Horizontal Mergers and Uncertainty

Nicolas Le Pape and Kai Zhao

Abstract
Some path-breaking work on mergers takes efficiency gains for granted, or assumes that firms have perfect knowledge when taking merger decisions. In practice, firms and competition authorities cannot know exact future efficiency gains, prior to merger consummation. This paper analyzes horizontal mergers when the output decision-making process is sequential. A key assumption is that mergers create uncertainty on productivity and informational asymmetry between firms. The paper also studies whether the merged firm has interest to reveal the information about its own cost to competing firms. In terms of "Merger Approval", the paper emphasizes the timing of regulatory intervention and distinguishes two different merger control interventions (ex ante or ex post enforcement). Since prudent competition authorities (using ex ante intervention) should take the restrictive policy, the framework illustrates why US Horizontal Merger Guidelines and EC Merger Regulation are biased in favor of the consumers’ interests.

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Keywords Merger; competition authorities; uncertainty; asymmetric information

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1 Introduction

This paper broadens the theory on horizontal mergers with uncertain efficiency gains in Stackelberg markets. In general, there are three phases in merger game: I. pre-merger, II. merger, III. post merger. Some path-breaking work on horizontal mergers takes efficiency gains for granted, or assumes that firms have perfect knowledge about the future merged entity when taking merger decisions. In practice, merging firms and Competition Authorities\textsuperscript{1} could not know the exact future efficiency gains (or losses) prior to merger consummation, in other words, the possibility that the merging firms become more efficient does not mean that these gains are actually realized once the operation has been cleared and has taken place. This is because merged firms are not just larger firms but more complex organizations. For instance, mergers create the uncertainty for employees because of the potential clashes of culture and management style, this uncertainty can lead to such dysfunctional outcomes as stress, job dissatisfaction, low trust in the organization, and increased intentions to leave the organization. These dysfunctions can, in turn, diminish productivity and increase the production cost (Morán and Panasian, 2005).

This paper analyzes the incentive to merge and the welfare effects of mergers in (quantity-setting) uncertain markets where output decision-making process is sequential. This framework is related to two strands of the merger literature. The first strand typically focuses upon the relationship between sequentiality (leader and/or follower) and merger incentive in a context of deterministic markets. Levin (1990) shows that in the absence of uncertainty, the private incentive to merge is higher and antagonism between the private and the collective advantage of the merger disappears, when a merged firm changes its behavior from a Cournot-Nash player to a Stackelberg leader player. In a game where asymmetric roles among the firms in the pre-merger situation (Stackelberg leader and follower compete in homogeneous good market) are introduced, mergers can also improve welfare and boost profits. For instance, when two followers decide to merge and when the newly merged entity behaves as a leader on the product market, the social welfare and merging firms’ profits increase even without cost savings following the merger (Daughety, 1990). In Stackelberg markets with $n$ rival firms and linear costs, two leaders rarely have an incentive to merge, nor do two followers when the new entity stays in the same category (Huck, Konrad and Mueller, 2001).

On the other hand, in the presence of uncertainty, to the best of our knowledge, all theoretical analyses are based on the key assumption that output (or price) decision-making process is simultaneous (Cournot or Bertrand). Amir et al. (2009) highlight the fact that the scope of profitable merger enlarges with uncertainty. The uncertain efficiency gains affect the ex ante beliefs on the merged firm’s cost by outsiders and elicit the competitive advantage to the merged firm from strategic aspects. Some authors investigate how cost uncertainty affects the incentives to merge, and they show that the incentives to merge depend on the information structure (Choné and Linnemer, 2008; Zhou, 2008a and 2008b). Banal-Estanol (2007) also investigates merger incentive under cost uncertainty. He concludes that uncertainty always enhances merger incentives if the signals are privately observed. The above-mentioned papers focus on the cost (or efficiency gains) uncertainty. There are also some frameworks which deal with the issue of merger under demand uncertainty, for instance, Gal-Or (1988) finds that demand uncertainty and asymmetric information may hinder mergers.

In the current paper, we turn our attention to cost uncertainty on merger with sequential output decisions in order to fulfill the gap of merger issue. The key assumption is that all firms face uncertainty as to the efficiency gains, in terms of variable costs that the merged firm could achieve, within the "Private incentive to merge" decision analysis (Merger phase); once the merger is consummated, insider first-to-

\textsuperscript{1}Merging firms in general have strong incentives to overestimate these gains in front of Competition Authorities.
knows its own actual cost, outsider-followers can perfectly observe the output level of merged firm (if insider behaving as leader) and infer the exact value of merged firm’s cost within the "Profitability of merger" (Post Merger phase). This information structure is different from the one proposed by Amir et al. (2009) where after merger no outsider is informed about the merged firm’s cost. The difference stems from the sequentiality of output decision. For instance, if the merged firm behaves as a leader, which makes the production decision firstly, the outsider-follower firms observe the production level of the insider and infer the actual cost of the insider. This design permits us to restudy the merger in the context of close relationship between the distribution of roles and the information structure. Concretely, the behavior of merged entity can alter the outsider firms’ information configuration: leader strategy chosen by insider generates the asymmetric information amongst non-merged firms (the outsider-follower is aware of insider’s cost, while the outsider-leader is not informed about it) and there will be the symmetric information amongst outsiders when insider behaves as a follower. In order to capture the impact of role distribution and information configuration, we take into account all possible two-firm mergers, such as merger between leaders (or followers), merger between leader and follower, and merger between followers resulting in merged leader firm.

Before the merger consummation, firms do not learn the actual cost of the merged entity, the incentive to merge for participants grows following the enlargement of variance. Till the extent of variance exceeds a certain threshold, the expected profit of the merged firm becomes larger than the sum of the pre-merger (participant) firms’ profits, and these firms facing cost uncertainty choose to merge. This finding highlights that even if there is neither efficiency gains nor informational advantage for merging firm, the cost uncertainty is able to induce the firms to merge. The relationship between cost uncertainty and merger incentives is also investigated by Banal-Estanol (2007) and Zhou (2008a). The former finds that cost uncertainty always enhances the incentives to merge and argues that the extra incentive is driven by information sharing. The latter shows that the extra merger incentives are reinforced by production rationalization. In our framework, the additional incentives are engendered by both role redistribution and lack of information.

Accordingly, we consider that mergers not only create market power, but also yield efficiency gains (or losses) of random magnitude. The merged entity’s cost information is private before the time of production, and the insider is able to signal its private information about the consequence of the merger through its market conduct. Since the firms outside of the merger (outsiders) are composed of leaders and followers, it is not only the merged firm (insider) that gets to recognize its cost, but also followers can actually observe the merged firm’s cost because of the second mover in case of the newly merged firm behaving as a leader. Meanwhile, each outsider-leader firm chooses one output level to maximize its ex ante expected profits for lack of information. Thus, when the merged firm behaves as a leader, there is the asymmetric information between outsider-leader and outsider-follower firms; when the merged firm plays the follower role, the gap of information among outsiders disappears, because all outsiders are uninformed about the real cost of merged entity. We analyze the profitability of merger in context of informational asymmetry. It is shown that the two-follower merger aiming to a leader strategy occurs more likely than the one satisfying statu quo. Furthermore, the merged firm has interests to pool the private signals to outsiders, in the absence of role redistribution. By contrast, in the presence of role redistribution, the concealment is more profitable from the viewpoint of insider.

Concerning "Merger Approval", we firstly study the case where Competition Authorities adopt the ex ante enforcement, in other words, they decide whether to approve or refuse the merger proposal with-

\footnote{The reason that we focus only on bilateral merger is explained by some illustrations in automotive domain, e.g. Daimler-Chrysler in 1998, Porsche-VW during 2004-2008, Chrysler-Fiat in 2009, etc. From the theoretical viewpoint, Zhou (2008a) demonstrates that two-firm mergers are far more frequent than three- or four-firm mergers.}
out knowing the actual cost of potentially merged entity. Under this circumstance, the merger between leaders always enhances welfare, as long as the participants have incentives to merge. This generates the unanimity of private and collective incentives, and it provides support for laissez-faire policy. Furthermore, enforcement practice in most countries (including the US and the EU) is closest to a consumer welfare standard. Thus, we carry on a separate analysis of consumer surplus in order to gain some insight into the relationship between distinct criterions of Competition Authorities and merger issue within sequential quantity-setting game.

Without loss of generality, the ex post policy intervention is also used by Competition Authorities to judge the implemented merger. The Synopsys decision and the Muris speech along with the recent FTC (Federal Trade Commission) enforcement actions against several consummated mergers all suggest that although the HSR (Hart Scott Rodino) statute makes ex ante merger enforcement possible, it does not proscribe the government from choosing ex post enforcement if the conditions suggest it is likely to be superior. According to Ottaviani and Wickelgren (2011), the Competition Authorities can employ a "wait and see" approach by letting the merger go through in order to have more accurate information about it. In particular, as Leibeskind (2004) has noted, because antitrust jurisprudence and recent industrial organization scholarship have both moved away from strong structural presumptions about what makes a merger anti-competitive, there is a stronger need for solid evidence of anti-competitive effects. Because these can be hard to prove ex ante, this explains the recent renewed interest in ex post merger enforcement and why to introduce the ex post enforcement in this framework.

By studying two alternative criterions under two different policy intervention timings, we find that the timing of policy intervention has important implication to the choice between the two welfare standards: the consumer welfare standard is more rigorous than the aggregate welfare standard in case of

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3See US Merger Guidelines Section 4. Merging parties, arguably, know more about potential efficiency gains than Competition Authorities. Firms have strong incentives to dissemble about efficiency.

4In merger control, the emphasis is now firmly on consumer surplus. It is worth reflecting on the rationale put forward in support of a consumer welfare policy standard in these areas (as opposed to a total welfare standard). In principle, economists advocate a total welfare standard that encompasses a balancing of rents to producers and consumers. Nevertheless, there are several arguments in support of entrusting a competition agency with a consumer surplus standard. These are based on the following considerations: (1) informational advantages, (2) merger selection bias, and (3) lobbying activities. In addition a consumer standard is considered to be easier to implement.

5The Federal Trade Commission's unanimous decision not to challenge in advance Synopsys, Inc. acquisition of Avant! Corporation in 2002 provides a good illustration of why the FTC has partially moved away from the dominant paradigm of ex ante merger enforcement. As with almost any merger investigation, the FTC had to determine whether, in the words of Commissioner Anthony, “efficiencies will be sufficient to outweigh any potential harm to competition.” Commissioner Anthony emphasized that there was a great deal of uncertainty regarding the answer in this particular case. Thus, while all Commissioners voted to close the investigation, Anthony and two other Commissioners also issued statements suggesting that the Commission should carefully monitor the market to consider a later, ex post, challenge to the merger. See more detailed in Statement of Commissioner Sheila F. Anthony and Commissioner Thomas B. Leary, Synopsys Inc./Avant! Corporation, FTC File No.021-0049.

6 Former Federal Trade Commission (FTC) Chairman Timothy Muris, however, has suggested a renewed interest in government enforcement of mergers after the fact: “If you have clients that are concerned with a transaction, let us know - whether or not it has been consummated. We are quite prepared to go after consummated mergers or mergers that are too small to require an HSR filing.” See Prepared Remarks of Timothy J. Muris, Antitrust Enforcement at the Federal Trade Commission: In a Word - Continuity, before American Bar Association Antitrust Section Annual Meeting, Chicago, IL, August 7, 2001.

7See Compton and Sher (2003) and Leibeskind (2004) for a discussion of these enforcement actions

8Prior to the passage of the Hart Scott Rodino (HSR) Act in 1976, the U.S. government could only challenge mergers after they had been consummated.

9In some theoretical papers, the distinction between ex ante and ex post enforcement has been emphasized, such as Besanko and Spulber (1990), Berges-Sennou et al. (2001), Pénard and Souam (2002). The ex post enforcement of competition authorities involves a comparison between the outcome of antitrust intervention and a counterfactual that describes what would have happened (or not happened) in its absence. It allows for an assessment of the benefits of enforcement and hence, when related to the cost of enforcement.

10See ex ante safety regulation and ex post tort liability in Shavell (1984) and Kolstad et al.(1990); ex ante versus ex post regulation of bank capital in Daripa and Varotto (2005); ex ante and ex post merger control in Ottaviani and Wickelgren (2009).
ex ante enforcement, while the consumer welfare standard is more lenient under ex post enforcement. Since prudent Competition Authorities (using ex ante intervention) should take the restrictive policy, our framework illustrates the reason why US Horizontal Merger Guidelines and EC Merger Regulation are biased in favor of the consumers’ interests.

The reminder of the paper is organized as follows. Section 2 presents the model and specifies the sub-game perfect equilibria for different types of mergers within uncertain markets. Section 3 analyzes the "Private incentive to merge" and the "Profitability of merger". Section 4 investigates the welfare implications of mergers and studies the relationship between private intention and collective incentive, this section is also devoted to some research about Competition Authorities’ distinct criterions (aggregate welfare standard or consumer welfare standard). Section 5 extends the model to allow Competition Authorities to adopt the ex post enforcement. Finally, section 6 discusses our main findings and concludes. The detail and some complicated expressions are in the Appendix.

2 Model

The timing of this game is summarized in the sketch map (Figure 1) which shows both decision structure and information structure in a time axis. Benchmark competition is modelled as a standard Stackelberg game with complete information to all active firms. The merger may generate either efficiency gains or losses, and there is some uncertainty on what will be the exact value of the insider’s marginal cost. Consequently, the merger not only gives rise to the productivity shock in newly merged entity at the time of merger, but also introduces a modification in the information structure of players, once the merger is implemented.

Figure 1: Game structure

At the point of "Private incentive to merge", all firms (including the merging firms) in industry face uncertainty as to the efficiency gains, in terms of variable marginal cost, that the merged firm could achieve. Thus, any merging firms must decide whether or not to merge without knowing the true cost of
the potentially merged firm in future.

Without loss of generality, we assume that there are two alternative timings of antitrust intervention: the one is \textit{ex ante} intervention, Competition Authorities decide whether the merger in question is approved, facing cost uncertainty; the other is \textit{ex post} enforcement, the insider recognizes its own production cost level after merger consummation and signals its private information through its market conduct, thus, Competition Authorities get to obtain the information\(^\text{11}\) about the production cost of insider, the advantage of \textit{post-hoc} review is that Competition Authorities can focus more on history than on predictions.

Once mergers are authorized, we turn to the \textit{post merger} game where insider \textit{first-to-knows} its own exact cost, and part of outsiders (outsider-followers) could be aware of the actual cost of insider during the "Profitability of merger". This information structure is different from the one proposed by Amir et al. (2009) where after merger none of outsiders are informed about the merged firm’s cost. The difference of the information structures stems from the sequentiality of output decision. For instance, if the merged firm behaves as a leader, which makes the production decision firstly, the outsider-follower firms observe the production level of insider and infer the actual productivity (cost) level of insider. This novel ingredient permits us to restudy the merger in the presence of the distribution of roles and the different information configurations. Concretely, the strategic behavior of merged entity can alter the outsider firms’ information configuration: leader strategy chosen by insider generates the asymmetric information amongst non-merged firms (because the outsider-follower firms are aware of insider’s cost, but the outsider-leader firms are not informed about it.) and there will be the symmetric information amongst outsiders when insider behaves as a follower. In order to capture the impact of the roles’ distribution in the \textit{pre-merger} situation and the impact of informational structure in the \textit{post merger} situation, we examine four alternative scenarios: a merger between two leaders (case A), a merger between two followers (case B), a merger between two followers resulting in a newly merged leader (case C) and a merger between one leader and one follower resulting in a newly merged leader (case D).

2.1 The benchmark situation

We consider an industry composed of \(n\) initially active firms producing homogenous products, who compete by setting quantity schedules. In the first stage, \(m < n\) firms act as Stackelberg leaders and independently decide on their individual supply. In the second stage, \(n - m\) Stackelberg followers decide upon their quantity after learning about the total quantity supplied by the leaders. Initially, we assume \(m > 2\) and \(n - m > 2\), the strict inequalities ensure that in every case the outsiders gather both leader and follower in the \textit{post merger} situation.\(^{12}\) All firms face the same constant average cost normalized to \(c\). The market price is determined by the linear inverse demand curve \(p = a - Q\) where \(a > c\). The aggregate industry output is given by \(Q = Q^l + Q^f\) with \(Q^l = \sum_{i=1}^{m} q^l_i\) and \(Q^f = \sum_{i=m+1}^{n} q^f_i\), \(q_i\) denotes the firm \(i\)'s individual quantity. The superscript "\(l\)" stands for a leader and "\(f\)" represents a follower.

The equilibria are obtained by backward induction. At the second (follower output decision) stage, each follower maximizes its profit (\(\pi^f_i\)) considering as given the production level of leader (\(Q^l\)). The best

\(^{11}\) According to timing, CAs with \textit{ex post} enforcement interfere after the “Output decision”, it is logical that CAs are aware of the actual production cost of insider. We suppose that there is no cost for acquiring the information. For instance, if the insider behaves as a leader, followers and CAs have the complete information on actual production cost of merged entity at no cost.

\(^{12}\) The particular cases: both \(m = 0\) and \(m = n\) correspond to a Cournot industry, the firms are in the simultaneous game. The Stackelberg and Cournot models are similar because in both competition is on quantity. However, as seen, the first move gives the leader in Stackelberg a crucial advantage. There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.
response function \((q_f^i)\) of a follower firm results from:

\[
\max_{q_f^i} \pi_f^i = (a - Q^f - Q^l - c)q_f^i
\]

At the first (leader output decision) stage, a leader selects its profit-maximizing output \((q_l^i)\) anticipating the best response function of each follower:

\[
\max_{q_l^i} \pi_l^i = [a - c - Q^f(Q^f)]q_l^i
\]

In the benchmark situation, the corresponding individual outputs and profits are:

\[
q_l^i(m) = \frac{a - c}{m + 1} \quad \pi_l^i(n, m) = \frac{(a - c)^2}{(m + 1)^2(n - m + 1)}
\]

\[
q_l^i(n, m) = \frac{a - c}{(n - m + 1)(m + 1)} = \frac{1}{n - m + 1}q_l^i
\]

\[
\pi_f(n, m) = \frac{(a - c)^2}{(m + 1)^2(n - m + 1)^2} = \frac{1}{n - m + 1}\pi_f^i
\]

Obviously, the distribution of roles among firms exhibits the first mover advantage\(^{13}\): each leader benefits from higher market share and earns higher profit in benchmark game.

### 2.2 The different merger scenarios

In this subsection, we focus upon a bilateral (two-firm) merger. When two firms make the decision whether to merge, all firms including the merging firms in the market are uncertain over the marginal cost of the newly merged entity. Thus, any two merging firms must decide whether or not to merge without knowing the actual cost of the merged firm in future. We suppose that the expected marginal cost of the merged firm is equal to the non-merged firm’s cost “s” which is the same as the benchmark firm’s one\(^{14}\).

The exact value of newly merged entity’s cost “c” is uncertain, it could be either higher or lower than this critical value c. Hence, we assume that \(a > \max\{c, c_i\}\) and the variance of this uncertain cost \(c_i\) is independently drawn from an identical distribution with \(Var(c_i) = \sigma^2\). The variance \(\sigma^2\) represents the degree of the uncertainty and captures marginal cost fluctuation. The merging firms can generate efficiency gains if \(c_i - c < 0\). This situation corresponds to the usual argument which puts forward to the increase in productive efficiency generated by the merger itself. Conversely, when \(c_i - c > 0\) the merger is assumed to cause efficiency losses (i.e. due to the clash of company culture).

#### Case A: Merger between two leaders

In this case, the industry is composed of \(m - 1\) leaders but still \(n - m\) followers since the newly merged entity behaves as a leader. Consider \(q_l^A\) as the merged firm’s quantity and \(q_o^A\) as outsider-leader firm’s output and \(q_o^A\) as outsider-follower’s output. From the standpoint of information structure, since insider first-to-knows its production cost (or productivity), its output level will depend on the actual cost \((c_i)\).

\(^{13}\)The leader’s profit under the sequential-game equilibrium will be higher than under Cournot equilibrium. Since follower firm reacts in a “Nash fashion”, leader firm could just choose to produce the Cournot output level. In this case, leader firm would earn exactly the Cournot profit. However, since in the sequential game leader firm chooses to produce a different output level, it must be increasing its profit compared with the Cournot profit level. The kind of reasoning is called a revealed profitability argument.

\(^{14}\)This assumption allows us to focus on the effect of uncertainty on mergers even without any uncertain efficiency gains.
namely \( q_{IA}(c_i) \); outsider-followers observe the output level of insider and then perfectly infer the merged entity’s cost, accordingly \( q_{IA}(c_i) \); as all leaders simultaneously decide the quantity level, outsider-leaders have no chance to observe the insider production, consequently, the outsider-leaders regard \( c \) as the insider’s productivity, we have \( q_{IA}(c) \).

By backward induction, we begin with the follower production stage. The optimizing question is

\[
\max_{q_{FA}} \pi_{FA} = (p^A - c)q_{FA} = [a - c - Q_{OA} - q_{FA}^{LA}(c) - q_{FA}(c)]q_{FA}
\]

From the first-order-condition, we derive the best response function of followers (See detail in Appendix A):

\[
(n - m + 1)q_{FA} = a - c - Q_{FA}(c) - q_{LA}(c_i)
\]

In the first (leader production) stage, outsider leaders are not aware of the actual cost of insider, thereby, they consider the insider’s cost as the expected value \( c \) and maximize the following profit function:

\[
\max_{q_{IA}} \pi_{IA} = (p^A - c)q_{IA} = [a - c - Q_{IA} - Q_{IA}^{LA}(c) - q_{IA} - q_{IA}(c)]q_{IA}
\]

For the insider, since it knows the real cost \( c_i \)

\[
\max_{q_{IA}} \pi_{IA} = (p^A - c_i)q_{IA} = [a - c_i - Q_{IA} - Q_{IA}^{LA}(c) - q_{IA} - q_{IA}(c)]q_{IA}
\]

We then obtain the following expressions for the equilibrium output (See detail in Appendix B):

\[
q_{IA}(c_i) = \frac{2(a - c) - m(n - m + 1)(c_i - c)}{2m}
\]

\[
q_{IA}(c) = \frac{a - c}{m}
\]

\[
q_{IA}(c) = \frac{a - c}{m}
\]

\[
q_{OA} = \frac{2(a - c) + m(n - m + 1)(c_i - c)}{2m(n - m + 1)}
\]

The aggregate quantity is expressed as

\[
Q^A = q_{IA}(c_i) + (m - 2)q_{IA}(c) + (n - m)q_{OA}(c)
\]

Both the equilibrium profits and the expected equilibrium profits of firms are given as follows (See detail in Appendix C).

**Insider:**

\[
\pi_{IA} = \frac{[2(a - c) - m(n - m + 1)(c_i - c)]^2}{4m^2(n - m + 1)}
\]

\[
E[\pi_{IA}] = \frac{(a - c)^2}{m^2(n - m + 1)} + \frac{n - m + 1}{4}\sigma^2
\]
decision”. \( \pi^A \) represents the exact value of merged firm’s profit which will be used to analyze the profitability of merger. In addition, the expected profit of merged firm is determined at the moment of “Private incentive to merge” where the actual cost of merged firm is concealed from all firms including merging parties, and this expected term is used to analyze the incentive to merge in the following section.

**Outsider-leader:**

\[
\pi^A_{lO} = \frac{(a - c)[2(a - c) + m(n - m + 1)(c_1 - c)]}{2m^2(n - m + 1)}
\]

\[
\mathbb{E}[\pi^A_{lO}] = \frac{(a - c)^2}{m^2(n - m + 1)}
\]

(5) (6)

Outsider-leader firms commit to quantities before the uncertainty is resolved, therefore, they possess zero information on merged entity’s cost, and only the expected value of the cost is relevant to them. A larger uncertainty, in the sense of an increased variance in the cost distribution with the same expected value, will not change the profit of outsider-leader firms. Consequently, uncertainty has no effect on them, and each outsider-leader’s expected profit is the same as when merged firm’s cost is deterministic \((c_1 = c)\).

**Outsider-follower:**

\[
\pi^A_{fO} = \frac{[2(a - c) + m(n - m + 1)(c_1 - c)]^2}{4m^2(n - m + 1)^2}
\]

\[
\mathbb{E}[\pi^A_{fO}] = \frac{(a - c)^2}{m^2(n - m + 1)^2} + \frac{1}{4}\sigma^2
\]

(7) (8)

It is worthwhile to note that, since both the merged firm and the outsider-follower firms know the exact marginal cost of merged entity, in addition, outsider-leader firms recognize no change in merged firm’s cost after merger, the asymmetric information about the merged entity’s cost not only does work in favor of the merged firm, but also is propitious to outsider-follower firms. This is because firms of both categories can adjust their production accordingly. In expected terms, the sensibility of firms’ gains to the uncertainty is not the same. The cost uncertainty effect affects more strongly the merged entity than the outsider (followers) group.

The consumer surplus (CS) and the social welfare (W) are easily found to be:

\[
CS^A = \frac{2[1 - m(n - m + 1)](a - c) + m(n - m + 1)(c_1 - c)]^2}{8m^2(n - m + 1)^2}
\]

\[
W^A = CS^A + \pi^A_l(c_1) + (m - 2)\pi^A_l(c) + (n - m)\pi^A_o(c)
\]

By simple calculation, we obtain the following expected values of CS and W.

\[
\mathbb{E}[CS^A] = \frac{(a - c)^2[1 - m(n - m + 1)]^2}{2m^2(n - m + 1)^2} + \frac{1}{8}\sigma^2
\]

(9) (10) (11)

\[
\mathbb{E}[W^A] = \mathbb{E}[CS^A] + \mathbb{E}[\pi^A_l] + (m - 2)\mathbb{E}[\pi^A_f] + (n - m)\mathbb{E}[\pi^A_o]
\]

\[
= \frac{(a - c)^2}{2} \left[ \frac{m^2(a - m + 1)^2 - 1}{m^2(a - m + 1)^2} \right] + \frac{1}{2}\sigma^2 + \frac{3}{8}\sigma^2
\]

(12)

Note that both consumer surplus and social welfare are increasing functions with respect to the variance \(\sigma^2\). Concretely, we have \(\frac{\partial E[CS^A]}{\partial\sigma^2} = \frac{1}{8}\) and \(\frac{\partial E[W^A]}{\partial\sigma^2} = \frac{a - m}{2} + \frac{3}{8}\). The extent of the uncertainty effect on
We restudy this scenario by introducing two elements: cost uncertainty and information structure, to study whether a merger might be advantageous from the viewpoint of social welfare in the absence of cost variation. Daughety (1990) found that the horizontal merger was potentially profitable for the merged firm and considered case C, where two followers merge and result in a firm behaving as leader.

Consider a special type of merger wherein two followers merge and result in a firm behaving as leader. From the viewpoint of informational structure, the private incentive to merge and ex ante enforcement merger control are studied by means of these expected values.

Social welfare evidently depends on the role distribution. The more leader firms, the lower impact of uncertainty on welfare.

**Case B: Merger between two followers**

In this case, we consider that two followers take part in the merger. The distribution of roles in the industry is assumed not to be altered by the merger decision in the way that merged entity behaves as a follower. The industry contains \( n - 1 \) firms with \( m \) leaders. From the viewpoint of informational structure, neither outsider-leader firms nor outsider-follower firms can infer the exact marginal cost of the merged firm, because this new second-mover entity and the non-merged followers simultaneously make the output decisions. Therefore, there is the informational symmetry between the outsider-leaders and the outsider-followers which are both unaware of the merged firm’s actual cost. The relevant equilibrium values are shown in Table 1. (See brief demonstration in Appendix D)

**Case C: Merger between two followers resulting in a leader**

Consider a special type of merger wherein two followers merge and result in a firm behaving as leader. As a result, there are \( m + 1 \) leaders and in contrast \( n - m - 2 \) followers. This case was examined by Daughety (1990) who found that the horizontal merger was potentially profitable for the merged firm and this merger might be advantageous from the viewpoint of social welfare in the absence of cost variation. We restudy this scenario by introducing two elements: cost uncertainty and information structure, to pro-

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**Table 1: Equilibrium values in case B**

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Actual terms(^a)</th>
<th>Expected terms(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q_i^B(c_i) )</td>
<td>( \frac{2(a-c)-(m+1)(-m)(c_i-c)}{2(m+1)(n-m)} )</td>
<td>( \frac{a-c}{(n+1)(n-m)} )</td>
</tr>
<tr>
<td>( q_j^B(c) )</td>
<td>( \frac{(a-c)}{m+1} )</td>
<td>( a-c )</td>
</tr>
<tr>
<td>( q_j^B(c) )</td>
<td>( \frac{(a-c)}{m+1} )</td>
<td>( a-c )</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_i^B )</td>
<td>( \frac{2(a-c)-(m+1)(-m)(c_i-c)}{4(m+1)(n-m)^2} )</td>
<td>( \frac{(a-c)^2}{(n+1)(n-m)} )</td>
</tr>
<tr>
<td>( \pi_j^B )</td>
<td>( \frac{1}{2(m+1)(n-m)^2} )</td>
<td>( \frac{1}{m+1} )</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>( CS_B )</td>
<td>( \frac{(a-c)^2(1+\sigma^2)}{(n+1)(n-m)^2} )</td>
</tr>
<tr>
<td>Social welfare</td>
<td>( W_B )</td>
<td>( \frac{(a-c)^2(1+\sigma^2)}{(n+1)(n-m)^2} )</td>
</tr>
</tbody>
</table>

\(^a\) Actual terms refer to the post merger game where the insider learns its own cost level. The merger profitability and ex post merger assessment are analyzed based on these values.

\(^b\) Expected terms refer to the pre-merger game where the (merger) participants do not know the future productivity level. The private incentive to merge and ex ante enforcement merger control are studied by means of these expected values.

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welfare, the same ranking is found. The cost uncertainty has the strongest impact on the merged firm’s expected profit when this
in the extreme situation where there is no uncertainty and the information is perfect and complete. The equilibrium values are displayed in Table 2.

Case D: Merger between one leader and one follower
Finally, we focus on the merger between one leader and one follower (the merged entity behaves as a leader). The number of leaders is the same as in the case B, and the number of leaders outside of merger equals to \( m - 1 \). This case without taking into account the issue of information sharing and uncertainty, was studied by Huck, Konard and Muller (2001), who were the first to observe that the merger between two firms from different categories increased the joint profits of firms. They compared the profitability of two-follower merger with that of leader-follower merger, and showed that mergers between a leader and a follower were unambiguously profitable. We derive the same outcome, if we suppose the merged firm’s cost is unaltered and equals to \( c \). The equilibrium values are shown in Table 3.

It is worth noting that the merged firm’s profit, the levels of consumer surplus and social welfare (prior to the merger consummation) are increasing functions with respect to the variance. Thus, the merged firm’s expected profit and the expected surpluses grow, as the uncertainty increases. By comparing the four aforementioned cases, we have the following remarks:

Remark 1 The cost uncertainty has the strongest impact on the merged firm’s expected profit when this entity is composed of two leaders, by contraries, the weakest effect on expected profit when two followers merge without role redistribution. More precisely, \( \frac{\partial E_W}{\partial \sigma} > \frac{\partial E_W}{\partial \sigma^2} > \frac{\partial E_W}{\partial \sigma^3} > \frac{\partial E_W}{\partial \sigma^4} \). In terms of social welfare, the same ranking is found \( \frac{\partial E_W}{\partial \sigma} > \frac{\partial E_W}{\partial \sigma^2} > \frac{\partial E_W}{\partial \sigma^3} > \frac{\partial E_W}{\partial \sigma^4} \). Furthermore, the intensity of uncertainty impact on merged firm’s profit and on the social welfare depends upon the distribution of uncertainty.

### Table 2: Equilibrium values in case C

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Actual terms</th>
<th>Expected terms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td>( q_C^I(c) = \frac{2(a-c)(m+2)(n-m-1)(c_1-c)}{2(m+2)} )</td>
<td>( q_C^E(c) = \frac{(a-c)}{2(m+2)} )</td>
</tr>
<tr>
<td><strong>Profit</strong></td>
<td>( \pi_C^I = \frac{2(a-c)(m+2)(n-m-1)(c_1-c)}{4(m+2)(n-m-1)} )</td>
<td>( \pi_C^E = \frac{(a-c)}{4(m+2)(n-m-1)} )</td>
</tr>
<tr>
<td><strong>Consumer surplus</strong></td>
<td>( C_S^I = \frac{2(a-c)(m+2)(n-m-1)(c_1-c)}{8(m+2)(n-m-1)} )</td>
<td>( C_S^E = \frac{(a-c)}{8(m+2)(n-m-1)} )</td>
</tr>
<tr>
<td><strong>Social welfare</strong></td>
<td>( W_C = C_S^I + \pi_C^I + m\pi_C^E + (n-m-2)\pi_C^E )</td>
<td>( E[W_C] = E[CS^E] + E[\pi_C^E] + mE[\pi_C^E] )</td>
</tr>
</tbody>
</table>

In terms of social welfare, the same ranking is found \( \frac{\partial E_W}{\partial \sigma} > \frac{\partial E_W}{\partial \sigma^2} > \frac{\partial E_W}{\partial \sigma^3} > \frac{\partial E_W}{\partial \sigma^4} \). Furthermore, the intensity of uncertainty impact on merged firm’s profit and on the social welfare depends upon the distribution of uncertainty.
Table 3: Equilibrium values in case D

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Actual terms</th>
<th>Expected terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>$q_{l,D}(c_i) = \frac{2(\alpha - \varepsilon)(m-n)l_{i+1}}{2(m+1)}$</td>
<td>$q_{f}^{D}(c) = \frac{\alpha - \varepsilon}{m+1}$</td>
</tr>
<tr>
<td>Profit</td>
<td>$\pi_{f}^{D} = \frac{(\alpha - \varepsilon)(m-n)l_{i+1}^2}{2(m+1)(m-n)}$</td>
<td>$E[\pi_{f}^{D}] = \frac{(\alpha - \varepsilon)^2}{(m+1)^2(m-n)} + \frac{n-m}{4} \sigma^2$</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>$CS^D = \frac{2(\alpha - \varepsilon)(m-n)l_{i+1}^2}{2(m+1)(m-n)^2}$</td>
<td>$E[CS^D] = \frac{(\alpha - \varepsilon)^2(n-m-1)}{2(m+1)^2(m-n)^2} + \frac{1}{2} \sigma^2$</td>
</tr>
<tr>
<td>Social welfare</td>
<td>$W^D = CS^D + \pi_{f}^{D} + (m-1)\pi_{O}^{D}$</td>
<td>$E[W^D] = E[CS^D] + E[\pi_{f}^{D}] + (m-1)E[\pi_{O}^{D}]$</td>
</tr>
<tr>
<td>$\frac{\partial E[W]}{\partial \sigma^2} = \frac{n-m}{2} - \frac{1}{4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

roles $(n,m)$ except for case B.

In cases A, C and D, the newly merged firm behaves as a leader, there is always asymmetric information between outsider-leaders and outsider-followers. The greater the number of followers $(n - m)$ in the pre-merger market, the larger the intensity of uncertainty (on merged firm’s profit and welfare). By contrast, when there is symmetric information between outsiders, the extent of uncertainty effect on merged firm’s profit and on welfare are constant, irrespective of the number of followers.

For outsider-followers firms, the extent of the cost uncertainty will be the same, except for case B. Namely,

$$\frac{\partial E[\pi_{O}^{A}]}{\partial \sigma^2} = \frac{\partial E[\pi_{O}^{C}]}{\partial \sigma^2} = \frac{\partial E[\pi_{O}^{D}]}{\partial \sigma^2} = \frac{1}{4}$$

**Remark 2** In all cases, welfare is more sensitive to the cost uncertainty compared to consumer surplus.

Concretely, $\frac{\partial E[W]}{\partial \sigma^2} > \frac{\partial E[\pi_{i}^{i}]}{\partial \sigma^2} > \frac{\partial E[CS]}{\partial \sigma^2}$ (i = A, B, C, D and j = {1, f})

In the following section, we provide a detailed account of the consequences of the merger on profits. By dealing with the effects of uncertainty, information structure and role redistribution, we analyze the firms’ incentives to merge and the profitability of merger.

### 3 Merger analysis

The merger incentive is investigated in a situation where the merger creates the productivity shock and all firms in industry therefore are uncertain about the merged firm’s cost. We examine the private incentive to merge which results from the comparison between the ex ante expected profit of the merged firm and the sum of merging parties’ profits in benchmark. This allows us to derive the conditions under which
firms have incentives to merge without recognizing the future real cost of merged entity.

The focus of the study shifts to how asymmetric information affects the profitability of merger, and whether the merged firm has interests to reveal its private information to outsiders or to conceal its real cost from competing firms. The expressions of profit in different scenarios displayed in the previous section, enables us to study the profitability of merger which is determined by the difference of the sum of profits of merging firms in benchmark and the actual profit earned by the newly merged entity.

### 3.1 Private incentive to merge

Let $\Delta_{i|x}$ ($i = \{A, B, C, D\}$) represent the private incentive to merge. The firms have incentive to merge when $\Delta_{i|x} \geq 0$. The relationship between merger incentive and cost uncertainty under different scenarios is shown in Table 4.

Firstly, we demonstrate that the merging firms in scenarios C and D always have incentives to merge, irrespective of the cost uncertainty. This finding is consistent with the existing literature where the cost uncertainty is not taken into account. According to Daughety (1990), for instance, when two followers decide to merge and the newly merged entity behaves as a leader on the product market, the firms have incentives to merge even without cost-saving (or efficiency gains). In addition, HKM (2001) show that the merger between one leader and one follower is profitable, in the absence of information issue and cost fluctuation.

**Proposition 1** If the cost uncertainty is sufficiently large, i.e., with $\sigma^2 \geq \sigma^2_{\pi A}$ or $\sigma^2 \geq \sigma^2_{\pi D}$, the merger without redistribution of roles can be accepted by merging parties. Moreover, if the number of leaders is greater than followers, the merger between leaders needs more uncertainty in order that these firms have incentives to merge; otherwise, the merger between followers requires more uncertainty.
Proposition 1 implies that even if the expected cost is the same before and after merger, as the cost uncertainty grows larger, firms have more incentives to merge. Therefore, the expected profit of the merged firm grows following the enlargement of variance. When the extent of the variance exceeds a certain threshold, such as $\sigma_{m}^2$ and $\sigma_{n}^2$, the expected profit of the merged firm becomes larger than the sum of the firm’s profits in the benchmark case, and firms facing cost uncertainty choose to merge. This proposition highlights that even if there is neither efficiency gains nor informational advantage for merging firm, the cost uncertainty is able to induce the firms to merge.

The relationship between cost uncertainty and merger incentives is also investigated by Banal-Estanol (2007) and Zhou (2008a). The former finds that cost uncertainty always enhances the incentives to merge and argues that the extra incentive is driven by information sharing. The latter shows that the merger incentives are reinforced by production rationalization and informational asymmetry.

In the deterministic approach\footnote{Salant et al.(1983) show that a merger is profitable only when more than 80% of the industry’s firms participate in the merger.}, unless the market share is sufficiently large, most of the horizontal mergers are unprofitable. In proposition 1, as the variance of merged entity’s cost is sufficiently close to zero, the firms without role redistribution have no incentive to merge, this outcome accords with the main result of SSR (1983). However, in our stochastic model, even when the firms have no incentive to merge in the traditional deterministic case, as the variance grows larger, the expected profit also increases because the gain of the optimal quantity adjustment enlarges, and the expected profit of merged firm can exceed the sum of profits of the pre-merger firms. Therefore, proposition 1 presents one of the explanations of the merger paradox.

### 3.2 Profitability of merger

In this subsection, we consider the difference between the merged firm’s exact cost ($c_i$) in case $i$ ($i \in \{A,B,C,D\}$) and expected firms’ costs ($c$) as "$\delta^*$". The profitability of merger derives from the sign of the variation in actual profits ($\Delta^*_A$). For instance, $\Delta^*_A = \pi^A_f(\delta^A, n, m) - 2\pi^l(n, m)$ in case A. The extent of the cost variation for merged firms interacts with the merger’s profitability. We define $\delta^A_{inf}$ the threshold value of $\delta^A$ which separates profitable from unprofitable mergers. When $\delta^A < \delta^A_{inf}$ (respectively $\delta^A > \delta^A_{sup}$) we have $\Delta^*_A > 0$ (respectively $\Delta^*_A < 0$). In addition, in order to avoid boundary problems in which some firms are inactive, we also define $\delta^A_{inf}$ as the value of $\delta^A$ below which outsiders are ruled out of the market. It is given by the conditions: $q_{iD}^{A} = 0$ and $q_{iO}^{A} = 0$. Note that when we have $\delta^A_{inf} < \delta^A < \delta^A_{sup}$, the merger is profitable and two categories of outsiders remain on the market.

Proof:

\[
\left\{\begin{array}{l}
\sigma_{A}^2 > \sigma_{B}^2 > 0, \quad \text{when } \frac{q}{2} < m \leq n - 3; \\
\sigma_{A}^2 > \sigma_{n}^2 > 0, \quad \text{when } 3 \leq m < \frac{q}{2}.
\end{array}\right. \]

"
By comparing $\delta$ formed about the exact value $c_i$, firms are aware of the actual cost of merged entity. In cases A, C and D, outsider-leader firms are uninformed about the exact cost $c_i$. In Table 5, we summarize the ranges of cost variation ($\delta$) in different scenarios wherein the merger is profitable.

To ensure that none of outsider firms exit the market and the merger is profitable, the potential cost change in different scenarios should satisfy the condition that $\delta$ lies in the interval $[\delta_{inf}, \delta_{sup}]$. Note that there is no constraint on the exit of outsider in case B.

### 3.2.1 Incomplete information

Under incomplete information, the merged firm knows its own marginal cost, whereas not all outsider firms are aware of the actual cost of merged entity. In cases A, C and D, outsider-leader firms are uninformed about the exact value $c_i$. However, the timing of the game implies that outsider-follower firms are aware of $c_i$. In Table 5, we summarize the ranges of cost variation ($\delta$) in different scenarios wherein the merger is profitable.

Table 5: Merger profitability and potential efficiency gains (or losses)

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$n \geq 6$ and $3 \leq m \leq n - 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A ($\delta_{inf}^A = \pi'^A - 2\pi'')$</td>
<td>$\delta_{inf}^A &lt; \delta \leq \delta_{sup}^A$</td>
</tr>
<tr>
<td>Case B ($\delta_{inf}^B = \pi'^B - 2\pi'')$</td>
<td>$\delta \leq \delta_{sup}^B$</td>
</tr>
<tr>
<td>Case C ($\delta_{inf}^C = \pi'^C - 2\pi'')$</td>
<td>$\delta_{inf}^C &lt; \delta \leq \delta_{sup}^C$</td>
</tr>
<tr>
<td>Case D ($\delta_{inf}^D = \pi'^D - (\pi' + \pi'')$)</td>
<td>$\delta_{inf}^D &lt; \delta \leq \delta_{sup}^D$</td>
</tr>
</tbody>
</table>

With

$$\delta_{inf}^A = \frac{2(n-m)}{m+n} < \delta < \frac{2n-m}{m+n} \leq \delta_{sup}^A$$

$$\delta_{inf}^B = \frac{2(n-m)}{m+n} < \delta < \frac{2n-m}{m+n} \leq \delta_{sup}^B$$

$$\delta_{inf}^C = \frac{2(n-m)}{m+n} < \delta < \frac{2n-m}{m+n} \leq \delta_{sup}^C$$

$$\delta_{inf}^D = \frac{2(n-m)}{m+n} < \delta < \frac{2n-m}{m+n} \leq \delta_{sup}^D$$

### Remark 3 By comparing $\delta_{sup}$ we obtain:

- $\delta_{sup}^C > \delta_{sup}^B > 0 > \delta_{sup}^A > \delta_{sup}^A$ if $m \in [3, \frac{n}{2})$
- $\delta_{sup}^C > \delta_{sup}^B > 0 > \delta_{sup}^B > \delta_{sup}^B$ if $m \in (\frac{n}{2}, n-3]$  

Since the values of upper bound $\delta_{sup}$ in case C and in case D are greater than zero, a merger with anticompetitive effects could also lead to efficiency losses. If the number of leaders is large enough (i.e., $m \in (\frac{n}{2}, n-3)$), a profitable merger between two leaders requires more marginal cost reduction in comparison with a profitable merger between two followers. In other words, the conditions on efficiency gains, under which the two-follower merger is profitable, are less restrictive. By contrast, if there are more follower firms in pre-merger market, a two-follower merger requires more efficiency gains to be profitable.

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16 In case B where two followers take part in the merger, all outsider firms are uninformed about the exact value $c_i$.  

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The higher $\delta^i_{\text{MPI}}$, the greater the allowed potential efficiency losses, the more likely mergers take place. Since the merger composed of two followers to form a leader (case C) generates potential efficiency losses higher than the merger between one leader and one follower (case D), to some extent that the merger in case C is less restrictive and takes place more likely.

The ceiling of $\delta^i$ depends upon the redistribution of roles. For instance, If we compare the profitable merger in case B to the one in case C, it is found that the resulting leader is less restrictive than the resulting follower. Though the merger leads to efficiency losses, the resulting leader can be profitable due to the effect of role redistribution. It is clear that the two-follower merger aiming to a leader strategy takes place more likely than the one satisfying the status quo.

### 3.2.2 Incomplete Vs complete information

Under complete information\footnote{The framework under complete information is studied in the working paper Le Pape and Zhao (2010).}, the information about merged firm’s real cost is no longer private, not only the merged firm is aware of its own marginal cost $c_i$, but also all outsider firms are informed about it. Using the deterministic case as a criterion, we study whether the merged firm has interests to reveal its own cost to competing firms\footnote{Under some circumstances (case A, C and D), outsider-follower firms can observe the insider’s output level, and then infer the exact value of its marginal cost.}.

Consider $\hat{\pi}^j_i (i = \{A, B, C, D\} \text{ and } j = \{l, f\})$ the merged firm’s profit in the situation where there is complete and perfect information (see expressions of $\hat{\pi}^j_i$ in Appendix E). It will be interesting to compare the profit of the insider under incomplete information scenario to that under complete information situation.

**Proposition 2** Within the range of $\delta^i \in \left[\delta_{\text{MPI}}, \delta_{\text{MPI}}^i\right]$, the profit realized by the merged firm will be greater under complete information than under incomplete information, when there is no redistribution of roles for the merging parties. The opposite outcome will be obtained if there exists a role redistribution.

**Proof:**

- $\pi^A_{l} < \hat{\pi}^A_{l}$ and $\pi^B_{f} < \hat{\pi}^B_{f}$
- $\pi^C_{l} > \hat{\pi}^C_{l}$ and $\pi^D_{l} > \hat{\pi}^D_{l}$ □

The acquisition of market power is usually the first motive for horizontal mergers. The argument is that horizontal mergers increase market concentration, which, by increasing market power, increases profitability. In the absence of the redistribution of roles (cases A and B), the equilibrium price is higher under complete information than incomplete information, the higher price gives rise to higher market power, in addition, the merged firm produces more under complete information. Because of these two above-mentioned reasons, the merged firm will be more profitable under complete information, and it has interests to reveal information about its own cost to competing firms. This outcome is consistent with the well-known conclusion in the information sharing literature\footnote{There are some important contributions to this information sharing literature without merger issue, such as, Novshek and Sonnenschein (1982), Clark (1983), Vives (1984), Gal-Or (1985), Li (1985), Shapiro (1986) and Ratl (1996).}, that, concentrates on a firm’s incentives to
share its private information with competing firms. In particular, it shows that firms competing in quantities are not willing to reveal their private information about market demand, but are willing to reveal their private information about production costs.

By contrast, in the presence of role redistribution (cases C and D), the strengthening of market power under incomplete information leads to more profitable merger compared to the one under complete information. This finding is in line with the conclusion of Zhou\textsuperscript{20} who delineates that "firms are less likely to merge when they possess more information" (Zhou, 2008a, p.68).

The insider is the first firm that is informed about its own exact marginal cost, and consequently may enjoy the "first-to-know". In Amir et al. (2009), the merged firm always benefits from "first-to-know". In sharp contrast, within market where both leaders and followers exist, we demonstrate that "first-to-know disadvantage" could appear, in particular, when the merged firm has the same strategic behavior as \textit{ex ante} merging firms. Under these circumstances, the informational asymmetry created by merger is detrimental to the merged entity. This reinforces and illustrates the conjecture of Gal-or\textsuperscript{21}, that "the merger may impose an informational disadvantage on each firm that colludes" (Gal-or, 1988, p.639).

Let $\hat{\delta}_i^{up}$, $\hat{\delta}_i^{inf}$ denote respectively the upper bound and the lower bound under complete information (see Appendix F). By comparison with the boundary under incomplete information, we derive the following lemma.

\textbf{Lemma 1 i).} In the absence of role redistribution, if and only if the merging firms generate efficiency gains, the merger could be profitable. Moreover, the ceiling of this potential efficiency gains under incomplete information $\hat{\delta}_i^{up}$ (with $i = A,B$) is smaller than that under complete information.

\textbf{ii).} In the presence of role redistribution, i.e. case C and case D, even though the merger leads to efficiency losses, this merger could be profitable. Furthermore, the threshold of potential efficiency losses is larger under incomplete information.

\textbf{Proof:}

Case A: $\hat{\delta}_A^{up} < \hat{\delta}_A^{inf} < 0 \quad 0 > \hat{\delta}_A^{up} > \hat{\delta}_A^{inf}$

Case B: $\hat{\delta}_B^{up} < \hat{\delta}_B^{inf} < 0$ \hspace{1cm} $\exists$

Case C: $\hat{\delta}_C^{up} > \hat{\delta}_C^{inf} > 0 \quad 0 > \hat{\delta}_C^{up} > \hat{\delta}_C^{inf}$

Case D: $\hat{\delta}_D^{up} > \hat{\delta}_D^{inf} > 0 \quad 0 > \hat{\delta}_D^{up} > \hat{\delta}_D^{inf}$ \hspace{1cm} $\square$

As shown in above proposition 2, incomplete information is beneficial to the merged entity in the presence of role redistribution, while it is detrimental to the merged firm in the absence of role redistribution. This permits us to explain the reason why we get to obtain the Lemma 1. In addition, it is obvious

\textsuperscript{20}The reason for Zhou (2008a) is that mergers are driven by production rationalization under cost uncertainty. When firms have more information, they are able to rationalize their production even without a merger, thus having less incentive to merge.

\textsuperscript{21}Gal-Or (1988) shows that the merged firms respond to market signals less aggressively, which induces non-merged firms to be more aggressive.
that under incomplete information, the condition that no firm exits the market, is more restrictive.

To sum up, in the current section, we analyze not only the private incentive to merge at the moment when no firm is informed about actual cost of merged entity, but also the profitability of merger at the moment when the merged firm learns its own cost. In addition, by comparing with the scenarios under complete information, the interesting outcomes are achieved. The existing literature explains profitable merger by uncertain efficiency gains or informational advantages. In contrast, we take a different approach to investigate whether increased uncertainty or different types of information structure can promote mergers in sequential Stackelberg game ($m > 2$) [or simultaneous Cournot game ($m = 0$)]. It is shown that without role redistribution, firms have incentives to merge when the uncertainty is sufficiently large, and only mergers generating efficiency gains could be profitable. In the presence of role redistribution, to some extent, the effect of role redistribution can substitute for the uncertainty effect, thereby firms always have incentives to merge even in the situation where the cost uncertainty is very tiny or equal to zero; besides, mergers leading to efficiency losses could be profitable.

4 Welfare analysis

We have so far examined firms’ incentives to merge and profitability of merger. In this section, we investigate the welfare implications of mergers. The relationship between private intention and collective incentive will also be studied.

The consumer welfare (CS) and social welfare (W) in benchmark are given as follows:

$$ CS = \frac{(a - c)^2(n + mn - m^2)^2}{2(m + 1)^2(n - m + 1)^2} $$

$$ W = \frac{(a - c)^2[(m + 1)(n - m + 1) + 1](n + mn - m^2)}{2(m + 1)^2(n - m + 1)^2} $$

4.1 Aggregate welfare

Since Competition Authorities intervene ex ante, they are not informed about the merged firm’s cost, it is logical to calculate the welfare implication based on the expected values. As we have demonstrated, both the aggregate surplus and merged entity’s profits are increasing functions with respect to the variance $\sigma^2$ in four alternative cases (see Remark 1). We compare the level of required uncertainty for profitable merger to that for welfare-enhancing so as to discover the relationship between private and collective incentives.

We model in a very simple way the decision of the Competition Authorities: a merger is approved whenever the expected change is positive. The standard presumption is that without synergies a merger significantly increasing market concentration leads to higher prices, lower aggregate output and lower social welfare. However, in the presence of synergies, welfare may increase. This is a well-known tradeoff between unilateral effects and efficiency gains, to be resolved by the Competition Authorities.

We want to address how this tradeoff is altered by the influence of cost uncertainty\(^\text{22}\). Consider

\(^\text{22}\) As Commissioner Anthony noted in her Synopsis statement, the degree of uncertainty about potential anti-competitive effects and efficiencies is an important factor. The importance of ex-ante uncertainty about the effect of mergers is also stressed by PricewaterhouseCoopers (2005) and Competition Commission (2008) in their evaluation reports of merger control policy in the UK.
Table 6: Comparison: incentive to merge Vs welfare-enhancing

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Threshold $\sigma_{T1}^2$</th>
<th>Comparison with $\sigma_{T1}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>$\sigma_{T1}^2 = \frac{4(a-c)^2(2m+1)}{\pi^2(n+m+1)^2(n(n-m)+3)}$</td>
<td>$\sigma_{T2}^2 &gt; \sigma_{T1}^2 &gt; 0$</td>
</tr>
<tr>
<td>Case B</td>
<td>$\sigma_{B1}^2 = \frac{4(a-c)^2(2m+1)(n-m+1)+2(m+1)(n(n-m)+2)(n-m+1)^2}{3(n+1)^2(n-m+1)^2}$</td>
<td>$\sigma_{B2}^2 &gt; \sigma_{B1}^2 &gt; 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case C</td>
<td>$\sigma_{C1}^2 = \frac{4(a-c)^2(2m+1)(n-m+1)+2(m+1)(n-n-3)}{3(n+1)^2(n-m+1)^2}$</td>
<td>$\sigma_{C2}^2 &gt; \sigma_{C1}^2 &gt; 0$</td>
</tr>
<tr>
<td>Case D</td>
<td>$\sigma_{D1}^2 = \frac{4(a-c)^2(2m+1)(n-m+1)}{3(n+1)^2(n-m+1)^2}$</td>
<td>$\sigma_{D2}^2 &gt; \sigma_{D1}^2 &gt; 0$</td>
</tr>
</tbody>
</table>

with $\Phi = \frac{2(m+1)(n-m+1)(n-m+1)^2}{3(n-m)^2-42(n-m)+1}$

$\Delta_{E[W]}^i = E[W^2] - W$ as the yardstick which judges whether the merger improves the social welfare. In case of $\Delta_{E[W]}^i > 0$, the merger enhances the welfare, and it will damage the welfare if $\Delta_{E[W]}^i < 0$.

In order that the merger generates welfare enhancement, the sufficiently large uncertainty is required. Table 6 enumerates the thresholds $\sigma_{T1}^2$, beyond which the merger always gives rise to welfare improvement.

**Proposition 3** Profitable mergers between leaders always constitute a welfare-enhancing merger, that generates the unanimity of private and social incentives. Besides, the merger in case B with large market size could achieve this unanimity.

**Proof:**

(a). In case of merger between leaders, the magnitude of variance guaranteeing the incentives to merge ensures the enhancement of social welfare without ambiguity. $\sigma_{T1}^2 > \sigma_{T2}^2 > 0$.

(b). Without role redistribution, whether the merging firms generate the amelioration of welfare depends upon the market configuration (n and m) and the size of market: if the market size is sufficiently large ($a > c + \Phi$), the magnitude of variance guaranteeing the incentives to merge ensures the welfare enhancement; otherwise, the latter covers with the former. $\sigma_{B1}^2 > \sigma_{B2}^2 > 0$ when $n > 6, m \in [3, n-3], a > c + \Phi$; otherwise, $\sigma_{B1}^2 > \sigma_{B2}^2 > 0$. 

19
(c). When two followers result in a newly merged firm behaving as leader, the uncertainty should be greater than the critical value $\sigma^2_{bC}$, to guarantee the enhancement of welfare. $\sigma^2_{bC} > 0 \ (\neq \sigma^2_{dD})$.

(d). In case of merger between one leader and one follower, as long as the variance is greater than the threshold $\sigma^2_{dD}$, this merger is always welfare-enhancing and the merging firms always have incentives to merge. $\sigma^2_{dD} > 0 \ (\neq \sigma^2_{bC})$.

See also Table 6. \( \square \).

4.2 Consumer welfare (two distinct antitrust criterions)

Although many analyses of mergers focus on an aggregate welfare standard, enforcement practice in most countries (including the US and the EU) is closest to a consumer welfare standard. So a separate analysis of consumer surplus is proposed in this subsection.

Using the similar methods ($A^t_{\text{i}, \text{CS}} = E[CS^t] - CS$), the thresholds $\sigma^2_{\text{CS}}$, beyond which the merger improves the consumer surplus are derived.

Proposition 4 Except for the case C, the consumer welfare standard is more rigorous than the total welfare standard.

Proof: If Competition Authorities act on the basis of consumer surplus,

(a). Profitable merger between leaders requires more uncertainty to guarantee the enhancement of consumer surplus compared to the welfare criterion, i.e.

$$\sigma^2_{\text{CS}A} > \sigma^2_{xA} > \sigma^2_{bA} \text{ with } \sigma^2_{\text{CS}A} = \frac{4(a-c)^2(2mn+2m^2n-2m^3-1)}{m^2(m+1)^2[(m-m+1)^2]}. $$

(b). In case of the merger between followers without role redistribution, the variance guaranteeing the consumer surplus enhancement ensures the welfare improvement and the private incentive to merge, when the market size is sufficiently large, i.e.

$$\sigma^2_{\text{CS}b} > \max \{\sigma^2_{bA}, \sigma^2_{bB}\} \text{ if } a > \Phi + c \text{ with } \sigma^2_{\text{CS}b} = \frac{4(a-c)^2[2(n-m)n(m+1-m)^2-1]}{(m+1)^2[n-m]^2(n-m+1)^2}. $$

(c). In case of merged leader firm composed of two followers, when there are enough active firms in market where the proportion of leaders is smaller than followers, the required uncertainty guaranteeing welfare enhancement covers with the one guaranteeing consumer surplus; otherwise, the reverse outcome appears, i.e.

$$\begin{cases} 
\sigma^2_{bC} > \sigma^2_{\text{CS}C} & \text{if } n > 12, \ 3 \leq m < \frac{4}{3} - 1 \\
\sigma^2_{\text{CS}C} > \sigma^2_{bC} & \text{otherwise}
\end{cases}$$

with

$$\sigma^2_{\text{CS}C} = \frac{4(a-c)^2(3m-n+3)(2(m+1)(m+2)n^2-2mn(2m(m+3)+5)+m^2[2m(m+1)(m+2)-3(n+1)])}{(m+1)^2(n+2)^2(3n-m)^2}. $$

(d). When the merger is composed of one leader and one follower, the uncertainty guaranteeing consumer surplus improvement ensures the one guaranteeing welfare enhancement without ambiguity, i.e.

$$\sigma^2_{\text{CS}D} > \sigma^2_{dD} \text{ with } \sigma^2_{\text{CS}D} = \frac{4(a-c)^2[2(n-m)n(m+1-m)^2-1]}{(m+1)^2[n-m]^2(n-m+1)^2}. \quad \square$$
As the antitrust decision on the basis of consumer surplus effectively guarantees both the welfare enhancement and the private intention of firms, to some extent, the severity of consumer surplus criterion can be regarded as the precision feature. This precision stems from the fact that the consumer surplus is less sensitive to uncertainty (see Remark 2). The economist’s natural reaction to a proposed merger goes something like the following: if a company proposes a takeover, or two companies propose a merger, then we can consider that this transaction will be at least privately profitable. This assumption will not, of course, turn out to be correct every time. When firms do not in fact forecast the profitability outcomes of mergers well (even as to the sign of the effects), for example, because of cost uncertainty, then the agencies should not adopt the default assumption that a merger would enhance the producer surplus portion of total welfare simply because the firms have proposed it. Nor should the agencies put much stock in the existence or magnitude of efficiencies claimed by merging parties in their negotiations with the agencies. As Porter (2005) summarizes, "we cannot assume that a merger will be efficient and profitable just because companies propose it." And this leads us to the conclusion that if the analysis of the impact of a merger on competition is implemented under (efficiency or merged firm’s productivity) uncertainty, consumer surplus is what agencies and courts do best.

5 Ex post merger control

When regulating the behavior of a private party which proposes a merger plan, the Competition Authorities are often uncertain about the sign and extent of the externality due to the shock caused by mergers. However, uncertainty will be disclosed and information on the magnitude of the externality typically becomes available once the merger is consummated. Clearly, the advantage of ex post merger enforcement is that it can focus more on (certain) history than on (uncertain) predictions.

In post merger game, the insider is able to first-to-know its cost and signal this private information through its market conduct. Therefore, when the intervention of antitrust agencies takes place ex post, Competition Authorities are aware of the real value of merged firm’s cost.

5.1 Ex post aggregate welfare criterion

Assume $\Delta_W$ the difference between the social welfare before and after merger.

$$\Delta_W = W_i - W$$

Making use of the similar method in merger’s profitability analysis, we try to find the ranges of $\delta_W$ wherein the merger improves the social welfare (see Appendix G). Furthermore, by comparing the upper bound of $\delta_W$ with the critical value $\delta_{iup}$ demonstrated in merger analysis section, we shed light on the following proposition.

Proposition 5

---

23Heyer (2006), supra note 2, at 38 ("Certainly the merging firms believe that they will be better off, as evidenced by the fact that they have chosen to merge, presumably, voluntarily.") See also Farrell and Shapiro (1990) ("Since any proposed merger is presumably privately profitable, it will also raise welfare if it has a positive external effect [i.e., on consumers and on nonparticipant firms.") and Kaplow and Shapiro (2007), supra note 7, at 83 ("The law implicitly presumes mergers to be advantageous to some degree… Setting the threshold of anticompetitive effects significantly above zero may be rationalized by the view that mergers typically generate some synergies, so they should not be prohibited unless the reduction in competition is sufficiently great.").
(a). If the merger is composed of two leaders, the welfare-enhancing merger is not always profitable, but the profitable merger improves social welfare without ambiguity.

(b). When two followers take part in the merger and the newly merged entity behaves as a leader, the welfare-enhancing merger is always profitable, however, the profitable merger could damage the aggregate surplus. Furthermore, when there is sufficiently less leader firms in the market, even the profitable merger generating the efficiency losses can enhance welfare.

(c). If the merger stems from firms of different types, the welfare-enhancing merger is always profitable.

Proof:

Case A: \( \delta_{\text{A}sup}^A < \delta_{\text{A}wsp}^A < 0 \)

Case B: Complicated (depending upon numerous parameters such as the market size "a", the marginal cost "c", the numbers of leaders and followers "n" and "m", etc.)

Case C: \( 0 < \delta_{\text{C}wsp}^C < \delta_{\text{C}sup}^C \), if \( n > 12 \) and \( m \in [3, \frac{2}{3} - 1] \)
\[ \delta_{\text{C}wsp}^C < 0 < \delta_{\text{C}sup}^C, \text{ otherwise} \]

Case D: \( \delta_{\text{D}wsp}^D < 0 < \delta_{\text{D}sup}^D \)

The first key point of this proposition is consistent with Farrell and Shapiro (1990, Proposition 5), Amir et al. (2009, Proposition 4) and Zhou (2008a, Proposition 5) finding that, under some conditions on demand and costs that are satisfied by the linear setting, if a merger with sure efficiency gains is profitable to the merging firms, it will also be welfare-improving.

The second point of proposition is counter-intuitive, it not only analytically demonstrates that the merger generating efficiency losses could be profitable, but also shows that the welfare could be possibly enhanced by the merger even leading to efficiency losses. The reason behind this is two-fold: 1. the role redistribution effect initiated by Daughety (1990), if the merger alters the behavior of the participants, the welfare can be improved by the merger in spite of the lack of synergies; 2. the informational advantage effect explained by several economists (i.e. Banal-Estanol, 2007; Amir et al., 2009). The combination of two same-direction effects can be sufficient to compensate the efficiency losses. Thus, it is possible that the "inefficient merger" (which generates efficiency losses) enhances the welfare.

Proposition 5 shows that when intervening ex post, Competition Authorities are aware of the merged firm’s cost. Under this circumstance, as long as the merger between leaders is profitable, it is always welfare-enhancing. By contrast, in the three other cases, welfare-improving mergers are unambiguously profitable.

We derive that the profitable merger between leaders is necessarily welfare-improving. It provides support for a laissez-faire policy if the decisive criterion rests on social welfare. By contrast, Competition Authorities must supervise more closely bilateral mergers which are consisted of either one or two followers.
5.2 *Ex post* consumer welfare criterion

Suppose Competition Authorities adopt the *ex post* consumer welfare criterion, we find the ranges of $\delta_{CS}$ wherein the merger improves the consumer surplus. And then we compare the upper bound of $\delta_{CS}$, namely $\delta_{CS}^{sup}$, with both $\delta_{sup}^A$ and $\delta_{sup}^W$ to achieve the following proposition.

**Proposition 6** If merger regulation occurs after a merger has been consummated, the consumer welfare standard is more lenient than the total welfare standard.

**Proof:** If Competition Authorities act on the basis of actual consumer surplus,

(a). In case A, when there are three or four leaders in pre-merger market, the profitable merger always improves the social welfare, but possibly damages the consumer surplus. When there are more than four leaders in the market, the profitable merger is unambiguously welfare-enhancing and consumer-surplus-improving, i.e.

$$
\begin{align*}
\delta_{CS}^A &< \delta_{sup}^A < \delta_{W}^A \\
\delta_{sup}^A &< \delta_{CS}^{sup} < \delta_{W}^{sup} \\
\delta_{W}^{sup} &< 0 \text{ if } m = 3 \text{ or } 4 \\
\delta_{W}^{sup} &< \delta_{CS}^{sup} < 0 \text{ if } m \geq 5
\end{align*}
$$

with $\delta_{CS}^A = \frac{-2(a-c)}{m(m+1)(n-m+1)}$.

(b). When there are sufficiently less leader firms in the market, the profitable merger generating efficiency losses can improve both consumer and aggregate surplus, and the welfare-enhancing merger ensures the rise of consumer surplus. Otherwise, the efficiency gains are necessary to guarantee the improvement of consumer surplus and welfare, and the merger improving consumer surplus enhances the welfare, i.e.

$$
\begin{align*}
0 &< \delta_{sup}^C < \delta_{CS}^{sup} < \delta_{W}^{sup} \text{ if } n > 12, m \in [3, \frac{n}{2} - 1] \\
\delta_{CS}^{sup} &< \delta_{sup}^W < 0 < \delta_{C}^{sup} \text{ otherwise}
\end{align*}
$$

with $\delta_{CS}^{sup} = \frac{2(a-c)(n-3m-3)}{(m+1)(m+2)(n-m)^2-1}$.

(c). When the merger is composed of one leader and one follower, the merger improving the consumer surplus is always profitable and welfare-enhancing, i.e.

$$
\delta_{CS}^{sup} < \delta_{W}^{sup} < 0 < \delta_{sup}^D \text{ with } \delta_{CS}^{sup} = \frac{-2(a-c)}{(m+1)(n-m)(n-m+1)}.
$$

□

The Propositions 4 and 6 gain some insight into the relationship between the distinct criterions of Competition Authorities and the timing of policy intervention. When Competition Authorities adopt *ex ante* enforcement, antitrust enforcers have less information about the merger, the consumer welfare standard is more restrictive than the aggregate welfare standard. By contrast, when Competition Authorities choose *ex post* enforcement, they are aware of the real cost of merged firm, the consumer welfare standard is more lenient than the aggregate welfare standard.
6 Concluding remarks

This paper extends the strand of literature on horizontal mergers in a homogeneous oligopoly where there are leaders and followers. Within sequential output decisions, we focus upon the cost uncertainty and the efficiency gains (or losses), in order to fulfill the gap of merger issue under uncertainty. In this model, the merger decision is made before firms learn the merged firm’s cost. We find that the expected profit of merged firm grows following the enlargement of variance. When the extent of variance exceeds a certain threshold, firms facing uncertainty choose to merge. On the other hand, if there is role redistribution, even in the absence of uncertainty effect, firms have incentives to merge.

In terms of profit, we analyze the profitability of merger in context of informational asymmetry. It is shown that the two-follower merger aiming to a leader strategy occurs more likely than the one satisfying status quo. Furthermore, the merged firm has interests to pool the private signals to outsiders, in the absence of role redistribution. By contrast, in the presence of role redistribution, the concealment is more profitable from the viewpoint of insider.

In terms of welfare, it is found that the merger between leaders always enhances welfare if participants have incentives to merge, this generates the unanimity of private and collective intentions. Nevertheless, the merger with role redistribution leads to the private-collective conflict. From the standpoint of Competition Authorities, after separately studying the two possible criterions: "aggregate welfare standard" and "consumer welfare standard", we find that the latter is more restrictive and more accurate than the former in an uncertain environment. In addition, by carrying on a separate analysis of ex post enforcement merger control, we gain some insight into the relationship between the distinct criterions of Competition Authorities and the timing of policy intervention.

We have restricted our analysis to a bilateral merger. A generalization would be to consider the merger composed of more than two firms, in order to relax the assumption and check the robustness of this framework. Another direction would be to take into account the Endogenous Stackelberg issue in the context of cost uncertainty.

Appendix:

A Best response function of followers

In the follower production stage. The optimizing question is:

\[
\max_{q^{fA}_O} \pi^{fA}_O = (p^A - c)q^{fA}_O = [a - c - Q^{fA}_O - q^{fA}_O - Q^{fA}_O(c) - q^{fA}_O(c)]\left[\frac{a}{c_i} - \frac{Q^{fA}_O(c)}{c_i}\right] (13)
\]

From the standpoint of information structure,

- \(Q^O(c)\): outsider leaders consider that the cost level of insider is equal to \(c\)
- \(q^{f}_O(c_i)\): first-to-know
- \(q^{f}_O(c_i)\): outsider followers observe the production level and perfectly infer the cost level of merged entity \(c_i\)
In the (first) leader production stage, outsider leaders are not aware of the actual cost of insider, thereby
to account the expected value \( c \)

\[
\text{max}_{q_O^A} \pi_O^A = (p^A - c)q_O^A = [a - c - Q_O^{-fA} - Q_O^{-lA}(c) - q_I^{lA}(c)]q_O^{lA}(c)
\]

plug the sum of follower quantity Eq. (15) into Eq. (16), the maximization problem becomes

\[
\text{max}_{q_O^A} \pi_O^A = \frac{1}{n-m+1} [(a-c) - Q_O^{-lA}(c) - q_O^{lA}(c) - q_I^{lA}(c)]q_O^{lA}(c)
\]

**FOC:**

\[
2q_O^{lA}(c) = (a - c) - Q_O^{-lA}(c) - q_I^{lA}(c)
\]

perfect symmetry for outsider leaders:

\[
Q_O^{-lA}(c) = (m-3)q_O^{lA}(c)
\]

reaction function of outsider leader is

\[
(m-1)q_O^{lA}(c) = a - c - q_I^{lA}(c)
\]

and note the sum

\[
Q_O^{lA}(c) = (m-2)q_O^{lA}(c) = \frac{m-2}{m-1} (a - c - q_I^{lA}(c))
\]

For insider (merged entity), when insider knows the real cost \( c_i \), the optimizing question is

\[
\text{max}_{q_I^{lA}} \pi_I^{lA} = (p^A - c_i)q_I^{lA} = [a - c_i - Q_O^{lA}(c) - Q_O^{-lA}(c)]q_I^{lA}(c_i)
\]

\[
= \frac{1}{n-m+1} [(a-c) + (n-m+1)(c-c_i) - Q_O^{lA}(c) - q_I^{lA}(c_i)]q_I^{lA}(c_i)
\]

**B Best response function of leaders and equilibrium output**

In the (first) leader production stage, outsider leaders are not aware of the actual cost of insider, thereby
FOC:  
\[ 2q_l^{lA}(c_i) = (a - c) + (n - m + 1)(c - c_i) - Q_{O}^{lA}(c) \]  
\[ \text{(19)} \]

when insider is not informed about the exact cost \( E(c_i = c) \)

\[
\max_{q_l^{lA}} \pi_l^{lA} = (p - c)q_l^{lA} = [a - c - Q_{O}^{lA}(c) - q_l^{lA}(c)]q_l^{lA}(c)
\]

\[
= \frac{1}{n - m + 1} [(a - c) - Q_{O}^{lA}(c) - q_l^{lA}(c)]q_l^{lA}(c)
\]

FOC with respect to expected value \( c \) is

\[ 2q_l^{lA}(c) = (a - c) - Q_{O}^{lA}(c) \]  
\[ \text{(20)} \]

then yield

\[ q_l^{lA}(c) + \frac{1}{2}(n - m + 1)(c - c_i) = q_l^{lA}(c_i) \]

It is straightforward that in case of \( c_i < c \), we obtain \( q_l^{lA}(c_i) > q_l^{lA}(c) \); otherwise, \( q_l^{lA}(c_i) < q_l^{lA}(c) \).

Based on Eqs. (18), (19) and (20), it is possible to derive leaders’ equilibrium outputs:

\[ q_l^{lA}(c_i) = \frac{2(a - c) - m(n - m + 1)(c_i - c)}{2m} \]

\[ q_l^{lA}(c) = \frac{(a - c)}{m} \]

\[ q_O^{lA}(c) = \frac{(a - c)}{m} \]

plugging them into follower’s reaction function Eq. (14), it yields

\[ q_f^{lA}(c_i) = \frac{2(a - c) + m(n - m + 1)(c_i - c)}{2m(n - m + 1)} \]

and then, we derive the aggregate output

\[ Q = q_l^{lA}(c_i) + (m - 2)q_f^{lA}(c) + (n - m)q_O^{lA}(c_i) \]

\[ = a - \frac{a}{m(n - m + 1)} - \frac{1}{2} - \frac{1}{m(n - m + 1)}(c - c_i) \]

C Real and expected profits

The profit of \( \text{insider} \):

\[ \pi_l^{lA} = (a - Q - c)q_l^{lA}(c_i) \]

\[ = \frac{a^2}{m^2(n - m + 1)} + \frac{m^2 + 2 - m(n + 1)}{4m^2(n - m + 1)}(c_i - c)^2 - \frac{2ac_i}{m^2(n - m + 1)} \]

\[ + \frac{c_i^2}{m^2(n - m + 1)} + \frac{a(c_i - c)(2 - m - m)}{m^2} + \frac{c_i(c_i - c)(m - 2)}{m^2} \]

\[ = \frac{2(a - c) - m(n - m + 1)(c_i - c)}{4m^2(n - m + 1)} \]
Knowing that $E[(c_i - c)^2] = \sigma^2$, $E[c_i] = c$, $E[c_i^2] = c^2 + \sigma^2$, $E[c_i - c] = 0$, $E[(c_i - c)c_i] = \sigma^2$, the expected profit of insider:

$$
E[\pi_i^A] = \frac{(n-m+1)\sigma^2}{4} + \frac{c^2}{m^2(n-m+1)} - \frac{2ac}{m^2(n-m+1)} + \frac{a^2}{m^2(n-m+1)}
$$

$$
= \frac{(a-c)^2}{m^2(n-m+1)} + \frac{n-m+1}{4}\sigma^2
$$

The profit of outsider-leader:

$$
\pi_O^A = (a - Q - c)q_O^A(c)
$$

$$
= \frac{(a-c)[2(a-c) + m(n-m+1)(c_i - c)]}{2m^2(n-m+1)}
$$

and then the expected profit of outsider leader is

$$
E[\pi_O^A] = \frac{(a-c)^2}{m^2(n-m+1)}
$$

The profit of outsider follower:

$$
\pi_O^A = (a - Q - c_i)q_O^A(c_i)
$$

$$
= \frac{[2(a-c) + m(n-m+1)(c_i - c)]^2}{4m^2(n-m+1)^2}
$$

the expected value is

$$
E[\pi_O^f] = \frac{(a-c)^2}{m^2(n-m+1)^2} + \frac{1}{4}\sigma^2
$$

## D Merger between two followers

Using the similar method (See Appendix A and B), the equilibrium outputs for followers are resolved on the basis of the following equations:

- $a - (n-m-2)q_O^{f,B}(c) - Q_O^{f,B}(c) - q_f^{f,B}(c) - c - q_f^{f,B}(c) = 0$ (outsider followers do not realize the insider’s real cost)

- $a - (n-m-2)q_O^{f,B}(c) - Q_O^{f,B}(c) - q_i^{f,B}(c) - c_i -q_i^{f,B}(c) = 0$ (insider know his own cost level)

- $a - (n-m-2)q_O^{f,B}(c) - Q_O^{f,B}(c) - q_i^{f,B}(c) - c - q_i^{f,B}(c) = 0$ (insider does not know his own cost level)

The expression of followers’ outputs can be found

$$
q_O^{f,B}(c) = \frac{(a-c) - Q_O^{f,B}(c)}{(n-m)}
$$

$$
q_i^{f,B}(c) = \frac{2(a-c) - (n-m)(c_i - c) + 2Q_O^{f,B}(c)}{2(n-m)}
$$

$$
q_f^{f,B}(c) = \frac{(a-c) - Q_O^{f,B}(c)}{(n-m)}
$$
and then, plugging them into leader’s profit function:
\[
\max q^l_B = (p^l - c)q^l_B = [a - c - (n - m - 2)]q^l_B(c) - \frac{q^l_B(c)}{q^l_B(c)}q^l_B(c)
\]

It is easy to calculate the leader output level:
\[
q^l_B(c) = \frac{a - c}{m + 1}
\]

Put the expression of \( q^l \) into the output for followers, we obtain
\[
q^f = \frac{2(a - c) - (m + 1)(n - m)(c_1 - c)}{2(m + 1)(n - m)}
\]

The equilibrium values in terms of price, profit, consumer surplus and social welfare, are displayed in Table 1. The other cases (case C and case D) can be resolved by the similar method.

E  Merged firm’s profit under complete and perfect information (\( \hat{\mathcal{R}}^j_i \))

\[
\hat{\mathcal{R}}^A_i = \frac{(a - 2c + c_i) + (c - c_i)(m - 1)n - (m - 2)n}{m^2(n - m + 1)}
\]

\[
\hat{\mathcal{R}}^B_i = \frac{(a - 2c + c_i) + (c - c_i)(n - m)(m + 1)}{(n - m)^2(m + 1)^2}
\]

\[
\hat{\mathcal{R}}^C_i = \frac{(a - 2c + c_i) + (c - c_i)[m(n - m) + (n - 2m)]}{(m + 2)^2(n - m - 1)}
\]

\[
\hat{\mathcal{R}}^D_i = \frac{(a - c + m(c - c_i))[(a - 2c + c_i) + (c - c_i)(n - m)(m + 1)]}{(n - m)(m + 1)^2}
\]

See also in Lepape and Zhao (2010)

F  \( \hat{\mathcal{S}}_{sup} \) and \( \hat{\mathcal{S}}_{inf} \)

\[
\hat{\mathcal{S}}_{sup} = \frac{a - c - (a - c)(m - 1)}{(m + 1)(n - m + 1)}
\]

\[
\hat{\mathcal{S}}_{inf} = \frac{a - c}{m + 1} + \frac{\sqrt{(a - c)(m + 1) + (a + c)(n - m) + (n - m)(m + 1)}}{(m + 1)(n - m + 1)}
\]
$$G \delta_{W_{sup}}^i = -\frac{2}{a(3 + 2m - 2n) + c(3 - 2m + 2n) + m(1 + 4m^2 + 7n + 4n^2 - m(7 + 8n))} \left\{ \frac{(a - c)^2\left[4a^5 - 4a^4(3 + 2m - 2n) + a(1 + 3m^2) + m^2 - 3m - 4n\right]}{m^2(1 + a^2 - 3n - 4m^2 + a(7 + 8n))} \right\}$$

$$\delta_{D_{W_{sup}}} = -\frac{2}{2} \left( \frac{(a - c)(2m - 2n - 1)}{(2 + m)(1 + 4m^2 + 8m - 3n - 9m^2)} \right)$$

$$\delta_{C_{W_{sup}}} = -\frac{2}{2} \left( \frac{(a - c)(2m - 2n - 1)}{(2 + m)(1 + 4m^2 + 8m - 3n - 9m^2)} \right)$$
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