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Information heterogeneity, housing dynamics and the business cycle

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Abstract
Empirical evidence shows that house prices are highly volatile and closely correlated with the business cycle, and the fact is at odds with the evidence that rental prices are relatively stable and almost uncorrelated with the business cycle. To explain the fact, we introduce information heterogeneity into a standard dynamic stochastic general equilibrium (DSGE) model with financial frictions. Agents are endowed with heterogeneous shocks, and rationally extract information from market activities. Since agents are confused by changes in average private signals about future fundamentals, the model generates an amplified effect of technology shocks on house prices, which accounts for the disconnect between house prices and the discounted sum of future rents. In addition, the model provides insights for the lead-lag relationship between residential and nonresidential investment over the business cycle. The solution method developed in this paper can be applied in other DSGE models with heterogeneous information.

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Keywords heterogeneous information, DSGE model, housing market

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1 Introduction

The recent financial crisis that started in the U.S. in December 2007 has demonstrated the importance of the housing sector in macroeconomic modeling. In response to the recession, a growing literature has tried to incorporate the housing sector into standard macroeconomic models to explain stylized facts in the housing market and the business cycle.[1] However, there are two facts that existing quantitative macroeconomic models have difficulty explaining: house prices are highly volatile and closely correlated with the business cycle, which is at odds with the evidence that rental prices are relatively stable and almost uncorrelated with the business cycle; and residential investment leads the business cycle while nonresidential investment moves contemporaneously with the business cycle.

The main goal of this paper is to present an alternative model to quantitatively explain these two facts. To incorporate the housing sector into the standard dynamic stochastic general equilibrium (DSGE) model, one usually assumes that firms need a collateral asset to secure their external financing as in Kiyotaki and Moore (1997), and specifies the collateral asset as houses, such as Iacoviello (2005), and Liu, Wang, and Zha (2013) et al. These types of models succeed in explaining either the close correlation between house prices and nonresidential investment or the close correlation between house prices and consumption, but fails in explaining the contrast between the high volatility of house prices and the low volatility of rental prices. Figure 1 illustrates the cyclical components of house prices and rental prices with the business cycle for the United States from 1975Q1 to 2010Q3[2] (all data are log-linearized and filtered using the Hodrick-Prescott filter). House prices

[1] Iacoviello (2010) is a recent survey. A inexhaustive reading list should include Iacoviello (2005), Davis and Heathcote (2005, 2007), Piazzesi and Schneider (2009), Iacoviello and Neri (2010), Sterk (2010), Caplin and Leahy (2011), Mian and Sufi (2011), Kiyotaki, Michaelides, and Nikolov (2011), Chaney, Sraer, and Thesmar (2012), Rupert and Wasmer (2012), Liu, Wang, and Zha (2013), Chatterjee and Eyigungor (2015), Burnside, Eichenbaum, and Rebelo (2016), and Favilukis, Ludvigson, and Nieuwerburgh (2017).

[2] In this paper, we collect the data of output, consumption, residential investment, and nonresidential investment from the St. Louis Fed.

are closely correlated with the business cycle and their correlation with U.S. GDP is around 0.52. In contrast, rental prices are almost uncorrelated with the business cycle and their correlation with U.S. GDP is less than 0.06. Furthermore, house prices are much more volatile than output and their standard deviation is around 1.55 times of the standard deviation of output. However, rental prices are much less volatile and their standard deviation is only 0.46 times of the standard deviation of output. To explain the difference between the volatility of house prices and the volatility of rental prices, in addition to incorporating financial frictions as in Liu, Wang, and Zha (2013), we further incorporate information frictions into the standard DSGE model, and demonstrate that information heterogeneity plays a key role in quantitative macroeconomic analysis of housing dynamics.

In the standard DSGE model with financial frictions, houses can be viewed as assets (see equation (20) in Liu, Wang, and Zha (2013)). If we define the rental prices as the marginal rate of substitution (MRS) between housing consumption and goods consumption, the asset pricing theory implies that house prices are determined by the discounted sum of future rents. With consumption smoothing, the model predicts that the volatility of house prices is much lower than the volatility of output (see Liu, Wang, and Zha (2013) for a detailed discussion). However, if households have heterogeneous information about the future average MRS between housing consumption and goods consumption, house prices will also be determined by households' expectations of other households' expectations of the future average MRS, households' expectations of other households' expectations of other households' expectations of the future average MRS, and so on. Therefore, higher-order expectations of the future average MRS play a potential role in determining the fluctuations of house prices. Our calibration exercise shows that information heterogeneity increases the relative volatility of house prices to output by more than 50% and explain the disconnect between house prices and the discounted sum of future rents compared with the full information case. However, our model still has a difficulty in predicting house prices having a higher volatility than output.

We assume households' information sets differ in two respects. First, households have dispersed information of the total factor productivity (TFP). Second, households have idiosyncratic information of the aggregate preferences on houses. When house prices rise, households are confused by whether this rise is driven by an improvement in TFP or an increase in the aggregate demand. Because of rational confusion, an improvement in TFP has an amplified effect on house prices[3]. Thus, information heterogeneity generates a higher volatility of house prices, and breaks down the close correlation between house prices and rental prices.

The other fact which standard macroeconomic models have difficulty in explaining is the lead-lag relationship between residential investment and nonresidential investment over the business cycle. Figure 2 displays the normalized cyclical components of residential and nonresidential investment over the business cycle for the United States from 1975Q1 to 2010Q3, and shows that residential investment leads the business cycle while nonresidential investment moves contemporaneously with the business cycle. The reason why standard real macroeconomic models have difficulty in explaining the lead-lag relationship is because nonresidential capital produces market consumption and investment goods, whereas residential capital produces only home consumption goods (e.g. Fisher, 2007). The asymmetry in how many goods to substitute away from residential capital provides a strong incentive to substitute away from residential capital toward nonresidential capital after a productivity shock. In our model, with incomplete information firms cannot fully observe the true TFP shocks, so the model generates a dampened response of nonresidential investment to TFP shocks. On the other side, since the amplified response of house prices mainly comes from the rising demand of real estate from households, the response of residential investment to TFP shocks is dampened, but to a smaller degree. In total, the correlation between lead residential investment

[3] The idea of rational confusion has long existed in the noisy rational expectation literature. For example, Bulow and Klemperer (1994) use this idea to explain the worldwide stock market crash of 1987. Bacchetta and van Wincoop (2006) claim that such rational confusion plays a key role in explaining the exchange rate disconnect puzzle and matching the evidence on micro trading activities.

and nonresidential investment increases, as does the correlation between lead residential investment and output. Our calibration shows that the correlation between lead residential investment and nonresidential investment increases from a negative value to a large positive value.

The paper is related to several strands of the literature. First, it is related to the literature incorporating financial frictions into models of business cycles (see Gertler and Kiyotaki, 2010, for a survey). Within this strand, there is a large body of work that specifies houses as a collateral asset, and investigates frictions in the house market affecting the business cycle[1]. For example, Iacoviello (2005) introduces collateral constraints tied to home values into a standard monetary business cycle model and shows that houses contribute to the amplification and propagation of demand shocks. In terms of the labor market, Rupert and Wesmer (2012) incorporate frictions in housing mobility into a standard searching and matching model to investigate the difference of unemployment rates between the U.S. and Europe. Sterk (2010) studies the effect of the housing bust in 2007 on the unemployment rate of the recent financial crisis. However, these models either do not consider the disconnect between house prices and rental prices or have difficulty in explaining it. Liu, Wang, and Zha (2013) estimate a real business cycle model with land as a collateral asset in firms' credit constraints, and claim that a shock originated from households' preferences on houses is important in determining land prices and the business cycle. In their model, the housing demand shock explains more than 90% of the observed fluctuations of land prices, and other shocks make almost no contributions, which seems counterintuitive[4].

In this paper, we investigate information frictions in explaining the high volatility of house prices. Trading with information frictions in the housing market has been considered in the literature for a long time (see Himmelberg, Mayer, and Sinar, 2005, for a survey). For recent evidence, Piazzesi and Schneider (2009) propose a search model with transaction costs and show that a small portion

[4] The other shocks include a patience shock, permanent and transitory shocks to neutral technology, permanent and transitory shocks to biased technology, a labor supply shock, and a collateral shock.

of momentum trades generates a high volatility of house prices. Burnside, Eichenbaum, and Rebelo (2016) develop a model with heterogeneous expectations and show that changes in expectations can generate the boom-bust cycles in the housing market. However, these models are not in a micro-founded general equilibrium framework, and therefore are not suitable for a quantitative analysis of the interaction of information frictions and the housing dynamics over the business cycle. To the best of my knowledge, this paper is the first to introduce imperfect information into the standard DSGE model with a housing sector, and shows information heterogeneity has the potential to explain the aforementioned puzzles in both the housing market and the macroeconomy. Our paper is also the first one to introduce information frictions to explain the lead-lag relationship between residential investment and business investment. Previous literature investigating the lead-lag relationship includes Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), Chang (2000), Gomme, Kydland, and Rupert (2001) Davis and Heathcote (2005), Fisher (2007), et al.

Finally, this paper also contributes to the growing interests in investigating imperfect information in macroeconomics[5]. In their seminal work, Phelps (1970) and Lucas (1972) demonstrate that the dispersion of information can help nominal shocks generate fluctuations in real variables. Recently, Morris and Shin (2002) investigate strategic interactions in a global game framework; Mankiw and Reis (2002) consider the case that agents update their information sets sporadically; and Sims (2003) formalizes the idea of information frictions by assuming limited capacity for processing information. Our work is more closely related with La'O (2010), which also studies the interaction of information frictions with financial frictions. However, our work differs from La'O's work in three aspects. First, our work directly investigates the information frictions in the housing market and the spillover

[5] Mankiw and Reis (2010) provide a recent survey. An inexhaustive list includes Phelps (1970), Lucas (1972), Townsend (1983), Mankiw and Reis (2002), Morris and Shin (2002), Sims (2003), Woodford (2003), Bacchetta and van Wincoop (2006), Nimark (2008), Lorenzoni (2009), Machowiak and Wiederholt (2009), Angeletos and La'O (2010), Graham and Wright (2010), and Guo and Shintani (2011), Crucini, Shintani, and Tsuruga (2010, 2015).

effects from the housing market to the business cycle. Second, we build our model in a dynamic stochastic general equilibrium framework and thus can quantitatively evaluate the contribution of information heterogeneity to both the housing market and the business cycle. Finally, La'O's work focuses on how the interactions of financial frictions and information frictions affect noise shocks as an independent source of the business cycle fluctuations.

The remainder of the paper is organized as follows. Section 2 provides empirical evidence about the two facts in the housing market and the business cycle. Section 3 introduces the model with both financial frictions and information frictions. Section 4 discusses the implications of our model regarding house prices, residential investment, and nonresidential investment over the business cycle. Section 5 presents additional evidence from the survey data. Finally, section 6 concludes.

2 Empirical Motivation

In this section, we empirically present the two facts that existing macroeconomic models have difficulty in explaining: the disconnect between house prices and the discounted sum of future rents; and the lead-lag relationship between residential investment and nonresidential investment. To investigate the disconnect between house prices and the discounted sum of future rents, we consider the user-cost approach, an approach commonly used in the literature (see Mayer, 2011, for a survey)[6]. This approach takes the simple non-arbitrage condition that the rent-price ratio should be equal to the user cost of housing, which is the sum of the after-tax equivalent-risk opportunity cost of capital and the expectation of future house prices appreciation excluding maintenance cost.

[6] There are three alternative approaches commonly used in the literature: the user-cost methodology which compares the present discounted value of future rents with house prices; the construction-cost approach that compares the cost of constructing a new home with house prices; and the affordability approach which compares the ability of potential buyers of the house with house prices.

This implies that the following relationship holds at each point in time:

$$\frac{R_t}{P_t} = \alpha_0 + \alpha_1 i_t + \alpha_2 \frac{(1-\delta_h)P_{t+1} - P_t}{P_t} + \varepsilon_t, \tag{1}$$

where R_t is the rental price for a representative home for one year at time t, P_t is the corresponding purchase price of the same home, i_t is the opportunity cost of capital, δ_h is the home depreciation rate, and ε_t is white noise.

We collect house prices and rent data from 1960Q1 to 2010Q3 from the Federal Housing Finance Agency (FHFA) home price index, and use the data with the same period from the Case-Shiller-Weiss (CSW) home price index as a robustness check. The FHFA series is well-known for its broad geographic coverage, but it covers only conventional mortgages. On the other hand, the CSW series covers both conventional and unconventional mortgages (see Davis and Heathcote (2007) for a detailed description of the data set). By assuming that the risk premium of house price fluctuations is constant, we take the federal funds rate to approximate the opportunity cost of capital. To introduce maintenance costs, we assume that houses depreciate at a constant rate $\delta_h = 0.01$ as in Iacoviello and Neri (2010). Table 1 presents the regression results of equation (1). The results show that appreciation in house prices has almost no explanatory power in the fluctuations of the rent-price ratio. One percent increases in house prices predict around 0.09 increases in rent-price ratio for the FHFA series, and around 0.02 increases for the CSW series. The null hypothesis $\alpha_2 = 1$ is rejected at any significance level for both of the two data sets. Thus, the regression results confirm the disconnect between house prices and the discounted sum of future rents.

Table 1: House price appreciation and rental prices

	α_0	α_1	α_2
The FHFA series	0.0449**	0.0022**	0.0899**
The CSW series	0.0439**	0.0024**	0.0191**

** indicates rejection at 1% significance level.

The second fact that we want to investigate is the lead-lag relationship between residential investment and nonresidential investment over the business cycle. The literature in home production has demonstrated that residential investment leads the business cycle and nonresidential investment lags the business cycle for the U.S. economy (see Davis, 2010, for a survey). In sharp contrast, Kydland, Rupert, and Šustek (2012) empirically show that the lead-lag relationship in the developed countries only holds for the two Western-Hemishpere countries: USA and Canada, and in other developed economies there is no such a clear feature of the lead-lag relationship between either residential investment or nonresidential investment and the business cycle. We reconsider the fact and calculate the correlations among the lead (lag) residential investment, the lead (lag) business investment, and the lead (lag) output for the following countries and periods: Austria (1988Q1-2012Q2), Finland (1975Q1-2012Q2), France (1978Q1-2012Q2), Netherlands (1988Q1-2012Q2), the U.K. (1970Q1-2012Q2), the EU (1988Q1-2012Q2), Australia (1959Q3-2012Q2), Canada (1981Q1-2012Q2), and the U.S. (1960Q1-2012Q2)[7]. All the data are logged and Hodrick-Prescott filtered. In Table 2, our main results confirm the leading (lagged) role of residential (nonresidential) investment over the business cycle in the U.S. and Canada. In other developed countries, there is no clear order among the second moments except Finland, which also shares this feature to some extent. One interesting thing in our calculation is that if we aggregate the five countries in the Europe together,

[7] The EU is aggregated by the five following countries: Austria, Finland, France, Netherlands, and the U.K.. We collect the data for the European countries from the Eurostat, for Canada from the OECD, for Australia from Australian Bureau of Statistics, and for the U.S. from the St. Louis Fed.

the aggregate will also somewhat perform like the U.S. and Canada.

Table 2: Second Moments - Empirical lead-lag correlations

	Austria	FIN	FRA	NET	UK	EU	AUS	CAN	US
$\rho(I^s_{t-1}, I_t)$	-0.359	0.453	0.576	0.227	0.210	0.301	0.355	0.398	0.503
$\rho(I^s_t, I_t)$	-0.268	0.378	0.618	0.567	0.094	0.288	0.267	0.228	0.289
$\rho(I^s_{t+1}, I_t)$	-0.161	0.202	0.448	0.138	-0.029	0.182	0.137	0.018	0.021
$\rho(I^s_{t-1}, Y_t)$	0.047	0.669	0.540	0.378	0.467	0.722	0.519	0.640	0.689
$\rho(I^s_t, Y_t)$	0.029	0.668	0.595	0.489	0.513	0.715	0.578	0.580	0.571
$\rho(I^s_{t+1}, Y_t)$	0.019	0.560	0.604	0.463	0.454	0.618	0.503	0.378	0.345
$\rho(I_{t-1}, Y_t)$	0.381	0.452	0.082	0.416	-0.063	0.495	0.335	0.491	0.498
$\rho(I_t, Y_t)$	0.473	0.653	0.186	0.584	0.007	0.596	0.479	0.662	0.724
$\rho(I_{t+1}, Y_t)$	0.484	0.737	0.261	0.610	0.089	0.621	0.510	0.745	0.797

$I^s_t, I_t,$ and $Y_t$denote residential investment, nonresidential investment and output respectively.

To further investigate the causality effect between residential and nonresidential investment, we conduct a bivariate vector autoregression (VAR) with a Granger-causality test for these two types of investment. To apply the Granger-causality test, we first test whether the two series have a unit-root process by the Dickey-Fuller test. If the two series are of $I(1)$, we further test whether the two are cointegrated. If we cannot detect a cointegration relationship between the two series, the following formulation is used in testing the null hypotheses:

$$\begin{aligned} \Delta I^s_t &= \alpha_0 + \sum_{i=1}^{k} \alpha_{1i} \Delta I^s_{t-i} + \sum_{\iota=1}^{k} \alpha_{2i} \Delta I_{t-i} + \varepsilon_{1t} \\ \Delta I_t &= \beta_0 + \sum_{i=1}^{k} \beta_{1i} \Delta I^s_{t-i} + \sum_{\iota=1}^{k} \beta_{2i} \Delta I_{t-i} + \varepsilon_{2t}. \end{aligned} \tag{2}$$

Failing to reject the H_0: $\alpha_{21} = \alpha_{22} = ... = \alpha_{2k} = 0$ implies that nonresidential investment does not Granger cause residential investment. Likewise, failing to reject H_0: $\beta_{12} = \beta_{12} = ... = \beta_{1k} = 0$ implies that residential investment does not Granger cause nonresidential investment. If the series are cointegrated, we need to incorporate an error correction term in testing the null hypotheses:

$$
\begin{aligned}
\Delta I_t^s &= \alpha_0 + \delta_1(I_t^s - \lambda I_t) + \sum_{i=1}^{k} \alpha_{1i} \Delta I_{t-i}^s + \sum_{\iota=1}^{k} \alpha_{2i} \Delta I_{t-i} + \varepsilon_{1t} \\
\Delta I_t &= \beta_0 + \delta_2(I_t^s - \lambda I_t) + \sum_{i=1}^{k} \beta_{1i} \Delta I_{t-i}^s + \sum_{\iota=1}^{k} \beta_{2i} \Delta I_{t-i} + \varepsilon_{2t},
\end{aligned} \tag{3}
$$

in which δ_1 and δ_2 denote speeds of adjustment. Failing to reject the H_0: $\alpha_{21} = \alpha_{22} = ... = \alpha_{2k} = 0$ and $\delta_1 = 0$ implies that nonresidential investment does not Granger cause residential investment. Likewise, failing to reject H_0: $\beta_{12} = \beta_{12} = ... = \beta_{1k} = 0$ and $\delta_2 = 0$ implies that residential investment does not Granger cause nonresidential investment.

The data we use in testing equation (2) or (3) are the same as in Table 2. However, we conduct the Granger-causality test for the period from 1984Q1 to 2005Q4 in the U.S. as a robustness check to avoid the potential problem of structural changes, since this period is well-known for its low volatility of the business cycle in contrast to other periods. The lag parameter k is selected by the Akaike information criterion (AIC). Table 3 shows the fact that in the U.S. and Canada residential investment Granger causes nonresidential investment and nonresidential investment does not Granger cause residential investment. This fact is very clear in Canada, but in the U.S., we can reject the null hypothesis that residential investment does not Granger cause nonresidential investment at any significance level, whereas we cannot reject the null hypothesis that nonresidential investment does not Granger cause residential investment for the period from 1984Q1 to 2005Q4 at 5% significance level, and for the period from 1960Q1 to 2010Q3 at 1% significance level. In other developed countries, there is no such feature similar as in the U.S. and Canada, except in Australia and the U.K. In contrast to the lead-lag relationship that the European aggregate shares with the U.S. and Canada, we cannot see such a similarity for the Granger causality of the two types of investment between the two regions.

Table 3: The causality test between residential and business investments

Country		$I_t^s \longrightarrow I_t$		$I_t \longrightarrow I_t^s$	
	Lag	χ^2 Value	p Value	χ^2 Value	p Value
Austria	4	4.120	0.390	8.199	0.085
Finland	6	13.63	0.034	12.318	0.055
France	6	116.52	0.000	99.495	0.000
Netherlands	4	5.311	0.257	7.454	0.114
UK	2	8.121	0.017	5.052	0.080
Euro	2	2.331	0.312	5.874	0.061
Australia	4	22.649	0.000	5.303	0.258
Canada	2	10.190	0.006	5.611	0.060
USA (1960Q1~2012Q2)	4	181.9	0.000	13.8	0.014
USA (1984Q1~2005Q4)	2	158.8	0.000	5.1	0.076

3 The Basic Model

To quantitatively explain the two facts in a dynamic general equilibrium framework, we build our model in the style of Liu, Wang, and Zha (2013) with real estate production and information heterogeneity. The model in Liu, Wang, and Zha (2013) is a variant of standard real business cycle models that include a feature of credit frictions (Kiyotaki and Moore, 1997). We add a real estate production sector into the model, and assume agents are endowed with heterogeneous information instead of perfect information. Following Iacoviello (2005), Iacoviello and Neri (2010), Kiyotaki, Michaelides, and Nikolov (2011), and Liu, Wang and Zha (2013), we assume two types of agents in the economy: a representative impatient entrepreneur and a continuum of patient households. The representative entrepreneur owns two types of firms: a continuum of residential firms and a continuum of nonresidential firms. The whole economy is segmented geographically and endowed with a continuum of islands. Each island $i \in [0, 1]$ contains one residential firm, one nonresidential firm, and one household. The residential firm hires labors from the household, and accumulates residential structures to build houses. The nonresidential firm also hires labor from the household, accumulates nonresidential capital, and combines with real estate input to produce final goods.

The household provides labor services, saves for next period, and consumes final goods and housing services. The final goods can be used to finance residential investment and nonresidential investment, whereas real estate can only be used for residence.

3.1 Entrepreneurs

The representative entrepreneur owns a continuum of residential firms and a continuum of nonresidential firms. On each island reside one residential firm and one nonresidential firm. The residential firm and the nonresidential firm maximize their expected profits and return the profits to the entrepreneur. The nonresidential firm i takes a Cobb-Douglas constant-to-scale technology that uses labor, capital, and housing as input, according to

$$Y_{it} = K_{it}^{\mu_k}(A_t A_{it} N_{it}^{\prime k})^{v_k} H_{it}^{\prime 1-\mu_k-v_k},$$

where Y_{it} is the output, A_t is the aggregate technology level, A_{it} is the firm-specific technology level, K_{it} is capital produced at the end of last period, $H_{it}^{\prime}$ is the real estate input, and $N_{it}^{\prime k}$ is the labor input in the nonresidential market. μ_k and $1 - \mu_k - v_k$ measure output share of capital and real estate respectively. The residential firm i also takes a Cobb-Douglas constant-to-scale technology that uses labor, residential structures, and land as input, according to

$$H_{it}^0 = S_{it}^{\mu_h}(A_t A_{it} N_{it}^{\prime h})^{v_h} L_{it}^{1-\mu_h-v_h},$$

where H_{it}^0 is newly built housing, S_{it} are residential structures, L_{it} is the land endowment, and $N_{it}^{\prime h}$ is the labor input in the residential market. μ_h and $1 - \mu_h - v_h$ measure output share of residential structures and land respectively. The representative entrepreneur borrows B_{it} from household i in the asset market, invests I_{it} in the nonresidential capital market and I_{it}^s in the residential structure market, produces consumption final goods by purchasing real estate input $\Delta(H_{it})$ and hiring workers N_{it}^k, constructs houses by using the land endowment L_{it}, the labor input N_{it}^h, and the residential structure S_{it}, and consumes $C_t^{'}$ to maximize its expected utility according to

$$\begin{aligned}
&\max E \sum_{t=0}^{\infty} \beta^{'t} \frac{C_t^{'1-\gamma}}{1-\gamma} \\
s.t. \qquad & C_t^{'} + \int_I [(N_{it}^{'k} + N_{it}^{'h})W_{it} - \frac{\pi_{it} B_{it+1}}{R_{it}} + P_t(H_{it}^{'} - (1-\delta_h)H_{it-1}^{'}) + I_{it} + I_{it}^s - \\
& K_{it}^{\mu_k}(A_{it}A_t N_{it}^{'k})^{v_k} H_{it}^{'1-\mu_k-v_k} - P_t S_{it}^{\mu_h}(A_{it}A_t N_{it}^{'h})^{v_h} L_{it}^{1-\mu_h-v_h} + B_{it}]di = 0
\end{aligned}$$

where $\beta^{'}$ is the discount factor of the entrepreneur, γ measures the relative risk aversion, W_{it} is the wage that the entrepreneur pays for workers from the household i, π_{it} is the island-specific bond-holding shock, R_{it} is the island-specific interest rate, δ_h is the discount factor of houses, and P_t is house prices. The island-specific bond-holding shock π_{it} serves one and only one role, to slow down the learning of agents in island i from the bond market. To replace the assumption of the island-specific bond-holding shock, one can introduce another aggregate shock, such as a patience shock to the entrepreneur, to serve a similar role. For simplicity, we do not consider adding another aggregate shock. As in Kiyotaki and Moore (1997), we assume the entrepreneur needs collateral to secure its borrowings

$$B_{it+1} \leq mE_{it}(P_{t+1}H_{it}^{'}), \tag{4}$$

where m indicates that if borrowers repudiate their debt obligations, lenders can liquidate the borrowers' real estate assets but have to pay a proportional transaction cost $(1-m)P_{t+1}H_{it}$. Allowing capital as an additional collateral asset will amplify the effect of credit constraints since the entrepreneur will be more leveraged. We will discuss this later as a robustness check. Nonresidential capital accumulation follows the law of motion

$$K_{it+1} = (1 - \delta_k)K_{it} + \Phi_1(\frac{I_{it}}{K_{it}})K_{it},$$

and similarly, residential structure accumulation follows the law of motion

$$S_{it+1} = (1 - \delta_s)S_{it} + \Phi_2(\frac{I^s_{it}}{S_{it}})S_{it},$$

where δ_k and δ_s are the discount factors of nonresidential capital and of residential structures respectively, and $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ denote the adjustment cost functions of nonresidential capital and of residential structures respectively.

3.2 Households

We assume one household resides on each island i. The household i consumes the final goods, utilizes the housing services, and provides labor services to the residential firm and the nonresidential firm. The household maximizes its expected discounted sum of utility conditional on its own information set Ω_{it} by

$$\max E_i \sum_{t=0}^{\infty} \beta^t [\ln C_{it} + \chi_0 \chi_t \chi_{it} \ln H_{it} - \psi N_{it}],$$

where C_{it} is goods consumption, H_{it} is the housing consumption, N_{it} is the labor services provided by the household, β is the discount factor, χ_t and χ_{it} denote the aggregate and the idiosyncratic housing preference shocks respectively, and χ_0 and ψ are constant parameters. We assume households' discount factor $\beta > \beta^{'}$, which indicates that households are more patient than the entrepreneur and inclined to save. The household i's budget constraint is given by

$$C_{it} + P_t(H_{it} - (1 - \delta_h)H_{it-1}) + \frac{B_{it+1}}{R_{it}} - W_{it}N_{it} - B_{it} = 0.$$

3.3 Market Clearing

The economy has four markets in total: goods, labor, bond and housing. To clear the goods market, we have

$$C_t' + \int_I [C_{it} + I_{it} + I_{it}^s] di = \int_I Y_{it} di.$$

We assume labor is immobile across islands, so in island i we have

$$N_{it} = N_{it}'^k + N_{it}'^h.$$

To clear the bond market, we have

$$B_{it} + B_{it}' = 0.$$

Finally, to clear the housing market, we have

$$\int_I [H_{it} + H_{it}' - (1 - \delta_h)(H_{it-1} + H_{it-1}')] di = \int_I H_{it}^0 di.$$

3.4 Shocks

Our model includes two aggregate shocks and three idiosyncratic shocks. The aggregate shocks follow $AR(1)$ processes in logs,

$$\begin{aligned} \log A_t &= \rho_a \log A_{t-1} + u_t^a \\ \log \chi_t &= \rho_\chi \log \chi_{t-1} + u_t^\chi, \end{aligned}$$

where $u_t^a \sim N(1, \sigma_a^2)$, and $u_t^\chi \sim N(1, \sigma_\chi^2)$. The idiosyncratic shocks also follow the $AR(1)$ processes in logs

$$\begin{aligned}
\log A_{it} &= \rho_{ai} \log A_{it-1} + u_{it}^a \\
\log \chi_{it} &= \rho_{\chi i} \log \chi_{it-1} + u_{it}^\chi \\
\log \pi_{it} &= \rho_{\pi i} \log \pi_{it-1} + u_{it}^\pi,
\end{aligned}$$

where $u_{it}^a \sim N(1, \sigma_{ai}^2)$, $u_{it}^\pi \sim N(1, \sigma_{\pi i}^2)$, and $u_{it}^\chi \sim N(1, \sigma_{\chi i}^2)$. We also assume the law of large numbers applies for the distribution of all the three types of idiosyncratic shocks, as is common in the literature.

3.5 The Information Structure and the Equilibrium

At each period t, the representative entrepreneur has full information. However, the final goods firm i, the real estate firm i, and the household i can only obtain information from their market activities: idiosyncratic preferences series on houses $\{\chi_{t-s}\chi_{it-s}\}_{s=0}^\infty$, wage series $\{W_{it-s}\}_{s=0}^\infty$, interest rate series $\{R_{it-s}\}_{s=0}^\infty$, and house prices series $\{P_{t-s}\}_{s=0}^\infty$. The information set for agents in island i is denoted as

$$\Omega_{it} = \{\{\chi_{t-s}\chi_{it-s}\}_{s=0}^\infty, \{W_{t-s}\}_{s=0}^\infty, \{R_{it-s}\}_{s=0}^\infty, \{P_{t-s}\}_{s=0}^\infty\}.$$

We assume the parameters and the model structure are common knowledge, which indicates our model is in line with the framework of noisy rational expectation models.

The equilibrium is defined as follows:

1. Given prices and information restrictions, the allocations solve the utility maximization problem of the entrepreneur and of the household i and the profit maximization problem of the final goods firm i and the real estate firm i.

2. All markets clear, and $\{P_{t-s}, R_{it-s}, W_{it-s}\}_{s=0}^{\infty}$ are the market clearing house prices, interest rates of bonds, and wages, respectively.

4 Economic Implications

In our model, we assume residential firms, nonresidential firms, and households do not have full information about the economic fundamentals and differ in their information sets for different islands. Instead of an ad hoc assumption of perfect information, we assume agents can only extract information about the true economic fundamentals from their idiosyncratic market activities. With information heterogeneity, agents make their decisions based on their forecasts of not just true economic fundamentals but also forecasts of other agents' actions, forecasts of other agents' forecasts of other agents' actions, etc. In this section, we show that higher-order beliefs play a potential explanatory role in the two facts: the disconnect between house prices and rental prices, and the lead-lag relationship between residential investment and nonresidential investment over the business cycle.

Solving a dynamic general model with dispersed information requires dealing with the well-known "infinite regress" problem (Townsend, 1983), since higher-order beliefs are crucial for the decisions of agents and depend on the entire history of shocks. The literature has solved this type of model by either truncating the dependence of equilibrium actions on higher order beliefs (Nimark, 2008) or by assuming private information is revealed after an ad hoc period T (Lorenzoni, 2009). We take the second approach, and assume that after $T = 30$ periods all of the shocks are observed by agents across islands. The choice of T is based on two considerations: saving computational time and not affecting the results significantly if the value of T is increased. We assume all the shocks are relatively small in magnitude, so the inequality in (4) is always binding. Without the problem of occasional binding, one can solve the model by log-linearizing around the steady state. After log-linearization, we solve the linear equations by combining Sims's (2001) method and the guess-verification approach. In the model economy, agents on island i are integrated into two aggregate markets: the final goods market and the housing market. Therefore, decisions of agents are affected by two aggregate variables: consumption of the representative entrepreneur $C_t^{'}$ and house prices P_t. In the first step, we guess the aggregate variables, $C_t^{'}$ and P_t, to be linear functions of aggregate shocks; in the second step, we plug these two variables into the equations and solve the equations using Sims's (2001) method; in the third step, we update expectation operators of agents on island i by their information set Ω_{it}; finally, we verify the guess of linear functions of $C_t^{'}$ and P_t by minimizing their distance with the updated variables $C_t^{'}$ and P_t. The appendix provides a detailed description of the method.

To calibrate the model, we choose the parameters commonly used in the literature (e.g. Iacoviello and Neri, 2010). β and $\beta^{'}$ are set to 0.9925 and 0.97 respectively. Relative risk aversion, γ, is set to 2. The housing preference parameter χ_0 is set to 0.1 and the disutility on labor ψ is set to 1. The entrepreneurial "loan-to-value ratio" m is set to 0.89 to match the empirical debt to GDP ratio in the U.S. data. The nonresidential capital share in the output production function is set to $\mu_k = 0.63$, and the house share is set to $1 - \mu_k - v_k = 0.05$. For the real estate production function, the share of residential structures is set to $\mu_h = 0.1$, and the share of land is set to $1 - \mu_h - v_h = 0.1$. The discount factors for houses, residential structures, and nonresidential capital are set to $\delta_h = 0.01$, $\delta_s = 0.25$, and $\delta_k = 0.03$ respectively. These three discount factors, combined with the capital share in the goods production function and real estate production function, imply that nonresidential investment accounts for around 30% of the total output, residential investment accounts for about 6% of the total output, and the value of house stocks is about 1.80 time the total output. The solution method does not require us to specify the functional form of Φ_1 and Φ_2, but needs us to set the values of Φ_1, Φ_1', Φ_1'', Φ_2, Φ_2', and Φ_2'' in the steady state. We choose $\Phi_1(\frac{I}{K}) = \delta_k$, $\Phi_1'(\frac{I}{K}) = 1$,

$\Phi_2(\frac{I^s}{S}) = \delta_s$, and $\Phi_1'(\frac{I^s}{S}) = 1$, so that the model with adjustment costs has the same steady state as the model without adjustment costs. We set the second-order derivative of the adjustment cost function of residential investment $\Phi_2''(\frac{I^s}{S}) = -2.5$, the same as that of nonresidential investment $\Phi_1''(\frac{I}{K}) = -2.5$. The later is chosen as in Christiano, Eichenbaum, and Evans (2005)[8].

There are two aggregate and three idiosyncratic $AR(1)$ shock processes in total. Two parameters are crucial for the $AR(1)$ processes: the persistence and the variance of the shocks. The persistence and the variance of the shocks affect the response of business cycle variables in two different ways: first, the shocks to the model are directly affected; second, the precision of agents' information and agents' information updating process are altered. For the aggregate technology shock process, we assume a persistent shock process and set $\rho_a = 0.95$ as in Fisher (2005). Similarly, the autocorrelation in the aggregate housing preference shock is assumed to be $\rho_\chi = 0.95$. We choose $\sigma_a^2 = 0.00984^2$ to match the volatility of output, and $\sigma_\chi^2 = \frac{1}{10^2}\sigma_a^2$ to weaken the effect of housing preference shocks and focus on technology shocks as a main driving force of the business cycle fluctuations[9]. Since our interest is in the role of information heterogeneity in matching aggregate business cycle variables, we choose the persistence and the variance of idiosyncratic shocks to maximize the effect of information heterogeneity on house prices, and ignore the empirical micro-level cross-sectional facts. For the idiosyncratic bond-specific shock processes, we set $\rho_{\pi i} = 0$ and $\sigma_{\pi i}^2 = \infty$ for one and only one reason: to screen the information contained by the real interest rate. For the idiosyncratic technology shock and the idiosyncratic housing preference shock, we set $\rho_{ai} = 0.001$, $\rho_{\pi i} = 0.001$, $\sigma_{ai}^2 = 100^2\sigma_a^2$ and $\sigma_{\chi i}^2 = 100^2\sigma_a^2$. The high magnitude of idiosyncratic shocks implies that agents extract information mainly from house prices instead of idiosyncratic variables, such as island-specific wages and island-

[8] In the literature, one usually pins down the parameters $\Phi_1''(\frac{I}{S})$ and $\Phi_1''(\frac{I^s}{S})$ by matching the volatility of nonresidential investment and residential investment in the data, Unfortunately, our solving procedure can find a convergence point only for certain ranges of parameters values. Of course, this is left for future work.

[9] In our model, a low magnitude of the housing preference shocks is enough to confuse the rational agents. Nimark (2008) makes a similar assumption that the variance of the transitory labor supply shock is $\frac{1}{100}$ of other aggregate shocks, such as the technology shock.

specific technology shocks. This assumption of a large magnitude of idiosyncratic shocks relative to aggregate shocks has been used in the literature (Maćkowiak and Wiederholt, 2009).

To evaluate the model's performance, we turn on all the shocks and simulate the model 1,000 times with 142 periods in each simulation. The simulated data are then filtered with the Hodrick-Prescott filter. The average second moments of all the simulations and their empirical counterparts are reported in Table 4. Our model confirms the main arguments in Liu, Wang, and Zha (2013) that collateral constraints in nonresidential investment play a key role in explaining the close correlation between house prices and other business cycle variables. All of the correlations between house prices and other business cycle variables for the simulated data are well above their empirical counterparts. In comparison with the model with full information, two facts stand out for the model with heterogeneous information: first, information heterogeneity amplifies the response of business cycle variables to technology shocks[10]; second, the correlation between lead residential investment and nonresidential investment increases significantly from a negative value to a large positive value, and exceeds the correlation between lag residential investment and nonresidential investment. Similarly, the correlation between lead residential investment and output increases significantly from a small positive value to a large positive value, and exceeds the correlation between lag residential

[10]Since the standard deviation of housing preference shocks is one-tenth of the standard deviation of technology shocks, the role of housing preference shocks in our calibration is limited.

investment and output.

Table 4: Business cycle statistics for the models

	U.S. Data	Full info.	Hetero info.
		A. Basic statistics	
σ_y	1.42	1.08	1.31
σ_c/σ_y	0.62	0.48	0.36
σ_i/σ_y	2.54	2.35	2.73
σ_{i^s}/σ_y	5.05	2.64	2.54
σ_p/σ_y	1.55	0.49	0.64
$\rho(y_t, p_t)$	0.52	0.77	0.87
$\rho(c_t, p_t)$	0.47	0.90	0.94
$\rho(i_t, p_t)$	0.59	0.97	0.98
		B. Investment dynamics	
$\rho(I^s_{t-1}, Y_t)$	0.77	0.17	0.58
$\rho(I^s_t, Y_t)$	0.73	0.83	0.66
$\rho(I^s_{t+1}, Y_t)$	0.32	0.65	0.44
$\rho(I^s_{t-1}, I_t)$	0.84	-0.04	0.51
$\rho(I^s_t, I_t)$	0.71	0.69	0.59
$\rho(I^s_{t+1}, I_t)$	0.29	0.58	0.38
$\rho(I_{t-1}, Y_t)$	0.75	0.42	0.43
$\rho(I_t, Y_t)$	0.89	0.97	0.98
$\rho(I_{t+1}, Y_t)$	0.60	0.20	0.47

4.1 What drives house prices fluctuations?

Table 4 shows that information heterogeneity amplifies the response of business cycle variables to technology shocks, especially for house prices, whose standard deviation in the model with heterogeneous information is about twice the standard deviation in the model with full information. In contrast, the standard deviation of goods consumption increases slightly. These two together indicate that our model might be able to explain the puzzle of the disconnect between house prices and rental prices, since the later is closely correlated with final goods consumption. As discussed in Liu, Wang, and Zha (2013), the main reason why standard DSGE models with a housing sector cannot predict a high volatility of house prices can be illustrated by the Euler equation of households,

$$P_t = \beta(1-\delta_h)E_{it}\frac{C_{it}}{C_{it+1}}P_{t+1} + \frac{\chi_0\chi_t\chi_{it}C_{it}}{H_{it}}.$$

If we define rental prices as the marginal rate of substitutions between goods consumption and housing service consumption as

$$R_{it}^h = \frac{\chi_0 \chi_t \chi_{it} C_{it}}{H_{it}},$$

house prices can be expressed as

$$P_t = \beta(1-\delta_h) \int_I E_{it} \frac{C_{it}}{C_{it+1}} P_{t+1} di + R_t^h, \tag{5}$$

where $R_t^h = \int_I R_{it}^h di$ denotes the aggregate rental prices. We further write house prices recursively,

$$P_t = \beta(1-\delta_h) \bar{E}_t \frac{C_{it}}{C_{it+1}} P_{t+1} + R_t^h = \sum_{k=0}^{\infty} \beta^k (1-\delta_h)^k \bar{E}_t^{(k)} R_{t+k}^h \frac{C_t}{C_{t+1+k}}.$$

where $\bar{E}_t^0(P_t) = P_t$, $\bar{E}_t^1(P_{t+1}) = \bar{E}_t(P_{t+1})$, and higher-order expectations are defined as,

$$\bar{E}_t^k(P_{t+k}) = \bar{E}_t \bar{E}_{t+1} \cdots \bar{E}_{t+k}(P_{t+k}).$$

Therefore, house prices at time t depend on rental prices at time t, the average expectation at time t of rental prices at time $t+1$, the average expectation at time t of the average expectation at time $t+1$ of rental prices at time $t+2$, etc. In the case of complete information, the average expectation at t of the average expectation at $t+1$ of rental prices at $t+2$ coincides with the average expectation at t of the average expectation of rental prices at $t+2$, i.e. $\bar{E}_t \bar{E}_{t+1} \cdots \bar{E}_{t+k}(P_{t+k}) = \bar{E}_t(P_{t+k})$, and therefore equation (5) collapses to

$$P_t = \sum_{k=0}^{\infty} \beta^k (1-\delta_h)^k E_t R_{t+k}^h \frac{C_t}{C_{t+1+k}}.$$

Since households smoothly allocate their consumption period by period, the model with full information fails to predict a high volatility of house prices. However, in the case of imperfect information, equation $\bar{E}_t\bar{E}_{t+1}\cdots\bar{E}_{t+k}(P_{t+k}) = \bar{E}_t(P_{t+k})$ does not hold. In other words, even though rental prices are relatively stable, house prices might still be volatile since house prices are also determined by higher-order expectations of future rental prices. Figure 3 displays the response of house prices to one positive standard deviation of technology shocks in the models with full information and in the model with heterogeneous information. Information heterogeneity initially dampens the technology shocks, but amplifies and propagates the technology shocks after three quarters. Unfortunately, our model still fails to generate a higher volatility of house prices than output. To illustrate how information heterogeneity affects house prices, we plot the average expectation of next-period house prices for both the full information case and the heterogeneous information case in Figure 4, since equation (5) shows that it is crucial in determining house prices in this period. The figure displays that the model with heterogeneous information is accompanied by higher average expectations of house prices.

Table 5: House price appreciation and rental prices in simulated data

	α_0	α_1	α_2
U.S. Data	0.0449**	0.0022**	0.0899**
Full info.	0.0148	1.3487	1.4195
Hetero info.	0.0141	1.8102*	0.4895*

** and * indicate rejection at 1% and 10% significance level respectively.

To rigorously prove that information heterogeneity can explain the disconnect between house prices and rental prices, we test the user-cost equation as in (1) using the simulated data. The results in Table 5 show that the null hypothesis $\alpha_2 = 1$ cannot be rejected by the model with full information, but is rejected by the model with heterogeneous information at 5% significance level. In sum, even though the model with heterogeneous information cannot predict house prices having a higher volatility than output, it explains the disconnect puzzle between house prices and rental prices to some level.

4.2 Implications for Investment Dynamics

The other prediction of our model is the lead-lag relationship among nonresidential investment, residential investment, and output. Empirical studies have documented that residential investment leads the business cycle, but nonresidential investment lags the business cycle, and the two types of investment are positively correlated with each other (see Gangopadhyay and Hatchondo, 2009, for a survey). However, standard real business cycle models with home production predict the opposite and even a large negative value for the correlation between the contemporaneous residential investment and nonresidential investment. To match the data, several different channels have been emphasized in the literature, including adjustment costs in capital accumulation (Chang, 2000), time-to-build in nonresidential investment (Gomme, Kydland, and Rupert, 2001), multiple-market sectors (Davis and Heathcote, 2005), and a direct role for household capital as an input in market production (Fisher, 2007). In this paper, we highlight the information channel and show that the presence of information heterogeneity has a potential to explain the lead-lag relationship between residential investment and nonresidential investment.

As emphasized by Fisher (2007), real business cycle models with home production can predict the lead-lag relationship between residential investment and nonresidential investment, if home product enters the production function of market goods with a reasonable share. In our model, real estate enters the production function of final goods in two different ways: first, it directly enters the production function with a share of output equal to $1 - \mu_k - v_k = 0.05$; second, it serves as collateral for nonresidential investment. Since the share in our model is lower than the share of 0.14 in Fisher (2007), our model with full information cannot explain the lead-lag relationship, but it does predict a positive correlation of 0.69 between the contemporaneous residential investment and nonresidential investment as shown in the panel B of Table 4. The panel also shows information heterogeneity plays a key role in generating the positive correlation between lead residential investment and nonresidential investment. When there is no information frictions, the model predicts a negative correlation of -0.04, which is much less than the correlation between lead nonresidential investment and residential investment of 0.58. In contrast, when there is information heterogeneity, the correlation between lead residential investment and nonresidential investment increases to a significantly positive value 0.51, larger than the correlation of 0.38 between the lead nonresidential investment and residential investment. However, our model still produces a larger correlation between the contemporaneous residential investment and nonresidential investment, which is at odds with the data.

In the standard real business cycle model with home production, firms increase their production and nonresidential investment immediately in response to TFP shocks, whereas here real estate firms increase residential investment gradually. Therefore, the model predicts a negative correlation between lead residential investment and nonresidential investment. In the model with information heterogeneity, both residential firms and nonresidential firms are partially informed about the size of TFP shocks, and therefore both firms postpone their investment in response to TFP shocks. However, if the amplified house prices are mainly caused by rising demand from households, real estate firms will have a stronger incentive to increase residential investment in response to TFP shocks since the marginal revenue of real estate production increases. As shown in last subsection, the main reason the response of house prices is amplified is the breakdown in households' Euler equation (5). In our calibration, we find aggregate housing demand from households $H_t = \int_I H_{it} di$ decreases by much less in the model with information heterogeneity compared with the model with full information. Accordingly, residential investment will decrease by much less, and the correlation between lead residential investment and nonresidential investment increases. With the delayed response of nonresidential investment, our model predicts a hump-shaped response of output to one standard deviation of TFP shocks as in Figure 5. In the case of imperfect information, the response of output initially increases at a slow speed and peaks in several periods. The hump-shaped response of output confirms the finding in Nimark (2008) that imperfect information provides a potential explanation for the contrast between a positive autocorrelation of output in the data and a negative autocorrelation of output in the real business cycle theory (Cogley and Nason, 1995). The one-period-lag autocorrelation increases from -0.10 to 0.04, although not significantly.

5 Empirical Evidence from Survey Data

A difficulty in the literature of imperfect information is that it is hard to provide empirical evidence to test the model. A prediction of our model is that if we define expectation errors of real variables as the difference between the average expectation of real variables and the corresponding realized variables, the expectation errors should be correlated with the business cycle. For instance, the model predicts that the forecast errors of output are positively correlated with the business cycle in response to TFP shocks with a correlation of 0.052, since firms are partially informed about the shocks and agents' expectations of output tend to underreact. As other variables, such as house prices, are also positively responded to TFP shocks, if one identifies an independent shock in the expectation errors of output, a vector autoregression (VAR) should perform as this shock positively causes other real variables, such house prices, output and investment.

To confirm this prediction, we run a three-variable VAR with expectation errors of output, output, and house prices to consider the partial derivatives of output and house prices at various horizons with respect to shocks in the expectation errors of output. We compare the results from an empirical VAR to those arising from application of the same VAR specification to data generated from our model with information heterogeneity. To measure the average expectation of output, we collect data from the Survey of Professional Forecasters (SPF). The data cover the period from 1975Q1 to 2010Q3. We take the median forecasts of real GDP in the coming quarter as the forecast of output. We define the expectation errors as the percentage deviation of the realized real GDP from the forecast of real GDP. To see how innovations in the expectation errors affect other variables, we run the VAR with four lags and the expectation errors ordered first. Figure 5 shows the empirical impulse responses to shocks in expectation errors of output from the trivariate VAR. The shaded areas represent one-standard-error bias-corrected bootstrap confidence bands of Kilian (1998). The figure shows that one percent increases in agents' expectation errors are followed by around 0.05 increases in house prices and 0.4 increases in real GDP.

To run a similar trivariate VAR for the model, we collect simulated data with a length of 142 observations. The average expectations of real variables are directly calculated, as agents' information sets are clearly defined. Similarly, we define the expectation errors of output as the percent deviation between the average expectation of output and the true output. The correlation between the expectation errors and output is also a positive value of 0.042. Figure 6 plots the impulse response to one positive standard deviation of shocks in expectation errors of output from the trivariate VAR for the simulated data. The responses in the simulated data are as similar as the responses in the empirical data, although they differ in magnitude. A one percent increase in agents' expectation errors is followed by around a 0.05 percent increase in house prices and a 0.05 percent increase in output. The main difference between the data sets is that in the simulated data both house prices and output respond with a hump shape, but in the empiricial data, we do not observe such a hump.

To check the robustness of the results, we have repeated the VAR exercise using different variables or different numbers of variables. For instance, we have replaced the expectation errors of output by the expectation errors of nonresidential investment, and replaced output by nonresidential investment. We have also extended the three-variable VAR to a five-variable VAR by adding consumption and nonresidential investment. All of the regressions report similar qualitative results.

6 Conclude

The recent standard real business cycle models with financial frictions succeed in explaining the close correlations among house prices, consumption, and investment. However, the models cannot explain two facts: the disconnect between house prices and rental prices, and the lead-lag relationship between residential investment and nonresidential investment. We introduce information heterogeneity into a standard real business cycle model with real estate production and financial frictions. By assuming that agents are rationally confused about the sources of shocks, the model generates an amplified response of house prices to technology shocks, which explain the disconnect puzzle. Since the amplified response mainly comes from the rising demand of real estate from households, the model potentially explains the lead-lag relationship between residential investment and nonresidential investment.

There are several directions in which our paper can be improved[11]. In our model, although we show information heterogeneity amplifies the response of house prices to technology shocks, the volatility of house prices is still much lower compared to the data. One can introduce monetary shocks into the model and investigate the confusion between real shocks and nominal shocks, since nominal shocks can also be viewed as pure demand shocks and therefore may serve a similar role to housing demand shocks in our model. Second, we could apply the method of minimization of distance between the simulated second moments and the empirical second moments to pin down parameters for our calibration instead of choosing ad hoc values. Third, our model extends the

[11] Our solution method can only solve the model using certain ranges of parameters values. Of course, this is the most central issue to address.

standard real business cycle model in three directions: residential production, financial frictions, and information frictions. It is more intuitive to extend the model step by step, so one can clearly discuss how each extension affects the model. All of these are left for future work.

7 Solving a DSGE model with heterogeneous information

The solving procedure consists of four steps in total.

- **Step one:** shut down all the shocks, solve the model in the steady state, and log-linearize the model around the steady state. In our model, there are two aggregate variables which affect agents' decisions: housing prices P_t and the aggregate consumption of the entrepreneur C_t. The later one also determines the stochastic discount factor. We assume the two aggregate variables are a linear function of aggregate shocks $\Xi_t = \{\{u^a_{t-i}\}^T_{i=1}, \{u^\chi_{t-i}\}^T_{i=1}\}$, $C_t = CC * [u^a_t, u^a_{t-1}, ..., u^a_{t-T}, u^\chi_t, u^\chi_{t-1}, ..., u^\chi_{t-T}]'$, and $P_t = PP * [u^a_t, u^a_{t-1}, ..., u^a_{t-T}, u^\chi_t, u^\chi_{t-1}, ..., u^\chi_{t-T}]'$.

- **Step two**: replace the goods market clearing condition and the housing market clearing condition by the two above equations of the definitions C_t and P_t, and solve the linear difference equations as a typical rational expectation model.

- **Step three**: from Step two, we have

$$Y_{it} = G_1 Y_{it-1} + \Theta_c + \Theta_0 z^*_{it},$$

  and then apply an expectation operator to both sides of the above equation conditional on the information set Ω_{it}

$$Y_{it} = G_1 Y_{it-1} + \Theta_c + \Theta_0 E_{it} z^*_{it}.$$

  To derive $E_{it} z^*_{it}$, we should first keep in mind that the signals s_{it} island i receives are linear functions of z_{it}, given by,

$$s_{it} = \Gamma z_{it}.$$

By Kalman filter updating, we have

$$E_{it}z_{it} \equiv E(z_{it}|s_{it}) = \Sigma\Gamma'(\Gamma\Sigma\Gamma')^{-1}s_{it} = \Sigma\Gamma'(\Gamma\Sigma\Gamma')^{-1}\Gamma z_{it}.$$

- **Step four**: plug the solved individual variables into the goods market clearing condition and the housing market clearing condition, derive the updated C_t^* and P_t^*, and match the distance between (C_t, P_t) and (C_t^*, P_t^*). If the distance is zero or close enough to zero, we solve the model. In our calibration, the square root of the distance is less than 10^{-3}, although we cannot find the exact solution.

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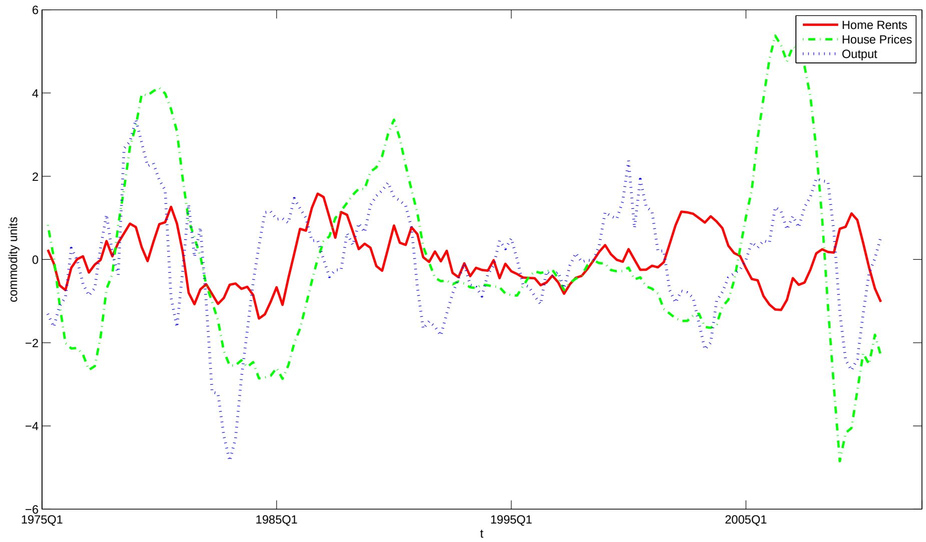

Figure 1: Home rents and house prices with the business cycle.

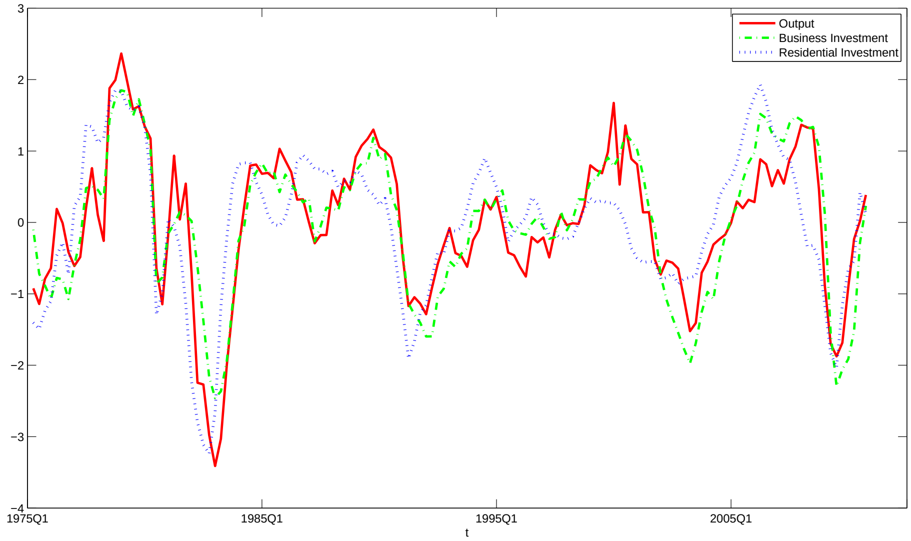

Figure 2: Residential investment and nonresidential investment with the business cycle

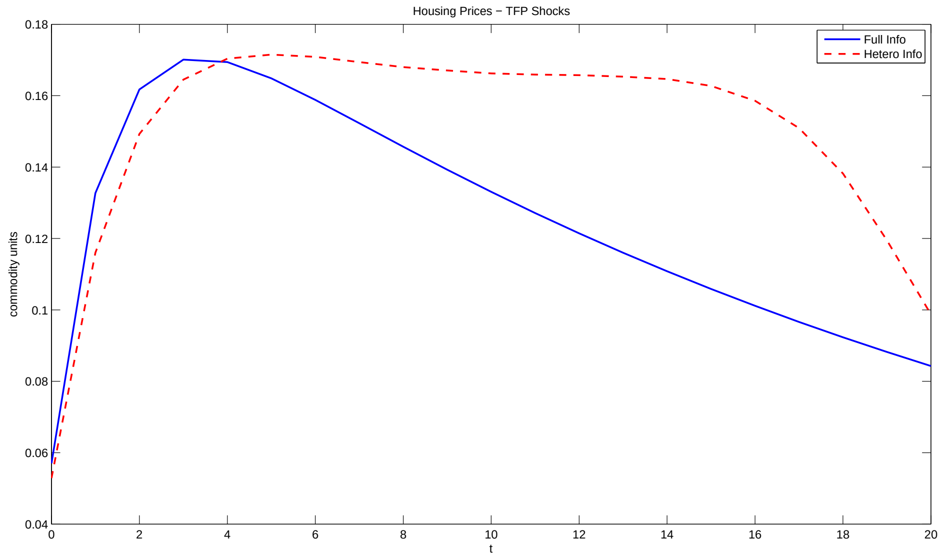

Figure 3: House prices in response to TFP shocks

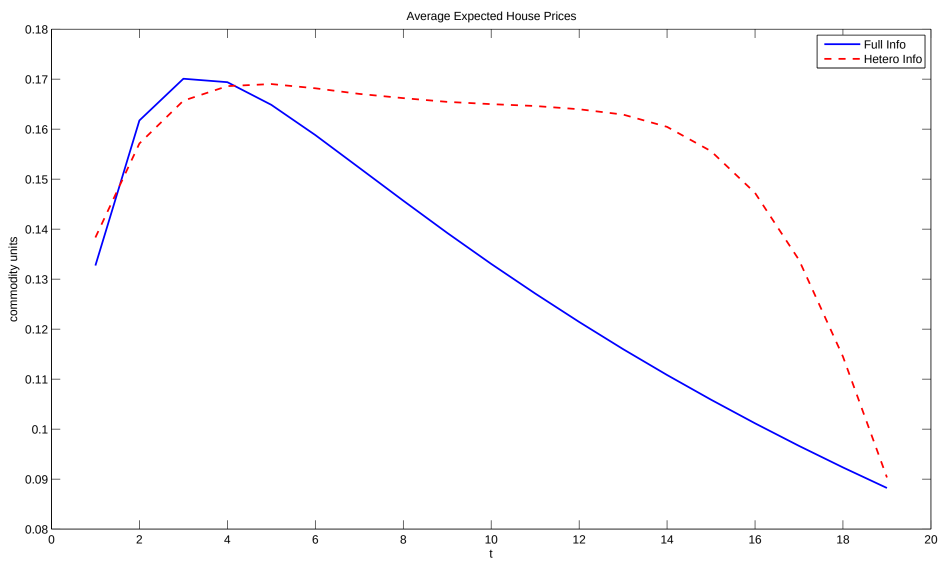

Figure 4: Average expectation of next-period house prices

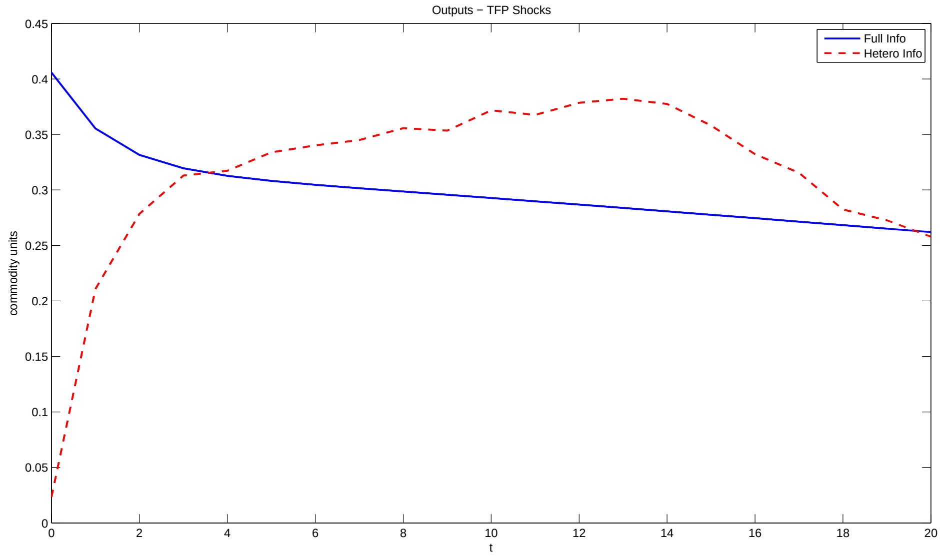

Figure 5: Outputs in response to TFP shocks

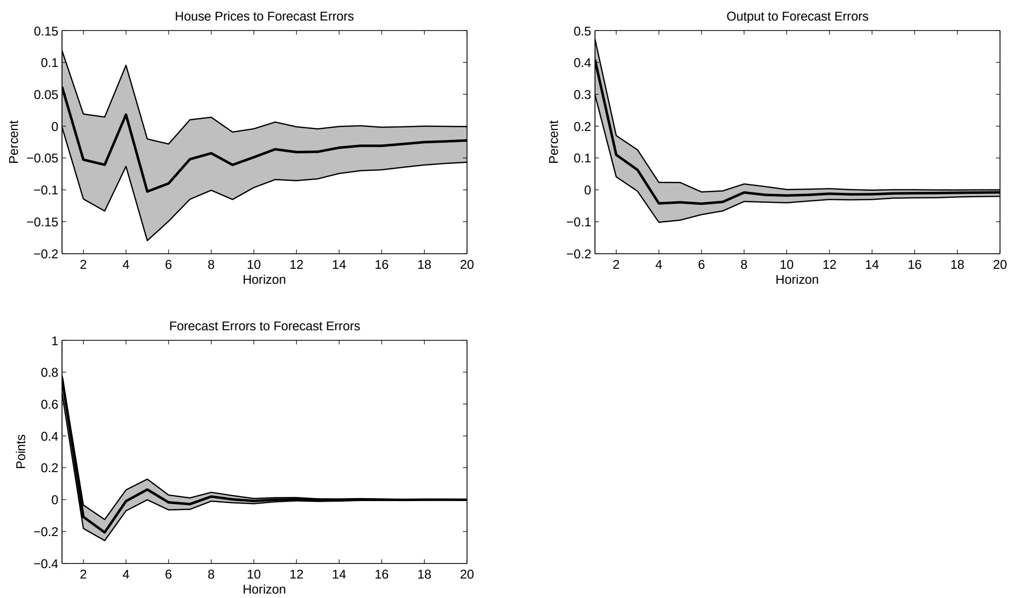

Figure 6: Empirical evidences from SVAR

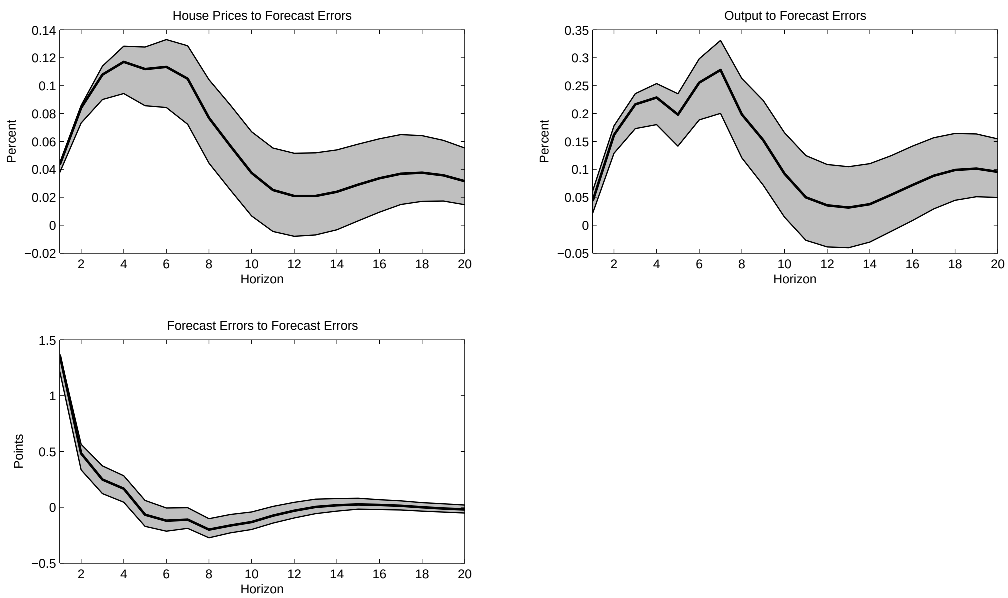

Figure 7: Simulation evidences from SVAR