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Discounting, beyond Utilitarianism

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Abstract
Discounted utilitarianism and the Ramsey equation prevail in the debate on the discount rate on consumption. The utility discount rate is assumed to be constant and to reflect either the uncertainty about the existence of future generations or a pure preference for the present. The authors question the unique status of discounted utilitarianism and discuss the implications of alternative criteria addressing the key issues of equity in risky situations and variable population. To do so, they introduce a class of intertemporal social objectives, named Expected Prioritarian Equally Distributed Equivalent (EPEDE) criteria. The class is more flexible than Discounted utilitarianism in terms of population ethics and it disentangles risk aversion and inequality aversion. The authors show that these social objectives imply interesting modifications of the Ramsey formula, and shed new light on Weitzman's "dismal theorem".

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1 Introduction

The current debate on the economics of climate change has focused primarily on the choice of the social discount rate, which is crucial because of the long term impacts of most greenhouse gases. The starting point of most analyses, despite their differences, remains the seminal work by Ramsey (1928), who showed in a standard discounted utilitarian model that the optimal consumption discount rate is given by the following equation, known as the Ramsey formula:

$$r = \delta + \eta g, \tag{1}$$

where r is the consumption discount rate, δ the utility discount rate, η the elasticity of the utility of consumption, and g the growth rate of consumption.

Most of the debate following the publication of the Stern review (Stern, 2006) has been about the appropriate value of δ. Some authors (Schelling, 1995; Stern, 2006) endorsed an 'ethical' approach supporting a low value of δ in line with the probability of human extinction, while others (Nordhaus, 2007, 2008; Weitzman, 2007) called for larger values based on the preferences revealed on financial markets. Several papers (Atkinson et al., 2009; Dasgupta, 2008; Anthoff, Tol and Yohe, 2009) have also stressed the key role of η in the analysis. It has been interpreted in at least three different ways: in terms of risk aversion; in terms of inequality aversion; in terms of the intertemporal elasticity of substitution. Unfortunately, the workhorse model of welfare economics behind the Ramsey formula, namely, the discounted expected utility model, is unable to distinguish between these three different notions.

Another difficulty behind the Ramsey formula, in addition to the ambiguous meaning of its key components, is that it leaves aside a key issue of the climate change problem, namely population size. There is a significant literature on population ethics discussing how to compare populations of different sizes (classical references in that field include Broome, 1991, 2004; Ng, 1989; Blackorby, Bossert and Donaldson, 2005). However, the literature on climate change has hardly taken stock of these contributions and to the best of our knowledge the impact of (uncertain) population size on social discounting has hardly been studied.[1] One reason is that population size does not affect the social discount rate if one

[1] A recent exception is Millner (2013), but the paper only considers variants of the utilitarian criterion.

remains within the limited scope of discounted utilitarianism.

The aim of this paper is to address these limitations of the Ramsey formula in a framework involving risk on future welfare and population size. We introduce a specific class of welfare models, the Expected Prioritarian Equally Distributed Equivalent (EPEDE) criteria, which distinguishes risk aversion and inequality aversion, and is explicit about how populations of different sizes are compared. The model admits the discounted utilitarian model as a special case. It is also explicit about the meaning of the different parameters, so that they can be precisely discussed. Doing so, we endorse a normative approach. We believe that this approach is appropriate for the problem of climate change which involves future generations whose interests cannot be taken into account by market mechanisms. At the very least, we think the approach is worth being pursued.

From the social welfare model, we can derive a social discount rate. We show that it involves three elements in addition to the standards components of the Ramsey formula: a term related to population size; a term related to the relative priority of the welfare of generations concerned by the investment; a covariance term. The term related to population size is likely to increase the social discount rate. This is so because the welfare of individuals is weighted by the population size so that they receive less weight when living in larger populations; given that population size refers to the overall number of people in all existing generations, future generations live in more populated societies on average so that their welfare has less social importance. We provide an example showing that the population term can be substantial. The effect of relative priority and covariance is less clear; we however indicate that they may reverse the conclusions of Weitzman's dismal theorem (Weitzman, 2009).

The present paper relates two distinct lines of research. The first one tries to incorporate equity considerations in frameworks involving risks. As discussed above, the standard utilitarian approach cannot distinguish risk aversion and inequality aversion. Actually, the utilitarian criterion has been advocated in a seminal paper by Harsanyi (1955), who argued that, in the presence of risk, the concavity of the utility function should represent the population's attitude about risk-taking. Harsanyi's result has been criticized for being unable to take into account equity considerations both ex ante and ex post (Diamond, 1967; Broome, 2004; Ben Porath, Gilboa and Schmeidler, 1997; Gajdos and Maurin, 2004). In the present paper, we adopt an ex post approach that makes it possible for the concavity of the utility function to depend on equity considerations (Fleurbaey, 2010; Grant, Kajii, Polak and Safra, 2012), while remaining in the realm of the expected utility paradigm to ensure social rationality, e.g., statewise dominance and time consistency.[2]

The second line of research on which the paper builds bears on the evaluation of policies involving a variable population. Classical references in this field are Broome (1991), Broome (2004), Ng (1989) and Blackorby, Bossert and Donaldson (2005). We particularly build on Blackorby, Bossert and Donaldson (2007), which addresses both issues of variable population and uncertainty. This paper is formally the closest to our work and inspires our approach to the variable population problem, but they introduce strong Pareto and separability principles which impose the additive structure of utilitarianism, whereas we consider more general possibilities. Bommier and Zuber (2008) address a similar question, but they focus on the risk on population size. They rely on a weaker Pareto principle than Blackorby, Bossert and Donaldson (2007) but even this version may not be satisfactory according to arguments in Fleurbaey (2010).

The paper is organized as follows. Section 2 introduces a family of social objectives that generalize the utilitarian criterion, whose limitations are discussed in more details. This family we propose makes it possible to introduce equity considerations while leaving some role for individual judgements in the evaluation of aggregate risks. Section 3 discusses properties of this family in terms of the evaluation of situations involving populations of different sizes. Section 4 derives the implications of this family for the social discount rate and shows that the Ramsey formula (1) needs to be supplemented with additional terms involving population size, the global social welfare and the correlation between the different components of the social discount rate. Section 5 derives the implications of these new criteria for the question of catastrophic risks which has been studied in the utilitarian context by Weitzman (2009). Section 6 concludes. An Appendix contains the proofs of the results of Section 3.

[2] An ex ante approach has been suggested by Diamond (1967) and investigated by Epstein and Segal (1992) but, as the latter contribution acknowledges, the ex ante approach may imply time consistency problems.

2 Intertemporal social evaluation: the limits of classical utilitarianism

2.1 The framework

We let $\mathbb{N}$ denote the set of positive integers, $\mathfrak{N}$ the set of non-empty finite subsets of $\mathbb{N}$, $\mathbb{R}$ the set of real numbers, and $\mathbb{R}_+$ the set of positive real numbers. For a set D and any $n \in \mathbb{N}$, D^n is the n-fold Cartesian product of D. Also, for two sets D and E, D^E denotes the set of mappings from E into D.

Our framework is adapted from Blackorby, Bossert and Donaldson (2007). The set of *potential* individuals (who may or may not exist) is $\mathbb{N}$. In the definition of a person, we include all her relevant characteristics and in particular the generation she belongs to. Hence there exists a mapping $T : \mathbb{N} \to \mathbb{N}$ that associates to each individual i the period she will exist provided she comes to life, $T(i)$. Individuals live for one period only, so that we call this period a generation.

In contrast with Blackorby, Bossert and Donaldson (2007), we work directly with utility numbers. Hence an alternative u is a collection of utility numbers, one for each individual alive in the alternative. Let X be an interval in $\mathbb{R}$, which corresponds to possible utility levels. We assume that $0 \in X$, where $x = 0$ denotes the *neutral* utility level such that individuals are indifferent between having this utility level and not existing. We denote $\underline{x} = \inf X$ and $\bar{x} = \sup X$ the minimal and maximal utility levels.

We let $\mathcal{U} = \bigcup_{\mathcal{N} \in \mathfrak{N}} X^{\mathcal{N}}$ denote the set of possible alternatives when at least one individual exists. An alternative then is a function assigning a utility number to each individual living in that particular alternative, where $\mathcal{N}$ is the subset of individuals living in that particular alternative. Note that we restrict attention to situations in which the population is always finite. In a variable-population framework, the size of the population may vary from one alternative to another. It is important to keep track of the population in an alternative. for any $u \in \mathcal{U}$, we let $\mathcal{N}(u)$ be the set of individuals in the alternative and $n(u) = |\mathcal{N}(u)|$ be the number of individuals in the alternative.

We consider situations where the final distribution of utilities as well as the set of individuals who will eventually exist may be uncertain. Hence we let $\mathcal{S}$ be the countable set of states of the world, with typical element $s \in \mathcal{S}$. A prospect $\mathbf{u}$ is a mapping $\mathbf{u} \in \mathbf{U} := \mathcal{U}^S$. For $s \in \mathcal{S}$, $\mathbf{u}(s)$ is therefore the alternative (the distribution of utility) induced by the prospect $\mathbf{u}$ in state s. A probability measure $\mathbf{p}$ is a mapping $\mathbf{p} \in \mathbf{P} := [0,1]^S$ such that $\sum_{s\in\mathcal{S}} \mathbf{p}(s) = 1$. For any utility function $G : \mathcal{U} \to \mathbb{R}$, any $\mathbf{u} \in \mathbf{U}$ and any $\mathbf{p} \in \mathbf{P}$, we denote $\mathbb{E}_{\mathbf{p}}\big(G(\mathbf{u})\big) = \sum_{s\in\mathcal{S}} \mathbf{p}(s)G\big(\mathbf{u}(s)\big)$. More generally, for any random variable K (that is any function $F : \mathcal{S} \to \mathbb{R}$), we denote $\mathbb{E}_{\mathbf{p}}\big(K\big) = \sum_{s\in\mathcal{S}} \mathbf{p}(s)K(s)$.

For an alternative $u \in \mathcal{U}$, whenever $i \in \mathcal{N}(u)$, $u^i \in X$ denotes the utility of individual i. For any $x \in X$ and $\mathcal{N} \in \mathfrak{N}$, we denote $x \cdot \mathbb{1}_{\mathcal{N}}$ the alternative $v \in \mathcal{U}$ such that $\mathcal{N}(v) = \mathcal{N}$ and $v^i = x$ for all $i \in \mathcal{N}$. For a prospect $\mathbf{u} \in \mathbf{U}$, $\mathbf{u}^i(s)$ denotes the utility of individual i in state of the world $s \in \mathcal{S}$ whenever $i \in \mathcal{N}\big(\mathbf{u}(s)\big)$. For a subpopulation $\mathcal{N} \in \mathfrak{N}$, we denote by $\mathbf{U}_{\mathcal{N}}$ the set of prospects such that, for every $\mathbf{u} \in \mathbf{U}_{\mathcal{N}}$, and every $s \in \mathcal{S}$, $\mathcal{N}\big(\mathbf{u}(s)\big) = \mathcal{N}$. These are the prospects such that the same individuals are present in all states of the world. In this case, we denote $\mathbf{u}^i$ the mapping $\mathbf{u}^i \in X^{\mathcal{S}}$ assigning in each state of the world to individual i her utility level induced by the social prospect $\mathbf{u}$. The mapping $\mathbf{u}^i$ represents the prospects of individual i.

Without loss of generality, for any $u \in \mathcal{U}$, we denote by u the prospect $\mathbf{u} \in \mathbf{U}$ such that $\mathbf{u}(s) = u$ for all $s \in \mathcal{S}$. These are sure prospects yielding the same alternative in all states of the world (they can be differentiated from uncertain prospects by the fact that they are not in bold font). Similarly, we denote $\mathcal{U} \subset \mathbf{U}$ the set of sure prospects.

A lottery is the combination of a probability measure $\mathbf{p} \in \mathbf{P}$ and a prospect $\mathbf{u} \in \mathbf{U}$. The set of lotteries is therefore $\mathbf{P} \times \mathbf{U}$. A social ordering is a transitive and complete binary relation R on $\mathbf{P} \times \mathbf{U}$. The notation $(\mathbf{p}, \mathbf{u})R(\mathbf{q}, \mathbf{v})$ will mean that $(\mathbf{p}, \mathbf{u})$ is at least as good as $(\mathbf{q}, \mathbf{v})$. We let P and I denote the corresponding strict preference and indifference relations.

2.2 Limitations of Classical Utilitarian criteria

Most of the existing literature on social discounting endorses the same basic welfare model, namely the Expected Discounted Utilitarian (EDU) criterion

$$\sum_{t=0}^{\infty} e^{-\delta t} n^t \mathbb{E}_{\mathbf{p}}\big(\mathbf{u}^t\big), \tag{2}$$

where n^t is the number of people in generation t (which is supposed to be known) and $\mathbf{u}^t(s)$ is, to simplify, the consumption of the representative agent of generation t in state s.

A first issue with this criterion is utility discounting, i.e. the fact that a decreasing weight $e^{-\delta t}$ is put on the expected utility of future generations. Many economists in the utilitarian tradition have denounced this feature of the discounted utilitarian criterion because it deviates from the ideal of equal treatment of all individuals. For instance, Frank Ramsey famously described discounting as a "practice which is ethically indefensible and arises merely from the weakness of the imagination" (Ramsey, 1928, p. 543). The Stern review (Stern, 2006) also emphasized this ethical flaw of the standard approach. Drawing on these criticisms, a prolific literature has studied whether it would be possible to combine an equal treatment of all generations with the Pareto principle in the context of infinite consumption streams. Although some positive results have been obtained, most of this literature stemming from Diamond (1965) has reached negative conclusions (Basu and Mitra, 2003; Zame, 2007; Lauwers, 2010). One way out of this dilemma is of course to consider a variable population framework and a risk on the population size, as proposed by Dasgupta and Heal (1979), Bommier and Zuber (2008) and Roemer (2011). This is the route that we follow in the present paper. We always assume that total (intergenerational) population is finite, so that all individuals in all generations can be treated in the same way, while the Pareto principle holds.

A second issue with EDU is the confusion between risk aversion and inequality aversion is formula 2. Society has to use individuals' risk preferences when making judgments about the distribution of goods between generations. This issue is well-known in the literature on social choice under uncertainty following Harsanyi's result (Harsanyi, 1955). One way to disentangle equity preferences and risk preferences while using an expected utility to assess social prospects is to endorse an ex post approach where equity considerations apply to final outcomes (Fleurbaey, 2010; Grant, Kajii, Polak and Safra, 2012). Another possibility to disentangle risk and equity preferences consists in using an alternative framework of dynamic choice proposed by Kreps and Porteus (1978), allowing a sequential resolution of uncertainty, as in the Epstein-Zin model of choice (Epstein and Zin, 1989). Several papers have investigated the consequences of such models for social discounting (Gollier, 2002; Traeger, 2014). One problem is that the Epstein-Zin model allows for non-monotonic preferences, which can lead to unfortunate conclusions (Chew and Epstein, 1990; Bommier and LeGrand, 2013). The expected utility model has normative appeal in terms of dynamic consistency. We see no reason (from a normative point of view) to abandon it if we can avoid doing so. We therefore stick to expected utility in this paper.

A second set of issues concerning the EDU criterion displayed in Eq. (2) relates to how it accounts for population change. It is almost never explicitly stated what assumptions are made in this respect. However, following a venerable tradition (Dasgupta and Heal, 1979; Stern, 2006), one can interpret the utility discount rate δ in Eq. (2) as accounting for the risk on the existence of future generations. Indeed, if generation t exist with probability $e^{-\delta t}$ (or equivalently if exactly T generations exist with probability $(1-e^{-\delta})e^{-\delta T}$), and the risk on population size is independent of the risk on individuals' utilities, Eq. (2) can actually be rewritten $\mathbb{E}_{\mathbf{p}}\left(\sum_{i\in\mathcal{N}(\mathbf{u})}\mathbf{u}\right)$.

The social criterion

$$V(u)=\sum_{i\in\mathcal{N}(u)}u^i,\qquad \forall u\in\mathcal{N}(u),$$

is known as the Total Utilitarian Criterion. The criterion has been criticized for yielding the Repugnant Conclusion (Parfit, 1984): for every population of significant size where all individuals enjoy excellent lives, there is a (larger) population with lives barely worth living that is better according to Total Utilitarianism, provided that the latter includes sufficiently many people.

To address this issue a key new concept was introduced, namely, the notion of critical level. It is defined as follows. Imagine adding a single individual to a population, holding the utility of all other members of the population constant. The critical level is the utility level of the additional individual which leaves total welfare unchanged, i.e. her existence is socially neutral if her utility is at the critical level. Several authors (Broome, 2004; Blackorby, Bossert and Donaldson, 2005) have argued that the critical level should be a positive utility level, the same whatever the population we start with. And they have proposed a social criterion named the *Critical-Level Utilitarian* criterion:

$$V(u)=\sum_{i\in\mathcal{N}(u)}\left(u^i-c\right),\qquad \forall u\in\mathcal{N}(u),$$

where c is the critical level. With a positive $\bar{u}$, the Repugnant Conclusion is avoided.

The Critical-Level Utilitarian criterion, however, has problems of its own. For instance it implies the Very Sadistic Conclusion (Arrhenius, 2014): for every population where all individuals have terrible lives not worth living, there is a population where all individuals enjoy lives worth living (but below the critical level) that is worse according to the Critical-

Level Utilitarian criterion, provided that the latter includes sufficiently many people.

In order to avoid these issues, one may look at context sensitive theory, in the sense that the critical level depends on the wellbeing of the existing population (Ng, 1989). This is what we propose in the present paper by considering Expected Prioritarian Equally Distributed Equivalent (EPEDE) social orderings. They are defined as follows.

Definition 1 (EPEDE social ordering) *A social ordering R on $\mathbf{P} \times \mathbf{U}$ is an* Expected Prioritarian Equally Distributed Equivalent (EPEDE) social ordering *if there exist real numbers $(\alpha_n, \beta_n)_{n\in\mathbb{N}} \in (\mathbb{R}_+ \times \mathbb{R})^{\mathbb{N}}$ and a concave continuous function ϕ such that, for all $(\mathbf{p}, \mathbf{u}), (\mathbf{q}, \mathbf{v}) \in \mathbf{P} \times \mathbf{U}$*

$$(\mathbf{p}, \mathbf{u})R(\mathbf{q}, \mathbf{v}) \Longleftrightarrow \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big)$$

with $V(u) = \alpha_{n(u)}E(u) + \beta_{n(u)}, \forall u \in \mathcal{U}$, where

$$E(u) = \phi^{-1}\left(\frac{1}{n(u)}\sum_{i\in\mathcal{N}(u)} \phi(u^i)\right). \tag{3}$$

The EPEDE criteria involve taking the expected value of an affine transformation of the equally-distributed equivalent (EDE) welfare level defined by Eq. (3). These criteria are therefore related to expected EDE criteria proposed by Fleurbaey (2010). Compared to the initial proposition in Fleurbaey (2010), the EDE function has a specific, namely additively separable, form. In fixed population frameworks, the criterion

$$V(u) = \sum_{i\in\mathcal{N}(u)} \phi(u^i), \qquad \forall u \in \mathcal{N}(u),$$

is known as the Prioritarian criterion, and we retain this label. Compared to Utilitarianism, Prioritarianism adds the idea that Pigou-Dalton transfers of utility are socially valuable, hence introducing inequality aversion.

The EPEDE criteria are hence able to disentangle inequality aversion (embodied in the concavity of function ϕ) and risk aversion (embodied in the utility number u^i). The function ϕ^{-1} is used to define function E so that individuals' risk attitudes are not transformed. Indeed, if we were to take the expected value of a Prioritarian social criterion, it would be as if the society was paternalistically increasing individuals' risk preferences so that individuals use a VNM utility function $\phi(u^i)$ instead of u^i. The EPEDE criteria enable us to respect individuals' risk preferences, at least in some cases. Indeed, when all individuals have the same u^i number and population is fixed, EPEDE criteria amount to maximizing individuals' expected utilities.

A last feature of EPEDE criteria is that they explicitly account for population change. Indeed, the critical level for any given population (if it exists) depends only on the parameters $(\alpha_n, \beta_n)_{n\in\mathbb{N}}$ and on the equally distributed equivalent level of welfare expressed by function E. For a given an alternative $u \in \mathcal{U}$, it is indeed straightforward to check that the critical level $c \in X$ for an additional individual k is given by the following equation:[3]

$$\phi(c) = (n(u)+1)\,\phi\left(\frac{\alpha_{n(u)}}{\alpha_{n(u)+1}}E(u) + \frac{\beta_{n(u)} - \beta_{n(u)+1}}{\alpha_{n(u)+1}}\right) - n(u)\phi\left(E(u)\right). \tag{4}$$

3 EPEDE criteria

3.1 Properties characterizing EPEDE criteria

In this Section, we introduce some properties of the social ordering R and show that they characterize EPEDE criteria.

We first want R to be as rational as one could be, given that it serves for a reasoned evaluation of social situations. The expected utility criterion, in spite of many criticisms, remains the benchmark of rational decision-making under risk and the following axiom requires R to take the form of expected (social) utility.

Axiom 1 (Social expected utility hypothesis) *There exists a continuous function* $V : \mathcal{U} \to \mathbb{R}$ *such that, for all* $(\mathbf{p}, \mathbf{u}), (\mathbf{q}, \mathbf{v}) \in \mathbf{P} \times \mathbf{U}$*:*

$$(\mathbf{p}, \mathbf{u})R(\mathbf{q}, \mathbf{v}) \Longleftrightarrow \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big)$$

One limitation implied by this axiom is that it prevents R from evaluating what happens in one state of the world taking into account what would have happened in other states. In this fashion, ex ante fairness in lotteries (Diamond, 1967) is ignored, unless the utility numbers in any given state do incorporate a measure of the chances that individuals had in other

[3] Indeed, it must be the case that c satisfies:

$$\alpha_{n(u)}E(u) + \beta_{n(u)} = \alpha_{n(u)+1}\phi^{-1}\left(\frac{n(u)}{n(u)+1}\phi\big(E(u)\big) + \frac{1}{n(u)+1}\phi(c)\right) + \beta_{n(u)+1}$$

states. It is formally easy to generalize the criterion and rewrite it as $\sum_{s\in\mathcal{S}}\mathbf{p}(s)V_s(\mathbf{p},\mathbf{u})$, but it is then difficult to come up with a precise proposal for the state-specific functions V_s that would evaluate the consequences in state s as a function of the whole lottery $(\mathbf{p},\mathbf{u})$ (see Fleurbaey, Gajdos and Zuber, 2014).

The next axiom is a fairness principle requiring utility redistributive transfers to improve social welfare. It also requires that the identity of people does not influence the way evaluate distributions of utility for populations of similar sizes. It thus implicitly makes an assumption of anonymity.

Axiom 2 (Anonymous Pigou-Dalton) *For all $u,v\in\mathcal{U}$ such that $n(u)=n(v)$ and for all $\mathbf{p}\in\mathbf{P}$, if there exists a bijection $\pi:\mathcal{N}(u)\to\mathcal{N}(v)$, $i,j\in\mathcal{N}(u)$ and $\varepsilon>0$ such that*

1. $v^{\pi(i)}+\varepsilon=u^i\leq u^j=v^{\pi(j)}-\varepsilon$;
2. $u^k=v^{\pi(k)}$ *for all* $k\in\mathcal{N}(u)\setminus\{i,j\}$,

then $(\mathbf{p},u)P(\mathbf{p},v)$.

The above Pigou-Dalton transfer principle only applies to sure prospects. It could easily be extended to any acts, so that simultaneous Pigou-Dalton utility transfers in all states of the world improve social welfare. We do not need this stronger version in our axiomatization, although EPEDE criteria do satisfy it. Note that transfers are made in utility levels and not in resources. The appeal of this property thus hinges on the assumption that the utility values are the correct measure of individuals' welfare.

The Pareto principle is the hallmark of social evaluation, but the principle of consumer sovereignty is normally invoked when the individuals are fully informed about the options. In the presence of risk, by definition the individuals do not know what will ultimately happen if they choose such or such option, so that respecting their ex ante preferences is less compelling than under full information. In particular, there are situations in which the distribution of final situations across individuals is known ex ante, while it is only the identity of winners and losers that is not known. In such situations, the ignorant individuals may all be willing to take a risk, but everyone knows that it is not in the interest of the ultimate losers and everyone knows that this ex ante unanimous preference for a risky lottery will break down as soon as uncertainty is resolved. In view of such considerations, we restrict the application of the Pareto principle to situations in which such a breakdown of unanimity with greater information cannot occur. Two cases are retained here. There is first the case of sure acts, in which full information about final utilities prevails.

Axiom 3 (Pareto for no risk) *For all* $u, v \in \mathcal{U}$ *and for all* $\mathbf{p} \in \mathbf{P}$*, if* $\mathcal{N}(u) = \mathcal{N}(v)$ *and* $u^i \geq v^i$ *for all* $i \in \mathcal{N}(u)$ *then* $(\mathbf{p}, u)R(\mathbf{p}, v)$*. If furthermore* $u^j > v^j$ *for some* $j \in \mathcal{N}(u)$ *then* $(\mathbf{p}, u)P(\mathbf{p}, v)$*.*

Second, there is the case in which all individuals share exactly the same fate in all states of the world. They may ultimately regret[4] having taken a risk if they are unlucky, but they will unanimously do so.

Axiom 4 (Pareto for equal risk) *For all* $\mathcal{N} \in \mathfrak{N}$*, for all* $\mathbf{p}, \mathbf{q} \in \mathbf{P}$*, for all* $\mathbf{u}, \mathbf{v} \in \mathbf{U}_{\mathcal{N}}$*, if for all* $s \in \mathcal{S}$ *and for all* $i, j \in \mathcal{N}$ $\mathbf{u}^i(s) = \mathbf{u}^j(s)$ *and* $\mathbf{v}^i(s) = \mathbf{v}^j(s)$*, then*

$$(\mathbf{p}, \mathbf{u})R(\mathbf{q}, \mathbf{v}) \Longleftrightarrow \forall i \in \mathcal{N}, \quad \mathbb{E}_{\mathbf{p}}(\mathbf{u}^i) \geq \mathbb{E}_{\mathbf{q}}(\mathbf{v}^i).$$

Pareto for Equal Risk is important because it allows social preferences to take into account individuals' risk preference. If only Pareto for No Risk was assumed, we might end up with criteria such as the expected (critical-level) prioritarian criteria that distort individual risk attitudes.

We last introduce a requirement of subpopulation separability. The motivation for separability axioms is primarily a matter of simplicity. Under separability it is possible to perform the evaluation of a certain change affecting a particular population (e.g., the present and future generations) independently of the rest of the population that is not concerned (e.g., the past generations). The following axiom, however, only applies to sure acts in which utility is the same in all states.

Axiom 5 (Separability for sure acts) *For all* $\mathcal{M}, \mathcal{N} \in \mathfrak{N}$ *such that* $\mathcal{M} \subset \mathcal{N}$*, for all* $\mathbf{p} \in \mathbf{P}$*, for all* $u, \tilde{u}, v, \tilde{v} \in \mathcal{U}$*, if* $\mathcal{N}(u) = \mathcal{N}(v) = \mathcal{N}$*,* $\mathcal{N}(\tilde{u}) = \mathcal{N}(\tilde{v}) = \mathcal{M}$ *and*

[4] The notion of regret used here corresponds to a comparison with the decision that would have been made under full information about the final state of the world. It does not mean that individuals would want to change their decisions if they had to do it again under the same informational circumstances.

$u^i = v^i$ *for all* $i \in \mathcal{N} \setminus \mathcal{M}$,

$u^j = \tilde{u}^j$ *for all* $j \in \mathcal{M}$,

$v^j = \tilde{v}^j$ *for all* $j \in \mathcal{M}$,

then $(\mathbf{p}, u)R(\mathbf{p}, v) \Longleftrightarrow (\mathbf{p}, \tilde{u})R(\mathbf{p}, \tilde{v})$.

We do not impose separability on risky prospects. One reason is that the separability principle may not be so attractive in that case. Consider the following prospects, described by matrices in which a cell gives the utility of an individual in a particular state of the world (rows are for two individuals, columns for two equiprobable states). It seems natural that the social ordering satisfies

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \text{ is preferred to } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

because individual expected utilities are the same and less inequality ex post is obtained in the preferred prospect. The second individual faces the same personal prospect in both social prospects. Separability in risky situations would imply that

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ is preferred to } \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

This conclusion is not appealing, because the two individuals still have the same expected utility in the two lotteries, but now the preferred lottery is one where inequality prevails ex post.

The following result identifies the EPEDE family as the one satisfying the axioms.

Theorem 1 *The social ordering R satisfies Axioms 1, 2, 3, 4 and 5 if and only if it is an EPEDE social ordering.*

Proof. See Appendix A. ■

3.2 Population ethics and critical-levels

As discussed in Section 2, when considering populations of different sizes we have to face the question of how we can compare them, and trade-off the level of wellbeing with the size of the population. A crucial concept in the literature on population ethics has been the concept of the *critical level.* Given a distribution of utilities $u \in \mathcal{U}$, the critical-level, if it exists, is $x \in X$ such that, for all $\mathbf{p} \in \mathbf{P}$, $(\mathbf{p}, u)I(\mathbf{p}, v)$ where $v \in \mathcal{U}$ is defined as $v^i = u^i$ for all $i \in \mathcal{N}(u)$ and $v^k = c$, where $k \in \mathbb{N} \setminus \mathcal{N}(u)$. Hence, the society is indifferent to adding an individual with this level of welfare c or keeping the existing population with the existing distribution of welfare.

With an EPEDE ordering, the critical level is defined by (4). It is therefore a function of $E(u)$ and $n(u)$. In this section, we examine some salient possibilities.

First, when $\alpha_{n(u)} = 1$ and $\beta_{n(u)} = \beta_{n(u)+1}$, which is the case with the simple EPEDE criterion $V(u) = E(u)$, one obtains $c = E(u)$. It is then indifferent to add new members at the equally distributed equivalent level. This critical level avoids the repugnant conclusion but it violates the Negative Expansion Principle (Blackorby, Bossert and Donaldson, 2005) stating that the addition of a person to a utility-unaffected population should be ranked as bad if the utility level of the added person is negative. However, such violations occur only when $E(u) < 0$, which may be considered a particularly bad case, since the distribution u is then considered as bad as a situation in which everyone is below zero. This can happen with a very strong inequality aversion, because $E(u)$ is then close to the lowest utility in u.

A constant critical level (i.e., that depends neither on $E(u)$ nor on $n(u)$), as in Critical Level Utilitarianism, is hard to obtain with an EPEDE. To show this, consider the weaker requirement that it should not depend on $E(u)$, but may depend on $n(u)$. This property is for instance satisfied by Number-sensitive critical-level Utilitarian criteria discussed by Blackorby, Bossert and Donaldson (2005). A special case is when the critical level is always 0, whatever the population size. Then any life with a positive welfare level is worth adding (assuming Pareto), which is known as the 'Mere Addition Principle'.

Axiom 6 (Number-dependent critical level) *There exists real numbers* $(c_n)_{n\in\mathbb{N}} \in X^{\mathbb{N}}$ *such that for all* $\mathbf{p} \in \mathbf{P}$, *for all* $u, v \in \mathcal{U}$, *and for all* $k \in \mathfrak{N} \setminus N(u)$, *if*
$\mathcal{N}(v) = \mathcal{N}(u) \cup \{k\}$,
$u^i = v^i$ *for all* $i \in \mathcal{N}(u)$,
$v^k = c_{n(u)}$,
then $(\mathbf{p}, u)I(\mathbf{p}, v)$.

With this Axiom, we obtain a very specific class of EPEDE social criteria, as described in the following Proposition.

Proposition 1 *If the social ordering R satisfies Axioms 1, 2, 3, 4, 5 and 6 then one of the following two statements must be true:*

1. *X is bounded below and there exists ϵ such that $0 < \epsilon < 1$ and for all $(\mathbf{p}, \mathbf{u}), (\mathbf{q}, \mathbf{v}) \in \mathbf{P} \times \mathbf{U}$*
$$(\mathbf{p}, \mathbf{u}) R (\mathbf{q}, \mathbf{v}) \Longleftrightarrow \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big)$$
where
$$V(u) = \left(\sum_{i \in \mathcal{N}(u)} (u^i - \underline{x})^{1-\epsilon} \right)^{1/(1-\epsilon)}.$$

2. *X is bounded above and there exists ϵ such that $\epsilon > 0$ and for all $(\mathbf{p}, \mathbf{u}), (\mathbf{q}, \mathbf{v}) \in \mathbf{P} \times \mathbf{U}$*
$$(\mathbf{p}, \mathbf{u}) R (\mathbf{q}, \mathbf{v}) \Longleftrightarrow \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big)$$
where
$$V(u) = -\left(\sum_{i \in \mathcal{N}(u)} (\bar{x} - u^i)^{1+\epsilon} \right)^{1/(1+\epsilon)}.$$

Proof. See Appendix B. ■

A first remark about Proposition 1 is that whenever $X = \mathbb{R}$ neither Case 1 nor Case 2 are possible, given that X is unbounded, so that we obtain an impossibility theorem.

Case 2 of Proposition 1 is not palatable, as it implies a strong form of anti-populationism. It is indeed never worth adding people to a population. Only situations where all individuals enjoy the highest level of welfare $\bar{x}$ is equivalent to the no population case.

Case 1 may seem more appealing, since largest population are better. However, it displays a strong form of populationism, in the sense that larger populations are always better, whatever the level of welfare of the additional people. If $\underline{x} < 0$ it displays a strong form of the Sadistic Conclusion (Arrhenius, 2014), where people with a negative level welfare, and thus a life not worth living, are always worth adding to the population, from the social welfare point of view. If $\underline{x} = 0$, this criterion implies the repugnant conclusion. It is also noteworthy that the social ordering in Case 1 has limited inequality aversion, as $0 < \epsilon < 1$.

The result of Proposition 1 is therefore largely negative. Note, however, that it would be possible to have $c_n = c > \underline{x}$ if the function ϕ in the Definition of EPEDE social orderings (Eq. (3)) was no longer required to be concave everywhere and the social ordering was inequality averse above the critical level but inequality prone below it, i.e., with a formula like

$$V(u) = \phi^{-1}\left(\frac{1}{n(u)}\sum_{i\in\mathcal{N}(u)}\phi\big(u^i - c\big)\right),$$

where $\phi(z) = z^{1-\epsilon}$ when $z > 0$, $\phi(z) = -(-z)^{1-\varepsilon}$ when $z < 0$, and $0 < \epsilon < 1$. Although this is a controversial form of social ordering, a possible justification is that it focuses on raising individuals above the critical level c, even if this means sacrificing those who cannot make it (this is a triage approach discussed in Roemer, 2009).

To avoid extreme comparisons of populations of different sizes, while retaining the assumption that Pigou-Dalton transfers of utility are always welfare improving from the social point of view, one has to make the critical level sensitive to social welfare in the incumbent population. Observe that the critical level c satisfies:

$$\phi(c) - \phi\left(E_n^*(u)\right) = n(u)\left[\phi\left(E_n^*(u)\right) - \phi\left(E(u)\right)\right],$$

where $E_n^*(u) = \frac{\alpha_{n(u)}}{\alpha_{n(u)+1}}E(u) + \frac{\beta_{n(u)}-\beta_{n(u)+1}}{\alpha_{n(u)+1}}$. In other words, c is always on the side of the transformed $E_n^*(u)$, away from $E(u)$. When $E_n^*(u) < E(u)$, $c < E_n^*(u)$ with some amplification by $n(u)$ and by the concavity of ϕ. When $E_n^*(u) > E(u)$, $c > E_n^*(u)$ with some amplification by $n(u)$ but some brake due to the concavity of ϕ.

A natural option consists in requiring that for some given level c_0, one should have $c = E(u)$ whenever $E(u) = c_0$, independently of $n\,(u)$. In particular, whenever all individuals have $u^i = c_0$, the population size becomes indifferent. Above this level, increasing the population should naturally appear desirable, and below this level it should appear undesirable. This property is satisfied by the EPEDE such that

$$V(u) = \alpha_n\left(E\,(u) - c_0\right),$$

for an increasing sequence α_n. The following analysis shows more precisely how one can derive this particular EPEDE from simple requirements.

We introduce two properties. The first one, reflecting the above discussion, says that the addition of a person at a given equally distributed welfare level is socially good when welfare is sufficiently large, but socially bad otherwise.

Axiom 7 (Egalitarian expansion principle) *There exists* $c_0 \in X$ *such that* $c_0 \geq 0$ *and for all* $\mathbf{p} \in \mathbf{P}$, *for all* $x \in X$*:*

- *if* $x < c_0$ *then* $(\mathbf{p}, x \cdot \mathbb{1}_n) P (\mathbf{p}, x \cdot \mathbb{1}_{n+1})$ *for all* $n \in \mathbb{N}$*;*
- *if* $x \geq c_0$ *then* $(\mathbf{p}, x \cdot \mathbb{1}_{n+1}) R (\mathbf{p}, x \cdot \mathbb{1}_n)$ *for all* $n \in \mathbb{N}$*.*

If $c_0 = 0$ in Axiom 7, we obtain a weakening of two well-known principles. The first one is the Mere Addition Principle stating that the addition of a person to a utility-unaffected population should be ranked as good if the utility level of the added person is positive. The second is the Negative Expansion Principle (Blackorby, Bossert and Donaldson, 2005) stating that the addition of a person to a utility-unaffected population should be ranked as bad if the utility level of the added person is negative. If $c_0 = 0$, Axiom 7 applies these principles to egalitarian allocations only.

The second principle is a regularity condition on critical-levels, when they exist. It states that the critical level for an allocation with higher welfare is always larger than the one for an allocation with lower welfare. This can be seen as expressing the idea that societies have higher standards for additional lives when they enjoy higher levels of welfare.

Axiom 8 (Increasing critical level) *For all* $x, \tilde{x} \in X$*,* $c, \tilde{c} \in X$*,* $\mathcal{N} \in \mathfrak{N}$*,* $u, \tilde{u} \in \mathcal{U}$*,* $\mathbf{p} \in \mathbf{P}$ *and* $k \in \mathbb{N} \setminus \mathcal{N}$ *satisfying the following conditions:*

- $\mathcal{N}(u) = \mathcal{N}(\tilde{u}) = \mathcal{N} \cup \{k\}$;
- $u^i = x$ *and* $\tilde{u}^i = x$ *for all* $i \in \mathcal{N}$;
- $u^k = c$ *and* $\tilde{u}^k = \tilde{c}$*;*
- $(\mathbf{p}x \cdot \mathbb{1}_{\mathcal{N}}) I (\mathbf{p}, u)$ *and* $(\mathbf{p}, \tilde{x} \cdot \mathbb{1}_{\mathcal{N}}) I (\mathbf{p}, \tilde{u})$;

one has $c > \tilde{c}$ *if* $x > \tilde{x}$*.*

With these two additional axioms, we obtain a more specific family of EPEDE social orderings.

Proposition 2 *If the social ordering R satisfies Axioms 1, 2, 3, 4, 5, 7 and 8 then there exists $c_0 \in X$ such that $c_0 \geq 0$, a continuous increasing and concave function ϕ and an increasing sequence $(\alpha_n)_{n\in\mathbb{N}}$ such that $\left(\frac{\alpha_n}{n}\right)_{n\in\mathbb{N}}$ is a decreasing sequence and for all $(\mathbf{p},\mathbf{u}),(\mathbf{q},\mathbf{v}) \in \mathbf{P}\times\mathbf{U}$:*

$$(\mathbf{p},\mathbf{u})R(\mathbf{q},\mathbf{v}) \Longleftrightarrow \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big)$$

where

$$V(u) = \alpha_n \left(E\left(u\right) - c_0\right).$$

Proof. See Appendix C. ■

The social ordering in Proposition 2 does not necessarily avoid the repugnant conclusion. For the conclusion to be avoided, we need that the sequence $(\alpha_n)_{n\in\mathbb{N}}$ be bounded, which is not necessarily entailed by the fact that $\left(\frac{\alpha_n}{n}\right)_{n\in\mathbb{N}}$ is a decreasing sequence. Also, the social ordering in Proposition 2 avoids the Very Sadistic Conclusion if and only if $c_0 = 0$.

The conclusion that $\left(\frac{\alpha_n}{n}\right)_{n\in\mathbb{N}}$ must be a decreasing sequence is important for the analysis of the discount rate in the next section and holds quite generally. Intuitively, this comes from the fact that requiring the critical level to be increasing in $E\left(u\right)$, when ϕ is differentiable, means that

$$(n(u)+1)\,\frac{\alpha_{n(u)}}{\alpha_{n(u)+1}}\phi'\left(\frac{\alpha_{n(u)}}{\alpha_{n(u)+1}}E(u) + \frac{\beta_{n(u)} - \beta_{n(u)+1}}{\alpha_{n(u)+1}}\right) - n(u)\phi'\left(E(u)\right) > 0,$$

i.e.,

$$\left(\frac{n(u)+1}{n(u)}\right)\frac{\alpha_{n(u)}}{\alpha_{n(u)+1}}\phi'\left(E^*_n(u)\right) > \phi'\left(E(u)\right).$$

If there exists a value of $E(u)$ for which the critical level is equal to it (intuitively, the critical level should not be below $E(u)$ when the latter is very low, and should not be above it when it is very high), then this value is such that $E^*_n(u) = E(u)$, implying

$$\left(\frac{n(u)+1}{n(u)}\right)\frac{\alpha_{n(u)}}{\alpha_{n(u)+1}} > 1.$$

4 Implications for the discount rate

In this section we derive the social discount rate for the family of EPEDE social welfare functions. In the discounted utilitarian approach yielding Equation (1), the discount rate on consumption is the simple addition of a discount rate on utility and a specific term relative to consumption, combining the growth rate of consumption and the rate of decrease of marginal utility.

With the alternative criteria proposed here, the expression of the social discount rate is substantially modified. First, in addition to the utility discount rate, a term related to population size appears. Second, the consumption term generally differs from the utilitarian approach due to equity concerns. Two additional terms appear: one has to do with global social welfare; the other is related to the covariance between the different components of the social discount rate.

4.1 Social and person-to-person discount rates

The vast majority of the research on the social discount rate considers generations rather than individuals. It computes the social discount rate to be applied to assess transfer from one generation to another. In the present paper which allows for inequalities within generations, we must consider individuals rather than generations. We therefore follow the approach that we have developed in a more general framework in Fleurbaey and Zuber (2014), and first compute person-to-person discount rates.

To define this concept, we first need to further specify our framework. Recall that we consider a social welfare function taking the general form

$$W(\mathbf{p},\mathbf{u}) = \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big), \qquad \forall(\mathbf{p},\mathbf{u}) \in \mathbf{P}\times\mathbf{U}.$$

In this formula, we now assume that utility is derived from a (random) consumption process, so that in each state $s \in \mathcal{S}$

$$u_s = \left(u\left(c_s^i\right)\right)_{i\in N(u_s)},$$

where $c_s^i \in C$ is the consumption of individual i in state s, and C is an interval of non-negative real numbers. We therefore assume here that all individuals have the same utility function u. Extending the analysis to the case of heterogeneous utility functions is cumbersome but straightforward. The set of all possible consumption processes is $\mathcal{C} = \cup_{\mathcal{N}\in\mathfrak{N}} C^{\mathcal{N}}$, and the set of random consumption processes $\mathbf{C} = \mathcal{C}^S$. Hence, for $c \in \mathcal{C}$ we denote $u(c)$ the vector of utility numbers such that $u^i = u(c^i)$ for all $i \in \mathcal{N}(c)$; and for $\mathbf{c} \in \mathbf{C}$ we denote $u(\mathbf{c})$ the function such that $u(\mathbf{c})(s) = u(\mathbf{c}(s))$ for all $s \in \mathcal{S}$. We will restrict attention to situations where the size of the current population ($t = 0$) and its consumption is known.

When individuals, not generations, are the constitutive elements of social welfare, the discount rate must be computed primarily between two individuals.

Definition 2 *The person-to-person discount rate from an individual i in period 0 to an individual j in period t, denoted $\rho_t^{i,j}$, is:*

$$\rho_t^{i,j} = \left(\frac{u'(c^i)\ \mathbb{E}_{\mathbf{p}}\Big(\frac{\partial V}{\partial u^i}\big(u(\mathbf{c})\big)\Big)}{\mathbb{E}_{\mathbf{p}}\Big(u'(\mathbf{c}^j)\ \frac{\partial V}{\partial u^j}\big(u(\mathbf{c})\big)\Big)} \right)^{\frac{1}{t}} - 1. \tag{5}$$

To understand this definition, imagine that today (period 0) individual i can make an investment whose sure rate of return is r for the benefit of individual j living in period t. In the margin, such an investment has no effect on social welfare if:

$$(1+r)^t \mathbb{E}_{\mathbf{p}}\Big(u'(\mathbf{c}^j)\ \frac{\partial V}{\partial u^j}\big(u(\mathbf{c})\big)\Big) = u'(c^i)\ \mathbb{E}_{\mathbf{p}}\Big(\frac{\partial V}{\partial u^i}\big(u(\mathbf{c})\big)\Big),$$

with the convention that $\partial V\left(u_s\right)/\partial u_s^j = 0$ if j does not exist in state s. Observe that, while the existence and consumption of i in period 0 is certain, the social marginal utility of her consumption $\partial V\left(u_s\right)/\partial u_s^i$ may vary across states of the world.

When many individuals from period 0 make an investment that benefits many individuals in period t, one can evaluate the investment with a social discount rate that aggregates the person-to-person discount rates, provided the shares of the individuals in the investment (either as investors or as beneficiaries) are fixed. Suppose that each donor i in period 0 bears a fraction σ_0^i of the marginal investment ε, and that each recipient j in period t receives a fraction σ_t^j, with $\sum_{i:T(i)=0} \sigma_0^i = \sum_{j:T(j)=t} \sigma_t^j = 1$. The social discount rate is again the sure rate of return on the marginal investment that leaves social welfare unchanged.

Definition 3 *The social discount rate from period* 0 *to period* t*, denoted* ρ_t*, is:*

$$\begin{aligned}\rho_t &= \left(\frac{\sum_{i:T(i)=0}\sigma_0^i\ u'(c^i)\ \mathbb{E}_{\mathbf{p}}\Big(\frac{\partial V}{\partial u^i}\big(u(\mathbf{c})\big)\Big)}{\sum_{j:T(j)=t}\sigma_t^j\ \mathbb{E}_{\mathbf{p}}\Big(u'(\mathbf{c}^j)\ \frac{\partial V}{\partial u^j}\big(u(\mathbf{c})\big)\Big)}\right)^{\frac{1}{t}} - 1 \\ &= \left(\sum_{i:T(i)=0}\sigma_0^i\left[\sum_{j:T(j)=t}\sigma_t^j\left(1+\rho_t^{i,j}\right)^{-t}\right]^{-1}\right)^{\frac{1}{t}} - 1. \qquad (6)\end{aligned}$$

In the sequel we shall focus on the computation of the person-to-person discount rate. But it is important to bear in mind that such person-to-person discount rates are only elements of the more general social discount rate, which are aggregated through the formula displayed in Equation (6). Such an aggregation may give rise to interesting intra-generational equity issues that are explored in Fleurbaey and Zuber (2014) but not in the present paper. In Fleurbaey and Zuber (2014), we also consider the possibility of risky investments, where the expression of the social discount rate on expected returns must be complemented by an additional terms reflecting the correlation between returns on the investment and the marginal social value of the investment.

4.2 The Ramsey formula revisited

Let us first briefly examine how Equation (5) applies in the utilitarian case. Note that the standard critical level utilitarian case is actually a special case of EPEDE criteria, namely when $\phi(u) = u$, $\alpha_n = n$ and $\beta_n = -nu_c$ where u_c is the critical level. In that case, $V(\mathbf{u}) = \sum_{i\in\mathcal{N}(u)}\big(u^i - u_c\big)$, and $\partial V/\partial u^i = \partial V/\partial u^j = 1$ for the individuals of period 0 and for the individuals of period t in the states in which they exist.

This considerably simplifies formula (5). Let the probability of j coming to existence be $p(j) = \sum_{s\in\mathcal{S}:j\in N(u_s)} p(s)$, and let $\pi^j = -\ln(p(j))$. For a random variable $K : \mathcal{S} \to \mathbb{R}$, denote $\mathbb{E}_{\mathbf{p}}^j(K) = \sum_{s\in\mathcal{N}:j\in N(u_s)}\big(p(s)/p(j)\big)K(s)$ its expected value conditional on j existing.

In the critical-level utilitarian case, one obtains[5]

$$\begin{aligned}\rho_t^{i,j} &= \left(\frac{u'\left(c^i\right)}{p(j)\mathbb{E}^j_{\mathbf{p}}\left(u'\left(\mathbf{c}^j\right)\right)}\right)^{\frac{1}{t}} - 1 \\ &\simeq \frac{\pi^j}{t} - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}\left(u'\left(\mathbf{c}^j\right)\right)}{u'(c^i)}\right).\end{aligned}$$

This expression of the discount rate is clearly very close to the usual Ramsey formula. Indeed, if we denote $\delta = \frac{\pi^j}{t}$ and assume that $u(c) = c^{1-\eta}/(1-\eta)$ and $\mathbf{c}^j(s) = e^{gt}c^i$ for all $s \in \mathcal{S}$, we directly obtain Equation (1). This provides a foundation for the rate of pure time preferences δ based on the risk on the existence of future generations, as proposed by Dasgupta and Heal (1979) and Stern (2006). Another noticeable feature of the above expression is that the critical level plays no role in the value of the discount rate: population ethics views do not affect discounting.

4.2.1 The EPEDE social discount rate

Let us now examine the social discount rate for the more general family of EPEDE criteria. To do so, we need to introduce additional notation. We first introduce the term $\mu^i = u'(c^i)\phi'^i))$ and the random variable $\mu^j : \mathcal{S} \to \mathbb{R}$ such that $\mu^j(s) = u'\left(\mathbf{c}^j(s)\right)\phi'\Big(u\left(\mathbf{c}^j(s)\right)\Big)$ for all $s \in \mathcal{S}$ such that $j \in \mathcal{N}(\mathbf{c}(s))$. Both μ^i and μ^j bear only on individual consumption levels: they are measures of the social priority of individuals' consumption.

In contrast, for any $c \in \mathcal{C}$, the function $\Big[\phi'\big(E(u(c))\big)\Big]^{-1}$ —where E is the equally-distributed equivalent function defined in Eq. (3)— is a measure of social welfare. Indeed, function ϕ is concave, so that the whole expression is an increasing function of the equally-distributed equivalent, which is itself a measure of social welfare. We denote ξ the random variable such that

$$\xi(s) = \Big[\phi'\big(E(u(\mathbf{c}(s)))\big)\Big]^{-1}, \qquad \forall s \in \mathcal{S}.$$

The last random variable refers to population size. It is the function $\nu : \mathcal{S} \to \mathbb{R}$ such that $\nu(s) = \alpha_{n(\mathbf{u}(s))}/n(\mathbf{u}(s))$.

[5]To obtain this approximation, we use:

$$\rho_t^{i,j} \approx \ln\left(1+\rho_t^{i,j}\right) = \ln\left[\left(\frac{u'\left(c^i\right)}{p(j)\mathbb{E}^j_{\mathbf{p}}\left(u'\left(\mathbf{c}^j\right)\right)}\right)^{\frac{1}{t}}\right].$$

We also need notation for covariance between variables. For two random variables K and L, $\mathbb{C}\text{ov}_{\mathbf{p}}(K,L) = \mathbb{E}_{\mathbf{p}}\big[(K-\mathbb{E}_{\mathbf{p}}(K))(L-\mathbb{E}_{\mathbf{p}}(L))\big]$. For three random variables K, L and M,

$$\begin{aligned}\mathbb{C}\text{ov}_{\mathbf{p}}(K,L,M) &= \tfrac{1}{3}\Big(\mathbb{C}\text{ov}_{\mathbf{p}}(K,L)\mathbb{E}_{\mathbf{p}}(M) + \mathbb{C}\text{ov}_{\mathbf{p}}(L,M)\mathbb{E}_{\mathbf{p}}(K) + \mathbb{C}\text{ov}_{\mathbf{p}}(K,M)\mathbb{E}_{\mathbf{p}}(L)\Big) + \\ &\quad \tfrac{1}{3}\Big(\mathbb{C}\text{ov}_{\mathbf{p}}(KL,M) + \mathbb{C}\text{ov}_{\mathbf{p}}(KM,L) + \mathbb{C}\text{ov}_{\mathbf{p}}(LM,K)\Big).\end{aligned}$$

We use notation similar to the one introduced before for the covariances conditional on the existence of j.

We are now able to state the following result:

Proposition 3 *For the EPEDE family of social welfare functions, the person-to-person discount rate can be approximated in the following way:*

$$\begin{aligned}\rho_t^{i,j} &\simeq \frac{\pi^j}{t} - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\mu^j)}{\mu^i}\right) - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\nu)}{\mathbb{E}_{\mathbf{p}}(\nu)}\right) - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\xi)}{\mathbb{E}_{\mathbf{p}}(\xi)}\right) \\ &\quad -\frac{1}{t}\ln\left(1 + \frac{\mathbb{C}\text{ov}^j_{\mathbf{p}}(\mu^j,\nu,\xi)}{\mathbb{E}^j_{\mathbf{p}}(\mu^j)\mathbb{E}^j_{\mathbf{p}}(\nu)\mathbb{E}^j_{\mathbf{p}}(\xi)}\right) + \frac{1}{t}\ln\left(1 + \frac{\mathbb{C}\text{ov}_{\mathbf{p}}(\nu,\xi)}{\mathbb{E}_{\mathbf{p}}(\nu)\mathbb{E}_{\mathbf{p}}(\xi)}\right). \qquad (7)\end{aligned}$$

Proof. For the EPEDE family of social welfare functions, we have that (if i exists)

$$\frac{\partial V}{\partial u^i}\big(u(c)\big) = \phi'\big(u(c^i)\big)\frac{\alpha_{n(c)}}{n(c)}\Big[\phi'\big(E(u(c))\big)\Big]^{-1},$$

and hence

$$\begin{aligned}1+\rho_t^{i,j} &= \left(\frac{u'(c^i)\ \mathbb{E}_{\mathbf{p}}\Big(\frac{\partial V}{\partial u^i}\big(u(\mathbf{c})\big)\Big)}{\mathbb{E}_{\mathbf{p}}\Big(u'(\mathbf{c}^j)\ \frac{\partial V}{\partial u^j}\big(u(\mathbf{c})\big)\Big)}\right)^{\frac{1}{t}} \\ &= \left(\frac{u'(c^i)\ \mathbb{E}_{\mathbf{p}}\Big(\phi'\big(u(c^i)\big)\frac{\alpha_{n(\mathbf{c})}}{n(\mathbf{c})}\Big[\phi'\big(E(u(\mathbf{c}))\big)\Big]^{-1}\Big)}{p(j)\mathbb{E}^j_{\mathbf{p}}\Big(u'(\mathbf{c}^j)\phi'\big(u(\mathbf{c}^j)\big)\frac{\alpha_{n(\mathbf{c})}}{n(\mathbf{c})}\Big[\phi'\big(E(u(\mathbf{c}))\big)\Big]^{-1}\Big)}\right)^{\frac{1}{t}} \\ &= \left(\frac{1}{p(j)}\right)^{\frac{1}{t}}\left(\frac{\mu^i\mathbb{E}_{\mathbf{p}}(\nu\,\xi)}{\mathbb{E}^j_{\mathbf{p}}(\mu^j\nu\,\xi)}\right)^{\frac{1}{t}}.\end{aligned}$$

Therefore,

$$\rho_t^{i,j} \simeq \ln(1+\rho_t^{i,j}) = \frac{\pi^j}{t} - \tfrac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\mu^j\nu\,\xi)}{\mu^i\mathbb{E}_{\mathbf{p}}(\nu\,\xi)}\right)$$

Then we use the fact that, by definition, $\mathbb{E}_{\mathbf{p}}(\nu\,\xi) = \mathbb{E}_{\mathbf{p}}(\nu)\mathbb{E}_{\mathbf{p}}(\xi) + \mathbb{C}\mathrm{ov}_{\mathbf{p}}(\nu,\xi)$, and $\mathbb{E}^j_{\mathbf{p}}(\mu^j\nu\,\xi) = \mathbb{E}^j_{\mathbf{p}}(\mu^j)\mathbb{E}^j_{\mathbf{p}}(\nu)\mathbb{E}^j_{\mathbf{p}}(\xi) + \mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu,\nu,\xi)$, so that:

$$\rho_t^{i,j} \simeq \frac{\pi^j}{t} - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\mu^j)\mathbb{E}^j_{\mathbf{p}}(\nu)\mathbb{E}^j_{\mathbf{p}}(\xi)\left(1+\frac{\mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\nu,\xi)}{\mathbb{E}^j_{\mathbf{p}}(\mu^j)\mathbb{E}^j_{\mathbf{p}}(\nu)\mathbb{E}^j_{\mathbf{p}}(\xi)}\right)}{\mu^i\mathbb{E}_{\mathbf{p}}(\nu)\mathbb{E}_{\mathbf{p}}(\xi)\left(1+\frac{\mathbb{C}\mathrm{ov}_{\mathbf{p}}(\nu,\xi)}{\mathbb{E}_{\mathbf{p}}(\nu)\mathbb{E}_{\mathbf{p}}(\xi)}\right)}\right).$$

■

We can compare formula (7) with the discounting formula obtained in the critical-level utilitarian case. The first term $\frac{\pi^j}{t}$ was already present in the utilitarian case and embodies the risk on the existence of generation t. We will now discuss the role of four elements in the social discount rate: the social priority of private consumption, population size, aggregate social welfare in states where people exist, and the covariance between the three first elements.

4.2.2 The role of social priority of private consumption

The second term of formula (7) is clearly related to the term $\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}\left(u'(\mathbf{c}^j)\right)}{u'(c^i)}\right)$ of the critical-level utilitarian formula in the sense that it compares the social priority of the individuals based on their levels of consumption. The difference is that the new formula adds an equity term $\phi'(u)$, which represent the social priority of individual welfare. This terms introduces society's equity preferences, while the marginal utility of consumption in the standard Utilitarian formula should be measured using individuals VNM utilities that represent their risk preferences. This extra term will provide extra reasons to discount future consumption if the future individual is richer. A simple illustration can be provided using the functional forms $u(c) = \frac{c^{1-\eta}}{1-\eta}$ and $\phi(u) = \frac{u^{1-\epsilon}}{1-\epsilon}$ with $0 < \eta < 1$ and $\epsilon > 0$.[6] We then obtain:

$$\phi'(u(c))u'(c) = (1-\eta)^{\epsilon}c^{-(\eta+\epsilon(1-\eta))}$$

In the case where c^j is certain and $c^j = e^{gt}c^i$, we readily obtain

$$-\frac{1}{t}\ln\left(\frac{\mu^j}{\mu^i}\right) = \eta + \big(\epsilon(1-\eta)\big)g$$

so that we discount the future more, the higher the social preference for redistribution ϵ (with the utilitarian case corresponding to $\epsilon = 0$), provided $g > 0$. The situation is more

[6] A similar result can be obtained when $\eta > 1$ by taking $\phi(u) = -\frac{(-u)^{1+\epsilon}}{1+\epsilon}$ if $\eta > 1$, with $\epsilon > 0$.

tricky when there is risk on the growth rate of consumption, so that g is a random variable. Indeed, the introduction of $\epsilon > 0$ is similar to increasing in η, the individual coefficient of relative risk aversion. Gollier (2002) has showed that such an increase may have an ambiguous effect on discounting when g is normally distributed, depending on the mean and variance of the distribution of g.[7]

4.2.3 The role of population size

The third term of formula (7) involves ν and captures the additional role of the risk on population size. One way to look at this term is that it compares the correlation between the existence of the concerned individuals and a 'value' of population size measured by function ν. Indeed one can write $\mathbb{E}^j_{\mathbf{p}}(\nu) = \mathbb{E}_{\mathbf{p}}(\nu) + \frac{\mathbb{C}\mathrm{ov}_{\mathbf{p}}(\mathbf{1}^j,\nu)}{p(j)}$ where $\mathbf{1}^j$ is a function such that $\mathbf{1}^j(s) = 1$ if $j \in N(u_s)$ and $\mathbf{1}^j(s) = 0$ otherwise. One has $\mathbb{E}_{\mathbf{p}}(\mathbf{1}^j) = p(j)$, so that $-\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\nu)}{\mathbb{E}_{\mathbf{p}}(\nu)}\right) = -\frac{1}{t}\ln\left(1 + \frac{\mathbb{C}\mathrm{ov}_{\mathbf{p}}(\mathbf{1}^j,\nu)}{p(j)\mathbb{E}_{\mathbf{p}}(\nu)}\right)$. The sign of this term will depend on the monotonicity of the sequence $(\alpha_n/n)_{n\in\mathbb{N}}$: if it is increasing, the term will be negative; if it is decreasing, the term will be positive. Indeed, the future individual belongs on average to larger (intertemporal) populations; if each individual of a larger population brings lower social value, then there is an additional reason to discount future consumption. Proposition 2 suggests that the case where (α_n/n) is a decreasing sequence may be more appropriate, at least if we accept Axioms 6 and 7.

To illustrate the implications of this term, consider a case in which there is a risk on the existence of future generations. Each period, with probability p the world survives to the next period, and with probability $1-p$ the human species (and any species relevant for welfare) disappears. We also consider that potential population (i.e. absent the extinction risk) grows, so that total (intertemporal) population when the world exists until generation t is $n_t = (1+n)^t n_0$. The real number n can be interpreted as a long-run population growth rate.[8] A last simplifying assumption is that $\alpha_n = n^\kappa$, so that $\nu = \alpha_n/n = n^{\kappa-1}$. Proposition

[7]Our framework only considers a countable set of states of the world. So it cannot accommodate normally distributed random variables, which are continuous. Such random variables can however be approximated by discrete random variables, so that results for normally distributed variables can be used here.

[8]Indeed, assume that we take past populations into account, and that from generation $-\tau$ on population grows at rate n. Hence population in period $T > 0$ is $N_T = N_{-\tau}(1+n)^{T+\tau}$ so that total population when

2 suggests that suggest $0 < \kappa < 1$. In this simple case, we obtain that

$$\begin{aligned}\mathbb{E}_{\mathbf{p}}(\nu) &= \sum_{T=0}^{+\infty} p(1-p)^T n_0^{\kappa-1}(1+n)^{(\kappa-1)T} \\ &= \frac{pn_0^{\kappa-1}}{1-(1-p)(1+n)^{\kappa-1}},\end{aligned}$$

provided that $0 < (1-p)(1+n)^{\kappa-1} < 1$. Also,

$$\begin{aligned}\mathbb{E}^j_{\mathbf{p}}(\nu) &= \sum_{T=t}^{+\infty} p(1-p)^{T-t} n_0^{\kappa-1}(1+n)^{(\kappa-1)t} \\ &= \frac{pn_0^{\kappa-1}(1+n)^{(\kappa-1)t}}{1-(1-p)(1+n)^{\kappa-1}}.\end{aligned}$$

Hence $-\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\nu)}{\mathbb{E}_{\mathbf{p}}(\nu)}\right) = (1-\kappa)\ln(1+n)$. When $0 < \kappa < 1$, we thus obtain a term which is almost proportional to population growth. The larger population growth, the more we will discount future costs and benefits.

4.2.4 The role of social welfare

The next term in Eq. (7), is $-\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\xi)}{\mathbb{E}_{\mathbf{p}}(\xi)}\right)$. It compares expected social welfare levels in states where the individuals exist. Much like the second term, it can be viewed as measures of the association between the existence of the concerned individuals and the social welfare level. Indeed one can write $\mathbb{E}^j_{\mathbf{p}}(\xi) = \mathbb{E}_{\mathbf{p}}(\xi) + \frac{\mathbb{C}\text{ov}_{\mathbf{p}}(\mathbf{1}^j,\xi)}{p(j)}$. The intuition is the following: if the future individual belongs on average to populations where social welfare is higher, we should discount her consumption less, because social welfare would be increased more in states where general welfare is already higher if we were to transfer money to her. Note that this term, like the first and third term is zero if the future person exists in all states of the world.

4.2.5 The role of covariance

The last two terms of Eq. (7) involve covariances between the different elements of the social discount rate discussed so far. These terms are more complicated to account for. The

the world exist until generation T is $n_T = n_{-\tau-1} + N_{-\tau}\sum_{s=-\tau}^{T}(1+n)^s = n_{-\tau-1} + N_{-\tau}\frac{(1+n)^{T+\tau+1}-1}{n} = (1+n)^T\frac{(1+n)^{\tau+1}N_{-\tau}}{n}\left(1 - \frac{1}{(1+n)^{T+\tau+1}} + \frac{nn_{-\tau-1}}{(1+n)^{T+\tau+1}N_{-\tau}}\right)$. If τ is sufficiently large, we have $n_0 \approx \frac{(1+n)^{\tau+1}N_{-\tau}}{n}$ and $n_t \approx (1+n)^t n_0$ for all $t > 0$.

first says that the discount rate is lower if individual j is more frequently worse off in states with smaller population and greater social welfare. This is intuitive, as in such states j can contribute much to improving social welfare.

The second says that the discount rate is greater if population size and social welfare are inversely correlated. This may seem counterintuitive because the future should count more when population size and welfare are inversely correlated, but the previous trivariate correlation already incorporates that. Therefore one must see this second term are providing the benchmark for the first term.

These expressions simplify in relevant cases. In particular, if the risk on population is independent of the risk on consumption,[9] the first covariance term simplifies to

$$-\frac{1}{t}\ln\left(1+\frac{\mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\xi)}{\mathbb{E}^j_{\mathbf{p}}(\mu^j)\mathbb{E}^j_{\mathbf{p}}(\xi)}\right)$$

because in such a case $\mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\nu,\xi)=\mathbb{E}^j_{\mathbf{p}}(\nu)\ \mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\xi)$. In addition, the second covariance term is nil because $\mathbb{C}\mathrm{ov}_{\mathbf{p}}(\nu,\xi)=0$. In this case, the covariance terms transparently complement the term depending on social welfare: in addition to the covariance between the *existence* of the individual and general welfare, the covariance between her social priority and general welfare matters. When this covariance is high for the future person, we want to discount her consumption less because she has a high priority in good social states, i.e., in states in which the value of the equally-distributed equivalent is sensitive to j's fate because it is sensitive to the less well-off.

Another interesting possibility is when ξ is certain and takes (almost) the same value in all states of the world. This may occur when the society has strong preferences for redistribution and there is a significant number of poor people in all states of the world (perhaps because these are many poor past people, or because we are not able to completely alleviate poverty in the future).[10] Then $\mathbb{C}\mathrm{ov}_{\mathbf{p}}(\nu,\xi)=0$ and $\mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\nu,\xi)=\mathbb{E}^j_{\mathbf{p}}(\xi)\ \mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\nu)$, so

[9]This may occur when we think that the extinction risk is purely exogenous and that fertility is not influenced by economic conditions. The assumption seems less sensible if we consider endogenous fertility, and the possibility that consumption and population size may be influenced by a common phenomenon, e.g. climate change.

[10]Indeed, when ϕ becomes very concave, function E defined in Eq. (3) is such that $E(u)\approx\min_{i\in\mathcal{N}(u)}u^i$. $E(u)$ is also closer to $\min^i_{i\in\mathcal{N}(u)}$ the larger the fraction of the population whose utility level is $\min^i_{i\in\mathcal{N}(u)}$.

that the first covariance term in Eq. (7) becomes

$$-\frac{1}{t}\ln\left(1+\frac{\mathbb{C}\mathrm{ov}^j_{\mathbf{p}}(\mu^j,\nu)}{\mathbb{E}^j_{\mathbf{p}}(\mu^j)\mathbb{E}^j_{\mathbf{p}}(\nu)}\right),$$

and the second covariance term is nil.

If in addition population size and consumption are independent, the covariance terms completely disappear and only the first three terms of (7) remain, i.e.

$$\rho^{i,j}_t \simeq \frac{\pi^j}{t}-\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\mu^j)}{\mu^i}\right)-\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\nu)}{\mathbb{E}_{\mathbf{p}}(\nu)}\right).$$

This provides a formula for the discount rate which would also be obtained with the following number-dependent prioritarian social criterion:

$$V(u)=\frac{\alpha_{n(u)}}{n(u)}\left(\sum_{i\in\mathcal{N}(u)}\phi(u^i)\right)+\beta_{n(u)}.$$

5 Catastrophic risk

In an influential recent paper, Weitzman (2009) suggests that, in the presence of a fat tail in the distribution of risk, the discount rate can approach -1, implying an absolute priority to future consumption (the "dismal theorem"). His argument relies on the utilitarian criterion. In this section we reexamine it in the context of the EEDE criteria introduced in Section 2.

Weitzman's basic line of reasoning is as follows. The utilitarian discount rate satisfies the equation (see above):

$$\ln\left(1+\rho^{i,j}_t\right)=\frac{-\pi^j}{t}-\frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}\left(u'(\mathbf{c}^j)\right)}{u'\left(c^i\right)}\right).$$

The critical term for Weitzman's argument is $\mathbb{E}^j_{\mathbf{p}}\left(u'(\mathbf{c}^j)\right)$, which, in the case of a CRRA function $u(c)=\frac{1}{1-\eta}(c)^{1-\eta}$, $\eta>1$, and a continuous distribution of c^j depicted by a probability distribution function (PDF) f,[11] is equal to $\int_0^{+\infty}(c^j)^{-\eta}f(c^j)dc^j$. If one changes variables so as to refer to a growth rate, $c^j=c_0e^{gt}$, the expression becomes

$$c_0^{-\eta}\int_{-\infty}^{+\infty}e^{-\eta tg}\hat{f}(g)dg, \tag{8}$$

[11] Again, our framework consider discrete random variables, but continuous random variables can be approximated.

where $\hat{f}$ is the PDF of the growth rate of consumption. Expression (8) is essentially the moment-generating function of $\hat{f}$, and it is infinite if $\hat{f}$ has a fat tail in the negative values representing catastrophic risks.

A fat tail means that $\hat{f}(g) \propto (-g)^{-k}$ for some $k > 0$ when $g \to -\infty$. The PDF f cannot have a fat tail in the low values of c^j because c^j is bounded from below. However, one has $f(c^j) = \hat{f}\left(\frac{1}{t}\ln\frac{c^j}{c_0}\right) \propto \left(-\ln c^j\right)^{-k}$ when $c^j \to 0$. Such a PDF, for instance, has the property that, conditional on $c^j < q$, the probability of $c^j < q/2$ remains around 50% when $q \to 0$.

Weitzman (2009) motivated the fat tail assumption by the example of climate change, arguing that scientific uncertainty about the functioning of the climate system may give rise to a fat tailed distribution of future temperature. In fact, we do not need a fat tailed distribution of *temperatures* to support an argument in favor of giving an absolute priority to the future. It is sufficient that: 1) whenever temperature T is above a certain threshold T^*, consumption is 0; 2) $u'(0) = +\infty$; and 3) there is a positive probability that $T > T^*$. The result is true more generally if there is a subsistence level $c_{\min}$ such that $u'(c_{\min}) = +\infty$, and future generations are back to the subsistence level when $T > T^*$.

Given the frightening worst-case scenarios involving temperature increase above 10°C or 20°C, it is not unreasonable to assign a positive probability to the event of having a substantial part of the population at the subsistence level in future generations —in fact, extinction would be more accurate description of the situation.[12] The weakness of the argument in the preceding paragraph is rather the assumption of an infinite marginal utility at the subsistence level so that it is an absolute priority to raise c above $c_{\min}$ at any period. A typical form for the utility function could be $u(c) = \frac{1}{1-\eta}\left(c^{1-\eta} - (c_{\min})^{1-\eta}\right)$, which has a finite marginal utility at $c_{\min} > 0$. With such a function, the utilitarian discount rate remains finite even when the probability of $c = c_{\min}$ is positive.[13]

Let us now examine how Weitzman's result change when using the EPEDE family of social welfare functions. To simplify things, first assume that the risk on population is

[12]One should not forget that widespread premature deaths and bare survival for large numbers of the population is already the case today, for reasons having little to do with the climate. As Schelling (1995) argued, if the possible poverty of future generations is the reason to give them priority, we should give a stronger priority to the poor who exist today with certainty.

[13]Actually, Weitzman (2009) considers the utility function $u(c) = \frac{1}{1-\eta}\left(c^{1-\eta} - (c_{\min})^{1-\eta}\right)$ in his paper, but he obtains a "dismal" discount rate by letting $c_{\min} \to 0$.

independent of the risk on consumption and that the equally distributed equivalent is almost the same in all states of the world. Hence, using the notation of Section 4.2, we assume that $\mathbb{E}^j_{\mathbf{p}}(\xi) = \mathbb{E}_{\mathbf{p}}(\xi)$, and $\mathbb{C}\text{ov}^j_{\mathbf{p}}(\mu^j, \nu, \xi) = \mathbb{C}\text{ov}^j_{\mathbf{p}}(\mu^j, \nu) = \mathbb{C}\text{ov}_{\mathbf{p}}(\mu^j, \xi) = \mathbb{C}\text{ov}_{\mathbf{p}}(\nu, \xi) = 0$. In that case, we have showed at the end of Section 4.2 that

$$\rho^{i,j}_t \simeq \frac{\pi^j}{t} - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\mu^j)}{\mu^i}\right) - \frac{1}{t}\ln\left(\frac{\mathbb{E}^j_{\mathbf{p}}(\nu)}{\mathbb{E}_{\mathbf{p}}(\nu)}\right).$$

The critical term here is $\mathbb{E}^j_{\mathbf{p}}(\mu^j) = \int_0^{+\infty} u'(c^j)\phi'\big(u(c^j)\big)f(c^j)dc^j$. And we may well obtain the dismal result that $\mathbb{E}^j_{\mathbf{p}}(\mu^j) \to +\infty$, even if $u'(c_{min}) < +\infty$. We only need that $\phi'(u(c_{min})) = +\infty$, so that it is an absolute priority to raise c above $c_{\min}$. The difference with the Weitzman result is that the absolute priority does not come from an infinite marginal utility of consumption but from an absolute social priority to raise people's welfare above the minimal level of welfare.

This line of argument, however, no longer works when the equally distributed equivalent utility level varies between states of the world. Consider the following example. Assume that there is no risk on population and no intra-generational inequality. The world exists for T generations, each generation has N individuals consuming $c^t(s)$ in state $s \in \mathcal{S}$. Assume that $c^t(s) = e^{g(s)t}c_o$, for all $s \in \mathcal{S}$, where the consumption growth rate g is a random variable. Assume also that $u(c) = c^{1-\eta}$, with $0 < \eta < 1$, and $\phi(u) = \ln u$. Using the expression of the equally-distributed equivalent function (3), we have:

$$\begin{aligned} E\big(u(s)\big) = \exp\left(\frac{1}{T}\sum_{t=0}^{T}\ln\Big((e^{g(s)t}c_0)^{1-\eta}\Big)\right) &= c_0^{1-\eta}\exp\left(\frac{1}{T}\sum_{t=0}^{T}(1-\eta)g(s)t\right) \\ &= c_0^{1-\eta}\exp\left(\frac{(T+1)(1-\eta)}{2}g(s)\right). \end{aligned}$$

Noting that $(\phi'(u))^{-1} = u$ when $\phi(u) = \ln u$, we have that

$$\left(\phi'\Big(E\big(u(s)\big)\Big)\right)^{-1} = c_0^{1-\eta}\exp\left(\frac{(T+1)(1-\eta)}{2}g(s)\right).$$

Also, $\phi(u(c)) = (1-\eta)\ln c$ so that $\phi'(u(c))u'(c) = (1-\eta)c^{-1}$. Denoting $\kappa = \frac{(T+1)(1-\eta)}{2}$, we thus obtain that the discount rate is defined by:

$$
\begin{aligned}
1+\rho_t^{i,j} &= \left(\frac{\mathbb{E}_{\mathbf{p}}\left(\frac{\phi'(u(c))u'(\mathbf{c}^j)}{\phi'(E(\mathbf{u}))}\right)}{\mathbb{E}_{\mathbf{p}}\left(\frac{\phi'(u(c))u'(\mathbf{c}^i)}{\phi'(E(\mathbf{u}))}\right)}\right)^{-\frac{1}{t}} = \left(\frac{\mathbb{E}_{\mathbf{p}}\left[(1-\eta)(c^j)^{-1}c_0^{1-\eta}\exp\left(\kappa g\right)\right]}{\mathbb{E}_{\mathbf{p}}\left[(1-\eta)(c_0)^{-1}c_0^{1-\eta}\exp\left(\kappa g\right)\right]}\right)^{-\frac{1}{t}} \\
&= \left(\frac{\mathbb{E}_{\mathbf{p}}\left[\exp\left((\kappa-t)g\right)\right]}{\mathbb{E}_{\mathbf{p}}\left[\exp\left(\kappa g\right)\right]}\right)^{-\frac{1}{t}}.
\end{aligned}
$$

If $\frac{(T+1)(1-\eta)}{2} = \kappa > t$, that is if the planning horizon is sufficiently long compared to the distance with the future individual, the expectation $\mathbb{E}_{\mathbf{p}}\left[\exp\left((\kappa-t)g\right)\right]$ may converge even in the presence of a fat (left) tail in the distribution of the negative values of g. In that case, what would be problematic is rather a fat (right) tail in the distribution of the positive values of g. Note that the expectation $\mathbb{E}_{\mathbf{p}}\left[\exp\left(\kappa g\right)\right]$ would also converge, so that the discount rate would have a finite value, in contrast to Weitzman's result.

More strikingly, in this case, society may focus mainly on the good outcomes, rather than on catastrophic states. To see that, assume that g is uniformly distributed on the interval $[\underline{g}, \bar{g}]$, with $\underline{g} < 0 < \bar{g}$. Then,

$$
\mathbb{E}_{\mathbf{p}}\left[\exp\left((\kappa-t)g\right)\right] = \frac{e^{(\kappa-t)\bar{g}} - e^{(\kappa-t)\underline{g}}}{(\kappa-t)\left(\bar{g}-\underline{g}\right)}
$$

and

$$
\mathbb{E}_{\mathbf{p}}\left[\exp\left(\kappa g\right)\right] = \frac{e^{\kappa\bar{g}} - e^{\kappa\underline{g}}}{\kappa(\bar{g}-\underline{g})}.
$$

Hence,

$$
1+\rho_t^{i,j} = e^{\bar{g}}\left(1-\frac{t}{\kappa}\right)^{\frac{1}{t}}\left(\frac{1-e^{\kappa(\underline{g}-\bar{g})}}{1-e^{(\kappa-t)(\underline{g}-\bar{g})}}\right)^{\frac{1}{t}}.
$$

If $\underline{g} \to -\infty$ and T large enough so that $\frac{(T+1)(1-\eta)}{2} = \kappa > t$, then:

$$
\rho_t^{i,j} \approx \ln(1+\rho_t^{i,j}) \approx \bar{g}.
$$

The main intuition behind this result is the following. In the EPEDE approach, the social priority of the consumption of both the current and the future generation is nil when the growth rate of consumption is negative. This is so because their relative social priority embodied in the term $\frac{\phi'(u^j)}{\phi'(E(u))}$ is zero, and this is sufficient to overcome their differences in terms of marginal utility of consumption.

6 Conclusion

In this paper, we have introduced a general framework in which the horizon is finite but uncertain, and uncertainty bears on future utility as well as on the composition of the future population. Doing so, we have characterized non-utilitarian criteria which embody a greater concern for equity than utilitarianism, at the cost of weakening the Pareto principle. We have thus been able to explicitly introduce concerns for population size and to disentangle risk aversion and inequality aversion.

We showed that the expression of the social discount rate should then be modified in several respects. First, the consumption term should be augmented to take into account equity concerns. Second, a population term appears, which will depend on ethical views regarding population size. Third, two correlation terms emerge: one between the existence of future generations and intertemporal welfare; another between individual priority, population size and intertemporal welfare.

The role of correlations between individual and social well-being as an important factor in evaluations is a key contribution to the refinement of the Ramsey formula, that may reverse the conclusions of Weitzman's dismal theorem. Benefiting an individual who is badly off when the population is well off has a greater impact on social welfare, on average, than benefitting an individual who is badly off when the population is also badly off. This may seem disturbing because it seems to give a bonus to the states of the world in which the population is relatively well off. This occurs, however, only in the very special trade-off between helping a poor with a positive correlation with social welfare and a poor with a negative correlation. But most policy issues affect broader populations. Suppose one invests in a public good that is useful mostly in bad states (e.g., flood protection). When a bad state occurs, the investment benefits more individuals who are badly off. Even if the correlation between their well-being and social welfare is high, the fact that the investment benefits many badly off individuals may be sufficient to give it a greater social value than a similar investment that would create a public good suited to good states (e.g., a new transportation infrastructure).

Concerning the effect of inequality aversion on social discounting, it is known that inequality aversion increases discounting when future generations are better-off. It is also known that when growth is uncertain, and there is a substantial risk of future generations being less well-off, a higher inequality aversion can on the contrary decrease the discount rate. Our more general approach adds that, if the investment helps the most vulnerable in future generations, inequality aversion further decreases the discount rate. In addition, inequality aversion magnifies the effect of the correlation on discounting when future consumption is uncertain.

In the end, this paper provides reasons to think that the specific features of climate policies may justify evaluating them with a lower discount rate than other policies. Indeed, they protect the vulnerable, whose fate may be inversely correlated to that of the rich, and they provide more benefits in states of the world in which damages hit the poorest. Further research is however needed to substantiate those intuitions. It would require a more precise description of the uncertainty (on consumption and the existence of future generations) as well as good scenarios describing the costs and benefits. Moreover, in order to assess climate policies, one may also go beyond the discount rate and evaluate the changes in the risks they induce, their non-marginal effects and their precise impact.

Another direction of research that we intend to pursue is to enrich the framework further so as to make it possible to discuss the measurement of individual well-being. In this paper the measurement of utility has been treated as exogenous. A more concrete description of the economic allocations would enable us to further specify the social evaluation criteria in relation to principles of fairness, and to provide more concrete indications for applications to the assessment of integrated scenarios describing the long-term evolution of the climate and the economy. In particular, the relative prices of different commodities (environmental goods vs consumption goods) change with time, yielding different discount rates or different valuations of climate damages (Sterner and Persson, 2008; Gollier, 2010). It may be important to take into account the relative scarcity of some goods when evaluating the welfare of future generations.

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Appendix

Appendix A: Proof of Theorem 1

It is easily checked that EPEDE social orderings do satisfy Axioms 1–5.

The proof that Axioms imply that the social ordering is an EPEDE social ordering proceeds in 3 steps.

Step 1: the social ordering restricted to same-population alternatives is additively separable and concave. For any $\mathcal{N} \in \mathfrak{N}$ such that $|\mathcal{N}| \geq 3$, define the ordering $\bar{R}_{\mathcal{N}}$ on $X^{|\mathcal{N}|}$ as follows.

Define r the mapping $r : \mathcal{N} \to \{1, \dots, |\mathcal{N}|\}$ such that for all $i, j \in \mathcal{N}$ $r(i) > r(j) \iff i > j$.[14] For any $(x_1, \dots, x_{|\mathcal{N}|}), (y_1, \dots, y_{|\mathcal{N}|}) \in X^{|\mathcal{N}|}$, any $\mathbf{p} \in \mathbf{P}$ and any $u, v \in \mathcal{U}$ such that $n(u) = n(v) = \mathcal{N}$ and for all $i \in \mathcal{N}$, $u^i = x_{r(i)}$ and $v^i = x_{r(i)}$,[15]

$$(x_1, \dots, x_{|\mathcal{N}|})\bar{R}_{\mathcal{N}}(y_1, \dots, y_{|\mathcal{N}|}) \iff (\mathbf{p}, u)R(\mathbf{p}, v). \tag{9}$$

By Axiom 1, the relation $\bar{R}_{\mathcal{N}}$ is transitive, reflexive, complete and continuous. By Axiom 3, the relation is monotonic. By Axiom 5, any subset of $\mathcal{N}$ is separable. Therefore, by Theorem 3 in Debreu (1960), as $|\mathcal{N}| \geq 3$, there exist continuous and increasing functions $\phi^i_{\mathcal{N}}$ such that, for all $(x_1, \dots, x_{|\mathcal{N}|}), (y_1, \dots, y_{|\mathcal{N}|}) \in X^{|\mathcal{N}|}$,

$$(x_1, \dots, x_{|\mathcal{N}|})\bar{R}_{\mathcal{N}}(y_1, \dots, y_{|\mathcal{N}|}) \iff \sum_{i=1}^{|\mathcal{N}|} \phi^i_{\mathcal{N}}(x_i) \geq \sum_{i=1}^{|\mathcal{N}|} \phi^i_{\mathcal{N}}(y_i). \tag{10}$$

By Lemma 2 in Fleurbaey and Zuber (2013), because the social ordering $\bar{R}_{\mathcal{N}}$ satisfies the Pigou-Dalton transfer principle (given that R satisfies Axiom 2), it must be the case that the $\phi^i_{\mathcal{N}}$ functions in Eq. (10) are all identical and concave. Hence given equivalence (9), we obtain that there exists a continuous, increasing and concave function $\phi_{\mathcal{N}}$ such that, for all $\mathbf{p} \in \mathbf{P}$ and all $u, v \in \mathcal{U}$ such that $n(u) = n(v) = \mathcal{N}$:

$$(\mathbf{p}, u)R(\mathbf{p}, v) \iff \sum_{i \in \mathcal{N}} \phi_{\mathcal{N}}(u^i) \geq \sum_{i \in \mathcal{N}} \phi_{\mathcal{N}}(v^i). \tag{11}$$

Step 2: The fonction $\phi_{\mathcal{N}}$ in Eq. (11) does not depend on $\mathcal{N}$.

Denote $\mathcal{N}_0 \in \mathfrak{N}$ the set $\mathcal{N}_0 = \{1, 2, 3\}$. Consider $\mathcal{N} \in \mathfrak{N}$ such that $|\mathcal{N}| \geq 3$ and let $\mathcal{M} \in \mathfrak{N}$ be any set such that $|\mathcal{M}| = 3$, $\mathcal{M} \subset \mathcal{N}$. Let $\pi : \mathcal{N}_0 \to \mathcal{M}$. Let $u, v, u', v', \bar{u}, \bar{v}, \widetilde{u}, \widetilde{v} \in \mathcal{U}$ have the following characteristics:

1. $n(u) = n(v) = n(u') = n(v') = \mathcal{N}_0 \cup \mathcal{N}$, $n(\bar{u}) = n(\bar{v}) = \mathcal{N}_0$, $n(\widetilde{u}) = n(\widetilde{v}) = \mathcal{N}$;
2. $u^i = \bar{u}^i$ and $v^i = \bar{v}^i$ for all $i \in \mathcal{N}_0$;
3. $u'^i = \widetilde{u}^i$ and $v'^i = \widetilde{v}^i$ for all $i \in \mathcal{N}$;

[14]The mapping r provides a ranking of individuals in $\mathcal{N}$ that corresponds to their ranking in the set of potential individuals $\mathbb{N}$.

[15]The choice of the probability $\mathbf{p}$ does not matter for the definition of $\bar{R}_{\mathcal{N}}$ because, by Axiom 1, for all $u, v \in \mathcal{U}$ and any $p, q \in \mathbf{P}$, $(\mathbf{p}, u)R(\mathbf{p}, v) \iff (\mathbf{q}, u)R(\mathbf{q}, v)$.

4. $u^i = v^i$ for all $i \in \mathcal{N}$ and $u'^i = v'^i$ for all $i \in (\mathcal{N} \setminus \mathcal{M}) \cup \mathcal{N}_0$.

5. $u^i = u'^j$, $v^i = v'^j$, $u'^i = u^j$ and $v'^i = v^j$ for all $i \in \mathcal{N}_0$ and $j \in \mathcal{M}$ such that $j = \pi(i)$; and $u^k = u'^k$, $v^k = v'^k$, for all $k \in \mathcal{N} \setminus \mathcal{M}$.

Hence u' is just a permutation of elements in u, v' a permutation of elements in u'. Thus, by the (symmetric) representation in Eq. (11), we know that $(\mathbf{p}, u)R(\mathbf{p}, v) \Longleftrightarrow (\mathbf{p}, u')R(\mathbf{p}, v')$.

In addition, $u, v, \bar{u}, \bar{v}$ satisfy the conditions of Axiom 5, so that $(\mathbf{p}, u)R(\mathbf{p}, v) \Longleftrightarrow (\mathbf{p}, \bar{u})R(\mathbf{p}, \bar{v})$. Similarly, $u', v', \widetilde{u}, \widetilde{v}$ satisfy the conditions of Axiom 5, so that $(\mathbf{p}, u')R(\mathbf{p}, v') \Longleftrightarrow (\mathbf{p}, \widetilde{u})R(\mathbf{p}, \widetilde{v})$.

So, in the end, we have that $(\mathbf{p}, \bar{u})R(\mathbf{p}, \bar{v}) \Longleftrightarrow (\mathbf{p}, \widetilde{u})R(\mathbf{p}, \widetilde{v})$. Using Eq. (11) and the construction of $\bar{u}, \bar{v}, \widetilde{u}, \widetilde{v}$, this implies that:

$$\begin{aligned} \sum_{i \in \mathcal{N}_0} \phi_{\mathcal{N}_0}(u^i) \geq \sum_{i \in \mathcal{N}_0} \phi_{\mathcal{N}_0}(v^i) \quad &\Longleftrightarrow \quad \sum_{i \in \mathcal{M}} \phi_{\mathcal{N}}(u'^i) \geq \sum_{i \in \mathcal{M}} \phi_{\mathcal{N}}(v'^i) \\ &\Longleftrightarrow \quad \sum_{i \in \mathcal{N}_0} \phi_{\mathcal{N}}(u^i) \geq \sum_{i \in \mathcal{N}_0} \phi_{\mathcal{N}}(v^i). \end{aligned}$$

By Debreu (1960, Theorem 3), we know that additively separable representations of an ordering over a product of connected separable sets are unique up to positive affine transformations. Hence, there must exist $a_{\mathcal{N}} \in \mathbb{R}_+$ and $b_{\mathcal{N}} \in \mathbb{R}$ such that $\phi_{\mathcal{N}}(x) = a_{\mathcal{N}} \phi_{\mathcal{N}_0}(x) + b_{\mathcal{N}}$ for all $x \in X$.

Denoting $\phi := \phi_{\mathcal{N}_0}$, we therefore obtain that, for all $\mathbf{p} \in \mathbf{P}$ and any $u, v \in \mathcal{U}$ such that $n(u) = n(v) = \mathcal{N}$, with $|\mathcal{N}| \geq 3$:

$$(\mathbf{p}, u)R(\mathbf{p}, v) \Longleftrightarrow \sum_{i \in \mathcal{N}} \phi(u^i) \geq \sum_{i \in \mathcal{N}} \phi(v^i). \tag{12}$$

If $|\mathcal{N}| < 3$, Axiom 5 yields a similar result. Indeed consider any $\mathcal{N} \in \mathfrak{N}$ such that $|\mathcal{N}| < 3$, and any $u, v \in \mathcal{U}$ such that $\mathcal{N}(u) = \mathcal{N}(v) = \mathcal{N}$. Then take any $\mathcal{M} \in \mathfrak{N}$ such that $\mathcal{N} \subset \mathcal{M}$ and construct $\bar{u}, \bar{v} \in \mathcal{U}$ such that $\mathcal{N}(\bar{u}) = \mathcal{N}(\bar{v}) = \mathcal{M}$, $\bar{u}^i = u^i$ and $\bar{v}^i = v^i$ for all $i \in \mathcal{N}$ and $\bar{u}^j = \bar{v}^j$ for all $j \in \mathcal{M} \setminus \mathcal{N}$. By Axiom 5, $(\mathbf{p}, u)R(\mathbf{p}, v) \Longleftrightarrow (\mathbf{p}, \bar{u})R(\mathbf{p}, \bar{v})$. By representation (12), $(\mathbf{p}, \bar{u})R(\mathbf{p}, \bar{v}) \Longleftrightarrow \sum_{i \in \mathcal{N}} \phi(\bar{u}^i) \geq \sum_{i \in \mathcal{N}} \phi(\bar{v}^i)$. Hence, using the definition of $\bar{u}$ and $\bar{v}$,

$$(\mathbf{p}, u)R(\mathbf{p}, v) \Longleftrightarrow \sum_{i \in \mathcal{N}} \phi(u^i) \geq \sum_{i \in \mathcal{N}} \phi(v^i).$$

Step 3: The social ordering is an EPEDE social ordering.

Let us define $E : \mathcal{U} \to X$ the mapping such that, for all $u \in \mathcal{U}$,

$$E(u) = \phi^{-1}\left(\frac{1}{n(u)} \sum_{i \in \mathcal{N}(u)} \phi(u^i)\right). \tag{13}$$

For any $\mathbf{u} \in \mathbf{U}$, define $\mathbf{u}_e \in \mathbf{U}$ the prospect such that, for all $s \in \mathcal{S}$,

- $\mathcal{N}\big(\mathbf{u}_e(s)\big) = \mathcal{N}\big(\mathbf{u}(s)\big)$, and
- $\mathbf{u}_e^i(s) = E\big(\mathbf{u}(s)\big)$ for all $i \in \mathcal{N}\big(\mathbf{u}_e(s)\big)$.

The prospect $\mathbf{u}_e$ is therefore an egalitarian prospect such that, in each state of the world, all existing individuals are provided with the equally distributed equivalent level of welfare. By definition of function E and Eq. (12), we know that $(\mathbf{p}, \mathbf{u}(s))I(\mathbf{p}, \mathbf{u}_e(s))$ for all $s \in \mathcal{S}$, so that, by Axiom 1, $V\big(\mathbf{u}(s)\big) = V\big(\mathbf{u}_e(s)\big)$.

Consider any $\mathbf{p}, \mathbf{q} \in \mathbf{P}$, any $\mathcal{N} \in \mathfrak{N}$ and any $\mathbf{u}, \mathbf{v} \in \mathbf{U}_{\mathcal{N}}$. Using Axiom 1, together with the fact that $V\big(\mathbf{u}(s)\big) = V\big(\mathbf{u}_e(s)\big)$ and $V\big(\mathbf{v}(s)\big) = V\big(\mathbf{v}_e(s)\big)$ for all $s \in \mathcal{S}$, and that $\mathbf{u}_e$ and $\mathbf{v}_e$ satisfy the conditions of Axiom 4, we obtain the following equivalences:

$$\begin{aligned}
(\mathbf{p}, \mathbf{u})R(\mathbf{q}, \mathbf{v}) &\iff \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big) && \text{(Axiom 1)}\\
&\iff \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u}_e)\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v}_e)\big) \\
&\iff \forall i \in \mathcal{N}, \quad \mathbb{E}_{\mathbf{p}}(\mathbf{u}_e^i) \geq \mathbb{E}_{\mathbf{q}}(\mathbf{v}_e^i) && \text{(Axiom 4)}\\
&\iff \mathbb{E}_{\mathbf{p}}\big(E(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(E(\mathbf{v})\big) && \text{(Definition of } \mathbf{u}_e, \mathbf{v}_e)
\end{aligned}$$

Von Neumann Morgenstern utility functions are unique up to an increasing affine transformation. Hence the equivalence $\mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{v})\big) \iff \mathbb{E}_{\mathbf{p}}\big(E(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{p}}\big(E(\mathbf{v})\big)$ implies that there must exist $\alpha_{\mathcal{N}} \in \mathbb{R}_+$ and $\beta_{\mathcal{N}} \in \mathbb{R}$ such that $V(u) = \alpha_{\mathcal{N}} E(u) + \beta_{\mathcal{N}}$ for all $u \in \mathcal{U}$ such that $\mathcal{N}(u) = \mathcal{N}$.

Now, consider any $\mathcal{N}, \mathcal{M} \in \mathfrak{N}$ such that $|\mathcal{N}| = |\mathcal{M}| = n$, and any $x \in Int(X)$, where $Int(X)$ is the interior of set X. Let $u \in \mathcal{U}_{\mathcal{N}}$ be such that there exists $i, j \in \mathcal{N}$ and $\varepsilon > 0$ satisfying $u^i = x - \varepsilon$, $u^j = x + \varepsilon$ and $u^k = x$ for all $k \in \mathcal{M} \setminus \{i, j\}$. Let $v \in \mathcal{U}_{\mathcal{N}}$ be such that there exists $i', j' \in \mathcal{M}$ and $\varepsilon > 0$ satisfying $v^{i'} = x - 2\varepsilon$, $v^{j'} = x + 2\varepsilon$ and $v^{k'} = x$ for all $k' \in \mathcal{M} \setminus \{i, j\}$. By Axiom 2, $V(x \cdot \mathbb{1}_{\mathcal{M}}) > V(u) > V(v)$, so that, using the representation above:

$$\alpha_{\mathcal{M}} x + \beta_{\mathcal{M}} > \alpha_{\mathcal{N}} \phi^{-1}\left(\frac{n-2}{n}\phi(x) + \frac{1}{n}\phi(x+\varepsilon) + \frac{1}{n}\phi(x-\varepsilon)\right) + \beta_{\mathcal{N}}$$
$$> \alpha_{\mathcal{M}} \phi^{-1}\left(\frac{n-2}{n}\phi(x) + \frac{1}{n}\phi(x+2\varepsilon) + \frac{1}{n}\phi(x-2\varepsilon)\right) + \beta_{\mathcal{M}}$$

By continuity of the function ϕ, and letting $\varepsilon \to 0$, we obtain $\alpha_{\mathcal{N}}\phi(x)+\beta_{\mathcal{N}} = \alpha_{\mathcal{M}}\phi(x)+\beta_{\mathcal{M}}$. This is true of any $x \in Int(X)$, which implies that it must be the case that $\alpha_{\mathcal{N}} = \alpha_{\mathcal{M}}$ and $\beta_{\mathcal{N}} = \beta_{\mathcal{M}}$. This is true for any $\mathcal{N}, \mathcal{M} \in \mathfrak{N}$ such that $|\mathcal{N}| = |\mathcal{M}|$, hence we can write (α_n, β_n) instead of $(\alpha_{\mathcal{N}}, \beta_{\mathcal{N}})$ whenever $|\mathcal{N}| = n$.[16]

To sum up, for all $(\mathbf{p},\mathbf{u}),(\mathbf{q},\mathbf{v}) \in \mathbf{P}\times\mathbf{U}$

$$(\mathbf{p},\mathbf{u})R(\mathbf{q},\mathbf{v}) \Longleftrightarrow \mathbb{E}_{\mathbf{p}}\big(V(\mathbf{u})\big) \geq \mathbb{E}_{\mathbf{q}}\big(V(\mathbf{v})\big)$$

with

$$V(u) = \alpha_{n(u)}\phi^{-1}\left(\frac{1}{n(u)}\sum_{i\in\mathcal{N}(u)}\phi\big(u^i\big)\right) + \beta_{n(u)}, \qquad \forall u \in \mathcal{U}.$$

Appendix B Proof of Proposition 1

Axioms 1–5 imply that the social ordering is an EPEDE social ordering.

Let $\mathcal{N} \in \mathfrak{N}$ be such that $|\mathcal{N}| = n$. For any $x \in X$, let $\mathcal{M} \in \mathfrak{N}_3$ be such that $\mathcal{M} = \mathcal{N}\cup\{k\}$ (where $k \in \mathbb{N}\backslash\mathcal{N}$) and $v \in \mathcal{U}$ be such that $\mathcal{N}(v) = \mathcal{M}$ and $v^i = x$ for all $i \in \mathcal{N}$ and $v^k = c_n$, as defined by Axiom 6. By Axiom 6, for all $\mathbf{p} \in \mathbf{P}$, $(\mathbf{p}, x\cdot\mathbb{1}_{\mathcal{N}})I(\mathbf{p}, v)$. Using the representation of EPEDE social criteria, this implies that

$$\phi\left(\tfrac{\alpha_n}{\alpha_{n+1}}x + \tfrac{\beta_n-\beta_{n+1}}{\alpha_{n+1}}\right) = \frac{n}{n+1}\phi(x) + \frac{1}{n+1}\phi(c_n). \tag{14}$$

Eq. (14) holds for all $x \in X$. Letting $a = \frac{\alpha_n}{\alpha_{n+1}} > 0$, $b = \frac{\beta_n-\beta_{n+1}}{\alpha_{n+1}}$, this yields the following functional equation:

$$\phi\left(ax+b\right) = \frac{n}{n+1}\phi(x) + \frac{1}{n+1}\phi(c_n), \quad \forall x \in X. \tag{15}$$

The equation implies $\phi\left(ac_n+b\right) = \phi(c_n)$, so that $ac_n + b = c_n$.

[16]The above reasoning applies only for $n \geq 2$. The case $n = 1$ can be treated by using a full fleshed Anonymity axiom. Such an axiom has been omitted to avoid slowing down the exposition of the axiomatic characterization. Alternatively, it may seem acceptable to focus on cases where $n \geq 2$.

Let $f : X \to \mathbb{R}$ be defined by $f(x) = \phi(x + c_n) - \phi(c_n)$ for all $x \in X$. Using Eq. (15) and $ac_n + b = c_n$, one obtains, for all $x \in X$:

$$\begin{aligned} f(ax) &= \phi(ax + c_n) - \phi(c_n) \\ &= \phi(a(x + c_n) + b) - \phi(c_n) \\ &= \frac{n}{n+1}\phi(x + c_n) + \frac{1}{n+1}\phi(c_n) - \phi(c_n) \\ &= \frac{n}{n+1}f(x). \end{aligned}$$

The general solution to $f(ax) = \frac{n}{n+1}f(x)$ for all $x \in X$ is (Polyanin and Manzhirov, 2007): $f(x) = \Theta(x)\,|x|^{\omega}$, for all $x \in X$, and $a^{\omega} = \frac{n}{n+1}$, where $\Theta(z)$ is an arbitrary periodic solution to the functional equation $\Theta(az) = \Theta(z)$, such that Θ is continuous (except possibly at 0).

By definition of f, one therefore has $\phi(x) = f(x-c_n)+\phi(c_n) = \Theta(x-c_n)\,|x - c_n|^{\omega}+\phi(c_n)$ for all $x \in X$. The case $z = c_n$ requires that $\omega \geq 0$. The fact that ϕ is increasing and concave implies that Θ is constant for $x > 0$ and $x < 0$ with $\Theta(z - c_n) = \bar{\Theta} > 0$ if $x > c_n$ and $\Theta(z - c_n) = \underline{\Theta} < 0$ if $x < c_n$. In addition, if there is $x \in X$ such that $x > c_n$, the concavity of ϕ imposes $\omega < 1$. If there is $x \in X$ such that $x < c_n$, the concavity of ϕ imposes $\omega > 1$. Therefore, only two cases are possible: 1) $0 < \omega < 1$ and for all $x \in X$, $x \geq c_n$, $\bar{\Theta} > 0$, and $c_n = \underline{x}$, or for all $x \in X$, $x \leq c_n$, $\underline{\Theta} < 0$, and $c_n = \bar{x}$.

Consider the first case, that is ϕ is such that $\phi(x) = \bar{\Theta}(x - \bar{x})^{\omega}$ for all $x \in X$, with $\bar{\Theta} > 0$ and $0 < \omega < 1$. Let $\epsilon = 1 - \omega$. The fact that $a^{\omega} = \frac{n}{n+1}$ in Eq. (15) implies that $\alpha_{n+1} = ((n+1)/n)^{1/1-\epsilon}\alpha_n = (n+1)^{1/1-\epsilon}\chi$, where $\chi = \alpha_2/(2^{1/1-\epsilon})$. We also know that $ac_n + b = c_n$ so that $\beta_n - \beta_{n+1} = (\alpha_{n+1} - \alpha_n)c_n = (\alpha_{n+1} - \alpha_n)\underline{x}$. A sum of such expressions yields $\beta_2 - \beta_{n+1} = (\alpha_{n+1} - \alpha_2)\underline{x}$ so that $\beta_{n+1} = -\alpha_{n+1}\underline{x} + \zeta$ where $\zeta = \beta_2 + \alpha_2\underline{x}$. Hence, for all $u \in \mathcal{U}$ such that $n(u) = n$, we obtain:

$$\begin{aligned} V(u) = E(u) = \alpha_n \phi^{-1}\left(\tfrac{1}{n}\sum_{i\in\mathcal{N}(u)} \phi(u^i)\right) + \beta_n &= \alpha_n\left(\left(\tfrac{1}{n}\Big(\sum_{i\in\mathcal{N}(u)} \left(u^i - \underline{x}\right)^{1-\epsilon}\Big)\right)^{\frac{1}{1-\epsilon}} + \underline{x}\right) \\ &\quad -\alpha_n \underline{x} + \zeta \\ &= \chi n^{1/1-\epsilon}\left(\tfrac{1}{n}\sum_{i\in\mathcal{N}(u)} \left(u^i - \underline{x}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} + \zeta \\ &= \chi\left(\sum_{i\in\mathcal{N}(u)} \left(u^i - \underline{x}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} + \zeta. \end{aligned}$$

The case where ϕ is such that $\phi(x) = \underline{\Theta}(\underline{x} - x)^{\omega}$ for all $x \in X$ (with $\underline{\Theta} < 0$ and $1 < \omega$) can be treated similarly.

Appendix C Proof of Proposition 2

If the social ordering R satisfies Axioms 1, 2, 3, 4 and 5, we know by Theorem 1 that it is an EPEDE social ordering.

The social ordering R also satisfies Axiom 7. Using the representation of EPEDE social orderings, this means that there exists $c \geq 0$ such that for all $x \in X$:

- if $x < c$ then $\alpha_n x + \beta_n > \alpha_{n+1} x + \beta_{n+1}$ for all $n \in \mathbb{N}$;
- if $x \geq c$ then $\alpha_{n+1} x + \beta_{n+1} \geq \alpha_n x + \beta_n$ for all $n \in \mathbb{N}$.

Hence, for all $x < c$, we have that

$$\alpha_{n+1} x + \beta_{n+1} < \alpha_n x + \beta_n < \alpha_n c + \beta_n \leq \alpha_{n+1} c + \beta_{n+1}.$$

We have that $\lim_{x\to c} \alpha_{n+1} x + \beta_{n+1} = \alpha_{n+1} c + \beta_{n+1}$, and therefore it must be the case that $\alpha_n c + \beta_n = \alpha_{n+1} c + \beta_{n+1}$.

For all $n \geq 3$, we thus obtain

$$(\alpha_n - \alpha_{n-1})c = \beta_{n-1} - \beta_n.$$

When $n \geq 3$, we can compute $(\alpha_n - \alpha_2)c = \sum_{k=3}^{n} (\alpha_k - \alpha_{k-1})c = \sum_{k=4}^{n} = \beta_{k-1} - \beta_k = \beta_2 - \beta_n$. Normalizing (without loss of generality) $\beta_2 = -\alpha_2 c$, we have $\beta_n = -\alpha_n c$ for all $n \geq 2$.

An implication of this result is that

$$\alpha_n x + \beta_n < \alpha_{n+1} x + \beta_{n+1} \Longleftrightarrow \big(\alpha_{n+1} - \alpha_n\big)(x - c) > 0.$$

Thus, for Axiom 7 to hold, we need $\alpha_{n+1} > \alpha_n$ for all $n \geq 2$.

Assume that for some $x \in X$ there exist $z \in X$ and $\mathcal{N}$ such that for $u \in \mathcal{U}$ satisfying the following conditions

- $\mathcal{N}(u) = \mathcal{N} \cup \{k\}$;
- $u^i = x$ for all $i \in \mathcal{N}$;
- $u^k = z$;

we have $(\mathbf{p}, x \cdot 1_{\mathcal{N}}) I(\mathbf{p}, u)$, for all $\mathbf{p} \in \mathbf{P}$. Letting $n = |\mathcal{N}|$ and using the above results, it must be the case that:

$$\phi\Big(\frac{\alpha_n}{\alpha_{n+1}} x + \frac{\alpha_{n+1} - \alpha_n}{\alpha_{n+1}} c\Big) - \frac{n}{n+1}\phi(x) = \frac{1}{n+1}\phi(z). \tag{16}$$

For $x = c$, it is clear that we must have $z = c$. The left hand-side of Eq. (16) is continuous in x and the right-hand side is continuous in z. So there must exist an interval $\bar{X}$ included in a neighborghood of c such for all $x \in \bar{X}$, $x < c$ and there exist $z \in X$ such that Eq. (16) is satisfied. Furthermore, ϕ is concave over an open set and thus is differentiable, except on a countable set (See for instance Nicolescu and Persson, 2006, Theorem 1.3.7 p. 23).

Axiom 8 then implies that the left-hand side of Eq. (16) is increasing in x over $\bar{X}$. This implies

$$\frac{\alpha_n}{\alpha_{n+1}}\phi'\Big(\frac{\alpha_n}{\alpha_{n+1}} x + \frac{\alpha_{n+1} - \alpha_n}{\alpha_{n+1}} c\Big) > \frac{n}{n+1}\phi'(x).$$

But, given that $x < c$, we also know that $\frac{\alpha_n}{\alpha_{n+1}} x + \frac{\alpha_{n+1}-\alpha_n}{\alpha_{n+1}} c < x$. By concavity of ϕ, $\phi'\Big(\frac{\alpha_n}{\alpha_{n+1}} x + \frac{\alpha_{n+1}-\alpha_n}{\alpha_{n+1}} c\Big) < \phi'(c)$, and therefore

$$\frac{\alpha_n}{\alpha_{n+1}}\phi'(x) > \frac{n}{n+1}\phi'(x).$$

This implies that $\frac{\alpha_n}{n} > \frac{\alpha_{n+1}}{n+1}$.