The Endogeneity of the Natural Rate of Growth: An Alternative Approach

Acikgöz Senay and Merter Mert

Abstract
This study estimates and tests the endogeneity of the natural rate of growth using the balance-of-payments consistent rate of growth (BPCRG) instead of the actual rate of growth. Our approach is also theoretically compatible with the one proposed by Thirlwall (2001). Following this idea, we first calculated the BPCRG and then tested the Thirlwall’s Law for the U.S. economy. We then proceeded to test the endogeneity using the BPCRG. Empirical results we obtained support Thirlwall’s Law and endogeneity.

JEL O40 E10 E23 C22

Keywords Economic growth, Thirlwall’s Law, the endogeneity of the natural rate of growth, the bounds testing approach, ARDL, FM-OLS

Authors
Acikgöz Senay, Gazi University, Ankara, asenay@gazi.edu.tr
Merter Mert, Gazi University, Ankara

Citation
1. Introduction

There is a growing literature on testing the endogeneity of the natural rate of growth since Léon-Ledesma and Thirlwall (2002) initially showed that the natural rate of growth is endogenous [see; Vogel (2009), Libânio (2009), Lanzafame (2010) Acikgoz and Mert (2010), and Dray and Thirlwall (2011)]. However, all the studies testing the endogeneity used the actual rate of growth as a dependent variable revealing a problem which can be based on the main idea of Thirlwall (2001: p. 86; 2002: p. 84). Accordingly, the boom periods (on which the endogeneity depends) should be defined by the periods in which the actual rate \( (g_a) \) is greater than the warranted rate \( (g_w) \). However, most of the previous studies tested endogeneity when the actual rate is greater than natural rate \( (g_n) \). More importantly, at certain conditions, the actual rate might exceed the natural rate in recession periods, not in boom periods without balance of payments difficulties arising (Thirlwall, 2001: 86). Moreover, although the labor force and labor productivity have to increase during the boom periods, these might not be the only indicators for the endogeneity. The labor force and labor productivity may also decrease, indicating endogeneity.

This paper’s suggestion is to use BPCRG (indicated by \( g_b \)) as a dependent variable while testing the endogeneity of the natural rate of growth. In the light of these deliberations, we first estimate the components of BPCRG, and then we calculate the hypothetical rate of growth representing the BPCRG, based on the quarterly U.S. data. Secondly, we test the Thirlwall’s Law using the hypothetical rate of growth and actual rate of growth. Later, we test the endogeneity using the hypothetical rate of growth. Finally we discuss the empirical results.

The paper is organized as follows. Section 2, discusses the theoretical background of the paper. Section 3, gives the methodology of the study. Section 4 gives the estimation
results of the model together with a discussion of the findings. Finally, we conclude the discussion in Section 5.

2. Theoretical background

Thirlwall (1969) estimates the natural rate of growth using Equation (1) written below, where the growth rate of real output \( g \) is a function of the change in the percentage level of unemployment \( \Delta %U \).

\[
g = \gamma - \lambda (\Delta %U)
\]  

(1)

where, \( \gamma \) and \( \lambda \) are parameters to be estimated. If \( \Delta %U = 0 \), then \( \gamma \) equals to the natural rate of growth. After measuring the natural rate of growth, the endogeneity is tested using a dummy variable \( D \). In this case \( D \) takes the value of 1 for the years whose actual rate of growth exceeds the natural rate of growth and 0 for the rest of the years.

\[
g = \theta + \phi D - \psi (\Delta %U)
\]  

(2)

When \( \Delta %U = 0 \) and if the parameter \( \phi \) is statistically significant, the natural rate of growth equals to \( \theta + \phi \). Furthermore, if the sum of \( \theta + \phi \) is greater than \( \gamma \) (i.e., \( \theta + \phi > \gamma \)) this will indicate that the natural rate of growth increases during boom periods, suggesting endogeneity.

The point in need of attention is the following: Although boom periods are defined by periods in which actual rate of growth is greater than warranted rate of growth (Thirlwall, 2002, p. 84), endogeneity is tested when the actual rate is greater than the natural rate (Thirlwall, 2002, p. 90). Thirlwall (2002, p. 84) in fact states that “... if the actual rate of growth diverges from the warranted growth rate in either direction forces come into play which widen the divergence – but divergence is bounded by ceiling and floors. The ceiling is the natural rate of growth because the level of output cannot exceed the full employment ceiling.”
In order to solve the above mentioned problem, let us explain the relation between the warranted growth rate \( g_w \), the natural rate of growth \( g_n \) and the balance-of-payments consistent rate of growth \( g_b \). According to Thirlwall (2001) there are six cases where three of them show the boom periods and the other three cases show the recession periods based on the relation between \( g_w, g_n \) and \( g_b \). In cases where \( g_w < g_n > g_b \) or \( g_w < g_b < g_n \) or \( g_w < g_n < g_b \) (i.e. cases iii, vi and iv in Thirlwall (2001)), there will be an inflationary pressure, since \( g_w < g_n \). In that case, these periods are labeled as boom periods. When \( g_w < g_n > g_b \) or \( g_w < g_b < g_n \) there will be an inflationary pressure and tendency to balance of payments deficit, since \( g_b < g_n \). Besides when \( g_w < g_n < g_b \) there will be inflationary pressure and tendency to balance of payments surplus, since \( g_b > g_n \). (Thirlwall, 2001, pp. 86-87).

On the other hand, the cases where \( g_w > g_n > g_b \) or \( g_w > g_n < g_b \) or \( g_w > g_b > g_n \) i.e. cases i, ii and v in Thirlwall (2001), respectively, explains the recession periods. Similarly, when \( g_w > g_n > g_b \) there will be recession and tendency to balance of payments deficit. Besides when \( g_w > g_n < g_b \) or \( g_w > g_b > g_n \) there will be recession and tendency to balance of payments surplus. Moreover, according to Thirlwall (2001, p. 86), as mentioned above, when \( g_w > g_n < g_b \) or \( g_w > g_b > g_n \) economy will be in recession period, but the actual rate can exceed the natural rate without balance of payments difficulties arising since \( g_b > g_n \). Indeed as Thirlwall (2001, p. 86) mentioned “if the economy is in recession, but \( g_b \) exceeds \( g_n \), the actual growth rate can exceed \( g_n \) without balance of payments difficulties arising, and this may increase \( g_n \) by increasing labor force participation and raising the growth rate of productivity…”.
The discussion above shows that there are problems in testing the endogeneity of the natural rate of growth. To solve these problems and to be compatible with the theory, we offer to use the growth rate to be consistent with the balance-of-payments as a dependent variable instead of the actual rate of growth. Hence, the following equations can be written such as below where \( g_b \) represents the growth rate consistent with the balance-of-payments:

\[
g_b = \gamma - \lambda (\Delta\%U) \tag{1'}
\]

\[
g_b = \theta + \phi D - \psi (\Delta\%U) \tag{2'}
\]

Equation (1') expresses that when \( \Delta\%U = 0 \), \( \gamma \) is equal to the natural rate of growth. The endogeneity is tested based on Equation (2') using a dummy variable \( (D) = 1 \) as a value 1 for the years in which the growth rate consistent with the balance-of-payments exceeds the natural rate of growth and \( D = 0 \) for the remaining years.

On the other hand, the endogeneity might also be tested using a dummy variable. We can use \( (D) = 1 \) for the years where the growth rate consistent with the balance-of-payments is below the natural rate of growth and 0 for the remaining years. With a decreasing labor force and labor productivity, the natural rate of growth might fall during these periods, indicating endogeneity. Decreasing labor force and labor productivity might occur because of an exact converse situations as indicated in Thirlwall (2002, pp. 86-88) which are the following: a) decrease in labor force participation rates, b) decreasing working hours, c) migration of unemployed workers to another country for work, d) decreasing production causing low labor productivity.

Now, we can briefly explain why the growth rate is consistent with the balance-of-payments equilibrium. Thirlwall (1979) and Thirlwall and Hussain (1982) start from the balance-of-payments identity,
\[ P_d X + C = P_f ME \] (3)

where \( X, M, C, P_d, P_f, \) and \( E \) are the volume of exports, the volume of imports, the value of net capital receipts in national currency, domestic prices, international prices, and exchange rate, respectively. The volume of exports and imports are depicted by the following functions:

\[ X = A \left( \frac{P_d}{P_f E} \right)^\psi Z^\epsilon \] (4)

\[ M = B \left( \frac{P_f E}{P_d} \right)^\eta Y^\pi \] (5)

where \( P_d / P_f E \) represents the “terms of trade”, \( Z \) is “world income”, \( Y \) is “domestic income”, \( \psi \) and \( \eta \) are “price of elasticity”, \( \epsilon \) and \( \pi \) are “income of elasticity”.

The above functions can be expressed as follow where the variables in lower-case denotes growth rates:

\[ p_d + x + c = p_f + m + e \] (6)

\[ x = \psi (p_d - p_f - e) + \epsilon z \] (7)

\[ m = \eta (p_d - p_f - e) + \pi y \] (8)

By substituting the volume of exports and imports equations into the balance-of-payments accounting identity, ignoring the net capital inflows in the long run, and leaving alone the growth rate of domestic income yields to Equation (9) below.

\[ y^* = \frac{(1 + \psi + \eta)(p_d - p_f - e) + \epsilon z}{\pi} \] (9)

where \( y^* \) is the rate of growth of income that keeps the balance-of-payments in equilibrium.
If it is assumed that \( p_d - p_f - e = 0 \), i.e. the purchasing power parity is valid, then the terms of trade effect becomes equal to zero, and the following equations can be expressed.

\[
y^* = \frac{\varepsilon z}{\pi} \tag{10}
\]

or \( y^* = \frac{X}{\pi} \tag{11} \)

Equation (11) is the expression of “Thirlwall’s Law” which implies that the growth of domestic income (consistent with the balance-of-payments equilibrium) depends on the elasticity of exports with respect to imports.

Thirlwall (2002: 73-74) explains that there are two methods to test his law known as the “Thirlwall’s Law”. The first method involves estimating the elasticity of the income of demand for imports that would make the growth of the income consistent with the balance-of-payments equilibrium (which is equal to the actual rate of growth). Then the elasticity of the income of demand for imports which is compared with the estimated elasticity of imports with respect to the income from the time series regression analysis of the import demand function. The alternative method is to test if the growth of income is consistent with the balance-of-payments equilibrium which is a good predictor of the actual rate of growth.
First, we estimate the import demand function given in Equation (5). We then calculate the hypothetical rate of growth representing the balance-of-payments consistent rate of growth given in Equation (11). Second, we test the Thirlwall’s Law using the hypothetical and actual rate of growth. Finally, we test the endogeneity of the natural rate of growth using the hypothetical rate of growth. While testing the endogeneity, we focused on the periods in which the balance-of-payments consistent rate of growth is below the natural rate. This is because the U.S. economy is characterized by increasing balance of payments deficit in the relevant period (see Figure 1). Note that, if there is an increasing balance of payments deficit, the balance-of-payment’s consistent rate of growth is below the natural rate. However, we do not define these periods as boom periods since the U.S. economy is a mature developed economy which fall under the ‘case i’ in Thirlwall (2001); i.e. $g_w > g_n > g_b$. Thus, we focused on the periods in which the balance-of-payments consistent rate of growth below the natural rate, indicating endogeneity.

**Figure 1.** Current Account and Trade Balance for the U.S. Economy (1990Q1-2007Q4, Billion U.S. Dollars)
3. Econometric methodology

In this study, the import demand function is estimated by autoregressive distributed lag (ARDL) model. Because most of economic time series exhibit a non-stationary process, the bounds testing approach to co-integration, developed by Pesaran, Shin, and Smith (2001) is used. Unlike other tests such as two stage estimation of Engle and Granger (1987) and full information method of Johansen (1988), Johansen and Juselius (1990), the bounds testing approach can be applied irrespective of whether the underlying regressors are purely \( I(0) \), purely \( I(1) \), fractionally integrated, or mutually co-integrated. The bounds testing approach also has better small sample properties.

The empirical model specification which relates the volume of import (\( l\text{realimp} \)), real gross domestic product (\( l\text{realgdp} \)) and terms of trade (\( ltt \)) is given in Equation (12) below.

\[
\ln l\text{realimp}_t = \alpha + \pi l\text{realgdp}_t + \eta ltt_t + \epsilon_t \tag{12}
\]

The bounds testing approach estimates an unrestricted error-correction model (UECM) given in Equation (13) taking each of the variables in turn as dependent variable.

\[
\Delta l\text{realimp}_t = c_0 + c_1 t + \delta_1 l\text{realimp}_{t-1} + \delta_2 l\text{realgdp}_{t-1} + \delta_3 ltt_{t-1}
+ \sum_{j=1}^{q} \lambda_j \Delta l\text{realimp}_{t-j} + \sum_{j=1}^{p} \omega_j \Delta l\text{realgdp}_{t-j} + \sum_{j=1}^{r} \psi_j \Delta ltt_{t-j} + \psi D_t + u_t \tag{13}
\]

Equation (13) is a general form in which \( D_t \) is a vector of exogenous variables such as the structural change dummies and \( \Delta \) indicates first difference operator. The null hypothesis of no co-integration (\( \delta_1 = \delta_2 = \delta_3 = 0 \)) against the alternative of a long-run levels relationship (\( \delta_1 \neq \delta_2 \neq \delta_3 \neq 0 \)) is performed as a Wald restriction test. The asymptotic distributions of the \( F \)-statistics are non-standard under the null hypothesis. Pesaran, Shin and Smith (2001) provide two asymptotic critical values sets; all variables
are I(0) as lower bound and all variables are I(1) as upper bound. Decision rules to reject the null hypothesis are as follows:

(i) Reject the null hypothesis of no co-integration and conclude that there exists a long-run equilibrium among the variables, if the computed $F$-statistics is greater than the upper bound critical value,

(ii) Accept the null hypothesis of no co-integration, if the computed $F$-statistics is less than the lower bound critical value, and

(iii) The bounds test is inconclusive, if the computed $F$-statistics falls within the lower and upper bound critical values.

If a long-run relationship has been established, the long-term income and relative price elasticity’s are estimated from a conditional autoregressive distributed lag model, $\text{ARDL}(p,q_1,q_2)$, given in Equation (14) below.

\[
\text{lrrealimp}_t = c_0 + \sum_{j=1}^{p} \alpha_j \text{lrrealimp}_{t-j} + \sum_{j=0}^{q_1} \theta_{1j} \text{lrrealgdp}_{t-j} + \sum_{j=0}^{q_2} \theta_{2j} \text{ltt}_{t-j} + \psi D_t + u_t
\]  

We also estimate the long-term elasticity’s by using Fully Modified Ordinary Least Squares (FM-OLS) estimation method of Hansen (1992) in order to estimate the income elasticity of import ($\pi$) robustly. Hansen (1992) extends the FM-OLS estimator of Phillips and Hansen (1990) to cover general models with stochastic and deterministic trends. The two conditions which are considered essential for estimating the parameters by FM-OLS are (a) that there is only one unique integrating vector and (b) the regressors are not cointegrated among themselves.

4. Empirical results

Various variables, their definitions and data source used for this study are given below:
realgdp: The real GDP values (IFS-line 11199b, dollar) are calculated by deflating the GDP deflator (11199bir, 2005 = 100).

realexp and realimp: Export and import price indexes (IFS-lines 11176 and 11176x, 2005 = 100) are used in order to obtain the real exports and imports of goods and services (IFS-lines 11190c and 11198c, dollar).

tt: Terms of trade is defined as the ratio of import price index to consumer price index (IFS-line 11164, 2005 = 100). For estimating the natural rate of growth and testing its endogeneity, unemployment (IFS-line 11167C, thousands) is used to obtain the \( \Delta \% U_t \) series. All the series are taken from the IMF’s International Financial Statistics (July 2009), covering the period 1990:1-2007:4. They are measured in logarithms and seasonally adjusted by applying X-12-ARIMA method followed by the Bureau of Labor Statistics (BLS).

4.1 Empirical results for the import demand function

In order to test for the existence of a long-run relationship among the variables, the UECM’s given in Equation (13) are estimated by taking each one of the variables of the import demand function as dependent variable.\(^1\)

To determine the appropriate lag length \( p \) and whether a deterministic linear trend is required, the UECM is estimated by OLS, with and without a linear time trend, for \( p =

\(^1\)Although Pesaran, Shin and Smith (2001) indicated that the ARDL approach for testing the existence of a relationship between variables in levels is applicable irrespective of whether the underlying regressors are purely I(0), purely I(1), or mutually cointegrated. The orders of integration of the variables are determined by carrying out unit root tests. We used the augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979) and the generalized least squares de-trended Dickey-Fuller (DF-GLS) test (Elliot, Rothenberg and Stock, 1996). We also checked the effect of structural changes on the unit root test results by using the structural break tests of Zivot and Andrews (1992, ZA) and Lee and Strazicich (2003, LS). The \( l_{realimp}, l_{realgdp} \) and \( ltt \) series are found to be first-difference stationary series at the conventional significance levels, according to the ADF and DF-GLS tests. The ZA test shows that these three series are I(1) but not I(2) when structural changes in the form of a break in intercept, in the form of a break in trend and in the form of a break both in intercept and trend. The LS test results shows that log of real import, log of real GDP and log of terms of trade are non-stationary when structural breaks are taken into account. In brief, all the unit root test result indicate that the series used to estimate the import demand function are I(1) series.
1, 2,..., 6. Both Akaike (AIC) and Schwarz Information Criteria (SIC) are used to determine the lag length $p$. We also tested the first and fourth order residual autocorrelations for each lag by using the Breusch-Pagan Lagrange Multiplier (LM) statistics, which are distributed as $\chi^2(1)$ and $\chi^2(4)$, respectively.

Table 1 summarizes the values of the $F$-statistics for testing the existence of a level import demand equation under three different scenarios for the deterministic: (a) the constraint of unrestricted intercept and no trend ($F$-iii), (b) the constraint of unrestricted intercept and restricted trend ($F$-iv) and (c) the constraint of unrestricted intercept and unrestricted trend ($F$-v).²

The lags selected by AIC and SIC differ significantly. The lags selected by AIC is much larger than that by SIC, as might be expected. For the $lrealimp$ equation, AIC selects $p_c = p_t = 6$ while SIC selects $p_c = p_t = 1$.

²Because we have relatively small sample ($T = 72$), the critical values of the $F$-statistics modified by Narayan (2005) to accommodate small sample sizes are reported in this study (see Table 1).
Table 1. ARDL Bounds Test Results for Long-run Levels Relationship

<table>
<thead>
<tr>
<th></th>
<th>Sig. Levels</th>
<th>F-iii</th>
<th>F-iv</th>
<th>F-v</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I(0)</td>
<td>I(1)</td>
<td>I(0)</td>
<td>I(1)</td>
</tr>
<tr>
<td>10%</td>
<td>3.250</td>
<td>4.237</td>
<td>3.505</td>
<td>4.198</td>
</tr>
<tr>
<td>5%</td>
<td>3.947</td>
<td>5.020</td>
<td>4.100</td>
<td>4.900</td>
</tr>
<tr>
<td>1%</td>
<td>5.487</td>
<td>6.880</td>
<td>5.448</td>
<td>6.435</td>
</tr>
</tbody>
</table>

\[ k = 2, T = 72 \]

<table>
<thead>
<tr>
<th></th>
<th>( p_c )</th>
<th>( \chi^2(1) )</th>
<th>( \chi^2(4) )</th>
<th>( p_t )</th>
<th>( \chi^2(1) )</th>
<th>( \chi^2(4) )</th>
<th>Calculated ( F )-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(lrealimp)lrealgdp, ltt ) (AIC)</td>
<td>6</td>
<td>0.143 (0.705)</td>
<td>3.728 (0.444)</td>
<td>6</td>
<td>0.079 (0.778)</td>
<td>3.583 (0.465)</td>
<td>4.769* ( ) 5.510** ( ) 7.313**</td>
</tr>
<tr>
<td>( F(lrealimp)lrealgdp, ltt ) (SIC)</td>
<td>1</td>
<td>4.064 (0.044)</td>
<td>18.057 (0.001)</td>
<td>1</td>
<td>4.480 (0.034)</td>
<td>18.104 (0.001)</td>
<td>5.944*** ( ) 4.391* ( ) 5.698*</td>
</tr>
<tr>
<td>( F(lrealgdp)lrealimp, ltt ) (AIC)</td>
<td>5</td>
<td>0.602 (0.438)</td>
<td>6.022 (0.197)</td>
<td>4</td>
<td>0.425 (0.514)</td>
<td>5.626 (0.229)</td>
<td>0.321 ( ) 4.190* ( ) 5.540*</td>
</tr>
<tr>
<td>( F(lrealgdp)lrealimp, ltt ) (SIC)</td>
<td>4</td>
<td>0.124 (0.724)</td>
<td>2.920 (0.571)</td>
<td>4</td>
<td>0.425 (0.514)</td>
<td>5.626 (0.229)</td>
<td>0.084 ( ) 4.190* ( ) 5.540*</td>
</tr>
<tr>
<td>( F(ltt)lrealimp, lrealgdp ) (AIC)</td>
<td>4</td>
<td>0.953 (0.329)</td>
<td>4.379 (0.357)</td>
<td>5</td>
<td>0.037 (0.847)</td>
<td>8.896 (0.064)</td>
<td>1.971 ( ) 2.718 ( ) 3.123</td>
</tr>
<tr>
<td>( F(ltt)lrealimp, lrealgdp ) (SIC)</td>
<td>2</td>
<td>0.844 (0.358)</td>
<td>6.845 (0.144)</td>
<td>4</td>
<td>0.012 (0.912)</td>
<td>4.394 (0.355)</td>
<td>0.874 ( ) 2.772 ( ) 3.317</td>
</tr>
</tbody>
</table>

Notes:

(.) denotes \( p \)-values of the test statistics. 
\( k \) and \( T \) indicate the number of explanatory variables and the number of observations, respectively. 
\( p_c \) and \( p_t \) indicate the number of lags in the model with constant and with trend, respectively. 
F-iii indicates the \( F \)-statistics under the constraint of unrestricted intercept and no trend. 
F-iv indicates the \( F \)-statistics under the constraint of unrestricted intercept and restricted trend. 
F-v indicates the \( F \)-statistics under the constraint of unrestricted intercept and unrestricted trend. 
The critical values are extracted from Narayan (2005). 
***, ** and * indicate the null of there is no long-run levels relationship can be rejected at the 1%, 5% and 10%, levels.
For the validity of the bounds tests, residuals of the test regressions need to be serially uncorrelated. The $\chi^2$ statistics calculated at the lags selected by SIC indicate that using a relatively high lag order. The $F$-iii, $F$-iv and $F$-v statistics reject the null hypothesis which exists for “no level import demand” equation with at the 10% and 5% significance levels.

$F$-iii statistics indicate that a “no level real GDP equation” exists with a traditional significances level. The null hypothesis in turn indicates that there is a “no level effects”within the real GDP equation and that it can be rejected by the $F$-iv and $F$-v statistics with a 10% significance level. The $F$-statistics show that there are no long-run level relations among the variables at 1% significance level when the $ltt$ is taken as a dependent variable.

These bounds test results indirectly indicate that the real GDP and terms of trade can be accepted as “weakly exogenous”. There is a unique long-run relationship among the variables, implying that the ARDL estimates of the import functions are robust the way they are given in Table 2. These results also support the use of FM-OLS in order to estimate the elasticity’s.

**Table 2.** Long-term Elasticity’s of the Import Demand Function with ARDL

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>AIC</th>
<th></th>
<th>SIC</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>lrealimp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.147</td>
<td>2.192</td>
<td>2.203</td>
<td>2.420</td>
</tr>
<tr>
<td>With trend</td>
<td>(47.987)*</td>
<td>(7.226)*</td>
<td>(47.090)*</td>
<td>(7.074)*</td>
</tr>
<tr>
<td>lrealgdp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.539</td>
<td>-0.527</td>
<td>-0.473</td>
<td>-0.331</td>
</tr>
<tr>
<td>With trend</td>
<td>(-7.303)*</td>
<td>(-4.994)*</td>
<td>(-6.062)*</td>
<td>(-3.419)*</td>
</tr>
<tr>
<td>ltt</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With trend</td>
<td>(-16.085)*</td>
<td>(-3.483)*</td>
<td>(-16.916)*</td>
<td>(-3.989)*</td>
</tr>
</tbody>
</table>

Notes:

( ) denotes statistics and * indicate it is significant at the 1% level.

4 The selected ARDL models are (5,0,6) with constant and (5,0,6) with trend by AIC and pass the diagnostic tests of no serial correlation, heteroscedasticity and normality of residuals.

5The selected ARDL model are (5,0,3) with constant and (3,0,0) with trend by SIC and pass the diagnostic tests of no serial correlation, heteroscedasticity and normality of residuals.

3We also check the long-run levels relationship between the regressors (lrealgdp and ltt) of the import demand equation. The calculated $F$-statistics at the selected lags chosen by both AIC and SIC change between 0.562 and 4.038 when the lrealgdp is taken as dependent and change between 2.483 and 3.462 when the ltt variable is taken as dependent. These calculated values are smaller than the lower bound critical values for $k = 1$ and $T = 72$ and indicate that the null hypothesis of there is no levels relationship between the log of real GDP and log of terms of trade cannot be rejected at the conventional significance levels. These results also support to use the FM-OLS procedure.
The signs of the long-run income and relative price elasticity of demand for imports are as expected and statistically significant. The effect of an augmentation in relative prices or terms of trade is negative on imports, with long-term elasticity estimated between 0.331 and 0.539, in absolute value. Relative price elasticity on import estimated by using FM-OLS method is 0.213 without trend and 0.321 with trend. These values are smaller than that estimated values using ARDL. The long-term income elasticity is estimated to be between 2.147 and 2.420 by ARDL and 2.159 and 2.237 by FM-OLS. This indicates that FM-OLS also yields higher estimate of income elasticity of imports (see Table 3).

**Table 3. Long-term Elasticity’s of the Import Demand Function with FM-OLS**

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>With constant</th>
<th>With trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>( lrealimp )</td>
<td>2.159</td>
<td>2.237</td>
</tr>
<tr>
<td></td>
<td>(21.614)**</td>
<td>(21.404)**</td>
</tr>
<tr>
<td>( lrealgdp )</td>
<td>-0.321</td>
<td>-0.213</td>
</tr>
<tr>
<td></td>
<td>(-1.996)*</td>
<td>(-1.271)</td>
</tr>
<tr>
<td>( ltt )</td>
<td>-13.938</td>
<td>-15.340</td>
</tr>
<tr>
<td></td>
<td>(-7.562)**</td>
<td>(-7.952)**</td>
</tr>
<tr>
<td><strong>Stability Tests of Hansen (1992)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( SupF )</td>
<td>25.882</td>
<td>2.472*</td>
</tr>
<tr>
<td>( MeanF )</td>
<td>8.668</td>
<td>1.214*</td>
</tr>
<tr>
<td>( L_c )</td>
<td>0.086*</td>
<td>0.070*</td>
</tr>
</tbody>
</table>

*Notes:* (. ) denotes -statistics and ** and * indicate it is significant at the 1% and 5% levels. The Bartlett kernel is used for bandwidth to estimate the elements of covariance matrix. Critical values of \( SupF \) are 18.6, 14.8 and 13.0 for the model with constant; 21.4, 17.3 and 15.3 for the model with trend at the 1%, 5% and 10% levels, respectively. Critical values of \( MeanF \) are 8.50, 6.17 and 5.18 for the model with constant; 10.3, 7.69 and 6.58 for the model with trend at the 1%, 5% and 10% levels, respectively. Critical values of \( L_c \) are 1.03, 0.690 and 0.556 for the model with constant; 1.13, 0.778 and 0.625 for the model with trend at the 1%, 5% and 10% levels, respectively. *indicates that the null of the estimated long-run parameters are stable cannot be rejected at 1% level.

Table 3 also reports the parameter stability test results. \( SupF \), \( MeanF \), and \( L_c \) are the test statistics which are used to test the null result of long-run parameter stability. The \( SupF \) and \( MeanF \) tests require truncation of the sample size. We use the subset of samples within the range \([0.15T, 0.95T]\) with \( T \) denoting the sample size. As it was noted in Hansen
According to SupF and MeanF tests results, the long-run elasticity of import estimated without trend are not stable, while those estimated with the trend are found to be stable. The result from the $L_C$ tests however indicate that the long-run elasticity are relatively constant over time. This also confirms that import demand maintains a long-run levels relationship with the real GDP and terms of trade.

### 4.2 Testing the Thirlwall’s Law

The hypothetical income elasticity of import demand is defined as $\pi_a = \bar{x}/\bar{y}$ is 2.159 for the period 1990-2007. This hypothetical value is used to test hypothesis by imposing restriction on the level of the import demand equation. These tests are repeated for six different import demand equations and are reported in Tables 2 and 3. The results show that the restriction, $\pi_a = \bar{x}/\bar{y} = 2.159$, is valid at the conventional significance levels, implying that the U.S. growth has been constrained by the balance-of-payments.

Alonso (1999) proposed an alternative view for the second approach (which has been discussed above) covering the co-integration relations between the actual growth rates and the hypothetical growth rates. Britto and McCombie (2009) also followed this approach and claimed that a stable long-term relationship between the actual and the hypothetical growth rate is a test for Thirlwall’s Law. The authors did found a stable long-term relationship between the actual and the hypothetical growth rates series in the case of Spain and Brazil.

Following these studies, we test the long-run level relationships between the actual and the hypothetical growth rates with the bounds testing procedure. According to Pesaran,

---

We use the average of the estimated $\pi$’s in order to produce the hypothetical growth rate series defined in Equation (11) while estimating the natural rate of growth and testing its endogeneity. We also produce different hypothetical growth rates series using each one of them separately and observe the results are not sensitive to the selected value of income elasticity of imports demand.
Shin and Smith (2001), the bounds testing procedure can be used when (a) all variables are I(1) and co-integrated, (b) all variables are I(1) but not co-integrated, and (c) all are I(0). The actual and hypothetical growth rate series are found to be stationary according to unit root tests used in this study. The $F_{-iii}$, $F_{-iv}$ and $F_{-v}$ statistics at the selected lags ($p_c = p_t = 3$) are 6.344, 5.152 and 6.385 indicating that the existence of a significant co-integrating vector between the actual growth rates and the predicted growth rates.

<table>
<thead>
<tr>
<th>Table 4. VAR: Actual and Hypothetical Growth Rates and Hypothesis Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarterly Growth Rates-VAR(4)</strong></td>
</tr>
<tr>
<td>$g_a$</td>
</tr>
<tr>
<td>$g_b$</td>
</tr>
<tr>
<td>Constant</td>
</tr>
</tbody>
</table>

$g_a = 0.0011 + 1.0374g_b$

Restrictions
$g_a = -1$ and $g_b = 1$

$x^2 = 1.967$ with 0.161 p-value.

Note: VAR(4) model passes all diagnostic tests of no serial correlation, heteroscedasticity and normality of residuals.

Britto and McCombie (2009) mentioned that the VAR regressions provide an elegant illustration of the way in which the Kaldorian notion of long-run equilibrium growth rate differs from that of the neoclassical framework. Following the authors, a VAR model is estimated. The results obtained from VAR(4) model is given in Table 4. For the estimated vector, the constant is very close to zero and the coefficient of the predicted or hypothetical growth rate is close to one. According to the likelihood ratio test result, the coefficient of $g_b$ or slope is equal to unity. In summary, these results confirm the existence of the balance-of-payments restriction in the case of U.S. during the period 1990-2007. This result is consistent with the related empirical literature that used the U.S. data [see; McGregor and Swales (1991), Atesoglu (1993), McCombie (1997), Atesoglu (1997), Hieke (1997)].

---

$^5$Johansen and Juselius (1990) test at the selected lag ($k = 3$) also indicate that the existence of a significant cointegrating vector between actual growth rates and predicted growth rates.
4.3 Testing the endogeneity of the natural rate of growth using hypothetical growth rates of output

In order to estimate the natural rate of growth of the U.S. economy, Equations (1’) and (2’) as defined in the second section were estimated. Parameter estimates are presented in Table 5, where \( g \) and \( \Delta%U \) are defined in section 2.

As pointed out in Léon-Ledesma and Thirlwall (2002), the change in the percentage of unemployment should be regarded as an endogenous variable that will bias for the coefficient uses in the estimates of Equation (1’). Under this endogenous explanatory problem, instrumental variable (IV) estimation method is performed to analyze whether the values obtained for the intercept term (i.e., the natural rate of growth) are biased or not. The IV method produces a consistent estimator in a situation in which a regressor is contemporaneously correlated with the error term. The most difficult aspect of IV estimation is, in general, to find instruments that are both relevant and exogenous. Therefore, the lagged values of the variables are used as instruments in this study. Following Davidson and MacKinnon (1993), the endogeneity of the change in the percentage unemployment was tested using these instruments.

Adding the fitted value of \( \Delta%U_i \) to the Equation (1’) and obtaining the \( t \)-statistic of its coefficient can be considered as sufficient to do the test under the following conditions (a) the null hypothesis where “the variables in instrument variables set are exogenous” is satisfied, (b) that there is only one endogenous variable and (c) that sample size is small.

In order to apply the testing procedure, we tried numerous combinations of the lagged variables. The \( t \)-statistics of the coefficients of the fitted values belonging to the \( \Delta%U_i \)

\[\text{t as noted in Peter Kennedy (2003, p. 162), “it may be possible to use as an instrument the lagged value of the independent variable in question; it is usually correlated with the original independent variable, and although it is correlated with the disturbance vector, because it is lagged it is not contemporaneously correlated with the disturbance – assuming the disturbance is not autocorrelated”.

\[\text{7 A lagged value of the endogenous regressor may not be a good instrument. For this reason, more than one lagged values of both growth rate and the } \Delta%U_i \text{ variable are used together in the estimation process.} \]
variable obtained in most of these specifications are statistically insignificant, implying that
the null hypothesis can be accepted at 1% and 5% significance levels. These results indicate
that $\Delta \% U_t$ variable is not endogenous. However, the failure to reject the null hypothesis at a
specific probability of a Type I error does not prove exogeneity. For this reason, Equation (1')
is also estimated by using the IV method. The IV estimates of the natural growth rate, when
the hypothetical growth rates ($g_b$) are used, the change is between 0.0058 and 0.0064. Briefly,
when performing the IV method, the values obtained for the natural rate of growth are not
quite different from those obtained while using least squares. According to these results, the
bias can be ignored in the study.

Table 5. Estimation Results for the Natural Rate of Growth

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Natural Rate of Growth (I)</th>
<th>Natural Rate of Growth when $g_n &gt; g_b$ (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.00639 (6.162)**</td>
<td>0.0121 (13.798)**</td>
</tr>
<tr>
<td>$\Delta % U_t$</td>
<td>$-0.08715$ ($-2.175$)**</td>
<td>$-0.0549$ ($-2.402$)**</td>
</tr>
<tr>
<td>$D_t$</td>
<td>$-0.0116$ ($-9.531$)**</td>
<td>$(-9.531)**</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.151</td>
<td>0.603</td>
</tr>
<tr>
<td>$F$-stat.</td>
<td>12.303**</td>
<td>51.789**</td>
</tr>
<tr>
<td>$DW$</td>
<td>1.611</td>
<td>1.544</td>
</tr>
<tr>
<td>$F_{BG}$-stat. ($k=2$)</td>
<td>1.415 [0.249]</td>
<td>2.293 [0.109]</td>
</tr>
<tr>
<td>$F_{BG}$-stat. ($k=4$)</td>
<td>2.278 [0.070]</td>
<td>1.146 [0.343]</td>
</tr>
<tr>
<td>$F_{BG}$-stat. ($k=8$)</td>
<td>2.468 [0.023]</td>
<td>1.012 [0.437]</td>
</tr>
<tr>
<td>$F_{BPG}$-stat.</td>
<td>3.583 [0.063]</td>
<td>4.344 [0.017]</td>
</tr>
<tr>
<td>$F_{WHITE}$-stat.</td>
<td>1.774 [0.177]</td>
<td>2.315 [0.067]</td>
</tr>
<tr>
<td>$JB$-stat.</td>
<td>1.931 [0.380]</td>
<td>12.659 [0.001]</td>
</tr>
</tbody>
</table>

Notes: $D_t = \begin{cases} 1 & \text{the hypothetical rate of growth is smaller than the natural rate of growth} \\ 0 & \text{the hypothetical rate of growth is greater than the natural rate of growth} \end{cases}$

$t$-statistics are given in parenthesis and are calculated by using heteroscedasticity-and-autocorrelation consistent standard errors.

***, ** and * indicate that the test statistic is statistically significant at the 1%, 5% and 10% levels, respectively.

$F_{BG}$ is the Breusch-Godfrey LM test for autocorrelation. Maximum lag length is taken 8; however, the results are for lags 2, 4 and 8.

$F_{BPG}$ is the Breusch-Pagan-Godfrey LM test for heteroscedasticity.

$F_{WHITE}$ is the White test for heteroscedasticity and it includes cross-term.

$JB$-stat. is the Jarque-Bera normality test statistic.

$p$-values of $F_{BG}$, $F_{BPG}$, $F_{WHITE}$ and $JB$ are given in square parenthesis.
According to Breusch-Godfrey LM test statistics, residuals from Equation (1′) using the actual rate of growth have a higher order autocorrelation problem at significance level of 5 and 10%. While Breusch-Pagan-Godfrey (BPG) indicate that residuals have an heteroscedasticity problem at 10% significance level, White tests indicated that residuals are homoscedastic.

According to Breusch-Godfrey LM test statistics, residuals from Equation (2′) have no higher order autocorrelation problem at the conventional significance levels. While Breusch-Pagan-Godfrey (BPG) test results imply that residuals estimated using the hypothetical growth rate have an heteroscedasticity problem at the 10% level, White test indicate that residuals do not have heteroscedasticity problem. For this reason, standard errors for the coefficient estimates given in the columns are computed using heteroscedasticity-and-autocorrelation consistent standard errors. Besides this, estimator did provide correct estimates for the coefficient covariance’s in the presence of heteroscedasticity and autocorrelation. Jarque-Bera normality test results indicate that residuals of Equation (1′) are normal; however, residuals of Equation (2′) are abnormal.\(^8\)

The OLS estimates of the natural rate of growth with \(g_b\) given in the first column of Table 5 are statistically significant at 1%. The coefficient of the \(\Delta\%U\) variable is also estimated to be statistically significant. \(F\)-statistic indicates that the models are significant entirely at the 1%. According to the coefficient estimates, a 1% increase in the \(\Delta\%U\) results, on average, to about 0.0549% statistically significant decrease in the rate of growth of the real GDP as expected. The effect of\(\Delta\%U\) on the hypothetical rate of growth is found to be \(-0.0872\%\).

Estimation results employing \(g_b\) indicate that the natural rate of growth of the U.S. economy is 0.639 percent (see column I). Since \(g_b\) can also be above or below the natural rate

---

\(^8\) As noted in Kennedy (2003, p. 60), for situations in which the errors are not distributed normally, it turns out that in most cases a traditional test statistics has an asymptotic distribution equivalent to one of the tabulated distributions using in hypothesis testing.
of growth, a dummy variable \((D_t)\) is defined and added to Equation (2’). \(D_t\) takes the value of 1 when the \(g_b\) is lower than the natural growth rate. These estimates are given in the second column of Table 5. All the coefficient estimates are statistically significant. The coefficient of dummy variable \((D_t)\) is significantly positive, and the sum of the constant term and the coefficient of the dummy variable indicate that the natural rate of growth is lower than the natural rate of growth when \(g_n > g_b\) by approximately 92.18 percent. These empirical results therefore support the endogeneity hypothesis for the natural rate of growth for the U.S. economy.

5. Conclusion

In this study, we propose that in order to find evidence which supports the endogeneity of the natural rate of growth, the BPCRG should be used instead of the actual rate of growth. Furthermore, we explain that the labor force and productivity do not have to increase in order to indicate endogeneity since they might also decrease.

For testing the endogeneity, we first calculate the BPCRG and test the Thirlwall’s Law for the U.S. economy. Empirical results shown in this paper support the Thirlwall’s Law. Then, we test the endogeneity using the balance-of-payments consistent rate of growth. Empirical results support the endogeneity of the natural rate of growth. The estimated natural rate of growth is 0.639 percent and it decreases by 92.18 percent when \(g_n > g_b\) if the balance-of-payments consistent rate of growth data are used. This finding is remarkable and implies that the growth of the U.S. economy is very sensitive to the demand conditions.

Finally, in this study we emphasize and find evidence to the argument that the BPCRG should be used to test the endogeneity of the natural rate of growth. We can conclude that this approach is theoretically compatible with that proposed in Thirlwall (2001).

Bibliography


Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:


The Editor