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The Simple Analytics of Helicopter Money: Why It Works – Always

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Abstract

The author provides a rigorous analysis of Milton Friedman's parable of the 'helicopter' drop of money – a permanent/irreversible increase in the nominal stock of fiat base money which respects the intertemporal budget constraint of the consolidated Central Bank and Treasury – the State. Examples are a temporary fiscal stimulus funded permanently through an increase in the stock of base money and permanent QE – an irreversible, monetised open market purchase by the Central Bank of non-monetary sovereign debt. Three conditions must be satisfied for helicopter money always to boost aggregate demand. First, there must be benefits from holding fiat base money other than its pecuniary rate of return. Second, fiat base money is irredeemable – viewed as an asset by the holder but not as a liability by the issuer. Third, the price of money is positive. Given these three conditions, there always exists a combined monetary and fiscal policy action that boosts private demand – in principle without limit. Deflation, 'lowflation' and secular stagnation are therefore unnecessary. They are policy choices.

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Keywords Helicopter money; liquidity trap; seigniorage; secular stagnation; central bank; quantitative easing

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The views and opinions expressed are those of the author alone and should not be attributed to Citigroup or to any other organization the author is associated with. The author would like to thank Larry Summers from prodding him to write a short(ish) note setting out the essence of the helicopter money argument.

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1 Introduction

"Let us suppose now that one day a helicopter flies over this community and drops an additional $1000 in bills from the sky, ... Let us suppose further that everyone is convinced that this is a unique event which will never be repeated," (Friedman [1969, pp 4-5].

This paper aims to provide a rigorous analysis of Milton Friedman's famous parable of the 'helicopter' drop of money. A helicopter drop of money is a permanent/irreversible increase in the nominal stock of fiat base money with a zero nominal interest rate, which respects the intertemporal budget constraint of the consolidated Central Bank and fiscal authority/Treasury—henceforth the State. An example would be a temporary fiscal stimulus (say a one-off transfer payment to households, as in Friedman's example), funded permanently through an increase in the stock of base money. It could also be a permanent increase in the stock of base money through an irreversible open market purchase by the Central Bank of non-monetary sovereign debt held by the public—that is, QE. The reason is that QE, viewed as an irreversible or permanent purchase of non-monetary financial assets by the Central Bank funded through an irreversible or permanent increase in the stock of base money, relaxes the intertemporal budget constraint of the State. Consequently, there will have to be some combination of current and future tax cuts or current and future increases in public spending to ensure that the intertemporal budget constraint of the State remains satisfied. QE relaxes the intertemporal budget constraint of the consolidated Central Bank and Treasury either if nominal interest rates are positive or because fiat base money is irredeemable. In our simple model, QE is the irreversible purchase by the Central Bank of sovereign debt funded through irreversible base money issuance. The same results would hold, however, if the Central Bank purchased private securities outright instead of sovereign debt, or expanded its balance sheet through collateralized lending.

There are three conditions that must be satisfied for helicopter money as defined here to always boost aggregate demand. First, there must be benefits from holding fiat base money other than its pecuniary rate of return. Only then will base money be willingly held despite being dominated as a store of value by non-monetary assets with a positive risk-free nominal interest rate. Second, fiat base money is irredeemable: it is view as an asset by the holder but not as a liability by the issuer. This is necessary for helicopter money to work even in a pure liquidity trap, with risk-free nominal interest rates at zero for all maturities. Third, the price of money is positive.

The paper shows that, when the State can issue unbacked, irredeemable fiat money or base money with a zero nominal interest rate, which can be produced at zero marginal cost and is held in positive amounts by households and other private agents despite the availability of risk-free securities carrying a positive nominal interest rate, there always exists a combined monetary and fiscal policy action that boosts private demand—in principle without limit. Deflation, inflation below target, 'lowflation', 'subflation' and the deficient demand-driven version of secular stagnation are therefore unnecessary.[1] They are policy choices.

[1] The term 'lowflation' is, I believe, due to Moghadam, Teja and Berkmen (2014). The term 'subflation' has been around the blogsphere for a while. I use it to refer to an inflation rate below the target level or lower than is optimal. 'Secular stagnation' theories go back to Alvin Hansen (1938). I refer here to the Keynesian variant, which holds that there will be long-term stagnation of employment and economic activity without government demand-side intervention. There also is a long-term supply side variant, associated e.g. with Robert Gordon (2014), which focuses on faltering innovation and productivity growth. Larry Summers (2013) marries the demand-side and

The feature of irredeemable base money that is key for this paper is that the acceptance of payment in base money by the government to a private agent constitutes a final settlement between that private agent (and any other private agent with whom he exchanges that base money) and the government. It leaves the private agent without any further claim on the government, now or in the future.

The helicopter money drop effectiveness issue is closely related to the question as to whether State-issued fiat money is net wealth for the private sector, despite being technically an 'inside asset', where for every creditor that holds the asset there is a debtor who owes a claim of equal value (see Patinkin (1956/1965), Gurley and Shaw (1960) and Pesek and Saving (1967)), Weil (1991). The discussions in Hall (1983), Stockman (1983), King (1983), Fama (1983), Sargent and Wallace (1984), Sargent (1987) and Weil (1991) of outside money, private money and the payment of interest on money ask some of the same questions as this paper, but do not offer the same answer, because they don't address the irredeemability of fiat base money. Sims (2000, 2003), Buiter (2003a,b), Eggertson (2003) and Eggertsson and Woodford (2003) all stress that to boost demand in a liquidity trap, base money increases should not be, or expected to be, reversed. None of these papers recognised that even a permanent increase in the stock of base money will not have an expansionary wealth effect in a permanent liquidity trap unless money is irredeemable in the sense developed here; without this, there is no real balance effect in a permanent liquidity trap. Ben Bernanke spent years living down the moniker "helicopter Ben" which he acquired following a (non-technical) discussion of helicopter money (Bernanke 2003). The issue has also been revisited by Buiter (2003, 2007) and, in an informal manner, by Turner (2013), by Reichlin, Turner and Woodford (2013).

The paper shows that, because of its irredeemability, State-issued fiat money is indeed net wealth to the private sector, in a very precise way: the initial stock of base money plus the present discounted value of all future net base money issuance is net wealth, an 'outside' asset to the private sector, even after the intertemporal budget constraint of the State (which includes the Central Bank) has been consolidated with that of the household sector.

The paper also demonstrates that fiat base money issuance is effective in boosting household demand regardless of whether there is Ricardian equivalence (debt neutrality).

2 The model

All important aspects of how helicopter money drops work and what makes helicopter money unique can be established without the need for a complete dynamic general (dis)equilibrium model. All that is needed is a complete specification of the choice process of the household sector in a monetary economy, the period budget identity and solvency constraint of the consolidated general government/Treasury and Central Bank—the State - and the no-arbitrage conditions equating (in principle risk-adjusted) returns on all non-monetary stores of value and constraining the instantaneous nominal interest rate to be non-negative.

I shall show that, as long as the price of money is positive, the issuance of fiat base money can boost household consumption demand *by any amount*, given the inherited stocks of financial

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supply-side secular stagnation approaches by invoking a number of hysteresis mechanisms. For a formal model see Eggertson and Mehrotra (2014).

and real assets, given current and future wages and prices, and given current and future values of public spending on goods and services. Whether such helicopter money drops change asset prices and interest rates, goods prices, wages and/or output and employment depends on the specification of the rest of the model of the economy—including, in more general models, the behavior of the financial sector and of non-financial businesses in driving investment demand, production and labor demand, the rest of the 'supply side' of the economy and the rest of the world, if the economy is open. The point of this paper is to show that, whatever the equilibrium configuration we start from, helicopter money drops will boost household demand and must disturb that equilibrium. What 'gives' ultimately, in a fully articulated dynamic general equilibrium model, is not my concern.

The model of household behavior I use is as stripped-down and simple as I can make it without raising concerns that the key results will not carry over to more general and intricate models. The continuous-time Yaari-Blanchard version of the OLG model is used to characterize household behavior (see Yaari (1965), Blanchard (1985), Buiter (1988) and Weil (1989)). This model with its easy aggregation and its closed-form aggregate consumption function includes the conventional (infinite-lived) representative agent model as a special case (when the birth rate is zero). With a positive birth rate, there is no Ricardian equivalence or debt neutrality in the Yaari-Blanchard model. With a zero birth rate there is Ricardian equivalence. This permits me to show that helicopter money drops boost household demand regardless of whether there is Ricardian equivalence or not. Apart from the uncertain lifetime that characterized households in the Yaari-Blanchard model (which plays no role either in Ricardian equivalence or the effectiveness of helicopter money drops), the model has no uncertainty. To save on notation I consider a closed economy.

2.1 The household sector

We consider the household and government sectors of a simple closed economy. The holding of intrinsically worthless fiat base money is motivated through a 'money-in-the-direct utility function' approach, but alternative approaches to making money essential (cash-in-advance, legal restrictions, money-in-the transactions-function or money-in-the production function, say) would work also. For expository simplicity, there is only private capital. The helicopter money we discuss could, however, be used equally well to fund government investment programs as tax cuts or transfer payments that benefit households, or boost to current exhaustive public spending.

2.1a Individual household behavior

At each time $t \geq 0$, a household born at time $s \leq t$ maximizes the following utility functional:

$$\max E_t \int_t^{\infty} e^{-\theta(v-t)} \ln\left[\overline{c}(s,v)^{\alpha}\left(\frac{\overline{m}(s,v)}{P(v)}\right)^{1-\alpha}\right]dv$$

$$\{\overline{c}(s,v),\overline{m}(s,v),\overline{b}(s,v),\overline{k}(s,v); s \leq t, v \geq t\} \tag{1}$$

$$\overline{c}(s,v),\overline{m}(s,v) \geq 0, \theta > 0, 0 < \alpha < 1$$

where E_t is the conditional expectation operator at time t, $\theta > 0$ is the pure rate of time preference, $\overline{c}(s,v)$ is consumption at time *v* by a household born at time *s*, $\overline{m}(s,v)$, $\overline{b}(s,v)$ and $\overline{k}(s,v)$ are, respectively, the stocks of nominal base money, nominal risk-free constant market value bonds and real capital held at time *v* by a household born at time *s*, and $P(v) \geq 0$ is the general price level at time *v*.[2]

Each household faces a constant (age-independent) instantaneous probability of death, $\lambda \geq 0$. The remaining expected life time λ^{-1} is therefore also age-independent and constant. The randomness of the timing of one's demise in the only source of uncertainty in the model. It follows that the objective functional in (1) can be re-written as:

$$\max_{\{\overline{c}(s,v),\overline{m}(s,v),\overline{b}(s,v),\overline{k}(s,v);v\geq t\}} \int_t^\infty e^{-(\theta+\lambda)(v-t)} \ln\left[\overline{c}(s,v)^\alpha \left(\frac{\overline{m}(s,v)}{P(v)}\right)^{1-\alpha}\right]dv \qquad (2)$$

Households act competitively in all markets in which the operate, and asset markets are complete and efficient, with free entry. In particular, there exist actuarially fair annuities markets that offer a household an instantaneous rate of return of λ on each unit of non-financial wealth it owns for as long as it lives, in exchange for the annuity-issuing entity claiming the entire stock of financial wealth owned by the household at the time of its death.

The household has three stores of value: fiat base money, which carries a zero nominal rate of interest and is an irredeemable financial instrument issued by the State (the consolidated general government and Central Bank, in this note), nominal instantaneous bonds with an instantaneous nominal interest rate i and real capital yielding an instantaneous gross real rate of return ρ. Capital goods and consumption goods consist of the same physical stuff and can be costlessly and instantaneously transformed into each other. Capital depreciates as the constant instantaneous rate $\delta \geq 0$. The real wage earned at time *v* by a household born at time *s* is denoted $\overline{w}(s,v)$ and the lump-sum tax paid (lump-sum transfer payment received if negative) at time *v* by a household born at time *s* is $\overline{\tau}(s,v)$. Labor supply is inelastic and scaled to 1.

Competition ensures that pecuniary rates of return on bonds and capital are equalized. With money yielding positive utility, there can be no equilibrium with a negative nominal interest rate. Let $r(t)$ be the instantaneous risk-free real interest rate and $\pi(t) = \frac{\dot{P}(t)}{P(t)}$ the instantaneous rate of inflation. It follows that.

$$i(t) \geq 0 \qquad (3)$$

$$\rho(t) - \delta(t) = r(t) = i(t) - \pi(t) \qquad (4)$$

The instantaneous budget identity of a household born at time $s \leq t$ that has survived till period *t* is:

[2] If a unit of real capital is interpreted as an ownership claim to a unit of capital (equity), then $\overline{k}$ can be negative, zero or positive. If it is interpreted as a unit of physical capital itself, $\overline{k}$ has to be non-negative.

$$\dot{\bar{k}}(s,v)+\frac{\dot{\bar{m}}(s,v)+\dot{\bar{b}}(s,v)}{P(v)}\equiv\left(\rho(v)-\delta+\lambda\right)\bar{k}(s,v)+\left(i(t)+\lambda\right)\frac{\bar{b}(s,v)}{P(v)}+\lambda\frac{\bar{m}(s,v)}{P(v)}+\bar{w}(s,v)-\bar{\tau}(s,v)-\bar{c}(s,v) \tag{5}$$
[3]

The real value of total non-human wealth (or financial wealth) at time v of a household born at time s is

$$\bar{a}(s,v)\equiv\bar{k}(s,v)+\frac{\bar{m}(s,v)+\bar{b}(s,v)}{P(v)} \tag{6}$$

The flow budget identity (5) can, using (4) and (6) be written as:

$$\dot{\bar{a}}(s,v)\equiv\left(r(v)+\lambda\right)\bar{a}(s,v)-i(v)\frac{\bar{m}(s,v)}{P(v)}+\bar{w}(s,v)-\bar{\tau}(s,v)-\bar{c}(s,v) \tag{7}$$

The no-Ponzi finance solvency constraint for the household is that the present discounted value of its terminal financial wealth be non-negative in the limit as the time horizon goes to infinity:

$$\lim_{v\to\infty}\bar{a}(s,v)e^{-\int_t^v (r(u)+\lambda)}du\geq 0$$

Because the instantaneous utility function is increasing in both consumption and the stock of real money balances, the solvency constraint will bind:

$$\lim_{v\to\infty}\bar{a}(s,v)e^{-\int_t^v (r(u)+\lambda)}du=0 \tag{8}$$

Note that base money is viewed as an asset by the holder (the household). The terminal net financial wealth whose present discounted value (NPV) must be non-negative includes the household's stock of base money.

The optimality conditions of the household's choice problem imply the following decision rules for the household:

$$\bar{c}(s,t)=(1-\alpha)\left(\theta+\lambda\right)\left(\bar{a}(s,t)+\bar{h}(s,t)\right) \tag{9}$$

$$\bar{h}(s,t)=\int_t^\infty\left(\bar{w}(s,t)-\bar{\tau}(s,t)\right)e^{-\int_t^v (r(u)+\lambda)du}dv \tag{10}$$

$$\frac{\bar{m}(s,t)}{P(t)}=\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{i(t)}\bar{c}(s,t) \tag{11}$$

3 The notational convention is that $\dot{\bar{k}}(s,v)\equiv\dfrac{\partial\bar{k}(s,v)}{\partial v}$.

The net present discounted value of household after-tax labor income, $\overline{h}(s,t)$, will be referred to as human capital. A shorter life expectancy (a higher value of λ) raises the marginal propensity to consume out of comprehensive wealth, *a* + *h*

2.1b Aggregation

We assume that there is a constant and age-independent instantaneous birth rate $\beta \geq 0$. The size of the cohort born at time t is normalized to $\beta e^{(\beta-\lambda)t}$. The size of the surviving cohort at time *t* which was born at time $s \leq t$ is therefore $\beta e^{(\beta-\lambda)s} e^{-\lambda(t-s)}$. Total population at time *t* is therefore given, for $\beta > 0$ by $\beta e^{-\lambda t} \int_{-\infty}^{t} e^{\beta s} ds = e^{(\beta-\lambda)t}$. For the case $\beta = 0$ we set the size of the population at *t* = *0* to equal 1, so population size at time *t* is again $e^{(\beta-\lambda)t} = e^{-\lambda t}$. For any individual household variable $\overline{x}(s,t)$, we define the corresponding population aggregate *X(t)* as follows:

$$
\begin{aligned}
X(\mathrm{t}) &= \beta e^{-\lambda t} \int_{-\infty}^{t} \overline{x}(s,t) e^{\beta s} ds \quad &\text{if } \beta > 0 \\
&= \overline{x}(0,t) e^{-\lambda t} \quad &\text{if } \beta = 0
\end{aligned}
$$

We assume that each household earns the same wage and pays the same taxes, regardless of age:

$$
\begin{aligned}
\overline{w}(s,t) &= \overline{w}(t) \\
\overline{\tau}(s,t) &= \overline{\tau}(t)
\end{aligned}
$$

It follows that each household, regardless of age, has the same human capital:

$$\overline{h}(s,t) = h(t)$$

Finally, there are neither voluntary nor involuntary bequests in this model, so

$$\overline{a}(s,\mathrm{s}) = 0 \tag{12}$$

By brute-force aggregation, if follows that aggregate consumption is determined as follows:

$$C(t) = (1-\alpha)(\theta+\lambda)\big(A(t)+H(t)\big) \tag{13}$$

$$\frac{M(t)}{P(t)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{i(t)} C(t) \tag{14}$$

$$\dot{A}(t) \equiv r(t)A(t) - i(t)\frac{M(t)}{P(t)} + W(t) - T(t) - C(t) \tag{15}$$

$$H(t) = \int_{t}^{\infty} \big(W(v) - T(v)\big) e^{-\int_{t}^{v}(r(u)+\beta)du} dv \tag{16}$$

$$A(t) = K(t) + \frac{M(t)+B(t)}{P(t)} \tag{17}$$

For future reference, the solvency constraint of the aggregate household sector is

$$\begin{aligned} &\lim_{v\to\infty} A(v)e^{-\int_t^v r(u)\mathrm{d}u} = 0 \\ &\text{or} \\ &\lim_{v\to\infty}\left(K(v) + \frac{M(v)+B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{d}u} = 0 \end{aligned} \tag{18}$$

Comparing the aggregate household financial wealth dynamics equation (15), with the individual *surviving* household financial wealth dynamics equation (7) shows that the return on the annuities, λA is missing from the aggregate dynamics. This is as it should be, because $\lambda A(t)$ is both the extra returns over and above the risk-free rate earned by all surviving households at time *t* and the amount of wealth paid to the annuities sellers by the (estates of the) fraction λ of the population that dies at time *t*.

Comparing the aggregate human capital equation (16)—describing the human capital of all generations currently alive but not of those yet to be born—and the individual surviving household's human capital equation (10), we note that if the households alive at time *t* were to discount *all* future after-tax labor income at the individually appropriate, annuity premium-augmented rate of return $r+\lambda$, they would fail to allow for the fact that the labor force to whom that after-tax labor income accrues includes the surviving members of generations born after time *t*. In the absence of the institution of "inherited slavery", those currently alive cannot claim the labor income of the future surviving members of generations as yet unborn. Population and labor force grow at the proportional rate $\beta-\lambda$, so the appropriate discount rate applied to the future aggregate streams of labor income is $r+\beta$.

2.2 The State

The State whose budget identity and solvency constraint we model is the consolidated general government (the Treasury in what follows) and Central Bank. Let G denote real public spending on goods and services (exhaustive public spending, current and or capital). The State's budget identity and solvency constraint are given in equation (19) and (20) respectively.

The implicit assumption that base money can be created at zero marginal real resource cost (and indeed that government bonds can be issued at zero marginal real resource cost) is reflected in the absence of terms like $\mu^M(t)\dot{M}(t), \mu^M(t)>0$ and $\mu^B(t)\dot{B}(t), \mu^B(t)>0$ on the RHS of equation (19). We also ignore any fixed cost of fiat base money issuance, although any fixed cost could be buried in *G(t)*.

$$\frac{\dot{M}(t)+\dot{B}(t)}{P(t)} \equiv i(t)\frac{B(t)}{P(t)} + G(t) - T(t) \tag{19}$$

Because of the irredeemability of base money, the solvency constraint of the State requires that the present discounted value of its terminal net *non-monetary* liabilities be non-positive, not that the present discounted value of its terminal net financial liabilities be non-positive.

$$\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} \le 0 \tag{20}$$

Equation (20) is the natural way to formalize the familiar notion that fiat base money is an asset (wealth) to the holder (the owner—households in this simple model) but does not constitute in any meaningful sense a liability to the issuer (the 'borrower'—the State or the Central Bank as an agent of the State). The owner of a $20 dollar Federal Reserve Note may find comfort in the fact that "This note is legal tender for all debts, public and private", but she has no claim on the Federal Reserve, now or ever, other than for an amount of Federal Reserve Notes adding up to $20 in value. UK currency notes worth £X carry the proud inscription *"… promise to pay the bearer the sum of £X"* but this merely means that the Bank of England will pay out the face value of any genuine Bank of England note no matter how old. The promise to pay stands good for all time but simply means that the Bank will always be willing to exchange one (old, faded) £10 Bank of England note for one (new, crisp) £ 10 Bank of England note (or even for two £ 5 Bank of England notes). Because it promises only money in exchange for money, this 'promise to pay' is, in fact, a statement of the irredeemable nature of Bank of England notes.

I believe that the irredeemability property of fiat currency—that it is an asset to the holder but not a liability of the issuer—extends also to the other component of base money (commercial bank reserves held with the Central Bank), but the simple theoretical model does not depend on this and does not make this distinction.

Until further notice, we assume, although unlike with the household sector, there is no optimizing justification for it, that the State satisfies its solvency constraint with strict equality. The case of the state as NPV creditor to the private sector, even in the long run, is considered briefly in Section 2.5.

Equation (19) implies that

$$\frac{M(t)+B(t)}{P(t)} \equiv \int_t^\infty \left(T(v)-G(v)+i(v)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)+B(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{21}$$

Because of the irredeemability of base money (equation (20)), assumed to hold with strict equality, the intertemporal budget constraint of the State is

$$\frac{M(t)+B(t)}{P(t)} = \int_t^\infty \left(T(v)-G(v)+i(v)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du} \tag{22}$$

Substituting the intertemporal budget constraint of the State into the aggregate consumption function (13), using (16) and (17), and rearranging yields:

$$C(t)=(1-\alpha)(\theta+\lambda)\begin{bmatrix} K(t)+\int\limits_t^{\infty}\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv \\ -\int\limits_t^{\infty}T(v)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\frac{1}{P(t)}\left(\int\limits_t^{\infty}i(v)M(v)e^{-\int_t^v i(u)du}dv+\lim\limits_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right)\end{bmatrix} \quad (23)^{4}$$

From integration by parts it follows that

$$\begin{aligned}&\int\limits_t^{\infty}i(v)M(v)e^{-\int_t^v i(u)du}dv+\lim\limits_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\\ &=\int\limits_t^{\infty}\dot{M}(v)e^{-\int_t^v i(u)du}dv+M(t)\end{aligned} \quad (24)^{5}$$

It follows that (23) can also be written as:

$$C(t)=(1-\alpha)(\theta+\lambda)\begin{bmatrix} K(t)+\int\limits_t^{\infty}\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv \\ -\int\limits_t^{\infty}T(v)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\frac{1}{P(t)}\left(M(t)+\int\limits_t^{\infty}\dot{M}(v)e^{-\int_t^v i(u)du}dv\right)\end{bmatrix} \quad (25)$$

---

4 Note that $\begin{aligned}&\int\limits_t^{\infty}i(v)\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}dv+\lim\limits_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}\\ &=\frac{1}{P(t)}\left(\int\limits_t^{\infty}i(v)M(v)e^{-\int_t^v i(u)du}dv+\lim\limits_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right)\end{aligned}$.

5 If instead of having a zero nominal interest rate, fiat base money carried the possibly time-varying nominal interest rate $i^M(t)$, equation (24) can be rewritten as

$$\begin{aligned}&\int\limits_t^{\infty}\left(i(v)-i^M(v)\right)M(v)e^{-\int_t^v i(u)du}dv+\lim\limits_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\\ &=\int\limits_t^{\infty}\left(\dot{M}(v)-i^M(v)\right)e^{-\int_t^v i(u)du}dv+M(t)\end{aligned}$$

with obvious modifications required in the intertemporal budget constraints of households and the State.

2.3 Debt Neutrality

When the birth rate is zero, the consumption function is equivalent to the consumption function of the representative agent model. From the perspective of pure fiscal stabilization policy - a cut in lump-sum taxes today accompanied by a credible commitment to an increase in future taxes equal in net present value to the up-front tax cut, will not boost household demand. With $\beta > 0$, an up-front tax cut and the credible announcement of a future increase in taxes of equal net present discounted value when discounted at the riskless rate *r* boosts the human capital of those currently alive because some of the deferred taxes will fall on as yet unborn generations. With $\beta = 0$ the wedge between the government's discount rate for future taxes, *r*, and the effective discount rate of the private sector for future taxes, $r + \beta$ disappears, and Ricardian equivalence or debt neutrality prevails. With $\beta = 0$, the aggregate consumption function (25) becomes

$$C(t) = (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^{\infty} \left(W(v) - G(v)\right) e^{-\int_t^v r(u)du} dv \\ + \dfrac{1}{P(t)}\left(\int_t^{\infty} i(v) M(v) e^{-\int_t^v i(u)du} dv + \lim_{v \to \infty} M(v) e^{-\int_t^v i(u)du} \right) \end{array}\right] \tag{26}$$

or, equivalently

$$C(t) = (1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t) + \int_t^{\infty} \left(W(v) - G(v)\right) e^{-\int_t^v r(u)du} dv \\ \\ + \dfrac{1}{P(t)}\left(M(t) + \int_t^{\infty} \dot{M}(v) e^{-\int_t^v i(u)du} dv \right) \end{array}\right] \tag{27}$$

Lump-sum taxes disappear from the aggregate consumption function once the intertemporal budget constraint of the State is used to substitute out the initial values of the private sector's holdings of monetary and non-monetary sovereign debt. The first line on the RHS of equations (26) and (27) shows the result, familiar from non-monetary representative agents models that the bite taken out of private comprehensive wealth by the government is measured by the net present discounted value of future exhaustive public spending.

2.4 Helicopter money with debt neutrality

Even in a representative agent model with debt neutrality/Ricardian equivalence, monetary injections will boost private consumption demand, holding constant the sequences of current and future spending on real goods and services $\{G(v); v \geq t\}$, prices, wages and interest rates. The path of lump-sum taxes and of non-monetary debt is irrelevant with $\beta = 0$, as long as the State satisfies its intertemporal budget constraint (22).

It is immediately obvious from equations (26) and (27) that, holding constant the sequence of current and future real exhaustive public spending constant, monetary injections will always boost consumption demand, as long as the price level $P(t)$ is positive. We can think of monetary injections, holding constant the path of current and future exhaustive public spending, as being introduced either through lump-sum transfer payments, *T*, or by purchasing non-monetary debt (sovereign bonds) from the private sector (QE or quantitative easing). If the State, starting at time *t*, increases the stock of base money by buying back non-monetary public debt from the public, say with $\dot{M}(v) = -\dot{B}(v) > 0$ for $t \le v \le t', t' > t$, it is clear from the intertemporal budget constraint of the State, equation (22), that, holding constant the current and future paths of the price level and interest rates, the State will have to raise the NPV of future public spending minus taxes to satisfy its intertemporal budget constraint. Permanent open market purchases of non-monetary public debt by the Central Bank (irreversible QE) are deferred helicopter money: future taxes will be cut and/or future public spending will have be raised if the State is to satisfy its intertemporal budget constraint.[6]

2.5 The creditor state

Remember that equation (20) does not have to hold with strict equality. The same holds for equations (20), (22), (23), (25),(26) and (27). Consider the case $\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} = Z < 0$, where the State is a net (non-monetary) creditor to the private sector, even in the very long run. We assume that *Z* is finite.[7]

The aggregate household solvency constraint (18) implies

$$-\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} = \lim_{v\to\infty}\left(K(v) + \frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} = -Z > 0 \quad \text{or}$$

$$-\lim_{v\to\infty} B(v)e^{-\int_t^v i(u)\mathrm{du}} = \lim_{v\to\infty}\left(P(v)K(v) + M(v)\right)e^{-\int_t^v i(u)\mathrm{du}} = p(t)Z > 0.$$

The state is a permanent creditor to the household sector, something it can do when the long-run growth rate of fiat base money is at least as high as the long-run nominal interest rate, since $\lim_{v\to\infty} M(v)e^{-\int_t^v i(u)\mathrm{du}} > 0$ requires $\lim_{v\to\infty}\frac{\dot{M}(v)}{M(v)} \ge \lim_{v\to\infty} i(v) \ge 0$.

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[6]Indeed, the State could choose to become a net non-monetary creditor to the private sector, with $B < 0$. The State's solvency constraint after all only requires the NPV of its terminal stock of *non-monetary* debt to be non-positive (equation (20)). It could be strictly negative in equilibrium, as long as the household sector satisfies its solvency constraint, that the NPV of the terminal value of its financial assets $K + \frac{M+B}{P}$ is non-negative.

[7] In a model with positive real growth in the long run, the ratio of real government bonds would be restricted to be finite.

$$C(t)=(1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^{\infty}\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv \\ -\int_t^{\infty}T(v)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\frac{1}{P(t)}\left(\int_t^{\infty}i(v)M(v)e^{-\int_t^v i(u)du}dv\right)-Z \end{array}\right] \tag{28}$$

Acting as a long-run NPV creditor state to the private sector therefore does not alter the capacity of the State to boost the comprehensive wealth of the household sector, after consolidation of the intertemporal budget constraints of the household sector. This is because, unlike the State, the household sector's NPV of *all* financial assets has to be non-negative in the long run.

From the government's intertemporal budget constraint (22) it is clear that the fiscal space created by $\lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}}>0$ can be used to cut future taxes or increase future public spending, but not to any greater degree than when the NPV of non-monetary sovereign debt in the long run was required to be zero.

2.6 Helicopter money in the 'normal' case

Consider what is perhaps the normal case, when, in the long run, the State grows the nominal stock of fiat base money at a proportional rate strictly below the instantaneous risk-free nominal interest rate, that is, $\lim_{v\to\infty} M(v)e^{-\int_t^v i(u)du}=0$. In the representative agent case ($\beta=0$) the consumption function becomes

$$C(t)=(1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^{\infty}\left(W(v)-G(v)\right)e^{-\int_t^v r(u)du}dv \\ +\frac{1}{P(t)}\left(\int_t^{\infty}i(v)M(v)e^{-\int_t^v i(u)du}dv\right) \end{array}\right]$$

The State can boost demand by monetary injections, for given sequences of exhaustive public spending, the general price level and interest rates. A larger future money supply will, ceteris paribus, increase the comprehensive wealth or permanent income of the household sector by boosting the NPV of the interest bills saved by borrowing through the issuance of zero-interest-bearing base money rather than through (positive) interest-bearing debt.

The same conclusion stares one in the face even more clearly when we use the equivalent expression for the seigniorage blessings of monetary issuance, shown in equation (27). The wealth-creating effect of seigniorage is the outstanding stock of base money plus the NPV of future base money issuance:

$$\frac{1}{P(t)}\left(M(t) + \int_t^{\infty} \dot{M}(v) e^{-\int_t^v i(u)du} dv \right).$$

Again this can be made arbitrarily large for given sequences of G, P and i

2.7 Helicopter money in a liquidity trap

Consider an economy stuck in the ultimate liquidity trap with the nominal interest rate at zero forever. With $i(v) = 0, v \geq t$, monetary injections lose none of their potency. Sure, the NPV of the current and future interest saved by issuing base money rather than non-monetary securities (bonds) is zero:

$$\int_t^{\infty} i(v) M(v) e^{-\int_t^v i(u)du} dv = 0 \text{ when } i(v) = 0, v \geq t.$$

But the NPV of the terminal stock of base money can be made anything the State (the monetary authority) wants it to be:

$$\lim_{v \to \infty} M(v) e^{-\int_t^v i(u)du} = \lim_{v \to \infty} M(v) \text{ when } i(v) = 0, v \geq t.$$

The alternative expression for the wealth represented by the seigniorage monopoly of the State:

$$M(t) + \lim_{v\to\infty} \int_t^v \dot{M}(\ell) e^{-\int_t^{\ell} i(u)du} d\ell = M(t) + \lim_{v\to\infty} \int_t^v \dot{M}(\ell) d\ell = \lim_{v\to\infty} M(v),$$

which encouragingly is the same as the one derived earlier, again shows that the authorities can use helicopter money to boost consumer demand even in the severest of all conceivable liquidity traps. What this means is that a fiat money economy where the State controls the issuance of fiat money, a liquidity trap is a choice, not a necessity. Most general equilibrium completions of a model with the consumption function used in this paper will have the property that if, in a perpetual zero nominal interest rate equilibrium, real demand is boosted by a sufficiently large magnitude, the permanent liquidity trap vanishes.

Equations (26) or (27) (or their more general versions without Ricardian equivalence) make it clear that it is also possible for the State to boost public spending on real goods and services, current or capital, and avoid any negative impact of the anticipation of higher future taxes on demand by monetizing the resulting public sector deficits.

2.8 Helicopter money without Ricardian equivalence

The way helicopter money affects household demand is the same in the overlapping generations model (the Yaari-Blanchard model with $\beta > 0$) as in the representative agent model ($\beta = 0$). A comparison of equations (23) and (25) with equations (26) and (27) shows that the comprehensive wealth term in the aggregate consumption function is augmented by base money issuance to the tune of

$$\int_t^{\infty} i(v)M(v)e^{-\int_t^v i(u)du}dv + \lim_{v\to\infty} M(v)e^{-\int_t^v i(u)du}$$

or, equivalently, $$M(t) + \int_t^{\infty} \dot{M}(v)e^{-\int_t^v i(u)du}dv.$$

It is clear from the model without Ricardian equivalence that permanent monetary base expansions of a given magnitude in NPV terms will now have different effects when they are implemented through up-front lump-sum transfer payments/tax cuts than through up-front QE (open market purchases of sovereign bonds) followed by deferred transfer payments or tax cuts. Because the deferred tax cuts will in part be enjoyed by generations not yet born today, the 'up-front QE and deferred transfer payment boost' version will be less expansionary, for a given NPV of base money issuance, than the version with the up-front transfer payment boost.

3 Some further considerations

3.1 Fiat base money is special

In this model unbacked fiat base money is unique for two reasons. First, it performs liquidity or transactions functions that cause it to be willingly held by private agents despite carrying a zero nominal interest rate, even when other safe assets are present that yield a positive nominal interest rates. I shoe-horned this uniqueness into the model by having money as an argument in the household's direct utility function. This is not very satisfactory. The only justification is simplicity and the robustness of the results of the paper to using other mechanisms for making fiat base money a superior asset (money in the production function, cash–in-advance or legal restrictions. What makes something (or some class of objects) desirable because of its unique transactions-facilitating properties differs in the many different approaches that have been adopted for generating a willingness to hold something that is pecuniary-rate-of-return-dominated as a store of value. It is the outcome of a collective, decentralized social choice. It may help if something is granted legal tender status by the State, but this not a necessary condition. Should fiat base money issued by the State lose this unique advantages it has in facilitating transactions, it will have to pay interest at the same rate as the other safe, liquid financial assets—bonds in this model, or it will not be held voluntarily by private agents. We are in the Wallace (1981, 1990) world of the Modigliani-Miller theorem for open market operations. The net present discounted value of future interest saved is, of course zero in this case. However, if the monetary asset is irredeemable, the NPV of the terminal base money stock would still be net wealth. For this to be positive, the growth rate of the nominal stock of base money would have to be at least equal to the nominal rate of interest in the long run. In the liquidity trap case, with a zero nominal interest rate forever, a helicopter money drop would still be effective in boosting household consumption demand, even though a helicopter bond drop would not be.

3.2 Fiat base money is net wealth

Fiat base money is net wealth for the consolidated private sector and State sector. Despite fiat money technically being inside money and an inside asset (issued by one economic agent and held by another), fiat base money behaviorally or effectively is like nature's bounty: an asset and wealth to the owner but not a claim on or liability of the issuer.

Indeed, looking at the version of the aggregate consumption function in equation (25) or (27), note that the term $\frac{1}{P(t)}\left(M(t) + \int_t^{\infty} \dot{M}(v) e^{-\int_t^v i(u)du} dv \right)$ could equally well represent true 'outside assets', like intrinsically worthless pet rocks or Rai, the stone money used on the Isle of Yap. The stock of rare bits of rock deposited on earth by meteorites, say, could be represented by *M(t)* and the net present value of future meteorite deposits could be represented by $\int_t^{\infty} \dot{M}(v) e^{-\int_t^v i(u)du} dv$. With some slight modifications, almost intrinsically worthless commodities like gold and intrinsically worthless virtual media of exchange like Bitcoin could also fit into our consumption function. Both are, of course, costly to produce or 'mine'. Helicopter drops of Rai, gold or Bitcoin would not share with fiat base money the property that they are issued by the State and can be used to fund the State. They don't roll off the printing presses but are gifts from nature (Rai and gold) and from human ingenuity (in the case of Bitcoin).

3.3 When is a helicopter money drop preferred to a bond-financed fiscal stimulus?

When there is no Ricardian equivalence, aggregate demand can be stimulated through sovereign bond-financed tax cuts (or through higher exhaustive public spending) as well as through helicopter money. Which method one prefers depends on how the model of the economy is completed and on policy preferences. The formal model of this note is not well suited to deal with problems like sovereign default risk or inflation risk, but richer models that permit a meaningful discussion of these issues would likely have the property that if (1) the sovereign has a high stock of non-monetary net debt outstanding and (2) there are political limits to its current and future capacity to raise taxes or cut public spending, adding to the stock of non-monetary debt through further sovereign bond issuance could raise sovereign default risk. That would call for monetary financing as the preferred funding method for a fiscal stimulus. The case for monetary financing would be stronger if inflation is below target and if one or more key financial markets are illiquid.

If the public finances are healthy (low sovereign debt and deficit, considerable political scope for cutting public spending or raising taxes) and inflation is above-target, using sovereign bonds to fund a stimulus would make sense.

In the current economic conditions faced by the euro area, Japan and, to a slightly lesser degree, by the US and the UK, with question marks behind the sustainability of the public finances and with inflation well below target, monetizing a fiscal stimulus would seem to be the obvious first choice.

3.4 The institutional implementation of helicopter money drops

In most contemporary advanced economies, the issuance of fiat base money (often with legal tender status) is performed by an agency of the State, the Central Bank, that has some degree of operational independence (and in a few cases even a measure of target independence) in the design and implementation of monetary policy. Some Central Banks can act as fiscal agents for the State (central government or federal Treasury/Ministry of Finance) but none that we know of can act openly as fiscal principals. Central Banks typically transfer their profits (over and above what they want to add to reserves or provisions) to their beneficial owner, the central government or federal Treasury.[8] Specifically, Central Banks cannot levy taxes, make transfer payments or pay overt subsidies to other domestic economic entities, nor can they engage in exhaustive public spending other than what is inevitably involved in the running of the Central Bank (payroll, capital expenditure on buildings and equipment, supplies, utilities etc.). The fact that many Central Banks have engaged in large-scale quasi-fiscal interventions, most recently during and after the North-Atlantic financial crisis of 2007-2008, does not change the basic legal and institutional reality that a Central Bank cannot implement helicopter money on its own.

Cooperation and coordination between the Central Bank and the Treasury is required for the real-world implementation of helicopter money drops. In practice, to implement the temporary fiscal stimulus permanently/irreversibly financed through the issuance of fiat base money that is closest to the original Friedman helicopter money parable—a lump-sum transfer payment households permanently funded through base money issuance -, the following coordinated fiscal-monetary actions would take place. There would be a one-off cash transfer to all eligible households by the Treasury. The Treasury funds these payments by selling Treasury debt to the Central Bank, which credits the account held by the Treasury with the Central Bank (which is not normally counted as part of the monetary base). As the Treasury pays out the cash to the eligible households, the Treasury's account with the Central Bank is drawn down. The monetary base increases because the transfer payment to the households either ends up as increased cash/currency held by households, corporates or banks or as increased bank reserves held with the Central Bank. A virtually identical story can be told if instead of a transfer payment to the household sector, the Treasury were to engage in a program of current or capital expenditure.

3.5 The irrelevance of the cancellation of Treasury debt held by the Central Bank.

From a fundamental economic perspective, it makes no difference whether the Central Bank cancels the sovereign bonds it buys (as proposed e.g. by Turner (2013)) or holds them indefinitely (rolling them over as they mature). This is because the Treasury is the beneficial owner of the Central Bank. The Treasury therefore receives the Central Bank's profits and is responsible for its losses. Their accounts (including balance sheets and P&L account) therefore can be—or indeed ought to be—consolidated to get a proper perspective on the flow of funds and balance sheet accounts that matter. The only reason to prefer cancellation of sovereign debt held

[8] The European Central Bank (ECB) is unique in that its shareholders are the national Central Banks (NCBs) of the 28 (as of May 2014) European Union member states. The profits of the ECB are distributed to the 18 (as of May 2014) NCBs of the EU member states that are also members of the euro area.

by the Central Bank over the Central Bank holding the sovereign debt permanently is that cancellation may be seen as a more credible commitment device.

The disaggregated period (instantaneous) budget identity, the intertemporal budget identity and the solvency constraint of the Treasury are given in equations (29), (30) and (31). Those of the Central Bank are given in equations (32), (33) and (34). As before, B stands for Treasury debt held outside the Central Bank. B^{cb} denotes Treasury debt held by the Central Bank. T is the real value of taxes paid by the private sector, T^{cb} is the real value of payments made by the Central Bank to the Treasury. The Central Bank is extremely frugal and does not spend on real goods and services. Because we are considering a closed economy, the Central Bank does not hold any foreign exchange reserves.

$$\frac{\dot{B}(t)+\dot{B}^{cb}(t)}{P(t)} \equiv i(t)\left(\frac{B(t)+B^{cb}(t)}{P(t)}\right)+G(t)-T(t)-T^{cb}(t) \tag{29}$$

$$\frac{B(t)+B^{cb}(t)}{P(t)} \equiv \int_t^{\infty}\left(T(v)+T^{cb}(t)-G(v)\right)e^{-\int_t^v r(u)du}dv+\lim_{v\to\infty}\left(\frac{B(v)+B^{cb}(t)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{30}$$

$$\lim_{v\to\infty}\left(\frac{B(v)+B^{cb}(t)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} \leq 0 \tag{31}$$

$$\frac{\dot{M}(t)-\dot{B}^{cb}(t)}{P(t)} \equiv T^{cb}(t)-i(t)\frac{B^{cb}(t)}{P(t)} \tag{32}$$

$$\frac{M(t)-B^{cb}(t)}{P(t)} \equiv \int_t^{\infty}\left(-T^{cb}(v)+i(v)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv+\lim_{v\to\infty}\left(\frac{M(v)-B^{cb}(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{33}$$

$$\lim_{v\to\infty}\left(\frac{B^{cb}(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}} \geq 0 \tag{34}$$

The Treasury's intertemporal budget identity and solvency constraint imply the Treasury's intertemporal budget constraint:

$$\frac{B(t)+B^{cb}(t)}{P(t)} \leq \int_t^{\infty}\left(T(v)+T^{cb}(t)-G(v)\right)e^{-\int_t^v r(u)du}dv \tag{35}$$

The Central Bank's intertemporal budget identity and solvency constraint, which recognizes the irredeemability of fiat base money implies the Central Bank's intertemporal budget constraint:

$$\frac{M(t)-B^{cb}(t)}{P(t)} \leq \int_t^{\infty}\left(-T^{cb}(v)+i(v)\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du}dv+\lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{36}$$

The Treasury, as the beneficial owner of the Central Bank, receives all the profits of the Central Bank. Ignoring changes in provisions and reserves this means that

$$T^{cb}(t) = i(t)\frac{B^{cb}(t)}{P(t)} \tag{37}$$

which implies that

$$\dot{M}(t) = \dot{B}^{cb}(t) \tag{38}$$

From (36) this implies that

$$\frac{M(t)-B^{cb}(t)}{P(t)} \leq \int_t^{\infty}\left(i(v)\left(\frac{M(v)-B^{cb}(v)}{P(v)}\right)\right)e^{-\int_t^v r(u)du}dv + \lim_{v\to\infty}\left(\frac{M(v)}{P(v)}\right)e^{-\int_t^v r(u)du} \tag{39}$$

Briefly, it does not matter whether the Central Bank today cancels an amount $B^{cb}(t)$ of debt owed to it by the Treasury and as a result does not pay out as profits to the Treasury an infinite future stream of central bank profits $\{i(v)B^{cb}(t), v \geq t\}$ (whose NPV is, of course, $B^{cb}(t)$), or whether it keeps its existing holdings of Treasury debt on its books and pays out as profits to the Treasury an infinite stream of future profits $\{i(v)B^{cb}(t), v \geq t\}$.[9]

3.6 Helicopter money drops and the ECB

Matters are slightly more complicated for the ECB, whose equity is held by the national Central Banks (NCBs) of the member States that are part of the euro area. Each NCB has its national Central Bank as its beneficial owner. Cancelling an amount $B_i^{cb}(t)$ of sovereign debt of euro area member state *i* (which has an equity stake η_i in the ECB), represents ultimately a wealth transfer of $(1-\eta_i)B_i^{cb}(t)$ to the Treasury of member State *i* from the Treasuries of all other member States. Holding $B_i^{cb}(t)$ indefinitely on the balance sheet of the ECB would result in an infinite stream of profits $\{i(v)\eta_i B_i^{cb}(t), v \geq t\}$ to the NCB of country *i*, and thus ultimately to the Treasury of country *i*, and $\{i(v)(1-\eta_i)B_i^{cb}(t), v \geq t\}$ to the NCBs of the remaining euro area member states and thus ultimately to their national Treasuries.

This real-world implementation of helicopter money drops is legal and easily implemented everywhere except in the euro area. Article 123.1 of the Treaty on the Functioning of the European Union States:

> *"Overdraft facilities or any other type of credit facility with the European Central Bank or with the Central Banks of the Member States (hereinafter referred to as 'national Central Banks') in favour of Union institutions, bodies, offices or agencies, central governments, regional, local or other public authorities, other bodies governed by public law, or public*

9 $\int_t^{\infty} i(v)B^{cb}(t)e^{-\int_t^v i(u)du}dv = B^{cb}(t)$

undertakings of Member States shall be prohibited, as shall the purchase directly from them by the European Central Bank or national Central Banks of debt instruments.[10]

This clause has commonly been interpreted as ruling out the financing of government deficits in the euro area through government debt sales to the ECB (or to the national Central Banks (NCBs) of the Eurosystem) and their monetization by the Eurosystem. Unless this can be fudged by the Eurosystem purchasing the sovereign debt in the secondary markets (as it did under the Securities Markets Programme and proposes to do under the Outright Monetary Transactions programme (should it ever be activated)), Article 123.1 deprives the euro area of the one policy instrument—a temporary fiscal stimulus permanently funded by and monetized by the Central Bank—that is guaranteed to prevent or cure deflation, "lowflation" or secular stagnation. It is time for Article 123 to be scrapped in its entirety if the euro area does not wish to face the unnecessary risk of falling into any of these traps.

4 Conclusion

4.1 The two funding advantages of fiat base money: zero nominal interest rate and irredeemability.

The fiat base money analyzed in this paper, which can be produced at zero marginal cost by the State (much like paper currency or bank reserves with the Central Bank in the real world), and which households are willing to hold at a zero nominal interest rate even when alternative stores of value with positive nominal interest rates are available, has two things going for it as a funding instrument for the State, compared to interest-bearing non-monetary debt. First, the State saves each period (instant in the continuous time model) the interest bill it would have paid had it issued bonds instead of money. Second, even if the nominal interest rate is zero and even if it is confidently expected to be zero forever, money is a more attractive funding instrument for the State because it is irredeemable. Fiat base money is net wealth to the private sector in the sense that its current stock plus the NPV of net future issuance is a component of the comprehensive wealth of the household sector.

4.2 Helicopter money drops always boost demand

A permanent helicopter drop of irredeemable fiat base money boosts demand both when Ricardian equivalence does not hold and when it holds. It makes the deficient demand version of secular stagnation a policy choice, not something driven by circumstances beyond national policy makers' control. It boosts demand when nominal risk-free interest rates are positive and when

10 http://www.lisbon-treaty.org/wcm/the-lisbon-treaty/treaty-on-the-functioning-of-the-european-union-and-comments/part-3-union-policies-and-internal-actions/title-viii-economic-and-monetary-policy/chapter-1-economic-policy/391-article-123.html

they are zero—and even in a pure liquidity trap when nominal interest rates are zero forever. A helicopter money drop always boosts demand when the price of money is positive.[11]

[11] In dynamic general equilibrium with flexible nominal prices, there always exists an equilibrium with a zero price of money in all periods and all States of nature – the barter equilibrium or non-monetary equilibrium. Obviously, helicopter money drops won't boost demand in such an equilibrium.

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