**Reply to Referee 1**

**Willem H. Buiter, August 3, 2014**

Those were very helpful comments and suggestions. My response follows:

**Re:** "In contrast to EW, the author does not specify an interest rate rule for monetary policy away from the zero lower bound and does not specify how the price level is determined. The partial equilibrium analysis conducted in this paper may account for the different conclusions reached from EW. In particular, a deeper discussion of how the author's results compare to the irrelevance proposition on quantitative easing in EW would be helpful."

The 'partial equilibrium' point is addressed (in the revised version of the paper)

On page 4: "All important aspects of how helicopter money drops work and what makes helicopter money unique can be established without the need for a complete dynamic general (dis)equilibrium model. All that is needed is a complete specification of the choice process of the household sector in a monetary economy, the period budget identity and solvency constraint of the consolidated general government/Treasury and Central Bank – the State - and the no-arbitrage conditions equating (in principle risk-adjusted) returns on all non-monetary stores of value and constraining the instantaneous nominal interest rate to be non-negative.

I shall show that, as long as the price of money is positive, the issuance of fiat base money can boost household consumption demand *by any amount*, given the inherited stocks of financial and real assets, given current and future wages and prices, and given current and future values of public spending on goods and services. Whether such helicopter money drops change asset prices and interest rates, goods prices, wages and/or output and employment depends on the specification of the rest of the model of the economy – including, in more general models, the behavior of the financial sector and of non-financial businesses in driving investment demand, production and labor demand, the rest of the 'supply side' of the economy and the rest of the world, if the economy is open. The point of this paper is to show that, whatever the equilibrium configuration we start from, helicopter money drops will boost household demand and must disturb that equilibrium. What 'gives' ultimately, in a fully articulated dynamic general equilibrium model nominal prices and wages, employment or output, is not our concern here."

On page 18: *"Neither is the difference in our results due to the fact that EW work with a general equilibrium model while I work with an incomplete or partial equilibrium model containing only the household and the State sectors. In this partial equilibrium framework I show that for any sequences of interest rates, (positive) price levels and other variables that are taken as exogenous by the individual households but will be endogenous in a fully-fledged general equilibrium model, household demand can be boosted through helicopter money drops by any desired amount. Whether such an increase in consumer demand raises prices and money wages alone, or employment and output as well, will depend on the general equilibrium 'closure' rule that is adopted. Establishing the point that household demand can be boosted does not require a general equilibrium model."*

**Re**: *"In contrast to EW, the author does not specify an interest rate rule for monetary policy away from the zero lower bound and does not specify how the price level is* *determined. The partial equilibrium analysis conducted in this paper may account for the different conclusions reached from EW. In particular, a deeper discussion of how the author's results compare to the irrelevance proposition on quantitative easing in EW would be helpful.*

*It seems possible that, by holding the path of nominal rates and inflation constant, the author is making implicit assumptions about the conduct of future monetary policy, and, therefore, the conclusions derived in this paper are in line with the analysis of EW. While partial equilibrium analysis is defensible if one is only concerned about the effect of money creation on desired consumption, it may be more problematic for relating these results to the broader literature."*

The fact that EW use a Taylor-type rule for the policy rate away from the ZLB and a monetary base rule at the ZLB is irrelevant for the differences between our results. This is stated on page 18: *"This (the asymmetric way in which base money enters the household sector's solvency constraint and the State's solvency constraint; WHB) is the only reason why we get a general effectiveness of permanent base money expansions result, away from as well as at the ZLB while EW in much of their paper (until equation 38 on page 196) get an ineffectiveness result at the ZLB. The fact that EW look at a specific Taylor-type interest rate rule away from the ZLB and a monetary base rule at the ZLB, while we establish the effectiveness result for all sequences of current and future interest rates, does not account for the differences in our results."* Our effectiveness result holds for all interest rate rules. When the policy rate is stuck at the ZLB, it holds as long as the stock of base money is increased sufficiently.

**Re:** *"In addition to holding nominal rates and inflation constant in the analysis, the author is not clear about why the zero lower bound would bind in his model. In the EW model, households are satiated in money at the zero lower bound – further increases in money have no utility value. The Cobb---Douglas preference specification adopted in this paper rules out satiation in money (see equation 14). Thus, the implications for money creation at the zero lower bound are not clear."*

I am happy to work with a more general money demand function that gives satiation at a finite value of real money balances. Since I don't want to lose the simplicity of the closed form solutions when I am discussing paths for the nominal interest rate that are always strictly positive, I have added the following subsection:

**2.1b The case of satiation in real base money balances**

The Cobb-Douglas instantaneous utility function does not have satiation in real money balances for finite holdings of real money balances. There is a material issue with the existence of a liquidity trap equilibrium or ZLB equilibrium when the demand for real money balances goes to infinity as the nominal interest rate goes to zero.[1] An infinite demand for real money balances can only be accommodated by a zero price level and/or an infinite stock of nominal money balances. In a Keynesian world (Old- or New-) the price level is predetermined and cannot drop to zero instantaneously. Even in a model

[1] This issue is considered at length and in depth in Eggertsson and Woodford (2003). I am indebted to an anonymous referee for pointing out the relevance of the issue.

with a perfectly flexible general price level, a zero general price level would hardly be an attractive or plausible equilibrium. Even the most QE-enamored monetary authority will have trouble coming up with an infinite stock of nominal base money. With a sticky general price level, what happens when *i = 0* and the demand for money becomes unbounded, depends on the rationing mechanism imposed by the monetary authorities on would-be holders of base money when their demand becomes unbounded (at the ZLB), and on the consequences of the rationing mechanism and the response of the private agents to this mechanism for the equilibrium configuration of prices and quantities in a fully articulated model. I do not propose to go there in this paper.

Instead I will consider a simple alternative instantaneous direct utility function that has satiation in real money balances at a finite level of the stock of real money balances. The model has the expositional advantage that, when the economy is stuck in an enduring liquidity trap (at the ZLB forever), it exhibits effectively the same behavior for aggregate consumption as the Cobb-Douglas utility function model does away from the ZLB. The model with satiation at the ZLB shares with the Cobb-Douglas model away from the ZLB the property that a permanent increase in the stock of base money always stimulates consumption demand.

Consider the case of an instantaneous utility function which, unlike the Cobb-Douglas function used thus far, has satiation in real money balances at a finite positive level of real money balances. We replace equation (2) with (2'):

$$\begin{aligned}
&\max_{\{\bar{c}(s,v),\bar{m}(s,v),\bar{b}(s,v),\bar{k}(s,v);v\geq t\}} \int_t^\infty e^{-(\theta+\lambda)(v-t)}\left[\ln \bar{c}(s,v) + u\left(\frac{\bar{m}(s,v)}{P(v)}\right)\right] \\
&u\left(\frac{\bar{m}(s,v)}{P(v)}\right) = \alpha\frac{\bar{m}(s,v)}{P(v)} - \frac{1}{2}\beta\left(\frac{\bar{m}(s,v)}{P(v)}\right)^2;\ 0 \leq \frac{\bar{m}(s,v)}{P(v)} \leq \frac{\alpha}{\beta};\ \alpha,\beta > 0 \\
&\qquad = \frac{1}{2}\frac{\alpha^2}{\beta};\ \frac{\bar{m}(s,v)}{P(v)} > \frac{\alpha}{\beta}
\end{aligned} \tag{2'}$$

The utility of real money balances increases in real money balances for $0 \leq \frac{\bar{m}(s,v)}{P(v)} \leq \frac{\alpha}{\beta}$, reaches its maximum value of $\frac{1}{2}\frac{\alpha^2}{\beta}$ at $\frac{\bar{m}(s,v)}{P(v)} = \frac{\alpha}{\beta}$, and is constant at $\frac{1}{2}\frac{\alpha^2}{\beta}$ for $\frac{\bar{m}(s,v)}{P(v)} > \frac{\alpha}{\beta}$.

The first-order conditions for a household optimum now imply:

$$\begin{aligned}\overline{m}(s,v) &= \frac{\alpha}{\beta} - \frac{1}{\beta}\frac{i(v)}{\overline{c}(s,v)} \quad \text{if } i(v) > 0 \\ &\geq \frac{\alpha}{\beta} \quad\quad\quad\quad\quad \text{if } i(v) = 0\end{aligned} \tag{12'}$$

For $i(v) > 0, v \geq t$ household consumption demand at time *t* is determined from:

$$\frac{\overline{c}(s,t)}{\theta+\lambda} - \frac{1}{\beta\overline{c}(s,t)}\int_t^{\infty}\left(i(v)\right)^2 e^{-\int_t^v (2r(u)-\theta+\lambda)du} dv + \frac{\alpha}{\beta} e^{-\int_t^v (r(u)+\lambda)du} dv = \overline{j}(s,t) \tag{13}$$

Equation (13) defines individual household consumption at time *t* as an increasing function of comprehensive household wealth:

$$\begin{aligned}&\overline{c}(s,t) = f\left(\overline{j}(s,t)\right) \\ &f' = \frac{(\theta+\lambda)\beta\overline{c}(s,t)^2}{\beta\overline{c}(s,t)^2 + (\theta+\lambda)\int_t^{\infty} i(v)^2 e^{-\int_t^v [2r(u)-\theta+\lambda]du} dv} > 0 \quad \text{for } \overline{c}(s,t) > 0\end{aligned} \tag{9'}$$

This is hardly surprising, because both consumption and (until satiation sets in) real money balances are normal goods. From Engel aggregation we know that if we have two goods in the instantaneous utility function, they cannot both be inferior. Since, for $i(v) > 0, v \geq t$, real money balances and consumption are positively related (see (12')) consumption demand and money demand are both increasing in comprehensive wealth. So it suffices to show that helicopter money can increase the comprehensive wealth of every household to demonstrate its effectiveness. This we do below.

When $i(v) = 0,\ v \geq 0$ we are in a permanent liquidity trap and there is satiation in real money balances at each instant. We assume that real money balances remain finite. The household consumption function for this case is given by

$$\overline{c}(s,t) = \left(\theta+\lambda\right)\overline{j}(s,t) \tag{9''}$$

This is the same as the consumption function derived in (9) from a Cobb-Douglas utility function with $\alpha = 0$.

When $i(t) = 0$ households may end up holding real money balances in excess of $\frac{\alpha}{\beta}$. To do so does, of course, use up comprehensive wealth without increasing instantaneous utility today. With the utility of consumption increasing without bound in consumption, would a utility maximizing household take resources out of real money balances in excess of $\frac{\alpha}{\beta}$ and allocate them to current consumption instead? If current consumption were the only option it would, but this household has an expected lifetime of duration $\lambda^{-1}$ , so it would want to allocate more to future consumption as well, since optimal consumption over time is characterized, both in the Cobb-Douglas model and in the model with satiation in real money balances for finite stocks of real money, by

$$\overline{c}(s,v) = \overline{c}(s,t)e^{\int_t^v (r(u)-\theta)du}$$

So if faced with redundant real money balances (a level in excess of the satiation level), an optimizing household would want to raise current consumption and consumption in all future time periods. To increase future consumption total comprehensive wealth has to be higher, but the household will be indifferent between holding that wealth in the form of base money, bonds or real capital, as the nominal yield on all these stores of value is zero.

In what follows, I will, except when I deal with the permanent liquidity trap case, work with the Cobb-Douglas instantaneous utility function. It permits a simple closed-form solution - unlike the non-homothetic preferences that generate instantaneous utility functions capable of producing satiation for a finite stock of real money balances. When I consider the permanent liquidity trap special case, in Section 2.7, I will switch to the instantaneous utility function with satiation, which, in the special case under consideration only requires one to set $\alpha = 0$ in the Cobb-Douglas model.

**Re:** *"Finally, in contrast to the literature, the author assumes that the transversality condition for the government's intertemporal budget constraint only applies to government debt. In, for example, Leeper (1991) and EW, the transversality condition applies to total government liabilities (monetary base plus government debt). This assumption by the author is justified on the basis that money issued by the state cannot be exchanged for any real good or service. However, households hold money under the assumption that it retains value for purchasing goods or services even into the future. Greater discussion of this assumption and its importance for the results in this paper would be helpful."*

This is indeed the sole reason for the difference between my effectiveness results and EW's ineffectiveness result at the ZLB. To the best of my understanding, EW assume, implicitly, until they consider the "fiscal commitment" condition in equation 38 on page 196 of their paper, that base money enters the solvency constraints of households and the State symmetrically.

On pages 19-22, I now have a section: **Why is this result different from the ineffectiveness result of Eggertsson and Woodford?**

## Why is this result different from the ineffectiveness result of Eggertsson and Woodford?

Eggertson and Woodford (2003) hereafter EW argue that expansions of the monetary base, holding constant the Central Bank's interest rate rule, will have no effect at the ZLB.

The effectiveness of helicopter money (or permanent QE) in boosting household demand in the pure liquidity trap case where the safe nominal rate of interest is at the effective lower bound/ZLB at all maturities is due to the asymmetric treatment of the NPV of the terminal stock of base money in the household solvency constraint (equation (20)) and in the State solvency constraint (equation (22)). This asymmetry is a result of the irredeemability of base money, which implies that base money is an asset to the holder but not a liability to the issuer. If the State instead were to treat base money as a liability, that is, if equation (22) were to be replaced by

$$\lim_{v\to\infty}\left(\frac{M(v)+B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}}=0 \quad (31)$$

Then the aggregate consumption function (25) or (27) would be replaced, respectively, by equation (32) and equation (33).

$$C(t)=(1-\alpha)(\theta+\lambda)\left[\begin{array}{l}K(t)+\int_t^\infty\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv\\-\int_t^\infty\left(T(v)-\frac{D(v)}{P(v)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv\\+\frac{1}{P(t)}\left(\int_t^\infty i(v)M(v)e^{-\int_t^v i(u)du}dv\right)\end{array}\right]$$

(32)[2]

[2] Note that
$$\int_t^\infty i(v)\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}dv+\lim_{v\to\infty}\frac{M(v)}{P(v)}e^{-\int_t^v r(u)du}=\frac{1}{P(t)}\left(\int_t^\infty i(v)M(v)e^{-\int_t^v i(u)du}dv+\lim_{v\to\infty}M(v)e^{-\int_t^v i(u)du}\right).$$

$$C(t)=(1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^{\infty}\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv \\ -\int_t^{\infty}\left(T(v)-\dfrac{D(v)}{P(v)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv \\ +\dfrac{1}{P(t)}\left(M(t)+\int_t^{\infty}\dot{M}(v)e^{-\int_t^v i(u)du}dv\right)-\lim\limits_{v\to\infty}M(v)e^{-\int_t^v i(u)du} \end{array}\right]$$

(33)

So in the pure liquidity trap case ($i(u)=0,\ u\ge t$) these two equivalent versions of the aggregate consumption function reduce to:

$$C(t)=(1-\alpha)(\theta+\lambda)\left[\begin{array}{l} K(t)+\int_t^{\infty}\left(W(v)-G(v)\mathrm{e}^{\beta(v-t)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}dv \\ -\int_t^{\infty}\left(T(v)-\dfrac{D(v)}{P(v)}\right)e^{-\int_t^v\left(r(u)+\beta\right)du}\left[1-e^{\beta(v-t)}\right]dv \end{array}\right]$$

(34)

For those who are concerned about the unbounded demand for real money balances at the ZLB in the Cobb-Douglas case, equation (34) can be reinterpreted, by setting $\alpha=0$, as the aggregate consumption demand when the economy is stuck permanently at the ZLB for the model with satiation in real money balances at a finite level of real money balances.

In equation (34), current and/or future money stocks don't appear. Helicopter money is completely ineffective and so, of course, is any increase in the base money stock, even if it is permanent, say a permanent increase in the monetary base brought about through irreversible QE.

I assume that the difference in results is due to a symmetric treatment of money in the household and State solvency constraints by EW. I cannot be completely certain of this, as EW specify the intertemporal budget constraint of the household (the first equation (not numbered) on their p. 149) directly - without explicitly giving a no-Ponzi finance solvency constraint for the household. From the form of the intertemporal budget constraint that the household solvency constraint is that the NPV of the household's terminal financial wealth, including base money balances be non-negative – base money is perceived as an asset by the household sector..

The only time we see something that looks like a solvency constraint for the State is EW's equation 38 on page 196, which requires that the NPV of the State's non-monetary liabilities be zero. The intertemporal budget constraint of the State is not spelled out, so we cannot back out whether the solvency constraint of the State requires the NPV of the terminal stocks of all financial liabilities of the State (including the base money stock) to be non-negative or just the NPV of the terminal stocks of non-monetary liabilities. The ineffectiveness of base money expansions at the ZLB in the first part of their paper suggests that EW have, implicitly, until they arrive at equation 38 on page 196, adopted a symmetric role of the base money stock in the solvency constraints of the household sector and of the State in the first part of their paper.

However, when on page 196 EW impose $\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}}=0$ they have the same asymmetry as regards the way base money is viewed between households on the one hand and the State on the other hand, as does our paper. EW note that, with the asymmetric solvency constraint, base money expansion at the ZLB can be effective. "*Thus a commitment of this kind can exclude the possibility of a self-fulfilling deflation of the sort above as a rational expectations equilibrium. It follows that there is a possible role for quantitative easing – understood to mean the supply of base money beyond the minimum quantity required for consistency with the zero nominal interest rate – as an element of an optimal policy commitment".*(Eggertsson and Woodford (2003, p. 197).

EW, however, view $\lim_{v\to\infty}\left(\frac{B(v)}{P(v)}\right)e^{-\int_t^v r(u)\mathrm{du}}=0$ not as the solvency constraint of the State (the consolidated general government and Central Bank) but as a fiscal commitment, which need not hold all the time. This probably accounts for the ineffectiveness of base money expansions at the ZLB when the economy is in a self-fulfilling deflationary trap and the 'fiscal commitment rule' is not imposed.

This is the only reason why we get a general effectiveness of permanent base money expansions result, away from as well as at the ZLB while EW in much of their paper (until equation 38 on page 196) get an ineffectiveness result at the ZLB. The fact that EW look at a specific Taylor-type interest rate rule away from the ZLB and a monetary base rule at the ZLB, while we establish the effectiveness result for all sequences of current and future interest rates, does not account for the differences in our results.

Neither is the difference in our results due to the fact that EW work with a general equilibrium model while I work with an incomplete or partial equilibrium model containing only the household and the State sectors. In this partial equilibrium framework I show that for *any* sequences of interest rates, (positive) price levels and other variables that are taken as exogenous by the individual households but will be endogenous in a fully-fledged general equilibrium model, household demand can be boosted through helicopter money drops by any desired amount. Whether such an increase in consumer demand raises prices and money wages alone, or employment and output as well, will depend on the general equilibrium 'closure' rule that is adopted. Establishing the point that household demand can be boosted does not require a general equilibrium model.