

Report on  
“Testing for near  $I(2)$  trends when the signal to noise ratio is small”  
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This paper provides an analysis of possibly conflicting outcomes of univariate and multivariate unit root and cointegration tests. The emphasis is on processes whose first differences may be described by a (near) integrated drift plus noise, where the noise has a much larger variance than the drift (the signal). This makes it very hard for univariate tests to detect this signal; they will tend to reject a second unit root. On the other hand, the (near)  $I(2)$  behaviour is more easily recovered in a multivariate analysis, in particular when the near integrated drift in a differenced series is represented by another series (with small or zero noise), as illustrated in a Monte Carlo experiment. An empirical analysis of the US Dollar/D-mark exchange rate and US and German prices and bond rates illustrates the main point of the paper, and also provides a link with the “long swings” literature, as well as with Imperfect Knowledge Economics.

The paper is well written, and makes an important point about the usefulness of multivariate cointegration analysis relative to univariate unit root tests. The discussion below points out some alternative mechanisms that may explain the findings of the paper. After that, some additional comments and suggestions are given, intended to help with improving the presentation of the results.

## Discussion

Unit root tests in autoregressive models have a unit root as the null hypothesis, which may be rejected against the alternative of a stable root. The same applies to cointegration (trace) tests in vector autoregressive models: the number of unit roots is always higher under the null hypothesis than under the alternative. This appears to be at odds with the explanation offered on p. 2, on p. 10, and in the first paragraph of Section 2.5, p. 11. Dickey-Fuller type tests cannot “have low power to detect a second unit root”, they can only have low power to *reject* a second unit root, against the alternative of a single unit root. The paper makes the argument that Dickey-Fuller tests *reject too often*; this can never be a problem of power. In other words, although in general we may expect higher power of multivariate tests than of univariate test, this power difference cannot explain the conflicting outcomes that are the subject of this paper. Instead, the over-rejection of the Dickey-Fuller test suggests a *size* problem, which is discussed next.

The model (6)–(7) is directly recognised as a state space model, in which (7) is the transition equation for the state variable  $\mu_t$ , and (6) is the measurement equation for  $\Delta q_t$ . Such models, and their statistical analysis based on the Kalman Filter, are discussed extensively in Harvey (1989). He shows that such AR(1)/random walk plus noise models imply an ARMA(1,1) model for  $\Delta q_t$ :

$$\Delta q_t = \bar{\rho}\Delta q_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1},$$

where  $\theta = 0$  if  $\text{var}(\varepsilon_{qt}) = 0$ , and  $\theta$  decreases monotonically to  $-\bar{\rho}$  as the signal-to-noise ratio  $\text{var}(\varepsilon_{\mu t})/\text{var}(\varepsilon_{qt})$  decreases to zero. In particular, if  $\text{var}(\varepsilon_{\mu t})/\text{var}(\varepsilon_{qt}) = 0.15^2 = 0.0225$  as

in the Monte Carlo experiment in Section 3, then  $\theta = -0.861$  if  $\bar{\rho} = 1$ , and  $\theta = -0.850$  if  $\bar{\rho} = 0.95$ . The effect of moving average components on unit root tests in autoregressions has received considerable attention since Schwert (1989). In particular, large negative values of  $\theta$  as found above imply that tests based on an autoregressive approximation of the ARMA model tend to (strongly) over-reject the null hypothesis. This provides a direct explanation of the results in Table 5, top panel (the bottom panel refers to a local alternative to the  $I(2)$  null hypothesis).

The question therefore arises if multivariate tests are less prone to such size distortions than univariate tests. This is clearly so in Case 1 on p. 14, because this corresponds to an exact VAR(2) for  $x_t^{(1)} = (x_{1,t}, x_{2,t}, x_{3,t})'$ . Therefore, the columns labelled  $S_1$  in Table 4 refer to the outcome of likelihood ratio tests in a well-specified VAR model, which do not suffer from the size distortions of the Dickey-Fuller tests. In contrast, in Cases 2 and 3 the state variable  $\mu_t$  is observed with noise, which implies that the vector processes  $x_t^{(2)} = (x_{1,t}, x_{2,t}, x_{4,t})'$  and  $x_t^{(3)} = (x_{1,t}, x_{2,t}, x_{5,t})'$  follow a VARMA(2,1) model; the moving average effects are most pronounced in Case 3, with the largest noise. As a result, multivariate cointegration tests based on a vector autoregressive approximation will now suffer from similar size distortions as the Dickey-Fuller test. For example, the last column of Table 4, top panel, implies that, even with a sample size as large as 500, the correct null hypothesis  $H(1, 1)$  is rejected in 55% of the Monte Carlo replications.

These points suggest that the main conclusion of the paper (“multivariate tests find near  $I(2)$  trends where univariate tests do not”) holds only under the condition that a finite-order VAR model provides an accurate approximation. There is no guarantee that this condition is satisfied in practice. However, we do know that finite-order VAR models in general imply ARMA models for the individual time series, which suggests that size distortions implied by moving average dynamics are more likely to occur in univariate than in multivariate models. Furthermore, residual autocorrelation in low-order VAR models is often taken as an indication that important variables are missing (rather than an indication that the lag length should be increased), and if the search for such variables leads to an extended model in which the state variables are measured without error, then this will indeed lead to more accurate unit root inference.

## References

- Harvey, A. C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge: Cambridge University Press.
- Schwert, G. W. (1989), “Testing for unit roots: A Monte Carlo investigation,” *Journal of Business & Economic Statistics*, 7, 147–159.