Economics
The Open-Access, Open-Assessment E-Journal

Discussion Paper
No. 2017-47 | July 26, 2017 | http://www.economics-ejournal.org/economics/discussionpapers/2017-47

How costly are public sector inefficiencies? A theoretical framework for rationalising fiscal consolidations

Jorge Onrubia-Fernández and A. Jesús Sánchez Fuentes

Abstract
Fiscal adjustments consisting of spending cuts or tax increases are generally presented as the unavoidable way for achieving public finance sustainability in the long term. However, this view of fiscal consolidation processes is limited as it leaves out other aspects related to public sector performance which are relevant not only from the macroeconomic but also from the microeconomic perspective. This paper models Public Sector Performance (PSP) by proposing a theoretical framework that integrates the conventional methodology for measuring its productive efficiency and the monetary assessment of social welfare changes linked to public policy reforms. Two equivalent measures of social welfare change generated by improving (or worsening) productive efficiency are deduced using duality theory. The first is obtained from the cost function, while the second arises directly from the production function. The results reveal that taking advantage of budgetary savings obtained from this approach constitutes a valuable tool for designing welfare-enhancing fiscal consolidation packages, meanwhile promoting sound fiscal balances and growth prospects over the long term.

JEL D24 D60 D61 H40 H50
Keywords Public sector efficiency; technical efficiency; allocative efficiency; social welfare changes

Authors
Jorge Onrubia-Fernández, ✉ Universidad Complutense de Madrid, FEDEA and GEN, jorge.onrubia@ccee.ucm.es
A. Jesús Sánchez Fuentes, Universidad Complutense de Madrid, ICEI and GEN

Both authors acknowledge comments from D. Santín, M. Hortas-Rico, J.J. Díaz, S. Bach and participants at the Banca d'Italia 14th Workshop on Public Finance, European Central Bank Fiscal Policies Division Seminar and XIX Encuentro de Economía Pública. Both authors acknowledge the financial support of the Spanish Ministry of Economy and Competitiveness (project ECO2012-37572). Onrubia acknowledges the financial support of the Santander-UCM program (project GR35/10A) and Sánchez-Fuentes the financial support of the Regional Government of Andalusia (project SEJ 1512).

Citation Jorge Onrubia-Fernández and A. Jesús Sánchez Fuentes (2017). How costly are public sector inefficiencies? A theoretical framework for rationalising fiscal consolidations. Economics Discussion Papers, No 2017-47, Kiel Institute for the World Economy. http://www.economics-ejournal.org/economics/discussionpapers/2017-47

Received July 16, 2017 **Accepted as Economics Discussion Paper** July 25, 2017
Published July 26, 2017

1 Introduction

Nowadays, an essential issue to be analysed in depth is the relationship between the productive efficiency of the public sector and the potential budgetary savings associated with its improvement. This is true especially for advanced economies in which effects of the current crisis are affecting public finances in a more evident way. Measurement of these budgetary savings constitutes an alternative fiscal policy tool which goes beyond the traditional view of fiscal consolidation (spendings cuts or tax hikes). This measure is helpful not only for short-term consolidation but it is required in order to ensure a sound long-term growth path.

However, as advanced in Pestieau and Tulkens (1993), properly measuring the public sector performance it is not straightforward prospect. Evaluation of public sector policies, they argue, requires the use of a multidimensional approach, and it entails a number of other difficulties. First, the objectives assigned to production units may not be always compatible with one another. Second, measuring the degree to which those objectives are satisfied will force analytists to introduce some basic value judgements in order to weigh certain partial indicators. Finally, the trade-off between allocative and non-allocative objectives may affect management in these public sector units.

Initially, the literature on public sector performance focussed on several issues related to public choice theory (collective decision making, lack of competition and rent seeking), in order to understand the behaviour of public decision-makers (politicians, bureaucrats, lobbyists, etc.).[1] Afterwards, since the late 1980s, the measurement of productive efficiency has attracted increasing interest within the public economics area.[2] This trend is even more evident for some specific sectors typically provided by the public sector -health, education, social care, etc.-.[3] This growing literature has mainly focussed on developing quantitative methodologies (usually grouped into parametric and non-parametric methods) from which we may achieve empirical measures of technical, allocative or overall efficiency with which a number of units (assumed to be homogeneous) have produced the public good(s) and service(s). Thus, all these measures usually provide us with one scenario to compare, in relative terms, the performance of these above mentioned public units.

[1]See Wilson (1989), Hettich and Winer (1993), Wolf (1993), Horn (1995) and Mueller (2003), among others, for further explanations about the topics dealt with from the point of view of this literature -government size, budgetary incrementalism, X-inefficiency, etc.-.

[2]See, among others, Afonso et al. (2005, 2010a, 2010b), Borge et al. (2008) and Casiraghi et al. (2009) for cross-country quantitative analyses on overall public sector performance and efficiency.

[3]See Clements (2002) and Afonso and St. Aubyn (2005) for illustrative applications providing public policy-specific efficiency scores.

However, as Pestieau (2009) highlighted, it is not simple to move from theoretical concepts to empirical assessments, when dealing with issues related to the public sector efficiency. The difficulties derive from both the conceptual and feasibility perspectives. On the one hand, it has not thus far been established how to break up the sources of allocative and technical efficiency or the relationship that exists between them. For instance, a poor allocation of resources may prove unfavorable to achieving high scores for technical efficiency. On the other hand, the lack of full availability of data (on inputs prices, for instance) may condition how we measure public sector efficiency. In this respect, analysts compare ideal and real-world assumptions to deal with this issue and to determine what may be done under each scenario. Fundamentals of this literature underlie the construction of our theoretical proposal. Surely, these contributions measuring the productivity of public services are very useful to improving the management of public resources. Nonetheless, we believe there is lack of literature connecting these results with the potential budgetary gains that may arise from a reduction of public sector inefficiency.

In this vein, the OECD (2011) has highlighted the importance of implementing reforms to increase the efficiency of public spending, specially for governments that are currently facing extraordinary budgetary imbalances. Among others, the OECD refers to the need to improve the productivity of public spending on education and health. In the first case, it is estimated that the gradual adoption of best practices in primary and secondary education could save resources equal to around 0.5% of GDP (with different countries ranging from 0.2 % to 1.2%), without compromising current educational targets. In the case of health, the resources released by improvements in productive efficiency could be even higher, at around 2% of GDP (between 0.4% and 4.8% in the full range). Along those same lines, the OECD (2015) highlights how the Great Recession has created an increasingly complex environment for governance, limited fiscal space, and growing demands for transparency and accountability. Against this background, governments are continually challenged to demonstrate better performance and management of available resources.

Furthermore, we believe that monetary gains could be significant in terms of social welfare. In this respect, it is important to account for not only budgetary savings but also the monetary gains in terms of income and wealth derived from a better education and health practices. Moreover, from a marginal cost of public funds perspective, we should also consider the resources released due to the reduction in deadweight losses caused by distortionary.

The aim of this paper is to provide a theoretical framework which consistently allows for integration of the conventional methodology for measuring productive efficiency and the monetary assessment of social welfare changes related to the public sector performance. In particular, we deduce two measures of social welfare changes generated by an improvement (or decline) in productive efficiency associated with the procurement of a public good. The first measure is obtained from the cost function, or in other words, from the supply side, while the second arises directly from the production function. According to duality theory, both measures are equivalent and drawn from the same set of information.

The rest of the paper is organized as follows. In the second section, we introduce our theoretical framework, on the basis of conventional measures of productive efficiency using Farrell's radial approach. In the third section, we present our integrated approach which combines different dimensions typically involved in policy-makers decisions (welfare changes, measures of inefficiencies, etc.). Finally, the last section presents our conclusions.

2 Theoretical framework

2.1 Recent concerns around Public Sector Efficiency

The monitoring of public sector activity and the potential derivation of measures of Public Sector Efficiency (PSE) clearly justify the increasing interest of analyses related to Public Sector Performance (PSP). This section briefly discusses the recent evolution of literature focussed on PSE, which refers to the efficient allocation and production of the public good and services. The existing literature is comprised of alternative approaches with which to measure, and evaluate, the PSP and (consequently) the PSE. Here we present a non exhaustive description of how this literature has evolved. Firstly, a growing number of studies (Afonso *et al.*, 2005 and 2010b, Borge *et al.* 2008, and Clements, 2002, among others) translated the traditional approach used to analyse the productive efficiency of firms to the case of public sector units (countries, municipalities, schools, hospitals, etc.) with the aim of obtaining empirical measures of the PSE for a set of units and rank them. Secondly, certain studies (Borge *et al.* 2008, among others) have further explored the identification of determinants of these empirical measures. An alternative perspective is considered by other authors (see Afonso *et al.*, 2010a, Casiraghi *et al.*, 2009, among others) in order to include the distributional concerns traditionally linked to public sector activity into the efficiency analysis.

All in all, it can be observed that some caveats remain necessary. Most of these analyses have focussed on productive efficiency or technical efficiency (ψ) have left out of their analysis issues related to allocative efficiency (γ), a relevant component of overall efficiency (η). This latter measure is our main interest in this paper. Second, the distributional concern has not yet been fully incorporated into analysis, although it is a component involved in policy-maker decisions.

Our paper aims to supersede these caveats by combining the elements presented; (i) empirical measures of efficiency, (ii) welfare impact and distributional concerns, and (iii) a monetary valuation of inefficiencies measured.

2.2 The public sector

This section introduces the notation used in subsequent sections and models the Public Sector Performance according to a framework which could be adapted to very different analyses.

Our model can be briefly described as follows. The public sector produces a vector of goods and services ($Y = \left(y^1, ..., y^H\right)$) which we consider excludable unlike pure public goods[4]. Each y^h is produced by a public agency with the corresponding production function for the case of single output, such that,

$$y^h = f_h(X) \tag{1}$$

where $X = (x_1, ..., x_n)$ is a vector of n inputs including fixed capital required for the activity and $f_h \in S = \{(X, Y) : X \text{ can produce } Y\}$ with S representing the set of technologies.

The unitary price for each of these n inputs are included in the vector $W = (w_1, ..., w_n)$. Consequently, the total cost of producing y^h (c^h) is defined:

$$c^h\left(y^h\right) = \sum x_i w_i \tag{2}$$

Assuming $H = 1$, for the sake of clarity in the presentation, this theoretical framework allows us to introduce the notation used in posterior sections by formally defining all the standard concepts of efficiency mentioned above from the input-oriented perspective.[5] First, departing from Farrell's (1957) efficiency approach, given the minimum quantity of inputs needed for producing the level of output Y (X^*), *technical efficiency* (ψ) is defined as the ratio between X and X^*, such that,

$$\psi = \frac{\|X^*\|}{\|X\|} \tag{3}$$

[4]Rivalry and excludability are assumed to consistently reflect changes in the demand observed for each public good.

[5]Analogous definitions can be found in the literature according to the output-oriented measures (see Coelli, 2005) for a detailed comparison of both approaches). There are no divergences in the analyses carried out from both perspectives. Therefore, one of them can be excluded.

where $\|\cdot\|$ represents the euclidean norm.[6]

Second, given the combination of inputs producing Y at the minimum cost (X^{**}), the allocative efficiency (γ) is defined as the following ratio:

$$\gamma = \frac{\|X^{**}\|}{\|X^{*}\|} \tag{4}$$

Third, the overall efficiency can be defined as the product of expressions (3) and (4):

$$\eta = \frac{\|X^{**}\|}{\|X\|} \tag{5}$$

Finally, we derive the corresponding expression for η in terms of production costs[7]:

$$\eta = \frac{c^{**}}{c} \tag{6}$$

where c and c^{**} are, respectively, the actual level of production costs and the production costs corresponding to X^{**}, the efficient combination of inputs when producing Y, from the technical and the allocative perspective.

3 PSE analysis: an integrated approach

3.1 The "expenditure-efficiency" function

The framework described above can be observed from a different perspective, in order to consider the dual version of the same problem. Under these circumstances, a given level of public output ($\hat{y}^h$) may be explained by the corresponding expenditure function ($c(\hat{y}^h)$), and the degree of overall efficiency ($\eta(\hat{y}^h)$). In other words, given the vector input prices (W), we can define an *"expenditure-efficiency" function* (ϕ) which is implicit in the conventional production function with productive factors:

$$\hat{y}^h = f(X)|_W \rightarrow \hat{y}^h = \phi(c,\eta)|_W \tag{7}$$

First of all, from (6), we can express the budgetary cost of producing a quantity of public goods ($\hat{y}^h$) from the vector of inputs (X^{**}) and the degree of overall efficiency reached in the productive process, η:

$$c(\hat{y}^h) = \eta^{-1}\sum_{i=1}^{n} x_i^{**} w_i \tag{8}$$

[6] Alternative approaches to the concept of productive efficiency would imply consideration of other definitions of norm. However, the subsequent formalization is valid for any alternative distance definition (and its respective norm).

[7] A proof of this equivalence between output and cost-oriented efficiency approaches can be seen in Coelli (2005).

Second, by applying the inverse function theorem to the optimal technology f_h^{**}, the optimal quantities of each input (x_i^{**}) to produce $\hat{y}^h$ are obtained. Note that these values only depend on factor prices and technological parameters of the production function:

$$x_i^{**} = f_h^{**-1}\left(\hat{y}^h, W\right), i \in \{1, 2, ..., n\} \tag{9}$$

Next, by combining (8) and (9), and solving for $\hat{y}^h$ we derive the expenditure-efficiency function ϕ, as proposed:

$$\hat{y}^h = \phi\left(c\left(\hat{y}^h\right), \eta\right)|_W \tag{10}$$

The latter expression shows that the optimal provision of this public good depends not only on the cost of producing it (actual public expenditure), but also the degree of efficiency with which it is produced (by the public agency). Empirical studies that analyse Public Sector Performance (PSP) from a public expenditure approach must bear this fact in mind.

3.2 Changes in the PSE, welfare impact and monetary valuation

This section presents an integrated approach which allows us to integrate the different dimensions involved in the evaluation of Public Sector Performance; (i) changes in the degree of efficiency, (ii) welfare impacts linked to public policies, and (iii) monetary valuation of effects. The latter may facilitate the understanding of the inefficiency costs. Moreover, an improvement in the degree of efficiency will help to provide the same public good or service but with a lower level of spending.

For the sake of clarification, we detail our assumptions. First, in the following analysis it is assumed that any change in the degree of efficiency is exogenous. Nevertheless, as Gibbons (2005) posits, the existence of internal disturbances within the organizations (miscoordination, lack of incentives, etc.) may be the source of inefficiencies. Second, the social welfare generated by consumption of public good (y) is measured in monetary value in the conventional way; that is, by computing the area under the curve of demand for the good and subtracting the cost of the inputs used in its production[8]. Additionally, to obtain accurate measurements of changes in consumer welfare we assume the demand functions involved to be compensated[9]. All in all, this theoretical framework contributes to measure welfare impacts linked to changes (improvement/worsening) in

[8] Note that, as in previous sections, the notation is simplified to a single public good y to highlight the underlying intuitions.

[9] See Willig (1976) for a discussion of the accurate measurement of these areas.

the degree of efficiency (η) with which the public good is produced.[10] Starting from Myrick-Freeman and Harrington's (1990) analysis framework for private goods, we next introduce productive efficiency concerns about public production in the evaluation of its social welfare impact in monetary terms.

Thus, using our "expenditure-efficiency" function defined in (10), we can define the following social welfare function:

$$\Omega = \Omega\left(X, W, \eta\right) = \int_0^y p(u)du - \sum\nolimits_{i=1}^n x_i w_i \tag{11}$$

where $p\left(\cdot\right)$ is the compensated demand function specified in its inverse form.

From (11) we obtain the first order conditions with respect to the inputs used (x_i), such that,

$$\frac{\partial \Omega}{\partial x_i} = p(y)\frac{\partial y}{\partial x_i} - w_i = 0, i = 1, ..., n \tag{12}$$

which determine the input demand functions $x_i^{**}\left(w_i, \eta\right)$ for all i. This should be noted here that these values are precisely those corresponding to the optimal vector of production factors, X^{**}. It allows us to compute the optimal output level of public good for a given level of overall efficiency,

$$y^{**}\left(\eta\right)|_W = \varphi\left(x_i^{**}\left(w_i, \eta\right), \eta\right) \tag{13}$$

Likewise, we can define the social welfare function associated with the production of this public good by considering the overall efficiency (η) as main argument:

$$\Omega\left(\eta\right)|_W = \varpi\left(x_i^{**}\left(w_i, \eta\right), \eta\right) \tag{14}$$

Applying the envelope theorem to the algebraic analysis described above, we obtain the following proposition.

Proposition 1 *The net welfare gain is the value of the marginal contribution, in monetary terms, brought about by a reduction (or increase) of overall inefficiency in the production function, so that,*

$$\frac{\partial \Omega\left(\eta\right)}{\partial \eta} = p(y^{**})\frac{\partial y^{**}\left(\eta\right)}{\partial \eta} - \sum\nolimits_{i=1}^n w_i \frac{\partial x_i^{**}\left(\cdot, \eta\right)}{\partial \eta} = p(y^{**})\varphi_\eta\left(x_i^{**}\left(w_i, \eta\right), \eta\right) \tag{15}$$

Some interesting implications follow. First, this result defines a relationship between the production function and the changes in welfare computed in the light of modification of the degree of efficiency. Second, it can be observed that, under full productivity of all inputs, the value generated

[10] Hereafter, we consider a generic public good in order to simplify the notation. Thus, we avoid the superscript "h" used so far to refer to different public goods.

by an infinitesimal improvement in productive efficiency is explained by the increase in the output generated. Third, from a different perspective, this gain could be seen as a closer approximation (φ_η) to the optimal technology (x_i^{**}).

Next, the dual version of this result is achieved. To do this, from (13) one can define the cost functions related to this production as a function of the optimal level of public good, the vector of inputs associated with the optimal technology, and the degree of overall efficiency reached, so that,

$$c(\eta) = c(y^{**}(\eta), \eta) \tag{16}$$

Accordingly, considering the difference between consumer surplus and producer quasi-rents, we can rewrite (11) as,

$$\Omega(y^{**}, \eta) = \int_0^{y^{**}} p(u)du - c(y^{**}, \eta) \tag{17}$$

Note that y^{**} guarantees that social welfare is maximized given that the price equals the marginal cost of public good (equilibrium first order condition).[11]

$$p(y^{**}) = \frac{\partial c(y^{**}, \eta)}{\partial y^{**}} \tag{18}$$

Again, combining (17) and (18), the following proposition emerges.

Proposition 2 *The net welfare gain (loss) is the value of the marginal contribution, in monetary terms, brought about by the reduction (increase) of production cost as a consequence of an improvement (worsening) of the degree of overall inefficiency*

$$\frac{\partial \Omega(y^{**}, \eta)}{\partial \eta} = -\frac{\partial c(y^{**}, \eta)}{\partial \eta} \tag{19}$$

Proof. Given (17), we compute the total derivative with respect to the degree of efficiency (η). That is,

$$\frac{d\Omega(y^{**}, \eta)}{d\eta} = \frac{\partial \Omega(y^{**}, \eta)}{\partial y^{**}}\frac{\partial y^{**}}{\partial \eta} + \frac{\partial \Omega(y^{**}, \eta)}{\partial \eta} \tag{20}$$

where:

$$\frac{\partial \Omega(y^{**}, \eta)}{\partial y^{**}} = p(y^{**}) - \frac{\partial c(y^{**}, \eta)}{\partial y^{**}} \tag{21}$$

and

$$\frac{\partial \Omega(y^{**}, \eta)}{\partial \eta} = p(y^{**})\frac{\partial y^{**}}{\partial \eta} - \left(\frac{\partial c(y^{**}, \eta)}{\partial y^{**}}\frac{\partial y^{**}}{\partial \eta} + \frac{\partial c(y^{**}, \eta)}{\partial \eta}\right) \tag{22}$$

[11]Hereafter, for the sake of simplicity, we will not explicitly notate the dependence between the optimal level of output (y^{**}) and the degree of overall efficiency (η).

Firstly, as a consequence of (18), we could identify $\frac{d\Omega(y^{**},\eta)}{d\eta}$ and $\frac{\partial\Omega(y^{**},\eta)}{\partial\eta}$. Next, from (22), grouping conveniently and again using (18), we obtain the proposition. ■

Corollary 1 *An improvement in the degree of overall inefficiency always involves an increase in social welfare.*

From this set of results, some interesting conclusions can be derived. First, this result defines a relationship between the cost function and the changes in welfare computed when the degree of efficiency is modified. Second, the infinitesimal improvements in productive efficiency obtained lead to a reduction in the cost of production and, consequently, they are welfare enhancing. Third, combining propositions 1 and 2 we obtain that the two welfare measures proposed must coincide due to the duality in the relationship between the production function and the cost function, which is underlying in (7).

3.3 Distributional issues

In this subsection, we analyse how the welfare gains from increased efficiency affect consumers of public goods and public sector itself as the producer. In this respect, we first identify the efficiency gains effects on consumer welfare. Let Ω^C be the measure of consumer surplus used (usually equivalent or compensatory variation), so that,

$$\Omega^C(\eta) = \int_0^{y^{**}(\eta)} p(u)du - p(y^{**}(\eta))y^{**}(\eta) \tag{23}$$

Then, the consumer's marginal gain is,

$$\frac{\partial\Omega^C}{\partial\eta} = p(y^{**}(\eta))\frac{\partial y^{**}}{\partial\eta} - \frac{\partial p(y^{**})}{\partial y^{**}}\frac{\partial y^{**}}{\partial\eta}y^{**}(\eta) - p(y^{**}(\eta))\frac{\partial y^{**}}{\partial\eta} \tag{24}$$

And simplifying,

$$\frac{\partial\Omega^C}{\partial\eta} = -\frac{\partial p(y^{**})}{\partial y^{**}}\frac{\partial y^{**}}{\partial\eta}y^{**}(\eta) \tag{25}$$

Now, from the producer's perspective, we repeat a similar strategy. First, we define the producer's surplus in terms of η:

$$\Omega^S(\eta) = p(y^{**}(\eta))y^{**}(\eta) - c(y^{**},\eta)\sum\nolimits_{i=1}^{n} x_i^{**}w_i \tag{26}$$

where x_i^{**} is determined by the n input demand functions, $x_i^{**}(w_i,\eta)$.

Again, the producer's marginal gain can be obtained by differentiating the previous expression:

$$\frac{\partial \Omega^S}{\partial \eta} = \frac{\partial p(y^{**})}{\partial y^{**}} \frac{\partial y^{**}}{\partial \eta} y^{**}(\eta) + p(y^{**}(\eta)) \frac{\partial y^{**}}{\partial \eta} - \frac{\partial c(y^{**}, \eta)}{\partial y^{**}} \frac{\partial y^{**}}{\partial \eta} - \frac{\partial c(y^{**}, \eta)}{\partial \eta} \tag{27}$$

Taking into account equation (18), we find that

$$\frac{\partial \Omega^S}{\partial \eta} = -\frac{\partial c(y^{**}, \eta)}{\partial \eta} + \frac{\partial p(y^{**})}{\partial y^{**}} \frac{\partial y^{**}(\eta)}{\partial \eta} y^{**}(\eta) \tag{28}$$

In light of these definitions, the following can be stated.

Proposition 3 *An improvement in the degree of overall inefficiency always leads to an increase in consumer welfare. By contrast, this welfare gain is not guaranteed in the case of producers of public goods.*

Proof. On the one hand, for consumers, this proof can be reduced to check the signs of the expressions mentioned above. As $\frac{\partial p(y^{**})}{\partial y^{**}} \leq 0$ and $y(\eta) > 0$, depending on the sign of $\frac{\partial y^{**}(\eta)}{\partial \eta}$ the consumer's net welfare gain will be positive or negative. The optimal vector of inputs (from the technological and minimisation of cost perspectives) is taken as presented in (13). As a consequence, in equilibrium a reduction of inefficiency may lead to a decreased level of output. To clarify this latter statement, we differentiate the first order conditions mentioned in (18) to achieve the following expression:

$$\frac{\partial p(y^{**})}{\partial y^{**}} \frac{\partial y^{**}(\eta)}{\partial \eta} = \frac{\partial^2 c(y^{**}, \eta)}{\partial y^{**2}} \frac{\partial y^{**}(\eta)}{\partial \eta} + \frac{\partial^2 c(y^{**}, \eta)}{\partial y^{**} \partial \eta} \tag{29}$$

Grouping conveniently:

$$\frac{\partial y^{**}(\eta)}{\partial \eta} = \frac{\frac{\partial^2 c(y^{**}, \eta)}{\partial y^{**} \partial \eta}}{\frac{\partial p(y^{**})}{\partial y^{**}} - \frac{\partial^2 c(y^{**}, \eta)}{\partial y^{**2}}}$$

On the one hand, looking at the denominator, it is straightforward to establish that $\frac{\partial p(y^{**})}{\partial y^{**}} - \frac{\partial^2 c(y^{**}, \eta)}{\partial y^{**2}} < 0$. On the other hand, any improvement in η lead to reductions in costs. Thus, $\frac{\partial^2 c(y^{**}, \eta)}{\partial y^{**} \partial \eta} < 0$ and, consequently, $\frac{\partial y^{**}(\eta)}{\partial \eta}$ is always positive.

All in all, we have proven that increases in consumer welfare can be derived from the response in the production costs to an improvement in overall efficiency.

On the other hand, for producers, using the price-elasticity of public good demand, defined as $\varepsilon = \frac{p(y^{**})}{y^{**}\frac{\partial p(y^{**})}{\partial x}}$, which is negative by definition, we can prove that $\frac{\partial \Omega^S}{\partial \eta}$ will be negative if and only if $\frac{\partial y^{**}(\eta)}{\partial \eta} > \varepsilon \frac{\frac{\partial c(y^{**},\eta)}{\partial \eta}}{p}$.

Under these conditions, the difference between the social welfare change and the variation in the consumer surplus would be negative. Therefore, there is no guarantee of obtaining welfare gains for producers. ■

From Proposition 3, the distribution of welfare gains derived from an improvement in the degree of efficiency may be established. Our results indicate that the determinants are the optimal output response to this increase and the price-elasticity of demand. In short, two different possibilities are achieved:

$$\text{(i) } 0 < \frac{\partial y^{**}(\eta)}{\partial \eta} < \varepsilon \frac{\frac{\partial c(y^{**},\eta)}{\partial \eta}}{p} \Leftrightarrow \frac{\partial \Omega^C}{\partial \eta} > 0, \frac{\partial \Omega^S}{\partial \eta} > 0 \tag{30}$$

$$\text{(ii) } \varepsilon \frac{\frac{\partial c(y^{**},\eta)}{\partial \eta}}{p} < \frac{\partial y^{**}(\eta)}{\partial \eta} \Leftrightarrow \frac{\partial \Omega^C}{\partial \eta} > 0, \frac{\partial \Omega^S}{\partial \eta} < 0 \tag{31}$$

3.4 Social Welfare changes over time

In order to show a different perspective on the conclusions described so far, we now consider an example to illustrate (and reinforce) the underlying intuition. We consider a scenario in which the overall efficiency to produce the public good y improves between two moments in time ($t = 0$, $t = 1$), from η_0 to η_1. To quantify the value of social welfare generated by the change in the degree of efficiency, we may choose to integrate one of the two welfare change measures presented in Propositions 1 and 2, respectively, and use $[\eta_0, \eta_1]$ as integration interval:

$$\Delta\Omega = \int_{\eta_0}^{\eta_1} p(y^{**}(\eta))\varphi_\eta\left(x_i^{**}(w_i,\eta),\eta\right)d\eta = -\int_{\eta_0}^{\eta_1} c(y^{**}(\eta),\eta)d\eta \tag{32}$$

The direct quantification of $\Delta\Omega$ from either of the two alternatives shown in (32) requires determination of the changes in the equilibrium output and in the optimal combination of inputs caused by the change in the degree of efficiency.

Contrarily, this computation may be simplified when information is available on production levels of the public good before and after the change analysed. To do this, using (11), we simply need to calculate the difference between initial and final social welfare values

$$\Delta\Omega = \int_0^{y_1} p(u)du - c(y_1,\eta_1) - \int_0^{y_0} p(u)du + c(y_0,\eta_0) \tag{33}$$

By using this quantification, one can observe how the potential welfare gains resulting from improved efficiency come from the displacement of the supply curve (as there is a reduction in the cost function). In other words, the marginal cost of producing public good goes from $\frac{\partial c(y,\eta_0)}{\partial y}$ to $\frac{\partial c(y,\eta_1)}{\partial y}$.

We can obtain an alternative expression for (33) by incorporating the change experienced by the cost function. To do this, we use the line integral of its gradient along any path between (x_0, η_0) and (x_1, η_1), and integrate along the line connecting them, such that[12]:

$$\Delta\Omega = \int_{y_0}^{y_1} p(u)du - \int_{\eta_0}^{\eta_1} \frac{\partial c(y_0, \eta)}{\partial \eta} d\eta - \int_{y_0}^{y_1} \frac{\partial c(y_1, \eta)}{\partial y} dy \tag{34}$$

Figure 1 shows in the green area the net social welfare gain expressed in (34). For the sake of simplicity, we assume linearity for all the curves involved; both compensated public good demand, and marginal cost functions (pre and post).

According to the analysis presented above, we could additionally define welfare changes experienced by consumers and the public sector as public good supplier. On the one hand, consumers enhance their welfare by increasing the area under the compensated demand curve, as a consequence of the equilibrium price decrease, from p_0 to p_1.

Figure 2 shows the consumers welfare gain, which is represented by the total blue area. On the other hand, the blue squared area represents the net change in producer welfare, which results from compensating for the decrease in the initial surplus due to a lower resulting price, whereas the red area refers to the new surplus caused by the reduction of costs charted in the new marginal cost function.

As a consequence, combining this graphical evidence with propositions presented above, we conclude that:

(i) For any $\eta > 0$, $\Delta\Omega = \Delta\Omega^C + \left(\Delta\Omega^S - \nabla\Omega^S\right) > 0$.

(ii) We have no guarantee implying that $\left(\Delta\Omega^S - \nabla\Omega^S\right) > 0$.

4 Potential empirical implementation

One purpose of this paper is to provide an integrated theoretical framework which may support comprehensive empirical analyses measuring public sector efficiency. This section aims to discuss how the theoretical framework described in previous sections may be translated to future empirical

[12]See Myrick-Freeman and Harrington (1990) for further details on the underlying method, which is outside the scope of this paper.

applications. However, as Pestieau (2009) pointed out, it is not straightforward to move from theoretical concepts to empirical assessments, when dealing with issues related to public sector efficiency.

The difficulties derive from both conceptual and feasibility perspectives. On the one hand, it has not been so far established how to break down the sources of allocative and technical efficiency, or the relationship that exists between them. For instance, a poor allocation of resources may prove unfavorable to achieving high scores of technical efficiency. On the other hand, the lack of full availability of data (on input prices, for instance) may condition how we empirically quantify the public sector efficiency. In this respect, they compare both ideal and real-world assumptions to deal with this issue and finally determine which may be done under each scenario.

Interestingly for a policy-oriented exercise, the potential empirical implementation of the proposal presented in this paper would lead to monetary valuations measuring the changes in the overall efficiency. Moreover, our framework integrates elements related to efficiency as wells as those related to distribution (to the extent to which this paper revisits the efficiency-equity trade-off). We consider that the current state of the art could adequately deal with these empirical challenges, both when measuring the performance of public units and when estimating the functions involved. Particularly, the following information would be required: (i) a vector containing the unitary price for each input, (ii) an estimation of the demand function and, finally, (iii) an estimation of the production function (cost function) in the primal (dual) approach.

Logically, we are aware that implementation of our proposal requires some "extra" information about prices. In this sense, recent studies have wisely dealt with this caveat (i.e. Gronberg *et al.,* 2012).[13]. In this paper the authors estimate a translog stochastic cost frontier model using panel data for charter campuses and traditional public campuses in Texas over the five-year period 2005-2009. The authors model expenditures per pupil as a function of three output indicators (q), two measures of input prices (w) and five environmental factors (x).

Furthermore, another decision that must be made is which efficiency measurement methodology is preferable according to the characteristics of the analysis. In this regard, Data Envelopment Analysis (DEA), Free Disposal Hull (FDH), econometric/stochastic, and semi-parametric approaches have been extensively applied although we focus here on information requirements as a key ele-

[13]The description of specific alternatives to overcome this difficulty goes beyond the scope of this theoretical analysis.

ment.[14]

5 Concluding remarks

In light of the current economic situation, the near future points to intense (supra-/intra-) national social debates on the monitoring of public sector performance (health, education, etc.). In particular, advanced economies are currently facing issues related to the reorganization of their welfare state. Within this framework, quantifying these budgetary savings strongly constitute an alternative fiscal policy tool which goes beyond the traditional view of fiscal consolidation (spending cuts or tax hikes). This measure is helpful not only for short-term consolidation but it is required in order to ensure a sound long-term growth path.

In this respect, important policy implications can be derived from our results. First, this paper has presented an integrated approach which combines different dimensions involved in the usual policy-maker decisions (efficiency in the production of a given public good, welfare impacts, and monetary valuation). This proposal satisfies additional features in comparison to the usual methodologies. Mainly, our approach would allow translation of measures of (in)efficiencies into to a monetary value. Second, our proposal may be adapted for use within a wide variety of empirical applications monitoring and/or evaluating public sector performance. For instance, this approach may provide guidance to the design of fiscal consolidation programs, so that they are compatible with a more efficient use of public resources. Finally, we have derived some analytical results which assist in the understanding of the underlying intuitions and their linkages.

To conclude, some interesting lessons can be extracted regarding the application of this approach to empirical analyses. First, the final results would lead to monetary valuations of the changes in the overall efficiency, which would prove a very interesting tool from the policy-maker perspective. Second, our approach integrates elements related to efficiency with others related to equity, allowing us to explore this classic trade-off. Third, this approach requires an estimate of production function as well the cost function, which may limit its application when information on the production procedure and/or the production cost is limited. However, as mentioned in section 4, existing literature discusses how to deal with major data requirements.. All in all, each of these issues should be decided taking into account the specific features of the public service provision under analysis.

[14]See Adam *et al.* (2011) as an illustration of the combined application of these methods.

References

[1] Adam, A., Delis, M and Kammas, P. (2011). Public sector efficiency: leveling the playing field between OECD countries, *Public Choice*, 146, pp. 163-183.

[2] Afonso, A. and Scaglioni, C. (2005). Public Services Efficiency Provision in Italian Regions: a Non-Parametric Analysis, Working Papers 2005/02, Department of Economics at the School of Economics and Management (ISEG), Technical University of Lisbon.

[3] Afonso, A., Schuknecht, L., and Tanzi, V. (2005). Public sector efficiency: An international comparison, *Public Choice*, 123 (3), pp. 321-347.

[4] Afonso, A., Schuknecht, L. and Tanzi, V. (2010a). Income distribution determinants and public spending efficiency, *Journal of Economic Inequality*, 8 (3), pp. 367-389.

[5] Afonso, A., Schuknecht, L. and Tanzi, V. (2010b). Public sector efficiency: Evident for new EU member states and emerging markets, *Applied Economics*, 42 (17), pp. 2147-2164.

[6] Afonso, A. and St. Aubyn, M. (2005). Non-parametric approaches to education and health efficiency in OECD countries, *Journal of Applied Economics*, pp. 227-246.

[7] Borge, L.E., Torberg F. and Tovmo P. (2008). Public sector efficiency: the role of political and budgetary institutions, fiscal capacity, and democratic participation, *Public Choice*, 136, pp. 475-495.

[8] Casiraghi, M., Giordano, R. and P. Tommasino (2009). *Behind public sector efficiency: the role of culture and institutions*, in: *The quality of public finances and economic growth*, edited by S. Barrios, L. Pench and A. Schaechter, European Commission, Occasional Paper No. 45. March.

[9] Charnes, A., Cooper, W. W., and Rhodes, E. (1978). Measuring the efficiency of decision making units, *European Journal of Operational Research*, 2 (6), pp. 429-444.

[10] Clements, B. (2002). How Efficient is Education Spending in Europe?, *European Review of Economics and Finance*, 1(1), pp. 3-26.

[11] Coelli, T.J., Prasada Rao, D.S., O'Donnell C.J. and Battese, G.E. (2005). *An introduction to efficiency and productivity analysis* (Second Edition). Springer

[12] Debreu, G. (1951). The Coefficient of Resource Utilization, *Econometrica*, 19, pp. 273–292.

[13] Fakin, B. and de Crombrugghe, A. (1997). *Fiscal adjustment in transition economies: social transfers and the efficiency of public spending: a comparison with OECD countries*, Policy Research Working Paper 1803. Washington, DC: World Bank.

[14] Farrell, M. J. (1957). The Measurement of Productive Efficiency, *Journal of the Royal Statistic Society*, 120, pp. 253–281.

[15] Figlio, D.N. and Kenny, L.W. (2009). Public sector performance measurement and stakeholder support, *Journal of Public Economics*, 93, pp. 1069–1077.

[16] Gibbons, R. (2005). Four formal(izable) theories of the firm?, *Journal of Economic Behavior & Organization*, 58, pp. 200–245.

[17] Gronberg, T.J., Jansen, D.W. and Taylor, L.L. (2012). The relative efficiency of charter schools: A cost frontier approach, *Economics of Education Review*, 31, pp. 302-317.

[18] Gupta, S. and Verhoeve, M. (2001). The efficiency of government expenditure Experiences from Africa, *Journal of Policy Modelling*, 23, pp. 433-467.

[19] Hettich, W. and Winer, S.L. (1993). Economic efficiency, political institutions and policy analysis, Kyklos, 46 (1), pp. 3-25.

[20] Horn, M.J. (1995). *The political of Public Administration. Institutional Choice in the Public Sector.* Cambridge, MA: Cambridge University Press.

[21] Myrick Freeman III, A. and W. Harrington (1990). Measuring Welfare Values of Productivity Changes, *Southern Economic Journal*, 56, pp. 892-904.

[22] Mueller, D. (2003). *Public Choice III.* New York: Cambridge University Press.

[23] OECD (2011). *Economic Policy Reforms 2011. Going for Growth.* OECD: Paris.

[24] OECD (2015). *Government at a Glance 2015*, OECD: Paris.

[25] Pestieau, P. (2009). Assessing the performance of the public sector, *Annals of Public and Cooperative Economics*, 80, pp. 133-161.

[26] Pestieau, P. and Tulkens, H. (1993). Assessing and explaining the performance of public enterprise, *FinanzArchiv*, 50, 293-323.

[27] Willig, R. D. (1976). Consumer's surplus without apology, *American Economic Review*, 66, pp. 589–597.

[28] Wilson, J.Q. (1989). *Bureaucracy: What Government Agencies Do and Why They Do It.* New York: Basic Books.

[29] Wolf, Jr. C. (1993). *Markets or Governments. Choosing between Imperfect Alternatives.* Cambridge, MA: The MIT Press.

6 Figures

Figure 1: *Welfare impact brought by a PSE gain.*

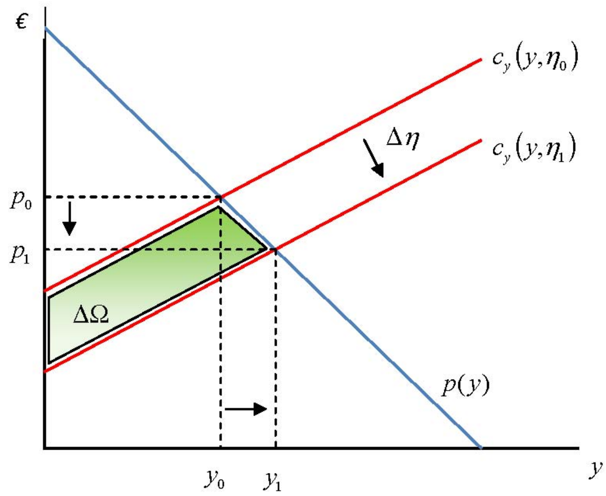

Figure 2: *Welfare impact brought by a PSE gain. Distributional issues.*