Patent Buyout in a Model of Endogenous Growth

Ravi Radhakrishnan

Abstract
This paper considers the prospect of a government patent buyout in a model of endogenous growth. To this end, the author modifies a standard quality ladder growth model by incorporating possibility of imitation, and rent protection activities (RPAs) by the innovator. The government finances the buyout by imposing a per unit sales-tax on the goods. The author shows that in this set-up, patent buyout by the government can lead to higher level of welfare without lowering an economy's growth rate along the balanced path. He highlights two sources of welfare improvement: elimination of monopoly pricing, and reduction in RPAs.

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1 Introduction

There is a wide consensus amongst economists that long run economic growth is driven by sustained technological progress resulting from R&D investment by firms (Romer (1986, 1990), Grossman and Helpman(1991), and Aghion and Howitt (1992)). The end product of R&D, however, is not a purely private good but shares characteristics of both private and public goods. That is, it is non-rival but partially excludable. These properties imply that there are externalities associated with R&D resulting in diminished incentives to innovate. Therefore, as Kremer (2010) notes, “institutions beyond competitive markets are required to promote innovation.” Intellectual Property Rights (IPR), such as patents, grant exclusive monopoly rights to innovators and enable them to appropriate the returns from R&D. The patent system has therefore been considered as an essential institutional arrangement to promote innovation.

The downside of patent protection is that it can cause welfare losses associated with monopoly pricing (Gilbert and Shapiro (1990), Deardoff (1991)). In addition, a large and growing strand of literature including Scotchmer (1991), Murray and Stern (2007), and Bessen and Maskin (2008) have shown that patent protection can deter other sequential and complimentary innovations. Boldrin and Levine (2013), in a scathing critique of the current patent system, argues that patent protection fosters neither innovation nor productivity. Instead, the system as it functions today, only leads to an increase in the number of patents filed and granted. Finally, patents do not even eliminate the possibility of imitation by other firms. This can be seen from the fact that in most cases a similar product emerges before the actual duration of a patent expires. For example, the first iPhone was launched in June 2007, while Samsung launched its android based competitor, the Galaxy S, in June 2010.\(^1\) By 2011, Apple and Samsung had entered into a prolonged legal battle over patent infringement which has come to be known as the “smartphone patent wars.” In fact, the smartphone patent wars are symptomatic of a wider problem of patent litigation resulting from cases of patent infringement.\(^2\) To the extent that the patent litigation expenses divert resources

\(^1\)In an earlier study Mansfield, Schwartz, and Wagner (1981) found that 60 %of the patented innovations studied were imitated within 4 years.

\(^2\)Lerner (1995) notes that patent litigation costs for the year 1991 accounted for more than 25 per cent of R&D
away from productive use, they can also be thought off as an additional waste associated with IPRs on top of monopoly pricing and deterring downstream complimentary innovations.

Given that potential innovators require an ex-ante incentive to innovate, and that IPRs, while providing those incentives, also lead to other welfare losses has led to a growing demand for a substantial reform to the current patent system. For example, Shapiro (2007) argues in favor of an “independent invention defense” in cases of patent infringement litigation. Under such a system the party accused of patent infringement would be absolved of all liability if it can show that the innovation was developed independently of the patented technology. On the other hand, Kremer (2010) argues in favor of prizes instead of patents, while Boldrin and Levine (2013) outline eight different proposals to reform the patent system. The list includes elimination of patents on federally subsidized research and instead making such research available to all market participants.

This paper considers another possibility of reforming the patent system in the form of patent buy-outs. A patent buyout is a mechanism by which the government buys a patent from an innovator and places it in the public domain. Such a mechanism still retains the incentive to innovate while eliminating the welfare distortions mentioned above. The idea of a patent buyout is not new; as early as 1839 the French government bought the patent on the first publicly announced photography process (the daguerrotype) and placed it in the public domain so that the discovery was “free to the world.” While more recent examples of patent buyouts are difficult to find, the potential impact of such a policy is apparent. The government buys the patent from the innovator by paying him his private valuation of the patent plus a mark-up that accounts for the divergence between private and social benefits of the innovation. The buyout is financed by placing a sales-tax on the good. As long as the tax is between the competitive price and the mark-up associated with monopoly pricing, such a policy can be potentially welfare enhancing. Additionally, the elimination of patent litigation expenses can also lead to an increase in the overall welfare.

expenditure that year. More recently, Besen and Meuer (2008) show that between 1985-2004 the attorneys fee alone for the patentee was on average $1.04 million (1992 dollars) per case.
I incorporate the patent buyout mechanism within an endogenous growth model, where growth is driven by firms undertaking uncertain R&D to come up with higher quality products. Once a higher quality product is innovated, it is immediately targeted for imitation at an exogenously given rate. Similar set up is employed in Segerstrom (2007), and Davis and Sener (2013). In my model, innovators also expend resources to protect the innovation from infringement by potential imitators. Therefore, the effective length of a patent is endogenously determined. The idea that innovators undertake expenditure to thwart potential imitators and innovators is motivated from the literature on “fully endogenous growth models,” which terms such expenditures as Rent Protection Activities (RPAs). Dinopolous and Syropolous (2007), Sener (2008), and Davis and Sener (2013) develop endogenous growth models where innovators do not passively stand-by waiting for other innovators or imitators to infringe upon their monopoly; instead they undertake substantial expenditure to ensure that their IPRs are safe. As mentioned above, these expenditures may take the form expensive patent litigation, or other expenses such as obtaining sleeping patents.

I contrast this set up with a scenario where the government offers to buyout the patent from the innovator by paying the innovator his private valuation of the patent. The motivation for the patent buyout is derived from Kremer (1993) which introduces an auction mechanism to implement such a buyout. In this mechanism, once a firm chooses to offer its patent for sale, the government solicits sealed bids from other private bidders for the patent. The government uses the information contained in these bids to come up with the private valuation of the patent and offers the patent holder the private valuation times a mark-up that is the ratio of a typical patent’s social to private valuation. Once the government buys the patent it is placed in the public domain. However, it also sells some of the patents to the highest private bidder to ensure that subsequent bidders have an incentive to reveal their true valuation of a patent. The idea is that there is a mechanism whereby the government can price the patent correctly. I will refer to this mechanism henceforth as the “Kremerian buyout” mechanism.

I derive the optimal tax to finance the buyout and show that there exists a tax rate such that the
resulting price is between the competitive price and the monopoly price level. By incorporating the Kremerian buyout in a growth model with innovation, imitation and RPAs, I am able to highlight two welfare enhancing effects of such a mechanism. First, the gain in welfare due to the elimination of monopoly pricing; and second, the gain in welfare due to the elimination of the RPAs. Finally, I am also able to show that such a policy does not lead to lower growth rate for the economy as compared to the one with strong IPR’s with high degree of RPAs.

The rest of the paper is organized as follows. In Section 2, I set up the model with endogenous patent length and derive the steady state equilibrium. Section 3 considers the possibility of patent buyout and derives the optimal tax rate to finance the buyout. Section 4 compares the welfare properties of the two different mechanisms and section 5 offers some concluding remarks.

2 Model

The model closely follows the quality ladders framework in Grossman and Helpman (1991a) and Segerstrom (2007).

2.1 Industry Structure

There is an economy with a continuum of industries indexed $\omega \in [0,1]$. The products in each industry can be supplied in a countable number of qualities $j=0,1,2,3...$ Firms within an industry are distinguished on the basis of the quality of the product they produce. Quality improvement takes place as a result of uncertain, and costly innovative R&D by firms. Every time a firm is successful in innovating, the quality of a good jumps from $j$ to $j+1$. At any given point of time there are two types of industries. First, industries where there is a single quality leader as a result of successful innovation. Second, since there is imitation in the model, there are industries in which successful imitation has taken place. In these industries there are multiple quality leaders.
2.2 Consumer Behavior

There are a fixed number of identical households. The members of each household are endowed with one unit of labor that is supplied inelastically for manufacturing, R&D and protection of property rights. It is assumed that labor is the only factor of production in each of these activities. Further, the labor market is assumed to be perfectly competitive so that all workers earn the same equilibrium wage rate. Finally, wage is assumed to be the numeraire and is constant through time.

The number of members of each household grows over time at an exogenous rate \( n > 0 \). Therefore, the supply of labor at time \( t \) is \( L(t) = L_0e^{nt} \). Each household maximizes the inter-temporal utility function given by:

\[
U = \int_0^\infty e^{-(\rho-n)t}lnu(t)dt,
\]

where \( \rho \) is the subjective discount rate, \( n \) is the population growth rate and \( u(t) \) is the consumer’s instantaneous utility at time \( t \). Instantaneous utility is given by

\[
u(t) = \left[ \int_0^1 \left( \sum_{j=0}^\infty \lambda^j d(j, \omega, t) \right)^\kappa \ d\omega \right]^{1/\kappa},
\]

where \( d(j, \omega, t) \) denotes the quantity consumed of a product of quality \( j \) produced by industry \( \omega \) at time \( t \), and \( \lambda > 1 \) represents the extent to which higher quality products improve on the lower quality products and \( \kappa \in (0, 1) \) determines the elasticity of substitution between consumer goods across industries given by \( \sigma = 1/(1 - \kappa) \). Given our assumption on the value of \( \kappa \), the goods are good substitutes (\( \sigma > 1 \)). The consumer maximizes the discounted utility subject to the intertemporal budget constraint

\[
\int_0^\infty e^{-R(t)}E(t)dt = A(0),
\]

where \( R(t) \) is the cumulative interest factor up to time \( t \), \( A(0) \) is the value of asset holdings at time \( t=0 \) plus the present value of future labor income, and \( E(t) \) is the consumer’s per capita
expenditure flow at time \( t \). The consumer’s time \( t \) expenditure flow is given by

\[
E(t) = \int_0^1 \sum_{j=0}^{\infty} p(j, \omega, t) d(j, \omega, t) d\omega,
\]

(4)

where \( p(j, \omega, t) \) is the price of a product of quality \( j \) produced by industry \( \omega \) at time \( t \).

Utility maximization involves two steps. First is a static problem where the consumer chooses the quantity of each good of quality \( j \) from industry \( \omega \) for a given expenditure \( E(t) \), and price vectors \( p(j, \omega, t) \). This involves maximizing the instantaneous utility function given in equation (2) subject to the budget constraint imposed in equation (4). The maximization assumes that goods of different quality within the same industry are perfect substitutes, therefore consumers only buy goods with the lowest quality adjusted price. Additionally, in cases where goods of different qualities have the same quality adjusted price, it is assumed that consumers buy goods with the highest quality. Given this, the demand for a good of quality \( j \) in industry \( \omega \) at time \( t \) is

\[
d(j, \omega, t) = \frac{q_j(\omega, t)p(j, \omega, t)^{-\sigma} E(t)}{\int_0^1 q_j(\omega, t)p(j, \omega, t)^{1-\sigma} d\omega}.
\]

(5)

Where, \( \sigma = 1/(1 - \kappa) \), \( \delta \equiv \lambda^{\sigma-1} \), and \( q_j(\omega, t) \equiv \delta^{j(\omega,t)} \); \( q_j(\omega, t) \) can be interpreted as the demand adjusted effective measure of the quality of a good on step \( j \) in industry \( \omega \) at time \( t \).

The second step of optimization is a dynamic problem in which the households decide on the allocation of expenditure \( E(t) \) over time. It is assumed households have access to capital markets where one can save by investing either in a portfolio of risky securities issued by R&D firms or in risk free bonds which bears a risk free interest rate. The equilibrium interest rate \( r(t) \equiv \frac{dR(t)}{dt} \) clears the capital market at each moment in time.

Substituting the demand function from equation (5) into equation (2) and substituting the resulting expression into equation (1), I get the inter temporal indirect utility function of the households. The dynamic optimization problem of the household then reduces to maximizing this indirect utility.
function, subject to the budget constraint in equation (3) given the rate of interest \( r(t) \). That is,

\[
\max_{E(t)} \int_{0}^{\infty} e^{-(\rho-n)t} \left[ \ln E(t) + \left( \ln \int_{0}^{1} \lambda j q_j(\omega,t)p(j,\omega,t)^{-\sigma} d\omega \right)^{1/\alpha} \right] dt,
\]

subject to \( \int_{0}^{\infty} e^{-R(t)} E(t) dt = A(0). \)

This problem yields the standard Euler equation

\[
\frac{\dot{E}}{E} = r(t) - \rho. \tag{6}
\]

Equations (5) and (6) describe optimal consumer behavior.

### 2.3 Producer Behavior

For each industry and each quality product there is constant returns to scale, with one unit labor producing one unit of the output. As mentioned earlier, labor is the only factor of production.

At any given point in time there are two types of industries: first, industries where imitation has occurred. In these industries there are multiple quality leaders. I assume that once imitation takes place there is Bertrand competition between the incumbent and the imitators, driving down the price of the good to its marginal cost and consequently the firms earn zero economic profit. In the second type of industries imitation has not taken place, and there is only one quality leader. In what follows I analyze producer behavior in each type of industry.

Once a higher quality good is innovated in a particular industry the innovator is awarded a property-right in the form of a patent and becomes the sole producer of the good in that industry, the demand for which is given by equation (5). However, the system of patents is not perfect and the good is targeted for imitation. The effective duration of the monopoly is determined by the RPAs undertaken by the innovator to protect the property-right. Thus the industry leaders’ problem is two fold: choose the price \( p_L \) given demand, and the amount of resources to be devoted toward RPAs.
I assume that the unit cost of RPAs increases as the product moves up the quality ladder; that is, RPAs becomes progressively more expensive. This makes sense because, as quality keeps upgrading several layers are added to the good. In such a scenario, protecting the property-rights over a good can become more costly for a variety of reasons. For instance, more resources need to be expended to keep the blue print of several components of the product a secret. Alternatively, one might need to apply for several patents for different components of the same product. Apple holds more than 400 patents for the technology relating to the I phone alone.\footnote{According to World Intellectual Property Organization, the number of claims on a patent is one the major factors determining the cost of patent application and prosecution} To capture this idea, I assume $a_p \delta^j$ is the unit cost of property right protection for a good of quality $j$ in industry $\omega$. The parameter $a_p$ may be interpreted as a fixed cost such as the hourly rate of a patent attorney.

Denoting $R$ to be the total intensity of RPAs, the total cost of property right protection at each time would be $R a_p \delta^j$. Given this, the instantaneous profit function of the single quality leader is given by

$$\pi_L = \begin{cases} (p_L - 1) \frac{q_j(\omega, t)p_L^{-\sigma}E(t)L(t)}{\int_0^1 q_j(\omega, t)p(j, \omega, t)^{1-\sigma}d\omega} - R a_p \delta^j & p_L \leq \lambda \\ 0 & p_L > \lambda \end{cases}$$

Define $Q(t) \equiv \int_0^1 q_j(\omega, t)d\omega$ (as in Segerstorm (2007)), as the average quality level across industries and

$$z(t) = \frac{Q(t)p_L^{-\sigma}E(t)}{\int_0^1 q_j(\omega, t)p(j, \omega, t)^{1-\sigma}d\omega}$$

as the per capita quantity demanded for a single quality leader’s product when the product is of average quality. Then using these equation the profit function can be rewritten as:

$$\pi_L = \begin{cases} (p_L - 1) \frac{q_j(\omega, t)}{Q(t)} z(t)L(t) - R a_p \delta^j & p_L \leq \lambda \\ 0 & p_L > \lambda \end{cases}$$

From the profit function given above, it is clear that the monopolist will charge a limit price $\lambda$.\footnote{According to World Intellectual Property Organization, the number of claims on a patent is one the major factors determining the cost of patent application and prosecution}
Thus the profit function can be rewritten as:

$$\pi_L = (\lambda - 1) \frac{q_j(\omega, t)}{Q(t)} z(t) L(t) - Ra_p \delta^j.$$

(8)

Note that the instantaneous profit flow decreases in the intensity of RPAs. However, by undertaking this expenditure the duration of monopoly pricing increases and this may increase the present discounted value of expected profits given by

$$V_I[j(\omega, t)] = E \left[ \int_0^\infty \pi_L e^{-\int_0^t r(s)ds} dt \right],$$

(9)

where $E$ in the above equation denotes the expectation operator. The monopolist will eventually choose $R$, such that these two effects offset each other. This is discussed in the next section.

2.4 R&D Races

I assume that innovation is uncertain and costly and only takes place in industries with multiple quality leaders, i.e, only goods that have been imitated are targeted for innovation\(^4\). This will certainly be the case if one considers a situation in which further innovation upon a good can only take place if the blueprint of the current highest quality good is available to other potential innovators. Until the time a successful imitation takes place the blueprint remains a secret and no other firms can innovate further. While this blueprint is available to the property right holder, it can be shown that for some reasonable parameter restrictions the leader has no incentive to go two steps above in the quality ladder. Instead, he can generate a higher profit by innovating a higher quality good in another industry.

Following Segerstorm (2007), I assume that innovation becomes progressively more difficult over time. The amount of labor required for each unit of innovative R&D in industry $\omega$ is given by $a_I \delta^j$, so as $j$ goes up, the per unit cost of innovation in industry $\omega$ goes up. If a firm invests $Ia_I \delta^j$

\(^4\)This is a simplifying assumption. The model can be extended to the case where goods with single quality leaders are also targeted for further innovation. Similar assumption is employed in Segerstorm (1991). In contrast, Segerstorm (2007), and Davis and Sener (2013) develop models where innovation takes place in all industries. In Davis and Sener(2013), the RPAs prevent both imitation, and innovation by other firms.
units of labor, then it is successful in discovering the next highest quality product \( j(\omega, t) + 1 \) with instantaneous probability \( Idt \). Since labor is the numeraire good, the total cost of innovation is also \( Ia_1 \delta^j \). Innovations therefore follow a Poisson process with arrival rate \( I \). Note that while innovation becomes progressively more difficult as one moves up the quality ladder, at each step of the quality ladder there is constant returns in innovation. That is, changing labor in a given proportion, yields a proportional change in probability of innovation.

There is an exogenous imitative intensity \( \overline{C} \) with which all goods are targeted for imitation (this assumption also follows Segerstorm (2007)).\(^5\) In case of a successful imitation, Bertrand competition ensues between the incumbent monopolist and the imitator and both of them end up making zero economic profit. This intensity of imitation does not vary across industries or over time. However, as mentioned earlier the single quality leader expends resources \( R \) to prevent imitation. Therefore one can envision a contest between innovators and imitators, with each side expending resources to make the outcome favorable to them more likely. I assume that the effective probability of imitation is given by

\[
P = \frac{\overline{C}}{\varphi R}, \tag{10}
\]

where \( \varphi \) can be interpreted as the extent to which the institutional set-up aids RPAs. Note that this probability is increasing in \( \overline{C} \) and decreasing in \( \varphi \) and \( R \). If the quality leader expends more resources or the IPR protection becomes stronger, then the instantaneous probability of imitation goes down.

**2.5 R&D Optimization**

In this section I analyze the firms’ choice of the optimal intensity of innovation and RPAs. Each firm engaged in R&D is assumed to maximize its expected discounted profits given by equation (9). To simplify notation I follow Segerstrom(2007) and use \( j_\omega \) to denote the state of the art quality level in industry \( \omega \) instead of \( j(\omega, t) \). For a single quality leader the relevant Hamilton-Jacobi-Bellman

\(^5\)There is a large literature that deals with the incentives to imitate, where the rate of imitation is endogenously determined along with the rate of innovation. See for example Segerstorm (1991), Davidson and Segerstorm (1998)
equation is given by:
\[
r V_I(j_\omega) = \max_R \{ \pi_L + P[V_{IC}(j_\omega) - V_I(j_\omega)] \},
\]
(11)
where \( V_I \) is the market value of the single quality leader (see equation (9)) and \( V_{IC} \) is the market value of the imitating firm. Since there are no profits to be earned as a result of imitation, the market value of the imitating firm must be zero at all times, that is \( V_{IC} = 0 \). The right side states that the monopolist makes a profit of \( \pi_L \) each time period and with instantaneous probability \( P \) loses his market value as a result of the imitation. Thus it is the maximized expected return on the stock of a monopolist. The left side is the return on an equal sized investment in a risk free bond. In equilibrium both have to be equal.

Solving the optimization problem in the right side of equation (11) yields
\[
a_p \delta j_\omega = \frac{C}{\varphi R^2} V_I(j_\omega).
\]
(12)
I now consider the optimization problem for the potential innovator in an industry where imitation has taken place. The potential innovator raises the resources for R&D in the capital market by issuing securities to households. Since innovation is an uncertain process these securities are risky. However, the risk is idiosyncratic and the households can earn a sure rate of return by investing in a diversified portfolio. The no arbitrage condition implies that the sure rate of return must equal the rate on risk free bonds. Given this the Hamilton-Jacobi-Bellman equation for the potential innovator is
\[
r V_P = \max_I \{-Ia_I \delta j_\omega + I[V_I(j_\omega + 1) - V_P]\},
\]
(13)
where \( V_P \) is the market value of the potential innovator. In each time period, a potential innovator incurs a cost of \(-Ia_I \delta j_\omega \) and succeeds in innovating the next higher quality good \( j_\omega + 1 \) with probability \( I \). The following proposition summarizes the result of the optimization problem of the potential innovator.
Proposition 1 Given free entry into R&D and constant returns to scale technology in innovation:

1. The maximized expected present value of the potential innovator, \( V_P = 0 \), and the optimal scale of innovative intensity is indeterminate.

2. The optimal intensity of RPAs (\( R \)), and the effective probability of imitation (\( P \)) are both constant across industries and over time given by

\[
R^* = \left( \frac{Ca_1}{\varphi \delta a_P} \right)^{1/2} ; P^* = \left( \frac{C \delta}{\varphi} \right)^{1/2} \left( \frac{a_P}{a_1} \right)^{1/2} .
\] (14)

3. \( P^* \) increases in the ratio \( a_P/a_1, \delta, \) and \( C \) and decreases in \( \varphi \).

Proof: In Appendix

Free entry and constant returns to scale in innovation imply that the optimal scale of innovative activity is indeterminate from the optimization problem of the firm. However, as will be shown in the next section, in steady state the innovative intensity is determined by the rate of growth of quality. Equation (14) determines the intensity of RPAs in terms of the parameters of the model. Since all parameters on the right side of the equation are constant \( R \) is constant across industries and over time. Equation (10) then implies that the effective probability of imitation is also a constant.

Substituting equation (10), (12) and (14) into equation (11) yields:

\[
r + 2P = \frac{\delta(\lambda - 1)z(t)}{a_f x(t)}, x(t) = \frac{Q(t)}{L(t)},
\] (15)

\( \frac{Q(t)}{L(t)} \) is a measure of the relative difficulty of R&D as in Segerstorm (2007). To see this note that an increase in \( Q(t) \) means the average quality is increasing and more labor is required to maintain the rate of innovation. An increase in \( L(t) \) implies that more labor is available for R&D. Thus \( x(t) \) is a measure of the relative difficulty of R&D.
2.6 Quality Dynamics

As mentioned in the previous section, the level of innovative intensity in the steady state is governed by the evolution of quality across industries. To see how quality evolves, let $\alpha$ be the measure of industries with a single quality leader and $\beta$ the measure of industries with multiple quality leaders.

$$Q(t) = \int_0^1 \delta j_\omega d\omega,$$

or

$$Q(t) = \int_\alpha \delta j_\omega d\omega + \int_\beta \delta j_\omega d\omega,$$

or

$$Q(t) = Q_L(t) + Q_C(t),$$

where $Q_L$ and $Q_C$ are the measure of qualities in $\alpha$ and $\beta$ industries. The time derivative of $Q(t)$ is

$$\dot{Q}(t) = \dot{Q}_I(t) + \dot{Q}_C(t).$$

(16)

The measure $Q_L$ in a $\alpha$ industry jumps from $j_\omega$ to $j_\omega + 1$ when innovation takes place in a $\beta$ industry. $Q_L$ drops when imitation occurs in a $\alpha$ industry and a good of quality $j_\omega$ leaves the industry. Therefore, $Q_L$ evolves as

$$\dot{Q}_L(t) = \int_\beta \delta j_\omega + 1 I d\omega - \int_\alpha \delta j_\omega P d\omega.$$

This gives us,

$$\dot{Q}_L(t) = \delta IQ_C - PQ_I.$$  (17)

Similar reasoning establishes the following

$$\dot{Q}_C(t) = PQ_L - IQ_C.$$  (18)
Substituting equations (19) and (20) into 18 yields

\[
\dot{Q}(t) = (\delta - 1)IQ_C,
\]

or

\[
\frac{\dot{Q}(t)}{Q(t)} = (\delta - 1)Iq_c, \tag{19}
\]

where \( q_c = Q_C/Q(t) \) is the relative quality of goods available through competitive markets. The above equation gives us the growth rate of average quality across industries. Note that the growth rate is proportional to the innovation rate over the measure of relative quality in the \( \beta \) industries. This makes sense because, in my model innovation occurs only in the \( \beta \) industries.

The level of difficulty of R&D is given by the variable \( x(t) \) (given in equation (16)). Taking the log differential of \( x(t) \) over time we get

\[
\frac{\dot{x}(t)}{x(t)} = \frac{\dot{Q}(t)}{Q(t)} - \frac{\dot{L}(t)}{L(t)}.
\]

or

\[
\frac{\dot{x}(t)}{x(t)} = (\delta - 1)Iq_c - n \tag{20}
\]

It can be seen from the above equation that in the steady state when \( x(t) \) and \( q_c \) are constant, \( I \) will also be a constant. This gives rise to the following proposition

**Proposition 2** In the steady state with constant \( x(t) \) and \( q_c \), \( I \) is constant; the steady state value of \( I \), \( q_c \), and \( q_l = 1 - q_c \equiv Q_L/Q \) are given by

\[
I = \frac{n(n + P)}{P(\delta - 1) - n}, \tag{21}
\]

\[
q_c = \frac{P(\delta - 1) - n}{(n + P)(\delta - 1)}; q_l = \frac{n\delta}{(n + P)(\delta - 1)}. \tag{22}
\]

Proof: In appendix
Note that a positive rate of innovation requires that, \( P > \frac{n}{(\delta - 1)} \). That is, a sufficiently high rate of imitation is required to sustain innovative activity. This is because innovation in this model only takes place in the \( \beta \) industries. The inequality \( P > \frac{n}{(\delta - 1)} \) is satisfied if \( a_p/a_I \) is sufficiently high. That is, the fixed cost of RPAs relative to cost of innovation is sufficiently high. I assume that to be the case.

### 2.7 Labor Market

In the model labor is used for manufacturing the final good in the \( \alpha \) and \( \beta \) industries, for innovative R&D and for property-right protection. The sum of labor used for each type of activity should be equal to the total labor in the economy. The total manufacturing employment is equal to total manufacturing employment in \( \alpha \) industries plus the total manufacturing employment in \( \beta \) industries. The manufacturing employment in an \( \alpha \) industry is given by:

\[
d_j(t) = \frac{q_j(t)}{Q(t)} \lambda z(t) L(t).
\]

The manufacturing employment in a \( \beta \) industry is given by:

\[
d_j(t) = \frac{q_j(t)}{Q(t)} \lambda^\sigma z(t) L(t).
\]

The total manufacturing employment is therefore given by:

\[
\int_\alpha d_j(t) L(t) d\omega + \int_\beta d_j(t) L(t) d\omega.
\]

This yields the following:

\[
L_M = z(t) L(t) \left[ (1 - q_c) + \lambda^\sigma q_c \right],
\]

where \( L_m \) is the total manufacturing employment. Innovative R&D only takes place in \( \beta \) industries, therefore the total labor used for this purpose is given by:

\[
L_I = \int_\beta I a_I \delta \lambda d\omega.
\]
Finally, since RPAs is only undertaken by the single quality leaders in the $\alpha$ industries, the total labor used for RPAs is

$$L_R = \int_\alpha^{\alpha} R a_p \delta^{j_{\omega}} d\omega,$$

or

$$L_R = R a_p Q q_l. \tag{25}$$

The employment in RPAs is an increasing function of $q_l$. This is because the per unit labor requirement on RPAs is increasing in the quality of goods. A rise in $q_l$ implies that quality of goods in the $\alpha$ industries goes up, therefore total labor employed in RPAs also goes up. Labor market equilibrium requires that total labor demand ($L_M + L_I + L_R$) equals the labor supply, $L(t)$. So,

$$z(t)L(t) [q_l + \lambda^q q_c] + I a_I Q q_c + R a_p Q q_l = L(t).$$

Dividing both sides of the equation by $L(t)$

$$z(t) [q_l + \lambda^q q_c] + I a_I x(t)q_c + R a_p x(t)q_l = 1. \tag{26}$$

The labor market equilibrium condition gives rise to the following proposition describing the behavior of interest rate($r(t)$), demand ($z$), and expenditure ($E(t)$) of the households in the steady state.

**Proposition 3** In the steady state the variables $r(t)$, $z$, and $E(t)$ are constant. Further, the interest rate is equal to the discount rate $\rho$.

**Proof**: In appendix.

Combining proposition 4 with equation (18) and dropping time subscript from $z$ and $x$ (since they
are constant), the steady state zero profit condition in R&D is

\[ z = \frac{(\rho + 2P)a_I x}{\delta(\lambda - 1)}. \]  

Equation (28) describes a positive linear relationship between the state variables \( z \) and \( x \) in the steady state. Further, the labor condition in equation (27) can be rewritten as

\[ z = \frac{1}{q_i + \lambda^\sigma q_c} - x \left[ \frac{Ia_I q_c + Ra_p q_i}{q_i + \lambda^\sigma q_c} \right]. \]  

(28)

The labor condition therefore implies a negative linear relationship between \( z \) and \( x \). Equation (28) and (29) can be solved simultaneously to pin down the steady state values of state variable \( z^* \) and \( x^* \). Once \( z^* \) and \( x^* \) are known we can solve for the expenditure \( E \) in the steady state, which in turn determines the demand for highest quality good in each industry.

We are now in a position to consider the possibility of a patent buyout and look at the implications of such a policy on the rate of innovation.

### 3 A Simple Model of Patent Buyout

As discussed earlier, I analyze the possibility of a Kremerian patent buyout as an alternative to the system of patent discussed in the previous sections. To keep model simple at this point, I simplify the buyout mechanism in two ways. In the Kremerian buyout, the government marks up the private value of a patent by a constant amount to account for the fact that the social returns from the patent are greater than the private returns. Further, the government, in order to elicit the true value of the patent from private buyers sells a certain fraction of the patents bought to the highest bidders. This way, the private bidders (from whose bids the government receives information on the value of a patent) have an incentive to reveal their actual evaluations. I do not consider these two possibilities at this moment. That is, I assume that the government pays the patent holder only the private value, and that the government makes all the patents publicly available. Therefore, in all industries the final good is traded in a competitive market, instead of a fraction of them.
The private value of a patent to the patent holder is the present discounted value of all future profits that the firm earns from the patent. In deriving the present value of the patent, the firm takes into account the fact that there is imitation at the rate of $\bar{C}$, and it has to spend an amount $Ra_p\delta_j^\omega$ to thwart imitation. This expenditure lowers the imitation probability to $Pdt$ in the time interval $dt$. That is, imitation is an exponential process with the arrival rate $P$. This fact in turn implies that the probability that the patent is not imitated upon until any time $t$ is $e^{-Pt}$. Given that the instantaneous profit of the firm as a result of innovation is $\pi_L$ (refer to equation 8), the interest rate in the steady state is equal to $\rho$, the present discounted value of expected profits is given by

$$v_I = \int_0^\infty \pi_L e^{(-\rho+P)t} = \frac{\pi_L}{\rho+P},$$

where $\pi_L$ is given in equation (10) as

$$\pi_L = (\lambda - 1) \frac{q_j(\omega, t)}{x(t)} z(t) - R^* a_p \delta^j, \text{ and } R^* = \left( \frac{C a_I}{\varphi a_p} \right)^{1/2}.$$ 

Since in the steady state $z$ and $x$ are constants given by $z^*$ and $x^*$, the present value of expected profits in equation (29) is equal to

$$v_I = \frac{(\lambda - 1) \delta_j^\omega z^*}{\rho + P} - R^* a_p \delta^j.$$ 

Equation (30) shows the present value of expected profits in the steady state for patent holder. As mentioned above, the government uses the Kremerian auction to discover this private value and offers the firm this amount to buyout the patent. If the firm agrees, then this represents the government expenditure ($G$) in an industry $\omega$. Once the government buys the patent, it places in the public domain so that the good can now be provided in a perfectly competitive market. The government raises the revenue for this expenditure by imposing a sales tax of an amount $\tau$ on the good. Now, since I have assumed that the marginal cost of production of the good is 1, the price of the good in the competitive market including the tax is $1 + \tau$. The total revenue of the government,
therefore will be $\tau$ times the demand for the good at the price $1 + \tau$. The demand for a good of quality $j$ in industry $\omega$ at the steady state is given by

$$d(j, \omega, t) = \delta^j \frac{z^*}{x^*}.$$  

Therefore, the tax revenue of the government from the sales tax is

$$T = \tau \delta^j \frac{z^*}{x^*}.$$  

(31)

I assume that the government chooses the tax rate so that the buyout does not lead to any additional government debt, that is, $T - G = 0$.

**Proposition 4** As long as $\rho > 1 - P$, there exists an optimal tax rate $\tau$ such that $1 + \tau < \lambda$ and $T - G = 0$.

Proof: In appendix

The above proposition implies that as long as the discount rate is high enough, the government can impose a tax such that the price of a good under patent buyout $(1 + \tau)$ will be less than the monopoly price $\lambda$. This is because, in the steady state the discount rate ($\rho$) is equal to the interest rate. Therefore a high value of $\rho$ implies a lower present discounted value of future stream of profits from an innovation. The government can therefore impose a tax that is low enough to generate the revenues required to buyout the patent by paying the present value of profits.

### 4 Welfare, and Output Growth with Patent Buyout

In this Section, I compare the steady-state growth and welfare implications of the conventional patent system to that of the patent buyout system. Since there are zero economic profits in both innovation and manufacturing, the total welfare in the economy is equivalent to the utility derived by the consumers. To compute the growth rate and welfare in the steady-state note that the
instantaneous utility in equilibrium is

\[ u(t) = \left( \int_0^1 [\lambda^j d(j, \omega, t)]^{\kappa} d\omega \right)^{1/\kappa}. \]  

(32)

This is based on the fact that only products that are consumed in equilibrium are the highest quality goods and the demand for all other goods is zero. Substituting the \( d(j, \omega, t) \) from equation (5), the instantaneous utility of the households under either system is given by

\[
\begin{aligned}
    u(t) &= \begin{cases} 
        EQ(t) \frac{1-\kappa}{\kappa} [q_l \lambda^{1-\sigma} + q_c] \frac{1-\kappa}{\kappa}, & \text{under conventional patent system} \\
        EQ(t) \frac{1-\kappa}{\kappa} [(1 + \tau)^{1-\sigma}] \frac{1-\kappa}{\kappa}, & \text{under patent buyout}
    \end{cases}
\end{aligned}
\]

(33)

In the model, economic growth is defined as the growth in utility of the consumers. Given that in the steady-state \( E, q_l, q_c, \) and \( \tau \) are constant (see proposition 3 and 4), it is easy to see that the steady-state growth rate in the economy under either system is determined by the rate of growth of quality in the steady-state. This leads to the following proposition

**Proposition 5** The steady-state growth rate of the economy is identical under the conventional patent system and the patent buyout system given by

\[ \gamma = \left( \frac{1 - \kappa}{\kappa} \right) n. \]  

(34)

**Proof:** Take natural logs of the instantaneous utility function in equation (33) and differentiate with respect to time and use the fact that in the steady-state \( \dot{Q} = n/Q \).

The above proposition implies that using the patent buyout mechanism does not diminish the rate of growth of an economy in the steady-state. To see the implications on the overall welfare we can compare the instantaneous utility functions under the two systems to see which one yields a higher welfare. Intuitively, this would depend on the relative quality of highest quality goods with monopoly pricing \( (q_l) \) under the conventional patent system. This is because in going from the conventional patent system to the patent buyout mechanism, while the price of goods sold in monopoly markets goes down (from \( \lambda \) to \( 1 + \tau \)), there is a corresponding increase in the price of
goods sold in competitive markets (from 1 to 1 + \(\tau\)). That is, there the relative price of goods sold in monopoly markets decreases. If the elasticity of substitution is high between the industries \((\sigma > 1)\), then consumers will switch demand from goods that were sold competitively to ones that were sold under monopoly. For their to be an increase in welfare, the relative quality of these goods needs to be to high. The following proposition formalizes this argument

**Proposition 6** The patent buyout system will generate a higher welfare compared to the conventional patent system if

\[
q_l > \frac{1 - (1 + \tau)^{1-\sigma}}{1 - \lambda^{1-\sigma}} \equiv \hat{q}_l \tag{35}
\]

**Proof:** Follows directly from equation (33).

As mentioned above, switching over to a patent buyout system will lead to a higher welfare if the relative quality of goods sold in monopoly markets \((q_l)\) is greater than the threshold value \(\hat{q}_l\).

Suppose the inequality in proposition 7 holds so that the patent buyout system yields a higher welfare than the conventional patent system. Let \(\Omega\) denote the difference in welfare between the two system so that

\[
\Omega = EQ(1 - \kappa) \left( \frac{(1 + \tau)^{1-\sigma}}{1 - \lambda^{1-\sigma}} \right) - [q_l \lambda^{1-\sigma} + 1 - q_l] \frac{1-\kappa}{1-\sigma} \tag{36}
\]

By assumption, \(\Omega > 0\) in the above equation. Note that in the above equation \(\partial \Omega / \partial q_l > 0\) and \(\partial \Omega / \partial \lambda > 0\). That is, the gain in welfare as a result of moving to a patent buyout system is greater, the greater the share of monopoly industries and the greater the monopoly price, \(\lambda\). It is clear from proposition (2) and (3) that \(q_l\) is an increasing function of the equilibrium intensity of RPAs, \(R^*\). Therefore, the gain in welfare of moving to a system of patent buyouts will be greater, the greater is the expenditure toward property right protection in the conventional patent system. The following proposition establishes this result

**Proposition 7** The higher the extent monopoly pricing \((\lambda)\) and RPAs \((R)\) under the conventional patent system, the higher is the gain in welfare from a system of patent buyouts.
5 Conclusion

In this paper, I modify the standard quality ladder model of growth by incorporating imitation and rent protection activities (RPAs) to analyze the growth and welfare impact of eliminating the patent system in favor of government patent buyouts. The government buys an innovator’s patent by paying the private value of the patent elicited through the auction mechanism highlighted in Kremer (1993). It then places the patent in public domain so that innovated good can be sold in competitive markets. The buyout is financed through a sales tax on the good. Using this framework, I show that if the discount rate of households is high enough, then there exists a tax rate such that the resulting competitive price plus tax is less than the monopoly price that would exist under the conventional patent system. I further show that the patent buyout mechanism can lead to higher overall welfare in the economy. The increase in welfare is due elimination of both monopoly pricing and rent protection activities. Finally, the buyout mechanism yields the same growth rate in the economy as the conventional patent system.

There is an increasing recognition amongst economists and legal professionals that the current patent system leads to substantial welfare losses and therefore must be reformed. The standard argument against such reform is that without patents innovation rate of an economy will decrease, which in turn will lower growth rates. My paper shows that this need not be the case. The paper also provides a framework to analyze other possible reforms to the patent system such as prizes, and elimination of patents on projects where R&D subsidies are granted. The framework is useful especially because it considers both the role of imitation, and RPAs in evaluating such policy proposals.
References


6 Appendix

Proof: Proposition 1

Note that the optimization problem of the potential innovator is linear in the innovative intensity $I$. If $a_I \delta j_\omega > V_I(j_\omega +1) - V_P$, then given free entry into innovation, an infinite amount of resources will be devoted to innovative activity or there will be an infinite demand for loans (supply of securities). This will cause interest rate to rise to infinity and would drive down the present expected market value of the firm to zero. Therefore, the above inequality cannot be sustained in equilibrium. A similar argument rules out $a_I \delta j_\omega < V_I(j_\omega +1) - V_P$ as an equilibrium condition. In this model the interest rate adjusts at each point in time to ensure that at equilibrium the following holds:

$$a_I \delta j_\omega = [V_I(j_\omega +1) - V_P].$$
Multiplying both sides of the equation with $I$ it can be seen that the total costs of innovation exactly equals the expected benefits from innovation, thus the present market value of the potential innovator is zero, i.e, $V_P = 0$. It follows then that the optimal innovative intensity $I$ will be such that

$$V_I(j\omega) = \frac{a_I \delta j\omega}{\delta}.$$  

(37)

Substituting for $V_I(j\omega)$ from equation (14) into equation (12) yields:

$$R^* = \left(\frac{C a_I}{\varphi \delta a_p}\right)^{1/2}$$

Since $C$, $a_I$, $\varphi$, $\delta \equiv \lambda^{\sigma-1}$, and $a_P$ are constant by assumption, $R^*$ is also a constant. Further, substituting $R^*$ in equation (10) it can be seen that

$$P = \left(\frac{C \delta}{\varphi}\right)^{1/2} \left(\frac{a_P}{a_I}\right)^{1/2},$$

is also a constant. Given the value of $P$ above, we have the following: $\partial P / \partial (a_P/a_I)$, $\partial P / \partial \delta$, $\partial P / \partial C > 0$ and $\partial P / \partial \varphi < 0$

**Proof: Proposition 3**

In the steady-state $x(t)$ is constant and we must have

$$I = \frac{n}{(\delta - 1)q_e}. \quad (38)$$

Thus the steady-state level of innovation intensity depends on population growth rate and the difficulty of innovation in the $\beta$ industries. An increase in $q_e$ implies that quality in the $\beta$ industries goes up relative to the average quality, therefore innovation gets more difficult at each step and innovation intensity must go down, unless there is positive population growth. We now need to determine $q_e$, to solve for the steady-state innovative intensity completely in terms of the parameters
of the model. We know from equation (21) that

\[
\frac{\dot{Q}_C(t)}{Q_C} = P \frac{Q_L}{Q_C} - I,
\]

further, since

\[
\frac{\dot{q}_c}{q_c} = \frac{\dot{Q}_C}{Q_C} - \frac{\dot{Q}}{Q}
\]

so it follows that in the steady-state when, \( \dot{q}_c = 0, \)

\[
q_c = \frac{P(\delta - 1) - n}{(n + P)(\delta - 1)},
\]

(39)

and further

\[
q_l \equiv Q_L/Q = 1 - q_c = \frac{n\delta}{(n + P)(\delta - 1)}.
\]

(40)

Substituting equation (39) into (40) yields

\[
I = \frac{n(n + P)}{P(\delta - 1) - n}.
\]

(41)

Proof: Proposition 4

From proof 3 we know that in the steady state \( x(t) \) and \( I \) are constant. We know from equation (17) that \( R \) is a constant. Therefore, equation (26) implies that in steady state \( z(t) \) is also a constant, its value given by:

\[
z = Q \frac{\lambda - \sigma}{q_l \lambda^{1-\sigma} + q_c} E(t).
\]

(42)

A constant \( z, q_l, \) and \( q_c \) implies that in steady state \( E(t) \) is also a constant and therefore from equation (6) the interest rate, \( r(t) \), is equal to the discount factor \( \rho \).

Proof: Proposition 5

Setting equation (30) equal to equation (31), and substituting for \( z^*/x^* \) from equation (32) to solve for \( \tau \) gives us

\[
\tau^* = \frac{\lambda - 1}{\rho + P} \left[ 1 - \frac{R a_P \delta}{(\rho + 2P) a_I} \right]
\]

(43)
It can be shown that if $\rho > 1 - P$, then $1 + \tau^* < \lambda$. 
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