at time $t$ and $\xi_i(t)$ an independent $U[0,1]$ random variable. If $W_i^{T}(t+1)$ is such that the profit of firm $i$ at time $t$ with this amount of wages would have been negative, $W_i(t+1)$ is chosen to be exactly at the equilibrium point where $P_i(t) = 0$; otherwise $W_i(t+1) = W_i^{T}(t+1)$. Finally, $g$ is a certain parameter modulating the way wages are indexed to inflation. We will assume in the following full indexation ($g = 1$), but choosing $g < 1$ can be useful to stabilize the Mark-0 economy in periods of hyper-inflation.

The above rules are intuitive: if a firm makes a profit and it has a large demand for its good, it will increase the pay of its workers. The pay rise is expected to be large if the firm is financially healthy and/or if unemployment is low because pressure on salaries is high. Conversely, if the firm makes a loss and has a low demand for its good, it will attempt to reduce the wages. This reduction is drastic if the company is close to bankruptcy, and/or if unemployment is high, because pressure on salaries is then low. In all other cases, wages are not updated. In essence, deeply indebted firms seek to reduce wages more rapidly, whereas flourishing firms tend to increase wages more quickly.

Note that within the model the productivity of workers is not related to their wages. The only channel through which wages impact production is that the quantity $\gamma R_u$ which wages impact production is that the quantity $W_i$.

Remarkably, the “native” state of the economy can display endogenous oscillations (or business cycles), as already noted in [17]. In the present case, such spontaneous oscillations occur in a small region around $(\rho^*, R = 0.75)$, see [17] for details.

III. THE “NATIVE” STATE OF THE ECONOMY

In [17, 19], we have shown that the Mark-0 economy, once set in motion, can settle in a variety of stationary macro-states, where the aggregate variables behave very differently. In our opinion, the strength of Agent Based modelling is precisely to show that micro-rules do matter, as they can lead to very different macro-states. We will not repeat such an analysis in full here, but focus on the role of a few variables, relevant to the topic of this paper. We start by analyzing the case where the CB does not react to inflation (i.e. the Taylor-rule Eq. (4) is with no inflation correlation).

As anticipated, a transition to a hyper-inflation, low output state (HYLO) occurs when expectations amplify inflation, more precisely
FIG. 1: Average unemployment (left) and inflation (right) in the \((ρ^*, R)\) plane. The HIHO phase in the top region of the graph is separated from the LILO phase in the bottom region by a critical line \(ρ^*(R)\). Other parameters are: \(τ^\text{ema} = τ^* = 0.5\) and \(ϕ_*=0\). Both inflation and natural rate are expressed as %/year, unemployment is expressed in % of the workforce.

when \(τ^\text{ema} > τ^1 \approx 1.1\).\(^7\) The full phase diagram is however quite complex, with a region of hyper-inflation but full employment (HYHO) when \(τ^\text{ema} \approx 1\) and low enough values of \(ρ^*\). For larger values of \(ρ^*\) (say, \(ρ^* = 3%\) represented by the dotted line in Fig. 2) one observes a sequence of transitions as \(τ^\text{ema}\) increases from zero: LILO \(\rightarrow\) HIHO \(\rightarrow\) HYHO \(\rightarrow\) LILO \(\rightarrow\) HYLO.\(^8\) This illustrates the highly non-trivial role of inflation expectations in our framework: all possible states of the economy can be reproduced within this simple framework, including a virtuous state of high output and relatively low inflation (LIHO), stagflation (HILO) or even deflation (HDLO).

FIG. 2: Average unemployment (left) and inflation (right) in the \((ρ^*, τ^\text{ema})\) plane. In the right panel, the grey area corresponds to the hyper-inflation region where inflation is not stationary, but constantly growing. The region \(τ^\text{ema} \gtrsim 1.1\) is a HYLO state (hyper-inflation, low output), but one can also end up in a HYHO state (hyper-inflation, high output) in a tongue-like grey region of the right plot. All other possibilities are present as well: HIHO, LILO, LIHO (bottom left region) and HILO (stagflation, see the “island” around the point \(ρ^* = 1, τ^\text{ema} = 1\)) or even deflation. Other parameters are: \(R = 0.8, τ^* = 0.5\) and \(ϕ_*=0\). Both inflation and natural rate are expressed as %/year, unemployment is expressed in % of the workforce.

IV. INFLATION TARGETING

We now pick two representative states of the economy, one with \(ρ^* = 1%/\text{year}\) corresponding to the the HIHO state, and the other with \(ρ^* = 3%/\text{year}\) corresponding to the LILO state. The inflation level of these native states is,

\(^7\) In the hyper-inflation phase, inflation itself grows with time, so the average inflation is undefined.
\(^8\) There is even a thin sliver of hyper-deflation in Fig. 2. We have checked that this is not a numerical artefact.
respectively, 4.7% and 0% while the unemployment rate is, respectively, 0.8% and 85%. The long run real return on savings ($\rho^d - \pi$) is, respectively, −3.7% and 0%. The HIHO state discourages long term savings while the LILO state is vastly inefficient in terms of output.

The CB steps in and modulates the interest rate according to the Taylor-rule, Eq. (4), with a standard value of $\phi_\pi = 2.5$ (i.e. an increase of inflation by 1% leads to the CB increasing the nominal base-line rate by 2.5%). We assume that firms and agents form their inflation expectations by giving an equal weight to the target inflation $\pi^*$ and the realised inflation $\pi^{\text{ema}}$; in other words we set $\tau^{\text{ema}} = \tau^* = \frac{1}{2}$. We will also report the results when agents fully trust the CB policy ($\tau^{\text{ema}} = 0, \tau^* = 1$). In the other extreme case ($\tau^{\text{ema}} = 1, \tau^* = 0$), the CB policy is, as expected, totally inefficient (results not shown).

The resulting states of the monitored economy are summarized in Figs. 3 and 4, where we show, as a function of the inflation target: the average unemployment $\langle u \rangle$, the average realised inflation $\langle \pi \rangle$, the probability $P_{\text{neg}}$ that the CB must impose negative rates and finally the average real interest rate paid on deposits $\langle \rho^d - \pi \rangle$.

### A. HIHO

Starting from a HIHO native state, one sees that targeting a low inflation rate has a destabilising effect and unemployment rockets to 95%, while realised inflation is below target (see Fig. 3 panel a). For our particular “from-the-hip” choice of parameters, realised inflation reaches target when $\pi^* \approx 2\%$ and overshoots beyond that point, but this allows unemployment, and the probability of negative rates, to be significantly reduced. For example, $P_{\text{neg}}$, plummets from $\approx 0.9$ for $\pi^* = 1.25\%$ to zero for $\pi^* > 2\%$. Note that the real interest rate on deposits goes from significantly negative (−3.7\%) in the native HIHO state to roughly zero when $\pi^*$ is in the range 2\% − 2.5\% in the monitored economy, while it remains negative outside that interval. Beyond $\pi^* = 2.5\%$, output continues to improve, but the real interest rates on savings dips. When $\pi^* = 3.25\%$, unemployment is however still around 20\% and only falls below 5\% when the target inflation exceeds 4\%.

The situation improves slightly when agents fully trust the ability of the CB to reach its target (i.e. $\tau^* = 1$ and $\tau^{\text{ema}} = 0$). Unemployment then falls below 5\% as soon as $\pi^* > 3\%$. In other words, stronger anchoring of inflation expectations is beneficial in our ABM setting, in agreement with the intuition gained from DSGE models. At variance with DSGE models, however, increased Taylor coefficients do lead to instabilities in our model (see [19]). Our setting highlights the difference between two policy transmission channels: behavioral biases, i.e. expectations based on historical data ($\tau^{\text{ema}}$), and rational anticipations resulting from a Taylor-rule-based intervention of the CB ($\phi_\pi$).

So overall, increasing the inflation target closer to that of the native state is favorable, at least up to a point beyond which realised inflation significantly overshoots the target and interest rates on savings become strongly negative. Note that a similar pattern also applies to the virtuous low inflation/high output (LIHO) state: with a natural state of the economy having an inflation of $\approx 2.5\%/\text{year}$, targeting a too low inflation (below $\approx 2\%/\text{year}$) induces a strong growth of unemployment, with a similar qualitative pattern as in Fig. 3.

### B. LILO

Let us now assume that the underlying economic mechanisms (described by the parameters of Mark-0) are such that the native state of the economy is LILO, for example when $R$ is small (firms are more reluctant to hire than to fire) or when $\rho^*$ or $\alpha_\pi$ are large (firms are more reluctant to take loans). In this case, the role of the CB is to kick start the economy by lowering the interest rate. The results of a Taylor-rule based policy are now shown in Fig. 4 as a function of the inflation target $\pi^*$. Surprisingly, the dependence of $\langle u \rangle$ on $\pi^*$ is non-monotonic. As long as $\pi^*$ is below 3\%, unemployment is in fact an increasing function of the inflation target, with very frequent periods of negative nominal rates. Unemployment reaches acceptable values only when $\pi^*$ is large enough. For example, when $\tau^* = 0.5$ and $\pi^* = 4\%$, unemployment is around 13\% (down from 85\% in the native state), long term real savings rate are close to zero and the probability of negative nominal rates is zero.

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9 Other values of $\phi_\tau$ have been studied as well, with similar conclusions. Larger $\phi_\tau$ lead to higher unemployment and more frequent negative nominal rates, while smaller $\phi_\tau$ lead to more abrupt (and hence less manageable) dependencies of $\langle u \rangle$ and $\langle \pi \rangle$ on $\pi^*$. It would be interesting to extend the present study to dual-mandate CBs.
FIG. 3: HIHO native state: Average unemployment (panel a), average realised inflation (panel b), probability that the CB must set nominal rates to negative values (panel c) and average real interest rate paid on deposits (panel d) as a function of the CB target inflation for $\tau_{ema} = \tau^* = 0.5$ and $\phi_\pi = 0$ (native state, red circles) or $\phi_\pi = 2.5$ (monitored state, blue triangles). Other parameters are: $\rho^* = 1\%$ and $R = 0.8$. Special symbol: phase transition point for an underlying native economy, inducing strong fluctuations (see Appendix A). Both inflation and natural rate are expressed as %/year, unemployment is expressed in % of the workforce.

C. Discussion

The results of this section suggest that independently of the nature of its native state, low inflation targets are detrimental to a CB-controlled economy. Interestingly, our results show that a situation where the realised inflation is lower than the target inflation cannot be optimal; in fact, realised inflation should rather overshoot target inflation on average (at least up to the point where the savings are wiped out by inflation). This is the case for example in the HIHO state discussed above, when the target inflation is 3%, and the realised inflation is 4%.

Note that the coefficients $\tau_{ema}$ and $\tau^*$, which here have been assumed to be constant for simplicity, must in reality be time dependent and related to other quantities in the model. For example, persistent inflation overshoots may result in a loss in the credibility of CB, which in the present model means an increase of the value of $\tau_{ema}$ and/or a decrease of $\tau^*$, as economic agents start looking for guidance in past realised inflation rather than in the official CB target. Such an increase may lead to a run-away inflation state that cannot be controlled anymore using Taylor-rule based policies. Within our model, the hyper-inflation scenario can be tamed if firms do not fully index wages on expected inflation (i.e. set the parameter $g$ in Eq. (12) to a value less than unity). This has the effect of reducing realised inflation (as households reduce consumption), pushing the hyper-inflation threshold $\tau^*$ to higher values (for example, $\tau^* \approx 1.4$ for $g = 0.5$). The smoothing parameter $\omega$ might also depend on inflation, as higher inflation fluctuations could make agents more short-sighted, i.e. $\omega$ would increase towards 1. It would therefore be interesting

\footnote{We again insist on the fact that the numbers quoted should not be taken at face value since no attempt has been made to calibrate Mark-0 on real data. In particular, the chosen elementary time scale of 6-months is reasonable but arbitrary, and directly scales all inflation and interest rate levels.}
to extend the present model to include a dynamic coupling between $\tau_{ema}$, $\tau^*$, and the target and realised inflation, as well as a dynamic dependence of $g$ and $\omega$ on inflation. We leave such a study for future work.

V. COMPARISON WITH DSGE & CONCLUSION

The issue of an optimal inflation target has only recently been considered within the mainstream DSGE macroeconomic model [7]. In this framework, the main cost of inflation comes from price dispersion and is a consequence of the following string of assumptions [24]: a) firms face friction costs and cannot update their prices as often as they would like; b) inflation leads to a stronger dispersion of (stale) prices across different sectors of the economy; c) stronger price deviations from equilibrium lowers economic efficiency.

However, while crucial in determining the optimal inflation rate within DSGE, such a dispersion induced cost has little empirical support [25]. Embracing the choice of parameters and welfare function made by Coibion et al. [7], the optimal inflation rate is found to be $\approx 1.5\%/year$. This number is however highly dependent on the assumption made about the subjective discount factor $\beta$ used by the representative household, i.e. how far in the future do economic agents assess the consequences of their present decision. In many DSGE calibrations, the discounting horizon is extremely long, for example 125 years (!) in the Coibion et al. paper [7]. Although rooted on rational arguments and based on the value of historical rates, such an enormous time scale is in our eyes totally unreasonable. In line with the behavioral arguments used to construct ABMs, where agents are assumed to be myopic, we believe that this time scale should be rather on the scale of a few – perhaps 5 – years. This substantially changes the conclusions of DSGE models, as the total output is now an increasing function of inflation up to 5.2% (see Fig. 5), more in line with the conclusions of our ABM.

The fact that a higher inflation tends to stabilize the Mark-0 economy is fully consistent with the results of our
FIG. 5: Effect of inflation on output in the DSGE model, following [7, 26]. All parameters are as in [7], but the subjective discount factor $\beta$ is changed from 0.998 to 0.95 (corresponding to a horizon of 5 years). In the former case, inflation causes output to decrease except for a very small window $\pi < 0.12\%$/year, invisible in the graph. A shorter horizon leads to a positive marginal effect of the inflation on output as long as $\pi < 5.2\%$.

previous studies [17, 19], where we showed that the bad states of the economy were often associated with a large amount of “inactive” money, stored in the agents’ and firms’ bank accounts. Increasing the inflation rate encourages investment and, to a smaller extent, consumption, thereby increasing the total amount of money circulating in the economy with the effect of lowering the unemployment rate and increasing the global output. Hence, Mark-0 emphasizes the benefits of inflation while it completely neglects all direct inflation costs, including the price dispersion induced cost present (but probably overestimated) in DSGE models.

More fundamentally, the most interesting difference between the DSGE and ABM modelling strategies is that the equilibrium state of the DSGE economy cannot be characterized. Only the dynamics of small perturbations around a God-given state can be computed. This strongly contrasts with the Mark-0 model (and more generally other ABM models), where the native state of the economy is itself an output of the model. This native state can change radically when the parameters characterizing the micro-behaviour of the different agents are only slightly modified. For example the LILO, HIHO or hyper-inflation states considered in this paper are emergent properties of the model, rather than postulated a priori. Not surprisingly, a rough knowledge of where the economy is “naturally” poised to go is needed to determine an adequate monetary policy. Trying to steer the economy too far from its native state can be detrimental and even lead to instabilities and crises (see [19]).

Mark-0 is a bare-bone ABM where many important effects are left out, that need to be considered in future studies. For example the network structure of firms [27, 28] and the dynamics of growth and innovation are clearly among the most urgent ingredients to be added in Mark-0. The difference with DSGE is that missing effects are straightforward to include in an ABM, while quite a bit of arm-twisting is usually necessary to include them in a DSGE framework without ruining the mathematical tractability of the model. In this sense, the much touted “micro-founded” nature of DSGE is quickly buried under a number of ad-hoc assumptions (such as Calvo’s sticky price assumption [29]), which are not much more convincing than the equally ad-hoc assumptions made in ABMs.

In any case, the main result of our study is that the optimal inflation rate could be somewhat higher than the currently accepted 2% target. One clear symptom of a too low target is a persistent under-realisation of inflation, perhaps similar to the current macroeconomic situation in the U.S. and in Europe. In our model, this predicament is alleviated by higher inflation targets, that are found to improve both unemployment/output and negative interest rate episodes, up to the point where erosion of savings becomes politically unacceptable. Although this conclusion is based on an arguably over-simplified model, it certainly militates for more work along these lines [11, 30]. After all, DSGE models are themselves over-simplified and, as recently emphasized by O. Blanchard [31], they have to become less imperialistic and accept to share the scene with other approaches to modelisation.
Acknowledgments

The input and comments of O. Blanchard, R. Bookstaber, H. Dawid, D. Delli Gatti, D. Farmer, C. Hommes, A. Kirman have been extremely useful.

Appendix A: Additional figures

We show here additional color plots that help clarify the origin of the transition points observed in Figs. 3 and 4. In Sec. III we have defined the “native” state of the economy (for a given parameter setting) as the state where $\phi_\pi = 0$. Since in this case the CB is not actively targeting an inflation level, we also posit that agents anticipate the inflation target to be equal to the CB interest rate, i.e. $\pi^* \equiv \rho^*$. This is why all curves with $\phi_\pi = 0$ in both Figs. 3 and 4 are flat.

However, as long as $\tau^* > 0$ this choice introduces an artificial “discontinuity” of the model since for arbitrarily small (but non zero) values of $\phi_\pi$ the CB is effectively inactive but agents do integrate the CB target inflation $\pi^*$ in their inflation expectation $\hat{\pi}$. We therefore show here additional plots in the $(\pi^*, R)$ and $(\pi^*, \rho^*)$ planes when $\phi_\pi \ll 1$, which one may consider as alternative “native” states that help understanding the dynamics of the model when $\phi_\pi = 2.5$. One could also consider a modification of the model where $\tau^*$ is a function of $\phi_\pi$, with $\tau^* = 0$ when $\phi_\pi = 0$, but we avoid this additional complication in the present work.

In Figs. 6(left) we show the phase diagram in the $(\pi^*, R)$ plane with $\rho^* = 1\%$. One now sees a dependence of the “native” state of the economy on the inflation target: when $R$ is well above its critical value $R_c$ ($R_c \approx 0.7$ for this parameter setting) output is high independently of $\pi^*$ while the realized inflation increases with $\pi^*$. When $R$ is below its critical value $R_c$ one sees the opposite situation where output decreases as a function of $\pi^*$ while inflation is relatively stable (although in the region around $R = 0.5$ and $\pi^* = 4\%$ we observe “business cycles”). One also sees the appearance of a “tongue” of low output and low inflation around $R = 0.5$ and $\pi^* = 0.3\%$ which extends the low output phase for $R < R_c$ to higher values of $R$ and corresponds to the special transition point observed in Fig. 3. This effect is much more prominent in Fig. 6(right) where we plot the same quantities with $\rho^* = 3\%$ (one can now see the transition point around $\pi^* = 3.5\%$ corresponding to the special transition point in Fig. 4).

We finally show an additional color plot in the $(\pi^*, \rho^*)$ plane for $R = 0.8$ which clearly shows a transition from a LILO state (for lower inflation targets) to a HIHO state (for higher inflation target) for any value of $\rho^*$ above approximately 1%. The transition line is slightly above the line $\pi^* = \rho^*$ for $\pi^* < 2\%$ and slightly below it for $\pi^* > 3\%$ with an inflection point around $\pi^* = 2.5\%$. This means that for smaller natural rates $\rho^*$ a comparatively smaller increase in the inflation target is sufficient to restore growth.

Appendix B: Pseudo-code of Mark 0 with inflation expectations

We provide here the pseudo-code for the Mark 0 code described in Sec. II. The source code is available on demand.
FIG. 7: Phase diagram in the \((\pi^*, \rho^*)\) plane. Here \(R = 0.8\) and \(\phi_\pi \ll 1\).
Algorithm 1 Mark 0

**Require:** $N_F(10000)$ Number of firms; $c_0(0.5), \beta(2), \gamma(0.1), \eta_u^c_{(Rq^c)}(t), \eta_u^b_{(Rq^b)}(0.2), \delta(0.02), \Theta(3), \varphi(0.1), f(0.5), \alpha_c, \phi_n, \alpha_r, \Gamma_0(0), \pi^*, \rho^*, \omega(0.2), g(1), \tau^{ema}, \tau^*$, $T(12000)$, $T_{CB}(5000)$; Numbers between parentheses indicate the value used for the present work besides what is specified in the text. We start computing averages after $T_{eq}(7000)$ time steps.

> **Initialization**

```
for (i ← 0; i < N_F; i ← i + 1) do
  p[i] ← 1 + 0.1(random − 1)
  Y[i] ← 0.5 + 0.1(random − 1)
  D[i] ← 0.5
  W[i] ← 1
  E[i] ← 2W[i]Y[i] random
  P[i] ← p[i] min(D[i], Y[i]) − W[i] Y[i]
  a[i] ← 1
end for
S ← N_F − ∑i E[i]
if ϕn == 0 then
  π* ← ρ*
end if
```

> **Main loop**

```
for (t ← 1; t ≤ T; t ← t + 1) do
  ε ← 1 ∑i Y[i]
  u ← 1 − ε
  p ← ε Y[i]
  w ← 1 − ε Y[i]
  u*[i] ← exp(W[i]Y[i])N F u
  x^{ema} ← ωx + (1 − ω)x^{ema} where x are π, ρ^*, ρ^*, u
  γ ← τ^{ema} π + τ^* π
  if t > T_{CB} then
    ρ0 ← ρ^* + φn(π^{ema} − π*)
  else if t ≤ T_{CB} then
    ρ0 ← ρ^*
  end if
  Γ ← max {ρ^*, ϕ_n(π^{ema} − π*)}, Γ_0
  D ← E[− E[− ] 0
  for (i ← 0; i < N_F; i ← i + 1) do
    if a[i] == 1 then
      if E[i] > −θW[i]Y[i] then
        E[+] ← E[+] + max {E[i], 0}
        E[−] ← E[−] − min {E[i], 0}
        Φ[i] ← E[i]
        η_+ ← [u^0 + (1 − Φ[i])^1/3]
        η_- ← [u^0 + (1 + Φ[i])^1/3]
      end if
    end if
  end for
  if P[i] > 0 then
    W[i] ← W[i][1 + γ(1 − Φ[i])] random
    W[i] ← min {W[i], (P[i] min {D[i], Y[i]} + ρ^) max {E[i], 0} + ρ^ min {E[i], 0}} / Y[i]
  end if
  if Y[i] < D[i] then
  W[i] ← W[i] Y[i][1 + γ(1 − Φ[i])] random
  W[i] ← min {W[i], (P[i] min {D[i], Y[i]} + ρ^) max {E[i], 0} + ρ^ min {E[i], 0}} / Y[i]
  end if
  if P[i] < 0 then
    W[i] ← W[i] Y[i][1 + γ(1 − Φ[i])] random
  end if
  if Y[i] > D[i] then
    W[i] ← W[i] Y[i][1 + γ(1 − Φ[i])] random
  end if
  if Y[i] < D[i] then
    if P[i] < 0 then
      W[i] ← W[i] Y[i][1 + γ(1 − Φ[i])] random
    end if
    if Y[i] < D[i] then
      if P[i] < 0 then
        W[i] ← W[i] Y[i][1 + γ(1 − Φ[i])] random
      end if
    end if
  end if
else if E[i] ≤ −θW[i]Y[i] then
  a[i] ← 0
  D ← D − E[i]
end if
end for
```
Algorithm 2 Mark0 (continued)

\( u \leftarrow 1 - \frac{1}{NF} \sum_{i} Y[i] \) \hspace{1cm} \( \triangleright \) Update \( u \) and \( \overline{p} \)

\( \overline{p} \leftarrow \frac{\sum_{i} p[i] Y[i]}{\sum_{i} p[i]} \)

\( \rho^f = p_0 + (1 - f)D/E^- \)

\( \rho^d = \frac{E^+ - D}{S + E^+} \)

\( S \leftarrow (1 + \rho^d) S + \sum_{i} W[i] Y[i] \)

\( c \leftarrow c_0 [1 + \alpha_c (\pi - \tilde{\rho}^{\lambda_{ema}})] \)

\( C_B \leftarrow c S \)

for \( (i \leftarrow 0; i < NF; i \leftarrow i + 1) \) do

\( D[i] \leftarrow \frac{C_B a[i] \exp(-\beta p[i] / \overline{p})}{\sum_{i} a[i] \exp(-\beta p[i] / \overline{p})} \) \hspace{1cm} \( \triangleright \) Inactive firms have no demand

\( E^+ \leftarrow 0 \)

for \( (i \leftarrow 0; i < NF; i \leftarrow i + 1) \) do

if \( a[i] == 1 \) then

\( S \leftarrow S - p[i] \min\{Y[i], D[i]\} \)

\( P[i] \leftarrow p[i] \min\{Y[i], D[i]\} - W[i] Y[i] + \rho^d \max\{E[i], 0\} + \rho^f \min\{E[i], 0\} \)

\( E^i \leftarrow E[i] + P[i] \)

if \( P[i] > 0 \) \&\& \( E^i > 0 \) then

\( S \leftarrow S + \delta E^i \)

\( E[i] \leftarrow E[i] - \delta E[i] \)

end if

end if

end for

\( R \leftarrow 0 \)

for \( (i \leftarrow 0; i < NF; i \leftarrow i + 1) \) do

if \( a[i] == 0 \) then

if \( \text{random} < \varphi \) then

\( Y[i] \leftarrow u \) random

\( a[i] \leftarrow 1 \)

\( P[i] \leftarrow \overline{p} \)

\( W[i] \leftarrow \overline{w} \)

\( E^i \leftarrow W[i] Y[i] \)

\( \delta \leftarrow \frac{E^+}{E^- + \max\{E[i], 0\}} \)

\( E^+ \leftarrow E^+ + \max\{E[i], 0\} \)

end if

end if

end for

for \( (i \leftarrow 0; i < NF; i \leftarrow i + 1) \) do

if \( a[i] == 1 \) then

if \( E^i > 0 \) then

\( E^i \leftarrow E[i] - R E[i] / E^+ \)

end if

end if

end for

end for
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The Mark I family of models was elaborated in a series of papers and books, in particular: E. Gaffeo, D. Delli Gatti, S. Desiderio, and M. Gallegati,


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