Conflict in the Wage-Led Growth Model

Romar Correa

Abstract
We model the interaction between capitalists and entrepreneurs as a dynamic game. The open-loop Nash equilibrium and the closed-loop Nash equilibrium are distinguished. The elasticity of intertemporal substitution as well as the level and responsiveness of the wage rate to the accumulation of capital are shown to be important for wage-led growth.

JEL  B51 C73
Keywords  capital accumulation; conflict: cooperation

Authors
Romar Correa, Department of Economics, University of Mumbai, Mumbai, romarcorrea10@gmail.com

1 Motivation

Some conundrums have arisen in the development of profit-led versus wage-led growth models. On the one hand, there are issues concerning the characterization of contemporary capitalism along these axes. For instance, the description of contemporary western economies as profit-led means that a fall in wages and a corresponding increase in the share of profits entails sufficient investment demand to compensate for the fall in consumption demand. However, in one appraisal, falling wage shares has not gone with high rates of growth of investment (Kapeller & Schütz, 2012). Furthermore, the increasing share of profits in national income is accompanied by growing consumption growth. Closer investigation, then, is required of the propensity to consume. Theoretically, the tension between wage-led and profit-led models of growth arises because of the combinations of coefficients in linear specifications of the Cambridge growth model. Thus, the net effect of the fall in the wage share on aggregate demand depends upon the magnitude of the elasticities of response to consumption and investment demand (Stockhammer & Onaran, 2012). The dynamics of our economy below is determined by the accumulation of capital in the classical tradition. However, Post Keynesians and others have gone beyond Marx’s proclamation that accumulation is the law of Moses and the prophets. The agenda was given by Kalecki: “the determination of investment decisions remains the central pièce de resistance of economics”. Scholars have been regularly experimenting with various reduced-form behavioral equations in the quest to explain investment.

We revert, on the other hand, to ‘first principles’, for the following additional reasons. One aspect of the discussions has revolved around operating with historical or with forward-looking definitions. Put differently, the choice is between closed-loop variables, those that depend on the state of the world or open-loop variables, concepts that are information-invariant. As an illustration of the latter, political economists have borrowed notions like the ‘rational attention’ of consumers (Cynamon & Fazzari, 2013). Expectations, according to this view, are sticky. Information is not updated because it is costly to do so. Post Keynesians often employ life-cycle models of consumption behavior. The individual’s consumption function is assumed to be open-loop. Behavioral theories, on the other hand, are closed-loop. Agents are believed to use information as it becomes available. Secondly, many believe that the typology of classes in the modern capitalist economy has to be reframed (Dünhaupt, 2013; Kim, Setterfield & Mei, 2013). For instance, workers have been divided into those on the shop floor and supervisors and managers. Workers who save derive some of their income from property. They own a stock of wealth which attracts profits, interest or rental income.

The conflict in the title of the paper is between capitalists who own the capital in the economy and entrepreneurs who represent capitalists in hiring workers, organizing production, and selling the output. At the same time, in light of the earlier remarks, the capitalist will not seem different from the representative agent of the textbooks. Only the potential employers are clearly separated. After meeting the wage bill, entrepreneurs return the residual revenue to the capitalists (the definitions and notations in the next section are from Foley & Michl, 1999). The two classes play a repeated game (the concepts and notations below are from Van Long, 2012). The open-loop Nash equilibrium (OLNE) is one in which the strategies of players do not depend on the state
of the game. A Markov-perfect Nash equilibrium (MPNE), in contrast, is one in which the strategies of the players depend on the information set available at every stage of the dynamic game. We set up and resolve the class conflict below. The final section is a summary.

2 The model and results

2.1 Preliminaries

The notations are as follows: real gross product is \( X \) and the number of employed workers is \( N \), \( x = X/N \) is a measure of labor productivity and \( k = K/N \) is a measure of capital intensity, \( \rho = x/k \) is the capital-output ratio. \( X = W + Z \), the sum of wages and profits. The average real wage is \( w = W/N \) and the rate of profit is \( v = Z/K \). Output is also the sum of consumption, \( C \), and gross investment \( I \). Consumption per worker is \( c = C/N \).

Recalling the definition of investment as the accumulation of capital, \( \dot{K} = I \), it is straightforward to combine the definitions to get the following differential equation:

\[
\dot{k}(t) = \nu(t)k(t) + w(t) - c(t)
\]

The problem of the capitalist is to choose a consumption stream \( c(t) \) to maximize

\[
\int_{0}^{\infty} e^{-\rho t} U(c(t)) \, dt
\]

(The discount factor is not to be confused with the capital-output ratio which was only used for the derivation of the dynamic equation). The entrepreneur chooses a wage stream \( w(t) \) to maximize her profits

\[
\int_{0}^{\infty} e^{-\rho t} \pi(w(t)) \, dt
\]

(A common degree of (im)patience is used for convenience). We work out the equilibria of the game. We draw attention to the connection between \( w \) and \( v \) as an interpretation of profit-led and/or wage-led growth.

2.2 The OLNE

The problem of the capitalist, given that the entrepreneur is playing an open-loop strategy, \( w^{OL}(t) \), is solved by the following Hamiltonian where \( \psi \) is the costate variable.

\[
H = U(c(t)) + \psi(\nu(t)k(t) + w^{OL}(t) - c(t))
\]

The first-order conditions are
\[ \frac{\partial \mathcal{H}}{\partial c(t)} = U'(c(t)) - \psi(t) = 0 \]  

1

\[-\dot{\psi}(t) + \rho \psi(t) = \frac{\partial \mathcal{H}}{\partial k(t)} = \psi(t) \nu(t) \]  

2

\[ \dot{k} = \frac{\partial \mathcal{H}}{\partial \psi(t)} = \nu(t)k(t) + w^{OL}(t) - c(t) \]  

3

Denote \(-\frac{u'''(c(t))c(t)}{u'(c(t))}\), the elasticity of intertemporal substitution of consumption, by \(\phi(t)\).

Combining equations 1 and 2, we get,

\[ c(t) = \frac{\phi(t)}{[\nu(t)-\rho]} \]  

4

Plugging 4 into 3, the steady state is given by

\[ \nu(t)k(t) + w^{OL}(t) = \frac{\phi(t)}{[\nu(t)-\rho]} \]  

5

In like manner, we work out the state-invariant strategy of the entrepreneur, given the open-loop consumption plan, \(c^{OL}(t)\), of the capitalist. Since the derivative of the profit function with respect to the wage rate is undefined, we assume that the corresponding costate variable is too. In that case, equation 2 vanishes from the entrepreneur solution set leaving just equation 3, whose steady-state properties have been included in our derivation of equation 5.

Our interest lies in a gently-sloping utility function which can deliver an arbitrarily ‘high’ value of the intertemporal elasticity of substitution and, consequently, a ‘high’ value of the left-hand side of 5. Wages and profits, in this sense, are not in conflict. At the same time, a ‘high’ value of the right-hand side can be delivered by a ‘low’ value of the denominator on the right-hand side. For instance, a low value of the discount rate can go with a low rate of profit. Player patience, the folk theorem, works against this outcome. In case the discount rate is greater than the rate of profit, a potential conflict between wages and profits will arise. All we have clearly is that

**Proposition 1:** Under open-loop strategies, wage-led growth is promoted by a high elasticity of intertemporal substitution.
2.3 The MPNE

We consider, now, strategies that are state-dependent. That is, the capitalist maximizes her utility function given that the entrepreneur is playing \( w^{FB}(k(t)) \) (FB stands for feedback).

The problem of the capitalist, in this case, is to choose a consumption stream \( c(t) \) to maximize

\[
\int_0^\infty e^{-\rho t} U(c(t)) dt
\]

subject to

\[
\dot{k}(t) = \nu(t)k(t) + w^{FB}(k(t)) - c(t)
\]

Her Hamiltonian is

\[
\mathcal{H} = U(c(t)) + \psi(\nu(t)k(t) + w^{FB}(k(t)) - c(t))
\]

The first-order conditions are

\[
\frac{\partial \mathcal{H}}{\partial c(t)} = U'(c(t)) - \psi(t) = 0 \tag{6}
\]

\[
-\dot{\psi}(t) + \rho \psi(t) = \frac{\partial \mathcal{H}}{\partial k(t)} = \psi(t)\nu(t) + \psi(t) \left( \frac{dw^{FB}}{dk(t)} \right) \tag{7}
\]

\[
\dot{k} = \frac{\partial \mathcal{H}}{\partial \psi(t)} = \nu(t)k(t) + w^{FB}(k(t)) - c(t) \tag{8}
\]

The difference between the earlier condition, equation 2, and the present equation 7 is the additional term on the right-hand side of the latter. We need to characterize the derivative. Its sign is given by the following

**Lemma:** \( \frac{dw^{FB}}{dk} \geq 0. \)

**Proof.** We know from the implicit function theorem that

\[
\frac{dw^{FB}}{dk} = - \frac{\partial \pi}{\partial k} \frac{\partial \pi}{\partial w^{FB}} \]

The problem is to derive the characteristics of the profit function on the right-hand side. We consider \( w' \geq w, k' \geq k \). The profit function is convex in input prices. In that case,
\[ 0 \geq \pi(w', k) - \pi(w, k) \geq \frac{\partial \pi(w, k)}{\partial w} (w' - w) \]

Assume that the entrepreneur is constrained in her level of capital to \( k \). Then a relaxation of the constraint to \( k' \) increases profits. In addition, the profit function of the ‘constrained’ producer is strictly concave in the level of capital. That is to say,

\[ 0 \leq \pi(w, k') - \pi(w, k) < \frac{\partial \pi(w, k)}{\partial k} (k' - k) \]

Putting \( b \) and \( c \) into \( a \), we get the result we seek. Hereafter, we denote the derivative by \( y(t) \). We proceed to the optimization problem of the entrepreneur. Recalling our remarks of the previous section, since we cannot characterize the costate function, only equation 8 is to be considered. Equation 4 is paralleled by

\[
c(t) = \frac{\varphi(t)}{[v(t) + y(t) - \rho]} \tag{9}
\]

Finally, the steady state parallel of 5 is

\[
v(t)k(t) + w^{OL}(t) = \frac{\varphi(t)}{[v(t) + y(t) - \rho]} \tag{10}
\]

Now, a fall in the rate of profit can be matched by a high discount rate as long as both the level and growth of wages (with respect to the capital stock) is ‘sufficiently high’ (Lavoie & Stockhammer, 2012). Proposition 1 continues to hold but we make the following addition.

**Proposition 2:** Under feedback rules, wage-led growth is promoted by both the level of the capital stock as well as degree of responsiveness of the wage rate to the accumulation of capital.

### 3 Conclusions

We develop a model of class conflict deriving from basic macroeconomic identities. In one configuration of equilibrium the protagonists are committed to antagonism in a ‘pure’ sense, that is independent of the state of the world. We show that a sufficiently high elasticity of intertemporal substitution is sufficient for a wage-led steady state of the system. In a situation of conflict when the parties monitor the state vector period by period, on the other hand, if both wages as well the response of wages to changes in the state of the world (summarized by the level of the capital stock) is sufficiently high, the economy is wage-led. Since workers might be capitalists, the politico-economic message is clear. Workers are better off aligning themselves with the production of goods and services rather than earning rents.
References


Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:

http://www.economics-ejournal.org/economics/discussionpapers/2014-41

The Editor