

This paper analyzes an oligopolistic industry where one firm is a regular private firm that maximizes its own profit while the second one is a (semi-) public firm that maximizes its own profits and some of the public's consumers' surplus. The innovation in this paper is that each firm dealing in cost reduction innovation which affects only its own costs. The main result is that the degree of the public ownership stimulates the output and innovation of the (semi-) public firm. Another interesting result is that the total output of the industry increases as the degree of the public ownership is increasing.

My comments are as follows:

1. Let's consider the two equations that follow eq. (9):

$$\begin{aligned} \text{Max}_{I_A} \Pi_A &= q_A^2 - \frac{\tau}{2} q_A^2 + \frac{\tau}{2} q_B^2 - \frac{1}{2} I_A^2, \\ \text{Max}_{I_B} \pi_B &= q_B^2 - \frac{1}{2} I_B^2. \end{aligned}$$

The q_A and q_B are with no asterisk. But, the above two equations are the outcome of replacing q_A and q_B [from (5) and (6)] by equation (9). In addition, elaboration of the mathematical process will be helpful.

2. The object function of the (semi-) public firm is:

$$\text{Max}_{I_A, q_A} \Pi_A = \pi_A + \frac{\tau}{2} (q_A^2 + q_B^2 - 2\gamma q_A q_B)$$

Where the term:

$$CS = \frac{1}{2} (q_A^2 + q_B^2) + \gamma q_A q_B$$

is the consumers' surplus.

It is not clear why the total consumers' surplus is multiplied by τ (the government share). It is reasonable that the government cannot affect the firm's decision when τ is less than 50%, and it is possible to consider the total consumers' surplus when τ is above 50%.

3. It is well known that public firm is less efficient than a private owned firm. In this paper it is assumed that the level of efficiency is the same in both types of firms. I wondered what are results when the private owned firm is more efficient in general and in the innovation processes in particular.
4. The variable "I" gets the title "innovation" but does not possess any characteristics of innovation, such as "patents", imitation, knowledge spill-over etc. It is actually investment in equipment to reduce the unit cost.

In conclusion: The paper is well written and the development of the model is done professionally. My main reservation is about the contribution of this paper to the existing theory. It is a nice extension of the current knowledge mainly by applying public firms' models into the well-known models of duopoly. My recommendation is to revise the paper and especially to consider my second and third remarks.