

Dear Romar Correa,

First of all, thanks for your positive reading and sorry for our delay in our reply.

Next, we have decomposed your comment into separated items in order to clarify the points you mentioned. Please, let us know if you have still any question related:

1. **X\*\* vs c\*\***: Firstly, two asterisks variables refer to globally efficient allocation. Upon this basis, X\*\* refers to quantity (units of inputs used to produce Y\*\*), whereas c\*\* refers to the minimum cost of producing Y\*\* (price\*quantity).
2. **Derivation from (5) to (6)**: Both expressions reflect equivalent definitions of global efficiency, depending on the approach used: cost/output oriented (see footnote 7).
3. **Why “expenditure-efficiency”?**: Expenditure functions has a long history within the duality theory. Our contribution here is to include an extra parameter (global efficiency) which clearly affects how the production process is carried out. That’s why we denoted it “expenditure-efficiency” function. This term is commonly employed in analyses related to public sector performance.
4. **“Social welfare function” definition**: It uses our expenditure-efficiency function as “ $\gamma$ ” and “ $x_i$ ” depend on the level of global efficiency. We chose this notation for the sake of readability. So we do not consider beneficial to extend the product of terms “ $x_i w_i$ ” given that it does not contribute to posterior algebraic developments.
5. **Proposition 1 proof**: We only derive this expression from equation (11). Thus, we thought that there were no room to include a “proof” statement.
6. **About equation (29)**: We consider our computations are right because the crossed derivation included in the last term of equation (29) is, in general, different from zero.