

Financial Stress, Regime Switching and Macrodynamics: Theory and Empirics for the US, EU and Non-EU Countries*

Pu Chen[†] and Willi Semmler[‡]

March 25, 2014

Abstract

Over-borrowing and financial stress has recently become an important issue in macroeconomic and policy discussions in the US as well as in the EU. In this paper we study two regimes of financial stress. In a regime of high financial stress, stress shocks can have large and persistent impacts on the real side of the economy whereas in regimes of low stress, shocks can easily dissipate having no lasting effects. In order to study the macroeconomic dynamics, with alternative paths resulting from financial stress shocks, we introduce a macromodel with a finance-macro link which uses multi-period decisions framework of economic agents. The agents can, in a finite horizon context, borrow and accumulate assets where however the above two scenarios may occur. The model is solved through nonlinear model predictive control (NMPC). Empirically then we use a Multi-Regime VAR (MRVAR) to study the impact of financial stress shocks on the macroeconomy in a large number of countries.

*We want to thank the ZEW for support of this study. Willi Semmler would also like to thank the Fulbright Foundation that helped to start this project while he was appointed Fulbright Professor at the University of Economics (WU), Vienna, in the Fall 2011.

[†]Melbourne Institute of Technology, Australia, email pchen@academic.mit.edu.au

[‡]Henry Arnhold Professor of Economics, Dept. of Economics, New School for Social Research, 79 Fifth Ave, New York, NY 10003, Research Associate, ZEW Mannheim, and Bielefeld University; email: semmlerw@newschool.edu

1 Introduction

The issues of over-borrowing and financial stress has recently become a concern in many countries. In the US the over-borrowing of households, financial firms and banks, and the government (States as well as Federal State) is at the forefront of the economic policy debate. In the EU it is the sovereign debt problem and the insolvency threat of the banking system in the periphery countries (Greece, Italy, Spain, Portugal) that threatens to become a cause of new financial meltdown. Over-borrowing and unsustainable private and government debt appear to be major predictors of a financial crises and prolonged financial stress also in other countries, with severe impact on the macroeconomy of the country – possibly with strong spillover effects to other countries.

As the empirics shows there is sometimes high leveraging, yet little financial stress, and positive shocks to stress do not affect the macroeconomy significantly. On the other hand, there is a financial shock that leads to considerable economic contractions. We want to study why the economies sometimes stabilize at a prolonged boom period with little effects from financial stress shocks, and sometimes, after financial shocks, tend to move toward low growth or negative growth rates. In order to study the debt dynamics, with alternative paths resulting from financial stress shocks, we introduce a finance-macro link which uses a multi-period decision framework of economic agents. The agents can, in a finite horizon context, borrow and accumulate assets where however two scenarios may occur.

In a first scenario, there is borrowing, but there is some path with considerable growth, and a stabilization at higher or moderate growth rates can occur. In spite of significant borrowing, interest rates remain relatively stable, or are only slightly rising due to higher leveraging and higher risk premia. Yet, slightly rising interest rates and credit spreads do not generate macro economic feedback mechanisms to produce strong contractions and positive financial stress shocks do not matter significantly.

On the other hand instability can arise in the sense that shocks can be amplifying, of the sort that macro feedback mechanisms generate also real downswings, for example through rising risk premia, credit constraints or extensive efforts of deleveraging by borrowers. High leveraging of economic agents, as well as factors creating mild nonlinearities, can drive up risk premia and credit spreads. Credit helps to create booms, but rising credit spreads can be destabilizing and create busts. In fact, often credit spreads by itself do not necessarily create macroeconomic instabilities, but once there is a significant impact on consumption and investment decisions this can result in a further decline in economic activities. This is likely to generate a vicious cycle, and the economy becomes unstable downward and may move to a low level economic activities.

In order to explore those two scenarios, we employ some model versions of the Blanchard-Fischer type (1989, ch.2) as it was designed for an open economy which allows, however, many agents in the economy to pursue debt finance. In principle the excess of absorption over production in an economy means borrowing – if not domestically then from abroad. So in our model households,

firms, banks and the State can borrow which will show up, if not domestically financed by savings, as borrowing from abroad, resulting in current account deficits.

Yet the excessive borrowing may show up in risk premia and thus in credit spreads, households, firms and banks or the sovereign have to pay. The risk premia and credit spreads may affect aggregate demand. Austerity policies that decrease demand, but drive up risk premia are likely to be self-defeating if they are not ending up in a decline of credit spreads: Economies are then in a trap where agents are income, liquidity and credit constraints, facing high interest rates, and they may show contractionary effects of austerity and not expansionary effects as some schools in economics are suggesting.

There are – as de Grauwe (2012) argues and empirically demonstrates – likely to be multiple equilibria, good ones and bad ones. Multiple equilibria are not new in macroeconomics. Multiple equilibria models have been used in macroeconomics since long. Yet, many of the models work with expectations dynamics, where a self-fulfilling prophecy can lead to the situation that countries can end up in a bad equilibrium with self-enforcing mechanisms working to stay there. De Grauwe (2012) for example shows that recently for the Euro zone there is a danger of such a self-enforcing mechanism where EU members can end up in a bad equilibrium. Those mechanisms are working for countries in the EU currency union, but may work less for stand-alone countries, for example US, UK and Japan. For countries in a loose currency union one might see such a mechanism.

Yet, instead of using a self-enforcing mechanism generated through the expectation dynamics one can also show that countries may face a vicious cycle, through non-linearities which eliminate the usual automatic stabilizers and multiplier effects. We want to show a real mechanism that also can create such a downward pushing force and can prevent recoveries from taking their path. A new perspective is taken here in the paper that allows for intertemporal behavior of agents within one regime, but simultaneously admits severe contractions and regime changes, though the agents intertemporally optimize. We want to explore the debt dynamics using two different model variants representing two different regimes. In the first model variant we keep the interest rate on borrowing constant or it is only slightly rising reacting to leveraging. Then we relax this assumption which leads us to another regime and dynamics.

The forces we discuss here, that can bring about instability, are basically working through a positive feedback effect of financial market and output. Rising borrowing from capital markets, bond issuing, and credit spreads, caused by previous excess leveraging, and other factors, can cause aggregate demand to fall. When aggregate demand falls utilization of capacity – and thus capital utilization rates – fall, and the lower income generates a lower surplus to payoff future liabilities, which in turn creates greater credit spreads, lower aggregate demand and so on. In our case there are dominantly real forces not expectational forces that accelerate downturns possibly creating lock-ins into a bad equilibrium. They are basically working as positive feedback mechanism

between credit spread and capital utilization.¹ This is likely to occur if there are vulnerabilities developed beforehand, that can trigger severe downturn through that mechanism.

Our model is similar to Hall (2010, 2011), Gilchrist and Zabrajssek (2011), and Mitnik and Semmler (2011, 2012) and allows not only the credit-macro feedback mechanisms in an multi-period model, but admits also to study the contractionary effect of deleveraging as discussed in Eggertson and Krugman (2011) and Kraine (2012).

After the presentation of a model variant of low and high stress regimes, we employ a multi-regime VAR (MRVAR) to demonstrate such dynamics in the data. We use a IMF Financial Instability index (FSI) recently provided for many countries that allow us to study the empirics of the impact of financial stress on the macrodynamics. We measure the real activities by a monthly industrial production index of the different countries.

As to the solution method, our model will not be solved locally through local linearization about the steady state, as used in DYNARE, or globally in an infinite horizon model by Dynamic Programming, as in Ernst and Semmler (2010), but we will solve the model by Nonlinear Model Predictive Control (NMPC), which has recently been developed by Gruene and Pannek (2012), and which allows to mimic a finite horizon decision making.² This new numerical method permits to approximate the accurate dynamic model by an N-period receding horizon model for a particular regime which will provide us with an approximate solution for the decision and state variables.³ This algorithm works not only when one introduces terminal constraints, but also without defining terminal constraints.

2 The model

We want to explore the leveraging-macro link for our above mentioned two scenarios by using two model variants. In the first model variant we keep the interest on debt constant or let it be affected only little by leveraging. Yet in a second variant stronger feedback effects to the real economy will be built in.⁴

¹Many recent DSGE models work with endogenous capital utilization and financial market, for example cost of capital when issuing bonds; see for example Sugo and Ueda (2016) . A relationship between capital utilization and the “user cost of capital” is also postulated by Keynes (1936).

²For details of the new numerical procedure and a number of computed examples, see Gruene et al (2012).

³The NMPC can viewed as a numerical method to approximate the corresponding infinite horizon model when N becomes large. On the hand hand it can be used in its own right by providing a solution for finite time approximate solutions in case the infinite time horizon solution requires too much information for the agents or cannot be computed or obtained. In that sense it can be interpreted as limited information solution – a solution with information constraints– as put forward by Sims (2011) in his theory of rational inattention, for details see Gruene at al. (2012).

⁴This second model can be seen as some extension of the paper by Mitnik and Semmler (2012) where there are now also feedback mechanisms affecting the macroeconomy included in a dynamic model.

2.1 Regime of Low Financial Stress

So we presume that we are in a – finite time– regime with low financial stress. The following model might hold:

$$V(k, d) = \max_{c_t, g_t} \int_0^N e^{-rt} U(c_t) dt \quad (1)$$

s.t.

$$dk_t = (g_t - \delta)k_t dt + \sigma_t k_t dZ_t \quad (2)$$

$$db_t = (rb_t - (y_t - c_t - i_t - \varphi(g_t k_t)))dt \quad (3)$$

In equ. (1) there is preference over log utility. The policy variables are consumption, and growth rate of capital stock, c_t, g_t .⁵ But note we have a model of finite time, operating in a regime of low financial stress. N will be the time horizon for the N –period receding horizon.

Equ. (2) represents the capital stock that increases due to investment but declines through the depreciation rate δ . There could be a stochastic shock occurring along the path, represented by the second term in equ. (2). This is the only stochastic shock we will consider. The equ (3) represents the dynamics of aggregate debt (households, firms, sovereign).⁶

Our debt dynamics is written here in a way which is standard if one allows for borrowing from abroad, see Blanchard and Fischer (1989, ch. 2). The interest payment on debt, rb_t , increases debt but the surplus $(y_t - c_t - i_t - \varphi(g_t k_t))$ – negative excess absorption– decreases debt. We have $i_t = g_t k_t$. Note that since consumption and investment are separate policy variables we allow here for external borrowing. Moreover, $\varphi(g_t k_t)$ is the adjustment cost for investment, which we presume to be a quadratic function. Overall the model has two decision variables and two state variables.

Assuming here $r = 0.04$, $\delta = 0.07$ and quadratic adjustment cost of investment, we obtain the following solutions using NMPC, setting the shock equal to zero. The numerical results are shown in figure 1.

The vertical axis shows the debt to capital stock ratio and the horizontal axis the capital stock. Here the paths are shown for different initial conditions. The upper end of the two paths represents the steady state which is unique where both the trajectories end up.

Next we can also let the credit spread risk moderately rise with leveraging. The credit spread (measured against a risk free interest rate) is made endogenous.

$$V(k, d) = \max_{c_t, g_t} \int_0^N e^{-rt} U(c_t) dt \quad (4)$$

⁵ Actually in the numerics we can take $c = C/k$, so that the two choice variables can be confined to reasonable constraints between 0 and 1.

⁶ We could also allow for sovereign debt here

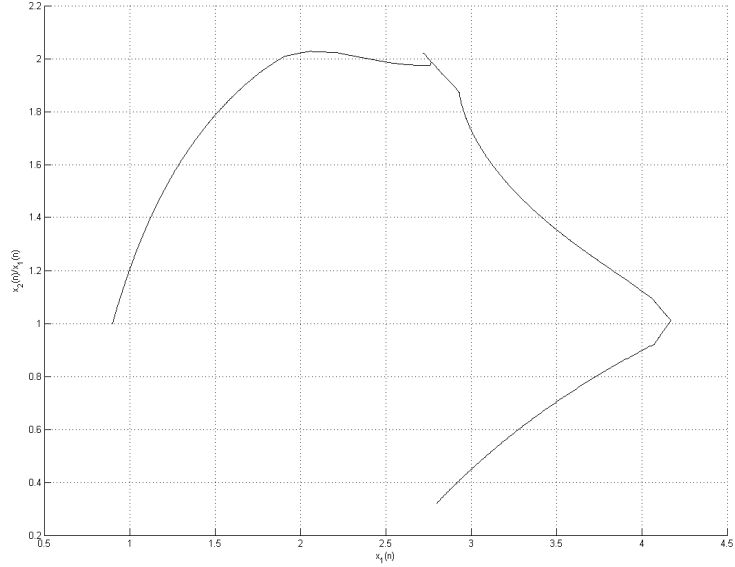


Figure 1: Dynamic paths of sovereign debt for constant interest rate, for two initial conditions, $N = 10$.

s. t.

$$dk_t = (g_t - \delta)k_t dt + \sigma_t k_t dZ_t \quad (5)$$

$$db_t = \beta(r(b_t/k_t)b - (y_t - c_t - i_t - \varphi(g_t k_t)))dt \quad (6)$$

The difference to the model (1)-(3) above is now that we assume that the credit spread maybe a nonlinear function of the debt to capital stock ratio. Our idea here is similar to Roch and Uhlig (2012) who have an on-off scenario: With a high probability of default bond prices are low and yields are high, and the reverse holds for a low probability of default. We smooth the on-off cases, and introduce a continuum of cases where the probability of default may steadily rise starting from a low level, then rising faster, and then leveling off.

Thus we want to let the credit spread rise with the debt to capital stock ratio, first slowly, then more rapidly, but it will finally be bounded. We use an arctan function, represented by $r(b_t/k_t)$, which gives us those properties:

$$r(b_t/k_t) = \arctan(b_t/k_t). \quad (7)$$

This is the function that has been used in Chiarella et al. (2009) and one can roughly also observe in de Grauwe (2012)⁷ The results of the debt dynamics is shown for $\beta = 0.1$ and $\beta = 0.2$ in figure 2.

⁷In de Grauwe representing there EU debt and bond yield data.

One would expect that with lower credit spreads, a lower steady state leveraging ratio is admissible. Again, debt is sustainable if the second term in equ. (6), the surplus, is equal to the first term, the interest payments on debt.

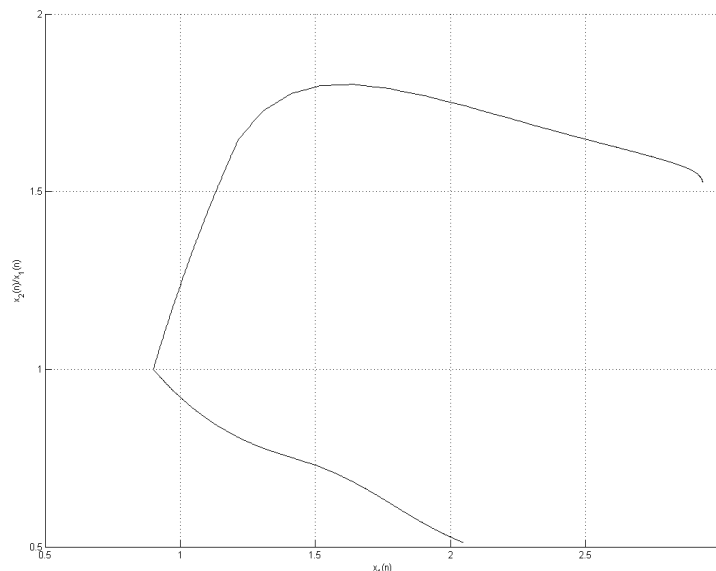


Figure 2: Debt dynamics for nonlinear interest payments on sovereign bonds; $\beta = 0.1$ lower graph and $\beta = 0.3$ upper graph, $N=10$.

As figure 2 shows the higher interest payments admits a higher steady state leveraging. Again, debt is sustainable if the second term in equ. (6), the surplus, is equal to the first term, the interest payments on debt. Actually in this version of a low financial stress and no macro feedback effects from slightly rising credit spreads, even a higher steady state debt is admissible and financial stress shocks would not be destabilizing.

Thus, in a low stress regime, with little feedback effect to consumption, investment and thus on demand and output, a positive financial stress shock would do little harm – one would expect a quick mean reversion, and not much lasting effects on output.

2.2 Regime of High Financial Stress

Now let us presume that we are in a regime of high financial stress, maybe with high leveraging but other factors also contributing to financial stress, see below. It is again a model in finite time — so we are in a receding decision horizon of N -periods. We now not only allow for credit spreads to be endogenous, but also for a feedback effect of leveraging on demand and output.

$$V(k, d) = \max_{c_t, g_t} \int_0^N e^{-rt} U(c_t) dt \quad (8)$$

s.t.

$$dk_t = (g_t - \delta)k_t dt + \sigma_t k_t dZ_t \quad (9)$$

$$db_t = \beta(r(b/k)b_t - (y_t^a - c_t^a - i_t^a - \varphi(g_t k_t)))dt \quad (10)$$

The difference to the first model variant above is here now that the credit spread maybe a nonlinear function of the leveraging, as before, but there is also an endogenous effect of this on demand, output and income. Thus the major difference to the first model variant is that the second variant has built in an endogenous utilization of capacity and thus has endogenized both credit spread and output. This is an important macroeconomic feedback mechanism that one sometimes can observe in a regime of high financial stress, see Hall (2010, 2011).

We can make the actual consumption and investment demand depending on rising interest rates, triggered by rising risky yields on bonds and rising credit spreads. This would affect consumption and investment demand in the following way:⁸

$$c_t^a = f(r(b/k))c^{opt} \quad (11)$$

$$I_t^a = g(r(b/k))I^{opt} \quad (12)$$

with the derivatives $\frac{df}{d(r(b/k))} < 0$ and $\frac{dg}{d(r(b/k))} < 0$. Though optimal consumption and investment plans might be targeted, actual consumption and investment decline due to rising risk premia and credit spreads. The cost of loans – if available at all – is rising. So, overall we may have :

$$y_t^a = u(r(b/k))y^{opt} \quad (13)$$

where again $\frac{du}{d(r(b/k))} < 0$. We take

$$u(r(b/k)) = (1 - r(b/k)) \quad (14)$$

and can use the rising credit spread as self-enforcing mechanism reducing demand, output and capacity utilization. We can write:

$$y^a = ((1 - r(b/k))k)^\alpha \quad (15)$$

Now if risk premia and credit spread might rise, but is bounded, y^a will decline due to higher credit cost, and thus we have lower consumption and investment demand and consequently capacity utilization falls. If capacity utilization falls, income, and thus tax revenue, as well as the surplus, to service the

⁸An analytical study of the following model, here only numerically solved, can be found in Mendoza and Semmler (2012).

debt, for all agents that borrow falls. This might make then borrowing more unsustainable – generating a further jump in credit spread or credit rationing.⁹

We expect here, starting with a leveraging roughly above normal, that the feedback mechanisms of higher yields, higher credit spreads and lower output may lead to a contraction of utilization of capital stock, and possibly capital stock itself, and to a rapidly increasing debt to capital stock ratio.

The debt dynamics with endogenous credit spread and endogenous output and surplus of system (7) -(9) is shown in figure 3, using NMPC.

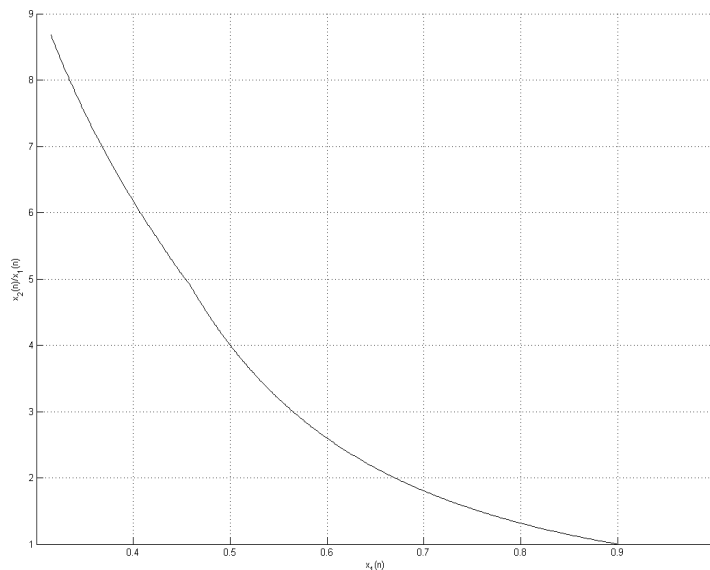


Figure 3: Debt dynamics with endogenous credit spread and endogenous output, $N = 10$.

Figure 3 shows, starting with a debt to capital stock ratio of roughly one, the feedback mechanisms of higher yields, higher credit spreads and lower output leading to a contraction of capital stock and to a rapidly increasing debt to capital stock ratio.

Note that in a high stress regime any financial stress shock could easily trigger a downturn and a protracted period of contraction. On the other hand, negative financial shocks in a regime of high stress, may give rise to strongly positive effects on demand and output. So monetary policy that not only reduces interest rates, but also reduces financial stress and credit spreads by other means, for example quantitative easing, might be particularly effective in a regime of high stress, see also Mittnik and Semmler (2012)

⁹This effect is also present in the model by Roch and Uhlig (2012) where it is shown that when income falls it will give rise to higher bond yields and thus higher credit spreads. The reverse effect is predicted to be seen if income rises.

In the case of high financial stress, however, the build-in stabilizer - the rising public deficit - and the multiplier effects of deficits spending and central bank's effort to reduce short term interest rates may not work so easily, since the possible rising risk premia and credit spreads might counteract fiscal as well as monetary policy actions. Though the interest rate may hit a zero bound, when in a recessionary period the interest rate is kept down by monetary policy¹⁰, the risk premia and credit spreads can still rise, possibly undercutting the positive effect of low or zero short rates set by the central bank. So, the policy of quantitative easing may become a reasonable policy.

So far, for both the regime version with low stress and high stress we have computed the macrodynamics staying within the regime, and not modeled a regime change itself. We have considered the effect of financial stress shocks in the different regimes. For the motivation of an empirical study, which will follow in the next section, this is sufficient. Actual mechanisms of regime changes are not easily modeled, since there are, as aforementioned, also expectational forces working, see for example as studied in de Grauwe et al (2012) and Roch and Uhlig (2012).

In the context of our new method, the NMPC procedure, we, however, can model regime changes. A regime change, from a regime of low to high stress with stronger contraction, could also occur when due to high leveraging and high financial stress credit contractions set in or credit demand falls, since consumers might be deleveraging, or the lender constrain credit, see Krainer (2012). The dynamics of this case when suddenly a new regime, a credit constrained regime, is arising is treated in Ernst and Semmler (2012). The subsequent sections will allow for regime switching in an econometric impulse-response study, in a multi-regime VAR (MRVAR).

3 Financial Stress and Output Measures

Given our model variants of low and high financial stress it is an important empirical issue to identify the high and the low financial stress regimes. What measures can one utilize to conduct empirical estimates? One issue is to measure the financial stress, the other is to track the interconnection of financial stress and output. This means that there is likely to be a high stress accompanying low output and high output accompanying a low stress regime. Yet, the effects of shocks of one affecting the other may be asymmetric with respect to regimes.

Our model variants above may suggest to take leverage ratios of economic agents to measure financial stress. So high leverage implying high financial stress and low leverage the reverse. However, there is an issue whether the ratio of net worth to capital assets, or the reverse measure, can be used as good measure of financial stress. This measure is greatly affected by the market valuation of assets as well as liabilities. In particular, asset valuation is heavily impacted by the confidence and estimate of income streams the asset generates, as well as

¹⁰See Mittnik and Semmler (2012)

presumed discount rates, and the liabilities such as credit instruments, short and long term loans, are strongly impacted by their corresponding risk premia.¹¹

Moreover, credit constraints, for example, as measured by the Fed index of changes in credit standards to determine the ease and tightness of obtaining credit as well as default premia and credit spreads and short term liquidity, are also important financial stress factors for economic agents. All this will affect credit demand and supply from the financial market and financial intermediaries. We thus need more extensive measures than only leverage to evaluate financial stress.

We therefore propose to measure financial stress empirically by taking the IMF (2011) financial stress index, the FSI. Note that the FSI has the following components:

$$FSI = \text{bankbeta} + TEDspread + \text{invertedtermspread} + \\ \text{corporatebondspread} + \text{stockmarketreturn} + \\ \text{stockmarketvolatility} + \text{exchangeratevolatility}$$

The FSI is available for a large number of EU countries.¹² The IMF's (2011) FSI¹³ refers to three major sources and measures of instability, namely: 1) a bank related index – a banking beta as 12-month rolling beta of bank stock index and a Ted or interbank spread, 2) a security related index – a corporate bond yield spread, an inverted term spread, and a monthly stock returns (measured as declines), six-month rolling monthly squared stock returns and finally, 3) an exchange rate index – a six-month rolling monthly squared change in real exchange rates. All three sets of variables are detrended and scaled with their standard deviations in order to normalize the measures.

¹¹This is implicit in Merton's risk structure of interest rates, see Merton (1974)

¹²The Federal Reserve Bank of Kansas City and the Fed St. Louis have also developed a general financial stress index, called KCFSI and STLFSI respectively. The KCFSI and the STLFSI, take into account the various factors generating financial stress. The KC index is a monthly index, the STL index a weekly index, to capture more short run movements, see also Hatzius et al (2010). Those factors can be taken as substitutes for the leverage ratios as measuring financial stress. See also the Bank of Canada index for Canada, i.e. Illing and Lui (2006). Both the KCFSI and STLFSI include a number of variables and financial stress is related to an: 1) increase the uncertainty of the fundamental value of the assets, often resulting in higher volatility of the asset prices, 2) increase uncertainty about the behavior of the other investors, 3) increase in the asymmetry of information, 4) increase to the flight to quality, 5) decrease in the willingness to hold risky assets, and 6) decrease in the willingness to hold illiquid assets. The principle component analysis is then used to obtain the FSI. Linear OLS coefficients are normalized through their standard deviations and their relative weights computed to explain an FSI index. A similar procedure is used by Adrian and Shin (2010) to compute a macro economic risk premium. We want to note that most of the variables used are highly correlated with credit spreads. The latter have usually the highest weight in the index, for details see Hakkio and Keeton (2009, tables 2-3.).

¹³This is published for advanced as well for developing countries, see IMF (2008) and IMF FSI (2011)

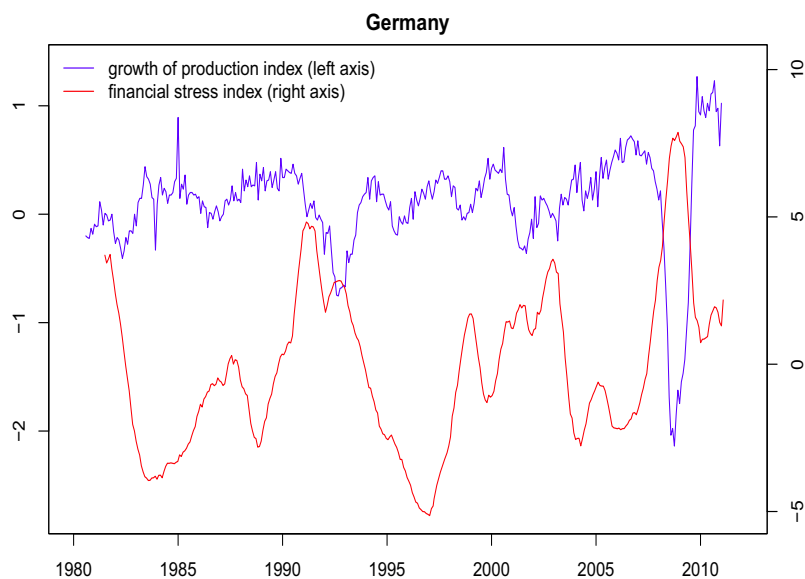


Figure 4: Financial stress and output for Germany: Financial stress index (IMFFSI, lower graph) plotted against growth rates of industrial production (3 month moving average, upper graph)

As measure for the performance of the macroeconomy we take a monthly production index for the different countries, or what is more proper in the context of our model, the growth rate of the monthly production index of the various countries we are considering. To measure output we use IP, the Industrial Production Index from the OECD (2012).

As concerning the IMF FSI, combining the three groups of variables with appropriate weight in a stress index and contrasting it with the monthly production index, one can observe clearly a counter-cyclical behavior. As an example, using Germany, this is illustrated in Figure 4, where the IP variable is shown for a three-month moving average.

As the comparison of the smoothed growth rate of the production index and the stress index in Figure 4 shows there is less financial stress that corresponds to good times and more financial stress in bad times. Financial markets and financial institutions are clearly doing better in economic booms than in recessions¹⁴. Given the apparent linkages between the FSI and economic activity, we would also expect a strong linkage between over-borrowing, financial stress and economic activity.¹⁵

A “one-regime VAR” has been used frequently to study the financial-macro

¹⁴This is also shown in an empirical study by Gorton (2010) who shows that there is more insolvency of financial institutions in bad times.

¹⁵We want to note that the financial stress index can also be linked to some broader index of economic activity, See Hakkio and Keeton (2009).

link, using the financial accelerator.¹⁶ Yet those “one-regime VAR” studies presume linear behavior of the variables, symmetry effects of shocks and mean reversion – mean reversion back to the same equilibrium–after the shocks. What we will pursue here is an multi-regime VAR(MRVAR). For applications of MRVAR modeling, see also Mittnik and Semmler (2012b) and Ernst, Mittnik and Semmler (2010). An potentially alternative approach is a Markov-switching VAR. We compare these two approaches and come to the conclusion that MRVAR is more proper for our purpose.

4 Empirics Using MRVAR

In order to empirically assess dynamic links between financial stress and output in different stress states that are predicted in two variants of the theoretical model of the high financial stress regime and the low financial stress regime, we need to accommodate different local dynamics in one model. MRVAR serves as a proper modeling framework for this purpose, because within MRVAR it is possible to test the existence of changing dynamics in different regimes and to quantify different regime-specific dynamics.

The MRVAR specification applied here is as follows

$$y_t = c_i + \sum_{j=1}^{p_i} A_{i,j} y_{t-j} + \epsilon_{it}, \quad \epsilon_{it} \sim (0, \Sigma_i), \text{ if } \tau_{i-1} < f_{t-d} \leq \tau_i \text{ for } i = 1, 2, \quad (16)$$

where f_{t-d} is the threshold variable observed at time $t-d$; and the regimes are defined by the prespecified threshold values $-\infty = \tau_0 < \tau_1 < \tau_2 = \infty$. Here we estimate a two regime VAR with the financial stress index as the threshold variable, intending to investigate the dynamics in high financial stress state and low financial stress state. The time horizon of our empirical investigation is chosen according to the maximum availability of data, which ranges for most countries from 1980 to 2012 (see Table 1 for details.). According to the theory developed in previous sections we expect that the financial stress index (FSI) and the output growth (GIP) are negatively correlated. Indeed with an exception of Norway the correlation coefficient between FSI and GIP are negative in all countries (see Table 1).

The determination of the threshold is key to the specification of a MRVAR model. The authors of IMF FSI¹⁷ used one standard deviation from the trend (HP filtered) of FSI to define high financial stress episode. Following their idea but to avoid the arbitrariness in determination of the threshold value, we will estimate the threshold value from data. Concretely, we will estimate the threshold value in the positive range of FSI and define those periods as being

¹⁶ Estimating the financial accelerator for the macroeconomy with a “one regime VAR”, see Christensen and Dib (2008). For the application of the financial accelerator to study financial intermediaries in a “one regime VAR”, see Hakkio and Keeton (2009) and Adrian et al. (2010).

¹⁷ See World Economic Outlook (WEO) Financial Stress, Downturns, and Recoveries, Oct. 2008

in the high financial stress state, when the FSI index for a country is higher than the threshold value and the other periods as being in the low financial stress state. According to the construction of FSI, a zero value of FSI implies neutral financial market conditions, the high financial stress state has to be beyond a certain deviation from the neutral situations. Therefore, we estimate the threshold value in the positive range of FSI.

Table 1: Descriptive Statistics of the Data

Variable	*FSI	*GIP	USA_FSI	USA_GIP
USA_FSI	1	-0.349	1	-0.349
USA_GIP	-0.349	1	-0.349	1
JPN_FSI	1	-0.316	0.505	-0.238
JPN_GIP	-0.316	1	-0.220	0.178
DEU_FSI	1	-0.212	0.575	-0.267
DEU_GIP	-0.212	1	-0.222	0.190
FRA_FSI	1	-0.091	0.457	-0.192
FRA_GIP	-0.091	1	-0.185	0.132
GBR_FSI	1	-0.162	0.766	-0.346
GBR_GIP	-0.162	1	-0.144	0.110
ITA_FSI	1	-0.114	0.439	-0.226
ITA_GIP	-0.114	1	-0.185	0.190
ESP_FSI	1	-0.018	0.261	0.015
ESP_GIP	-0.018	1	-0.123	0.115
DNK_FSI	1	-0.091	0.450	-0.141
DNK_GIP	-0.091	1	-0.090	0.038
FIN_FSI	1	-0.049	0.110	-0.116
FIN_GIP	-0.049	1	-0.130	0.102
NOR_FSI	1	0.046	0.477	-0.198
NOR_GIP	0.046	1	0.009	-0.059
SWE_FSI	1	-0.123	0.341	-0.160
SWE_GIP	-0.123	1	-0.198	0.195
NLD_FSI	1	-0.071	0.767	-0.367
NLD_GIP	-0.071	1	-0.068	0.026
BEL_FSI	1	-0.097	0.678	-0.355
BEL_GIP	-0.097	1	-0.100	0.025
AUT_FSI	1	-0.101	0.481	-0.312
AUT_GIP	-0.101	1	-0.115	0.060

Notes: Table 1 shows the correlation coefficients between the Financial Stress Index (FSI) and the growth of Industrial Output ($GIP=100\Delta \log(IP)$) of a country and their correlations with their counter parts in USA

Based an estimated threshold value, our two-regime VAR has a straightforward interpretation of the dynamic links between the financial stress and the

real output in the high and in the low financial stress states respectively.

We estimate a standard VAR and an MRVAR model for the FSI and the industrial output growth rate with $y_t = (FSI_t, GIP_t) = (FST_t, 100\Delta \log(IP_t))'$. We use AIC to discriminate between a standard VAR or an MRVAR. The AIC is given by

$$AIC(M, p_1, p_2) = \sum_{j=1}^M \left[T_j \log |\hat{\Sigma}_j| + 2n \left(np_j + \frac{n+3}{2} \right) \right], \quad (17)$$

where $M = 2$ is the number regimes; p_j is the autoregressive order of regime j ; T_j is the number of observations associated with regime j ; $\hat{\Sigma}_j$ is the estimated covariance matrix of the residuals of regime j ; and n denotes the number of variables in the vector y_t ¹⁸.

For the case of USA, AIC suggests a fourth-order standard VAR with $AIC = -45.793$. The threshold for the high financial stress state in MRVAR is estimated at 2.932. Accordingly, $FSI_t > 2.932$ is considered as the high financial stress state and the low financial stress state is when $FSI_t < 2.932$. For the estimated MRVAR we have $AIC(M = 2, p_1 = 3, p_2 = 2) = -202.965$. Based on the AIC values, MRVAR is a more proper specification than a one regime VAR. We have run the same model selection procedure for other 14 nations and the specification results are summarized in Table 2.

¹⁸The AIC takes into account for possible heterogeneity in the constant terms, c_j , and residual covariance, Σ_j , across regimes. This AIC criterion is also applied in Mitnick and Semmler (2011).

Table 2: Specifications of VAR and MRVAR models

Country	Sample	VAR		MRVAR		
		AIC	Lag	AIC	Lags	Threshold
USA	1980:12 2012:2	-45.7930	4	-202.9650	2,3	2.9320
JPN	1980:12 2012:2	832.6932	3	585.0516	1,3	2.5582
DEU	1980:12 2012:2	785.8416	5	624.0027	1,3	3.2239
FRA	1984:01 2012:2	443.5678	3	328.7833	1,3	3.2905
GBR	1980:12 2012:2	310.9635	2	182.8142	1,2	2.9402
ITA	1981:03 2012:2	620.6878	3	468.6478	3,3	2.5766
ESP	1980:12 2012:2	653.6477	6	498.7183	1,2	2.8680
DNK	1980:12 2012:2	1189.3758	5	1086.735	1,5	2.9650
SWE	1984:01 2012:2	704.0004	2	573.6479	4,2	2.4668
FIN	1980:12 2012:2	871.3759	3	741.3196	1,3	2.4148
AUT	1980:12 2012:2	740.5189	2	607.0639	2,3	2.5737
BEL	1980:12 2012:2	999.7599	4	864.4700	1,4	2.3635
NLD	1980:12 2012:2	1006.5427	6	859.0070	3,6	3.0776
CAN	1980:12 2012:2	401.1062	4	199.4409	2,3	2.9813
NOR	1980:12 2012:2	939.2660	4	752.6290	1,4	2.5553

Notes: Table 2 reports the results of specifications of VAR and MRVAR using AIC criterion. The left panel includes the specification results of VAR. The right panel are the results of MRVAR. The two numbers in the fifth column under the header *lags* are the lag order of the low financial stress regime and the high financial stress regime respectively. A key for the abbreviation of the country names is given in the appendix.

The AIC statistics in Table 2 show that MRVAR is a more proper specification not only for the USA but also for all other nations under investigation.

These clear statistical model selection results for all nations are also reflected in differences in dynamics of the respective two different regimes. We use here *within – regime* impulse-response functions to assess the dynamics of the two regimes. The *within – regime* impulse response function is a regime specific impulse response function that is calculated under an assumption that the system remains in the same regime. This is surely not a realistic assumption, as we observe that a system frequently migrates from one regime to the other. However, a regime-specific response analysis will help us to understand the short-run dynamic behavior associated with the respective regimes¹⁹.

¹⁹For a study of IRF allowing inter regime migrations see Mittnik, S. and W. Semmler (2012b).

Figure 5 shows the cumulative impulse response functions of the MRVAR for USA in the high financial stress regime and in the low financial stress regime, respectively. The upper 4 impulse response functions (IRF) are the IRF of the high financial stress regime, while the lower four IRF are those of the lower financial stress regime. The cumulative responses to one standard deviation shocks are significantly different in the two regimes. In the low stress regime, positive one standard deviation financial stress shocks have an effect of -0.36% on the growth of industrial production, while in the high stress regime the cumulative response to a one standard deviation financial shock will settle at a statistically significant level of -0.79% .

Taking into account of the difference in the standard deviations in the two regimes, for the same scale of financial shocks, the responses in the high financial stress regime is more than two times higher than that in the low financial stress regime. This difference provides an evidence that in a high financial stress state an increase in financial stress will have a much worse impact on the output than in a low financial stress state. For a period of high financial stress economic agents are usually income liquidity and credit constrained. So any further financial stress shock will reduce spending more, making demand and output declining further. This is unlikely to happen in a state of low financial stress and in a regime of higher growth where agents are less income, liquidity and credit constrained. In section 2, our theoretical model has elaborated different dynamics due to different financial stress situations. For a comparable empirical result, see Mittnik and Semmler (2012).

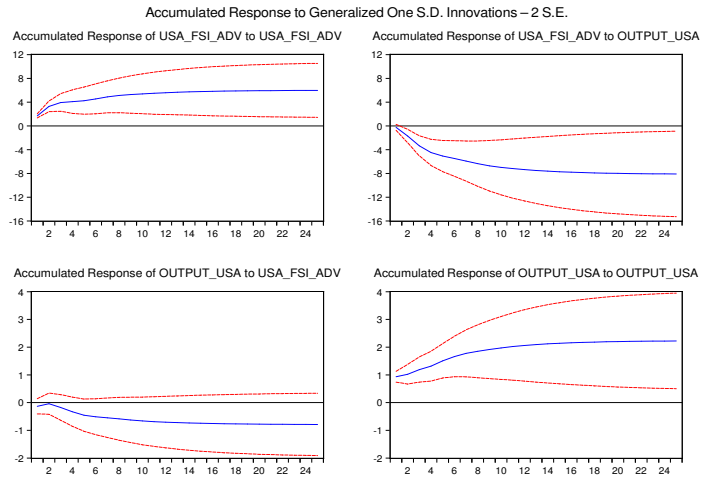
The impacts of output shocks on financial stress are also different in the two regimes (See Figure 5). In the high financial stress regime the cumulative response of the financial stress index to one standard deviation output growth shocks is negative and statistically significant, settling at a level of -8.1 . In other words, in the high financial stress regime a decrease in the industrial production growth will significantly worsen the financial stress situation. In the low financial stress regime, a one standard deviation output shocks have hardly any effects on the financial stress state.

The different responses in the high financial stress regime and in the low financial stress regime in the MRVAR show that, depending on the financial stress situation, the system may evolve to different equilibria, for which the theoretical model in the previous section has provided an economic explanation. Since the regime-specific VARs are stationary, the regime-specific mean values of the observed variables provide a rough estimate of the regime-specific equilibria.

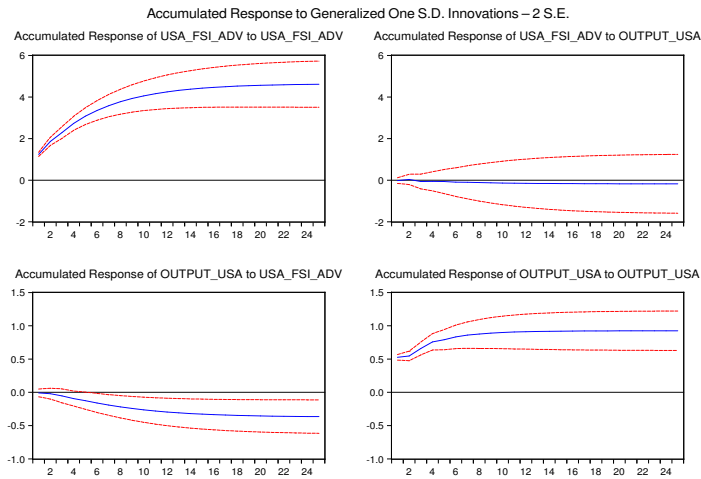
For USA, in the high financial stress regime the mean growth rate of the industrial production is -0.198% and the mean FSI value is 5.384 , while in the low financial stress regime these two values are 0.235% and -1.07 respectively, implying that the system would evolve to a state with contraction in output in the high financial stress regime and it would evolve to a state with positive output growth in the low financial stress regime.

It is to note that these regime-specific equilibria are conducted under the assumption of no inter regime migration. This is surely not a realistic assumption, since we observe frequent regime changes. Therefore, the quantified regime-

Figure 5: USA Multi Regime VAR



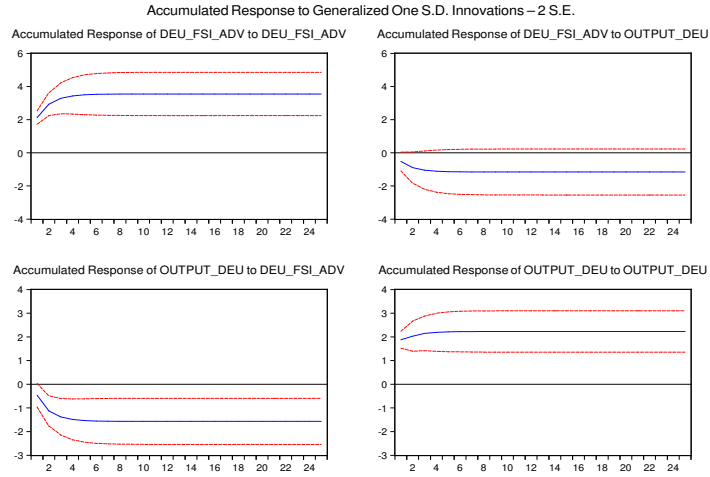
High Financial Stress Regime



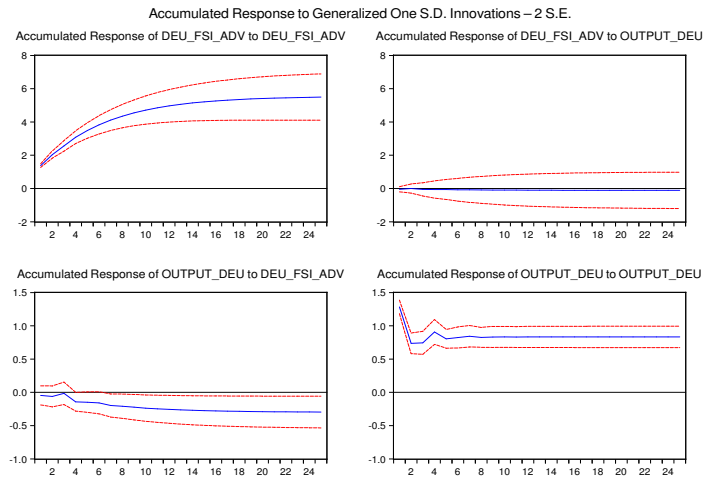
Low Financial Stress Regime

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime. *USA_FSI_ADV* is the name of the financial stress index of USA and *OUTPUT_USA* is the name of the growth rate of the industrial production of USA.

Figure 6: German Multi Regime VAR



High Financial Stress Regime



Low Financial Stress Regime

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime. *DEU_FSI_ADV* is the variable name of the financial stress index of Germany and *OUTPUT_DEU* is the variable name of the growth rate of the industrial production of Germany.

specific equilibria should only provide a hint to gauge how the system would evolve, if no exogenous shocks leading to regime switching.

The impulse-responses in two different regimes in the German MRVAR show similar patterns as those of the USA MRVAR (See Figure 6.) In the low stress regime, a positive one standard deviation financial stress shock has an effect of -0.296% on the growth of the industrial production after one year, while in the high financial stress regime the cumulative response to a one standard deviation financial shock will settle at a statistically significant level of -1.568% after 8 months. This implies that in the high financial stress situation an increase in financial stress has a much worse impact on the output than in the low financial stress situation.

In the high financial stress regime the cumulative responses of the financial stress index to a one standard deviation output growth shock are negatively increasing and they settle at the level of 1.159 after two years. In other words, in the high financial stress regime a decrease in the industrial production growth will worsen the financial stress situation. But in the low financial stress regime, a one standard deviation output shock has hardly any effect on the financial stress.

The different responses in the high financial stress regime and the low financial stress regime in the MRVAR of Germany show also that, depending on the financial stress situation, the system may evolve to different equilibria.

We summarize the MRVAR estimation results for all 14 nations in Table 4 in Appendix and the associated impulse-response functions are also given in the Appendix. The impulse response functions of these nations show certain common patterns but also some country-specific differences.

Regime Effects

1. In the high financial stress regime, across most nations the responses of output growth to financial stress shocks are negative, indicating that stressed financial conditions will have negative effects on output growth in these nations. Only in four nations: ESP, FIN, ITA and NOR, the responses are positive but negligibly small and statistically insignificant.
2. In most nations, positive output shocks will have negative financial stress responses in the high financial stress regime, implying that negative output shocks will intensify the financial stress situation in the most nations. Only in ESP, FIN, NLD and NOR the output shocks have positive, but small and statistically insignificant responses.
3. In the low financial stress regime, the response of output growth to financial stress shocks are negative across all nations except France and Italy, implying that improving financial conditions will have positive effect on the output growth in all nations. While in USA, DEU, FRA and ESP the responses are statistically significant, in other nations the responses are statistically insignificant (from zero).
4. In the low financial stress regime, the responses of financial stress indexes to output shocks are all statistically insignificant except in AUT, indicating

that the dynamic impact of real output on financial conditions are weak in the low financial stress regime. In DEU, FIN, SWE, ITA, AUT and NLD the responses are positive, while in other nations the response are negative.

5. Generally, the responses in a low financial regime are weaker than the responses in the respective high financial regime for shock of same scale. This is also reflected in the fact that in the low financial regime most impulse-response functions are statistically insignificant.

Country Heterogeneity

1. Across all large economies like USA, JPN, DEU and GBR the patterns of the dynamic responses between the output growth and the financial stress index are the same. In the high financial stress regime, the output growth responds negatively to the positive financial financial stress shock and the financial stress index responds also negatively to the positive output shocks. In the low financial stress regime, the financial stress index responses to output shocks are only negligible small, the output growth responds to the financial stress shocks are statistically insignificant. In BEL the dynamic responses between the real output and the financial index show the same pattern as the large economies.
2. FRA is a outlier among the large economies. Its dynamic responses between the real output and the financial stress index are weak and statistically insignificant. Other nations like ITA, NLD, NOR, FIN, DNK and SWE show also an insignificant dynamic responses between the real output and the financial stress index. This implying that the patterns of impulse response functions of the MRVARs of these nations are subject to considerable sample uncertainty. Hence they should not be over-interpreted.
3. ESP shows a stronger dynamic link between the real output and the financial stress index int the lower financial stress regime than in the higher financial stress regime. The dynamic responses between the real output and the financial stress index are negative and statistically significant in the lower financial stress regime, while the responses are small and statistically insignificant in the higher financial stress regime.
4. AUT shows the same dynamic patterns as the large economies in the higher financial stress regime. In the high financial stress regime, the output growth responds negatively to the positive financial financial stress shocks and the financial stress index responds also negatively to the positive output shocks. But it has statistically significant positive response of the financial index to the real output shocks.
5. All nations have a positive average output growth in the low financial stress regime. All nations except FRA, ESP, AUT and NOR have a negative average growth in the high financial stress regime. But the later ones have positive average growth in the high financial stress regime.

6. ESP, NOR and FRA are three nations in which the output growth is higher in the high financial stress regime than in the low financial stress regime.

Overall, the empirical analysis suggests that the stronger the position of an economy in the world in terms of output level the more autonomic is interaction between financial stress and output and henceforth the stronger is the evidence supporting the multi-equilibria scenario predicted by the theoretical model. Generally, in the low financial stress regime an increase in financial stress has weaker effects on the output growth than in the high financial stress regime. In some countries international spill-over effects may significantly affect their own financial stress and output growth, so that these countries show some heterogeneous response patterns.

5 Concluding Remarks

Often over-borrowing has led to financial stress and financial crisis. Historically, most severe economic crises have been preceded by a financial crisis which has amplified the decline in real economic activity. The latter in turn has often exacerbated the financial meltdown. On the other hand, there are many historical episodes where there were moderate or even strong financial stress shocks that, however, did not end up triggering real recessions.

In order to study the macroeconomic dynamics, with alternative paths resulting from financial stress shocks, we first have introduced a macromodel with a finance-macro link which uses a multi-period decision framework of economic agents. The agents can, in a finite horizon context, borrow and accumulate assets where the above two scenarios may occur. The model is solved through nonlinear model predictive control (NMPC). In contrast to studies of the financial accelerator model—which is locally amplifying but globally stable and mean reverting—our model can admit two basic regimes- a regime of low financial stress and convergence toward some growth path and a scenario of greater instability. In the latter scenario large contractionary effects can be expected. Whereas the financial accelerator leads, in terms of econometrics, to a single-regime VAR specification, the multi-regime dynamics studied here requires a multi-regime VAR (MRVAR) approach.

Using the IMF (2011) financial stress index and industrial production data for the US, the EU and Non- EU countries, our method of a MRVAR-based study enables us to conduct a regime specific response analysis. By using a MRVAR we could show that in a regime of high financial stress, stress shocks can have large and persistent impacts on the real side of the economy whereas in regimes of low stress, shocks can easily dissipate having no lasting effects. The same larger effects on financial stress and on output can arise in regimes of low output growth in contrast to periods of high output growth. Thus empirically, we find that financial-stress shocks and output shocks have asymmetric effects, depending on the regime the economy is in.

As we have shown, though there is heterogeneity across countries – with smaller countries showing weaker channels in the the financial-real interaction – there is also much similarity in larger economies. Across countries there are common features in the sense that in larger economies (for example in the US, Germany, France and Japan), large positive financial stress shocks in a high growth regime tend to have less of a contractionary effect than in a low growth and high stress regime. On the other hand, large reductions in financial stress tend to induce stronger expansionary effects in low– rather than in high–growth regimes.

The latter seems to be in particular important when evaluating “unconventional” monetary policy. The empirical analysis presented here strongly suggests that both the timing and the intensity of policy actions matter—findings that cannot be obtained by conventional, linear, single–regime analysis.

References

- [1] Adrian, T., A. Moench, and H. S. Shin (2010), “Macro Risk Premium and Intermediary Balance Sheet Quantities”, Federal Reserve Bank of New York, Staff Report no 428
- [2] O. J. BLANCHARD and S. FISCHER (2009): “Lectures on Macroeconomics”, Cambridge, Mass. and London: MIT Press.
- [3] Chiarella, C., P. Flaschel, R. Franke and W. Semmler (2009): “Financial Market and the Macro Economy”, Routledge.
- [4] Christensen, I. and A. Dib (2008), “ The Financial Accelerator in an Estimated New Keynesian Model”, *Review of Economic Dynamics*, vol 11: 155-178.
- [5] P. De Grauwe (2012), “The Governance of a Fragile Eurozone”, *Australian Economic Review*, Vol 45 Issue 3: 255-268.
- [6] G. B. Eggertsson and P. Krugman (2011), “Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo approach ,” http://www.frbsf.org/economics/conferences/1102/PKGE_Feb14.pdf
- [7] Ernst, E. and W. Semmler (2010), “Global Dynamics in a Model with Search and Matching in Labor and Capital Markets,” *Journal of Economic Dynamics and Control*, vol 34, no 9:1651-1679.
- [8] Ernst, E. S., Mittnik, and W. Semmler (2010), “Interaction of Labor and Credit Market Frictions: A Theoretical and Empirical Analysis, paper prepared for the Winter Meeting of the Econometric Society, Atlanta, January 2010.
- [9] Gilchrist, S., A. Ortiz, and S. Zagrajsek (2009), “Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model, manuscript, Boston University.

- [10] Gorton, G. B. (2010), "Slapped by the Invisible Hand", The Panic of 2007. Oxford, Oxford University Press.
- [11] Gruene, L. and J. Pannek (2011), "Nonlinear Model Predictive Control", The Panic of 2007. Springer, New York, Heidelberg.
- [12] Hakkio, C.S. and W.R. Keeton (2009), "Financial Stress: What is it, How can it be Measured, and Why does it Matter?" Federal Reserve Bank of Kansas City.
- [13] Hall, E. R. (2010): "Forward Looking Decision Making", Princeton, Princeton University Press.
- [14] Hatzius, J., P. Hooper, F. Mishkin, K. Schoenholtz, and M. Watson, Financial Condition Indexes: A Fresh Look after the Financial Crisis, www.Princeton.edu/~Watson
- [15] Illing, M. and Y. Liu (2006), "Measuring Financial Stress in a Developed Country: An Application to Canada, Journal of Financial Stability. vol. 2, no 3, October.
- [16] IMF (2011): "Financial Stress Index for Advanced Economies", <http://www.imf.org/external/pubind.htm>.
- [17] Merton, R.C. (1974), "On the Pricing of Corporate Debt. The Risk Structure of the Interest Rate", Journal of Finance, 2: 449-470.
- [18] Mittnik, S. and W. Semmler (2012a), "Regime Dependence of the Multiplier", Journal of Economic Behavior and Organization, forthcoming.
- [19] Mittnik, S. and W. Semmler (2012b), "Estimating a Banking-Macro Model for Europe Using a Multi-Regime VAR", SSRN <http://ssrn.com/abstract=2012106>.
- [20] OECD (2011): "OECD Monthly Economic Indicators".
- [21] F. Roch and H. Uhlig (2012), "The Dynamics of Sovereign Debt Crises and Bailouts", working paper, University of Chicago, Center, NBER and CEPR.

6 Appendix

6.1 Abbreviation of Country Names

Table 3: Abbreviation of Country Names

Abbreviation	Country
USA	The United States
JPN	Japan
DEU	Germany
FRA	France
GBR	United Kingdom
ITA	Italy
ESP	Spain
DNK	Denmark
SWE	Sweden
FIN	Finland
AUS	Austria
BEL	Belgium
NLD	Netherlands

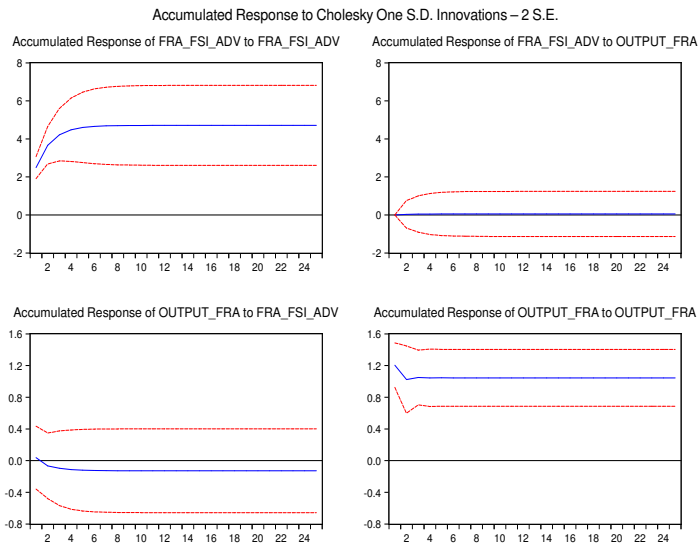
Notes: Table 3 shows the correlation coefficients between Financial Stress Index and Output of a country and their correlations with the variables in USA

6.2 Estimation Results

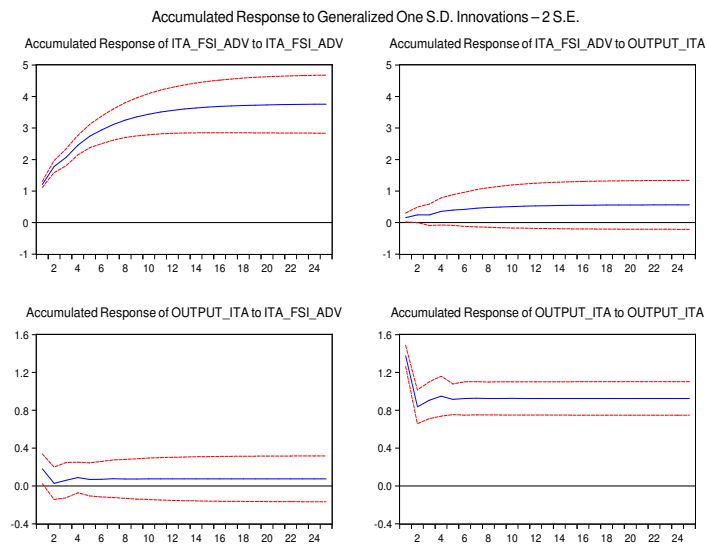
Table 4: Specifications of VAR and MRVAR models

	OBS_h	OBS_l	Threschhold	\overline{FSI}_h	$\overline{\Delta \log(IP)}_h$	\overline{FSI}_l	$\overline{\Delta \log(IP)}_l$
USA	54	321	2.9320	5.3836	-0.1976	-1.0699	0.2348
JPN	51	323	2.5582	5.2296	-1.1572	-0.7811	0.3054
DEU	56	319	3.2239	6.0396	-0.3634	-1.0818	0.2148
FRA	37	334	3.2905	6.6674	0.0780	-0.8374	0.0409
GBR	46	325	2.9402	5.8713	-0.2507	-0.8565	0.1070
ITA	70	303	2.0766	4.3830	-0.5752	-1.1013	0.1551
ESP	52	319	2.8680	4.9269	0.1960	-0.8137	0.0337
DNK	36	339	2.9650	5.4421	-0.8785	-0.6768	0.2454
SWE	49	288	2.0668	5.1691	-0.5124	-1.0215	0.3138
FIN	54	320	2.4148	4.3035	-0.0873	-0.7633	0.2624
AUT	75	300	2.5737	4.3297	0.1703	-1.1602	0.2814
BEL	53	322	2.3635	5.9382	-0.2664	-1.1234	0.2079
NLD	55	320	2.7776	6.0425	-0.6144	-0.9968	0.2476
CAN	59	316	2.9813	5.9065	-0.2820	-1.0358	0.2086
NOR	50	248	2.5553	4.9015	0.7562	-1.2444	0.0851

Notes: Table 4 reports the results of MRVAR. OBS_{hf} is the number of observations in the high financial stress regime and OBS_{lf} the number of observations in the low financial stress regime. *Threschhold* is the the value defines the high financial stress regime. \overline{FSI}_{hf} and $\overline{\Delta \log(IP)}_{hf}$ are the averages the variables in the high financial stress regime and \overline{FSI}_{lf} and $\overline{\Delta \log(IP)}_{lf}$ are the averages of the variables in the low financial stress regime.



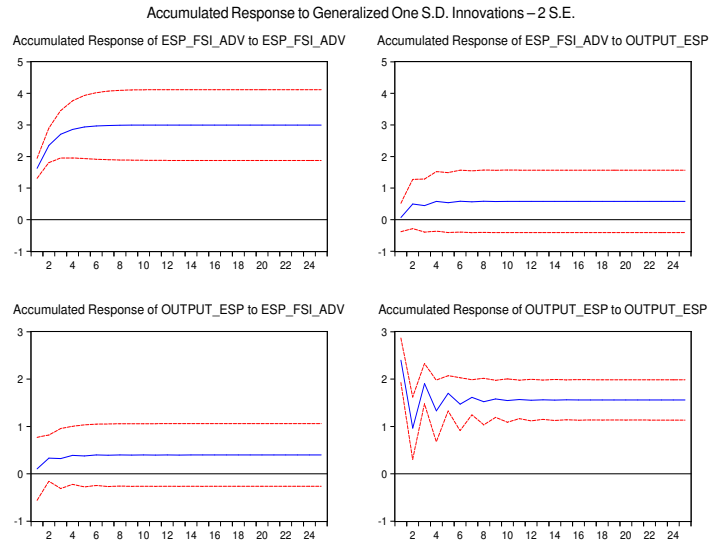
High Financial Stress Regime



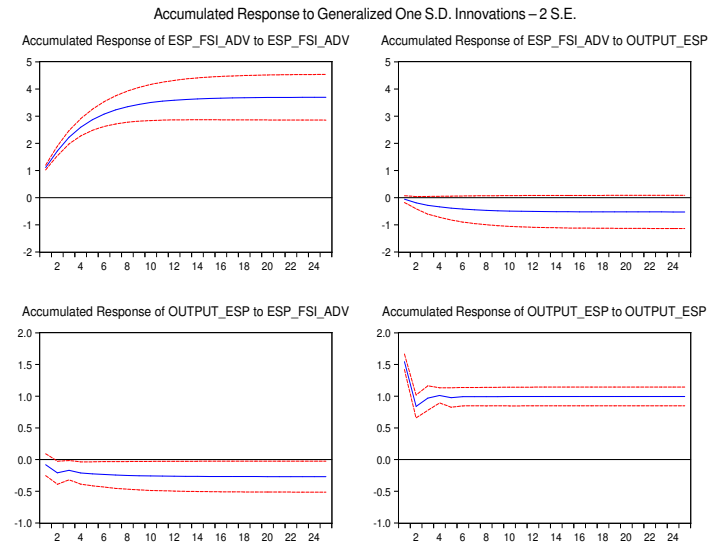
Low Financial Stress Regime

Figure 7: France Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



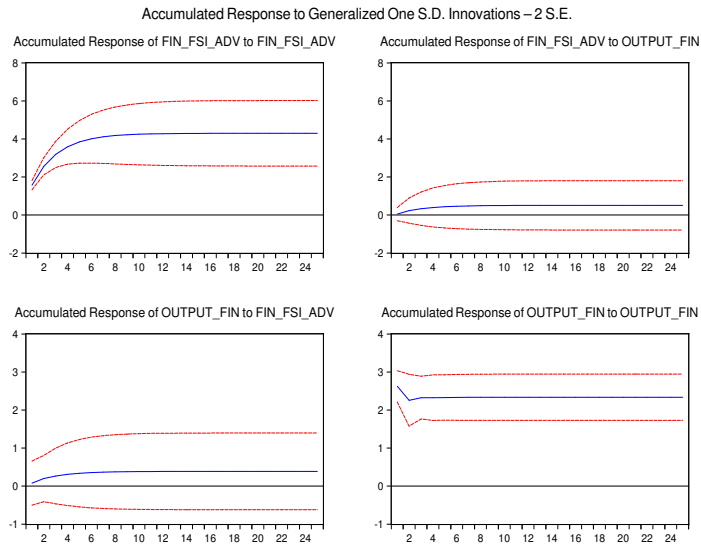
High Financial Stress Regime



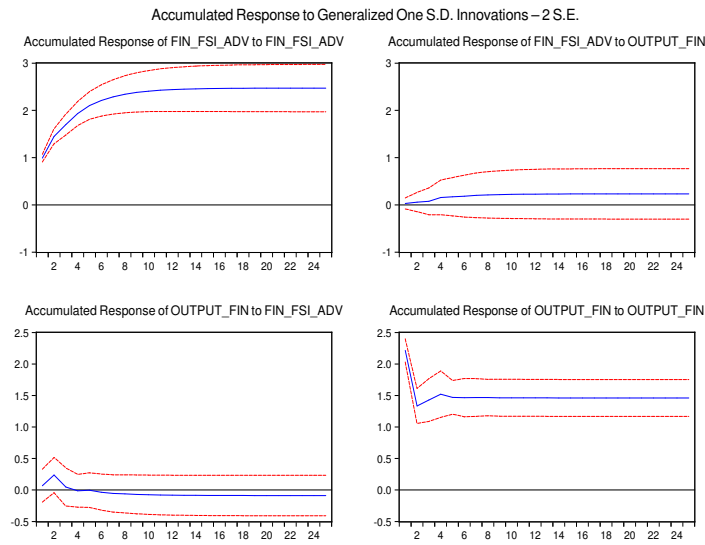
Low Financial Stress Regime

Figure 8: Spain Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



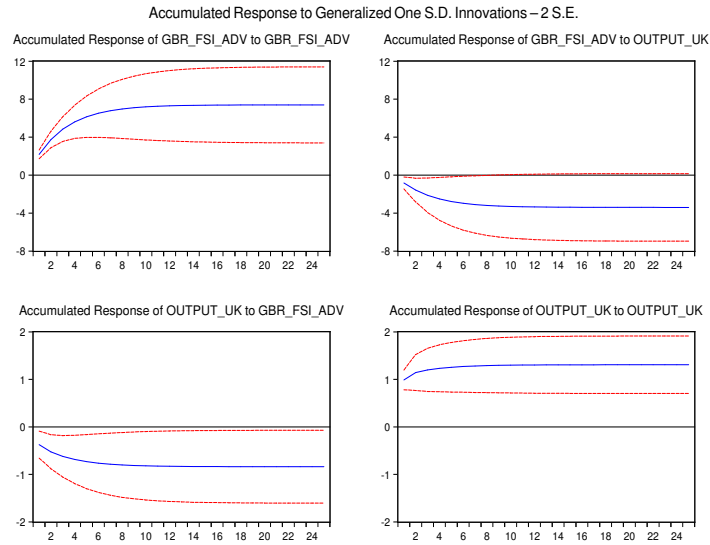
High Financial Stress Regime



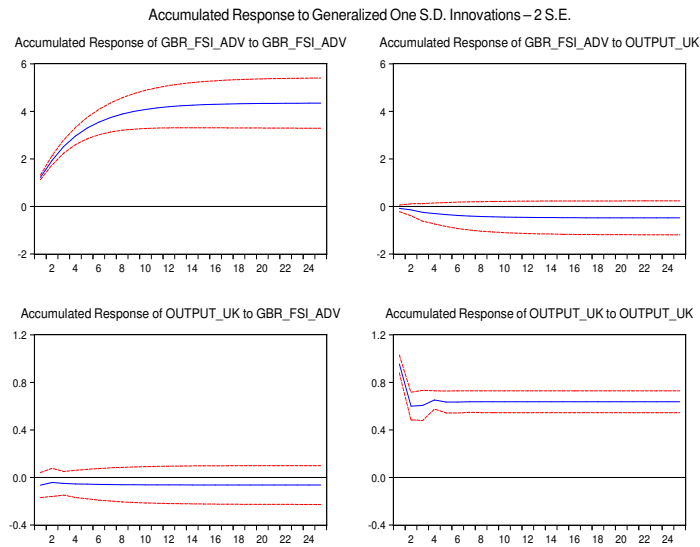
Low Financial Stress Regime

Figure 9: Finland Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



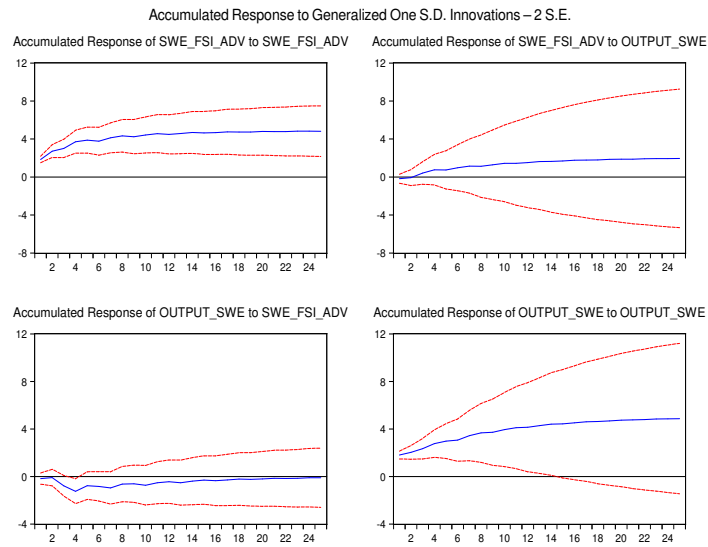
High Financial Stress Regime



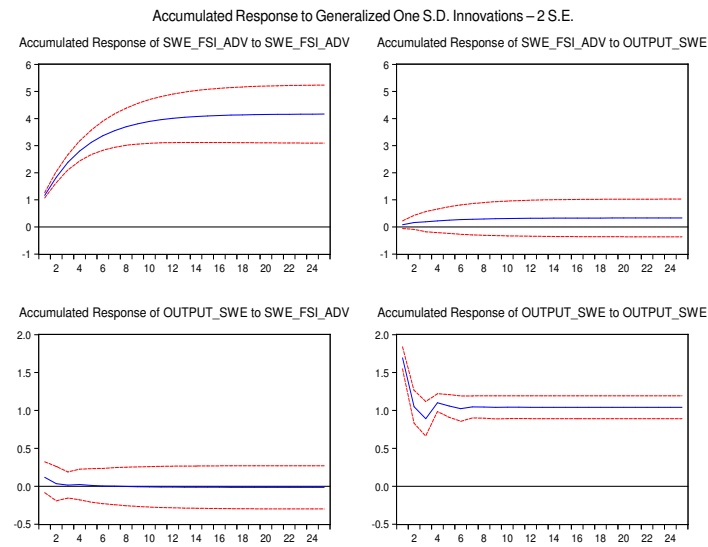
Low Financial Stress Regime

Figure 10: United Kingdom Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



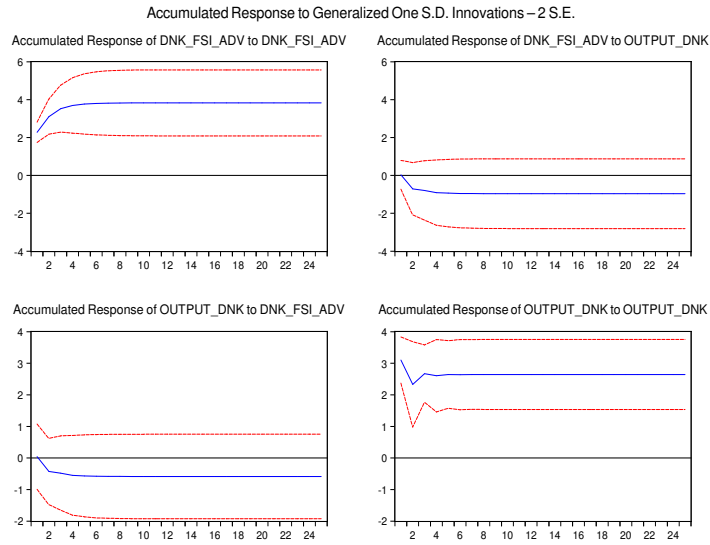
High Financial Stress Regime



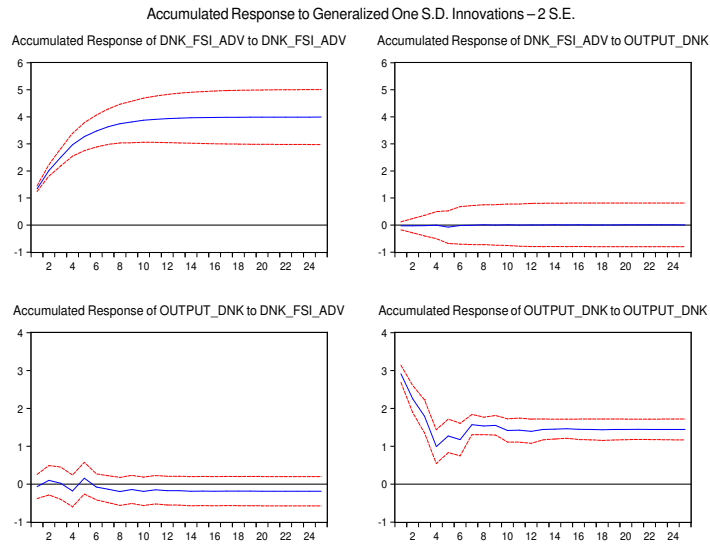
Low Financial Stress Regime

Figure 11: Sweden Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



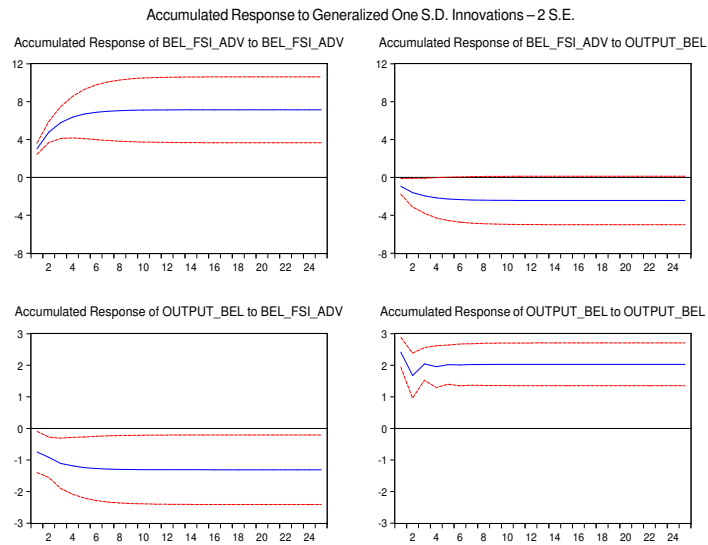
High Financial Stress Regime



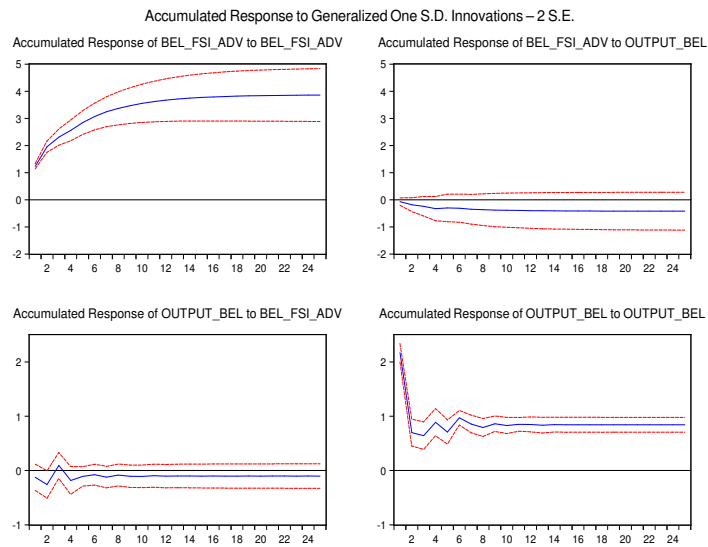
Low Financial Stress Regime

Figure 12: Denmark Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



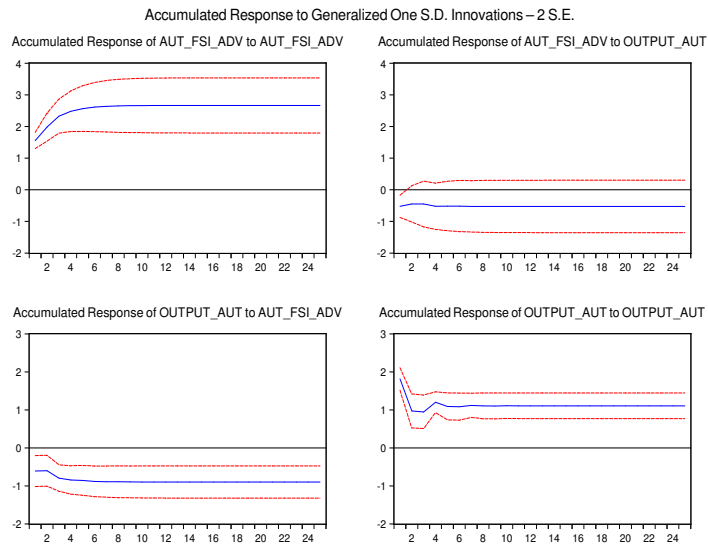
High Financial Stress Regime



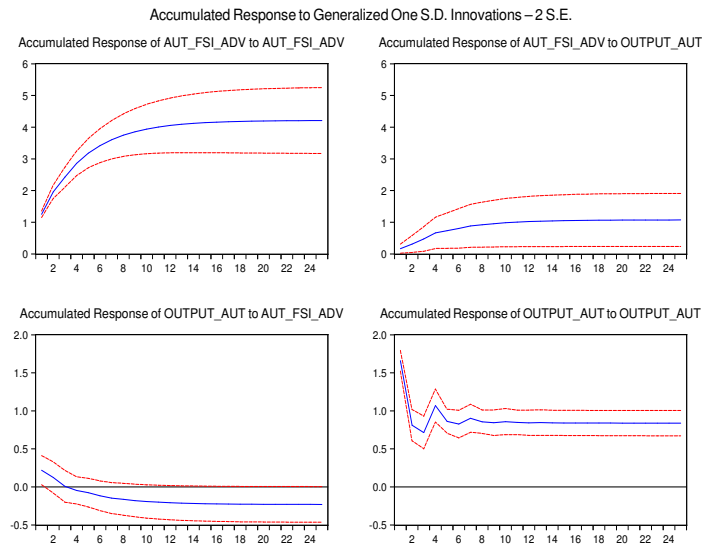
Low Financial Stress Regime

Figure 13: Belgium Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



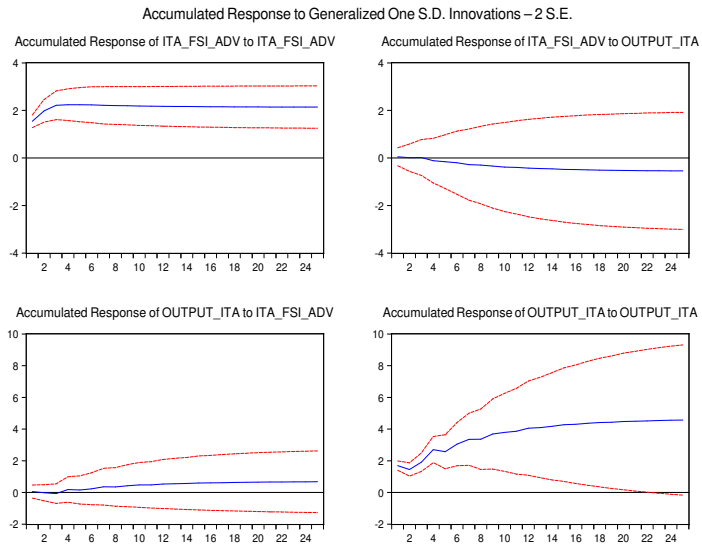
High Financial Stress Regime



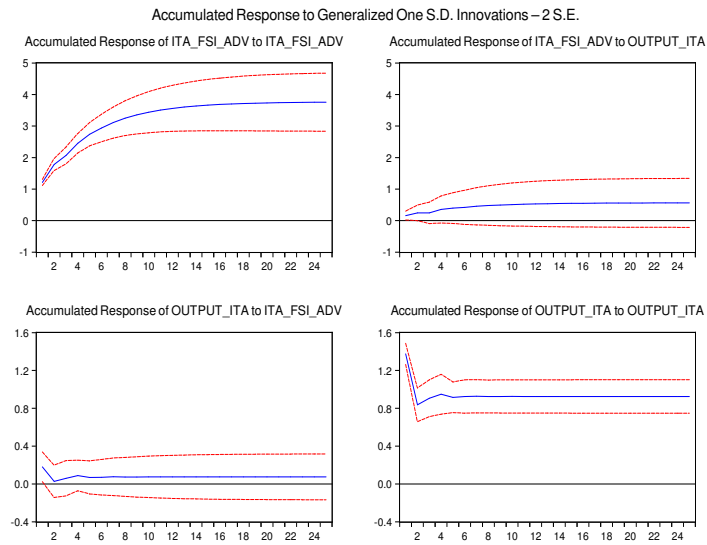
Low Financial Stress Regime

Figure 14: Austria Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



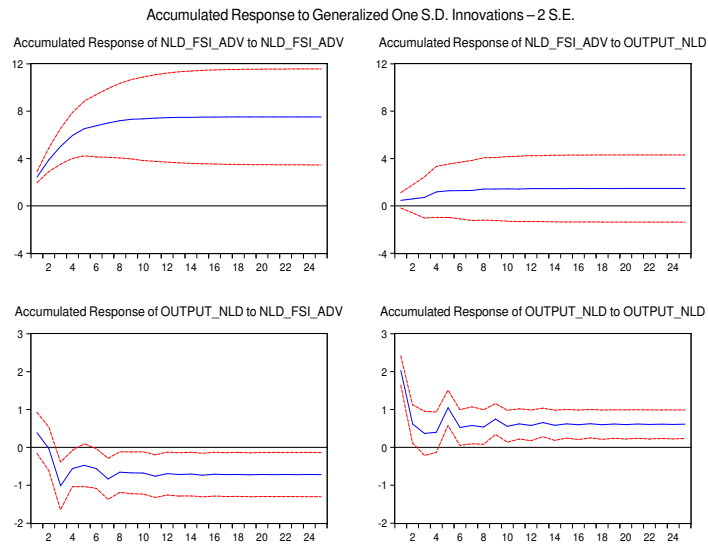
High Financial Stress Regime



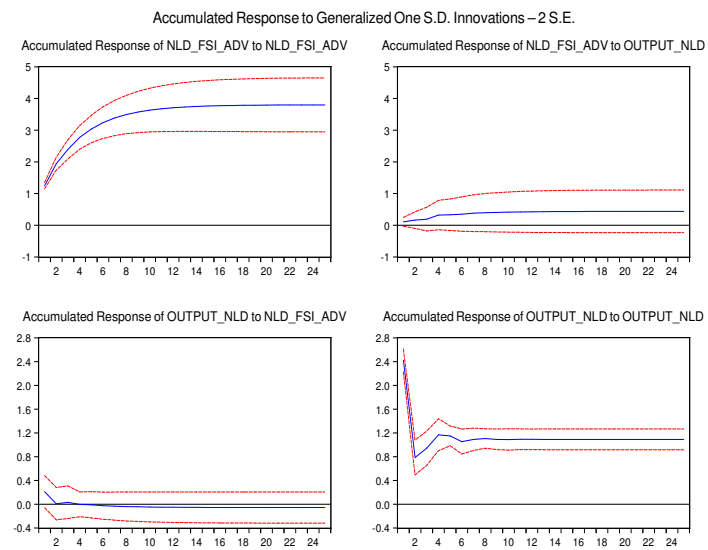
Low Financial Stress Regime

Figure 15: Italy Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



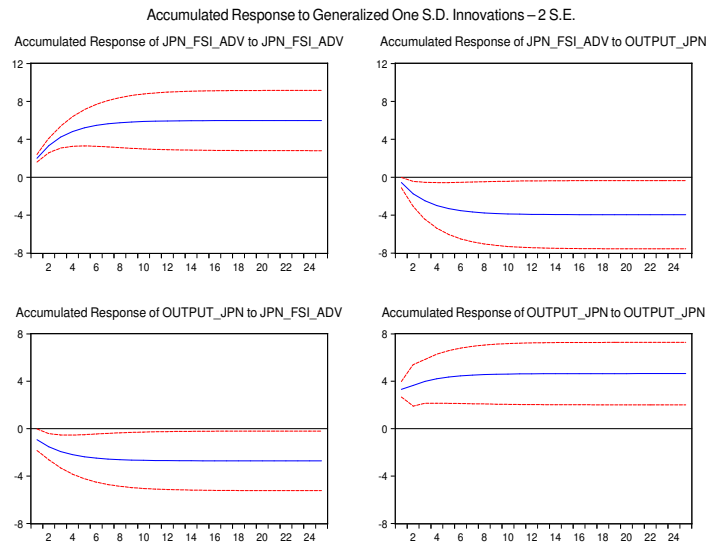
High Financial Stress Regime



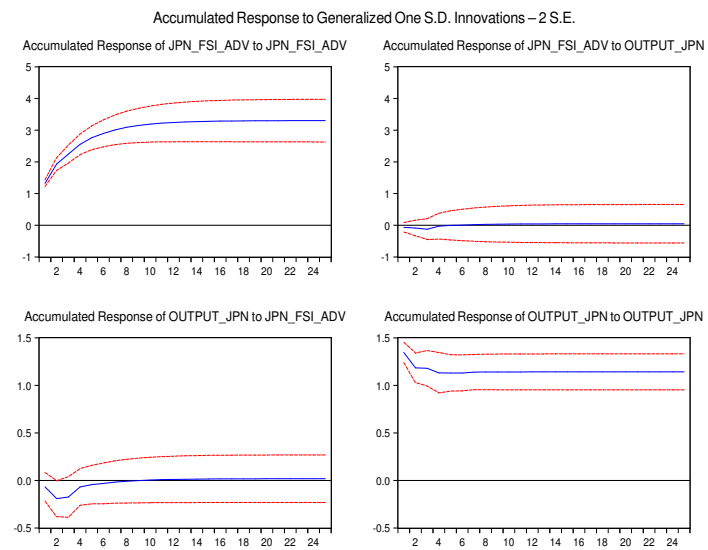
Low Financial Stress Regime

Figure 16: Netherlands Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



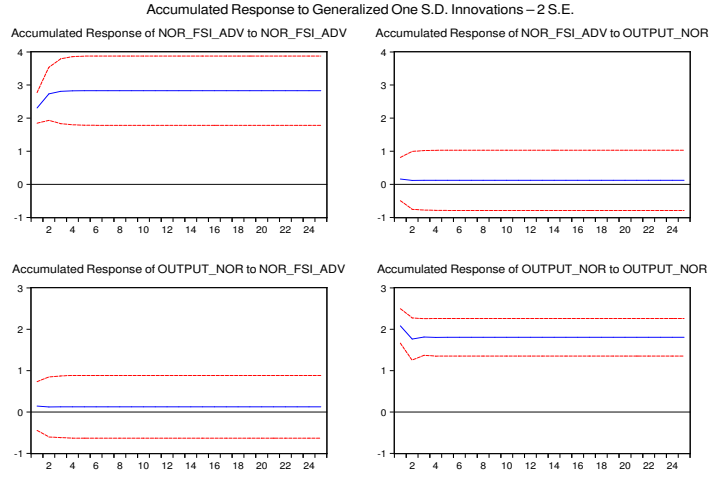
High Financial Stress Regime



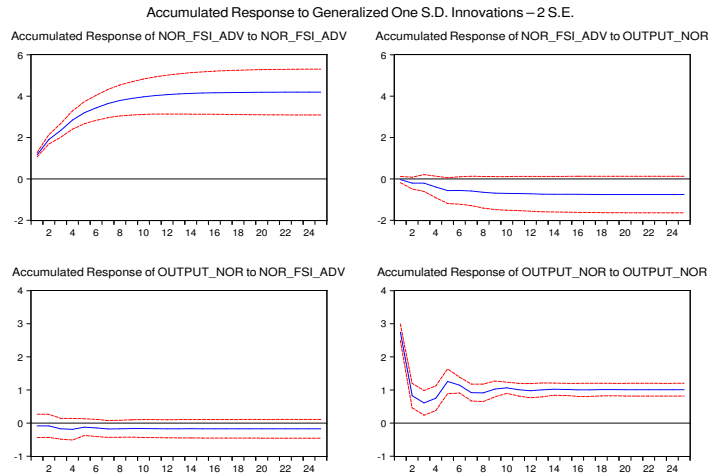
Low Financial Stress Regime

Figure 17: Japan Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.



High Financial Stress Regime



Low Financial Stress Regime

Figure 18: Norway Multi Regime VAR

Notes: The upper 4 diagrams are the regime-specific impulse-response functions of the high financial stress regime. The lower 4 diagrams are the regime-specific impulse response functions of the low financial stress regime.

6.3 MRVAR v.s. Markov Switching Models

In this paper we choose the MRVAR approach to model the regime changes. An often used approach in modeling regime changes is the Markov-switching VAR model. It is of interest to compare these two modeling approaches from both theoretical and empirical perspectives. We restrict our comparison to two regime cases, which is most relevant in this paper.

Formal Difference

A Markov-switching model with two regimes is defined as follows.

$$y_t = \begin{cases} c_1 + \sum_{j=1}^{p_1} A_{1,j} y_{t-j} + \epsilon_{1t}, & \text{for } s_t = 1 \\ c_2 + \sum_{j=1}^{p_2} A_{2,j} y_{t-j} + \epsilon_{2t}, & \text{for } s_t = 2 \end{cases} \quad (18)$$

where the variable s_t that controls the selection of the regime, follows a Markov process:

$$\Pr(s_t | \{s_{t-j}\}_{j=1}^{\infty}, \{y_{t-j}\}_{j=1}^{\infty}) = \Pr(s_t | s_{t-1}). \quad (19)$$

The variable s_t is usually assumed to be an unobservable. In our context s_t describes the state of the economy. $y_t = (FSI_t, 100\Delta \log(IP_t))'$ contains the two observables in our model.

A MRVAR model is a threshold VAR model, formally defined as follows.

$$y_t = \begin{cases} c_1 + \sum_{j=1}^{p_1} A_{1,j} y_{t-j} + \epsilon_{1t}, & \text{for } f_{t-1} < \tau^0 \\ c_2 + \sum_{j=1}^{p_2} A_{2,j} y_{t-j} + \epsilon_{2t}, & \text{for } f_{t-1} \geq \tau^0 \end{cases} \quad (20)$$

where $f_{t-1} = FSI_{t-1}$ is the threshold variable observed at time $t-1$; and the regimes are defined by the pre-specified threshold value $-\infty = \tau_0 \leq \infty$.

These two model are common in that given a choice of the regime $i = 1, 2$, the time series y_t is generated by a $VAR(p_i)$ process. They differ in the mechanism how a regime is chosen. While in the Markov switching VAR the switching from one state s_{t-1} to the other state s_t is a random process with a constant transition probability

$$\Pr(s_t = i | s_{t-1} = j) = h_{ji} \text{ with } i = 1, 2 \text{ and } j = 1, 2, \quad (21)$$

the regime switching in the MRVAR is governed by the value of the switching variable f_{t-1} . To facilitate the comparison we reformulate the MRVAR through defining a state variable for the MRVAR in the following way:

$$\mathcal{S}_t = \begin{cases} 1 & \text{for } FSI_{t-1} < FSI^0 \\ 2 & \text{for } FSI_{t-1} \geq FSI^0 \end{cases}.$$

Then the MRVAR (20) takes a similar form as the Markov switching model (18):

$$y_t = \begin{cases} c_1 + \sum_{j=1}^{p_1} A_{1,j} y_{t-j} + \epsilon_{1t}, & \text{for } \mathcal{S}_t = 1 \\ c_2 + \sum_{j=1}^{p_2} A_{2,j} y_{t-j} + \epsilon_{2t}, & \text{for } \mathcal{S}_t = 2 \end{cases} \quad (22)$$

Now we can calculate the transition probability in the MRVAR model:

$$\begin{aligned}
& \Pr(\mathcal{S}_t = 1 | \mathcal{S}_{t-1} = 1) \\
&= \Pr(FSI_{t-1} < FSI^0 | FSI_{t-2} < FSI^0) \\
&= \Pr(c_{1,1} + \sum_{j=1}^{p_1} A_{1,1..j} y_{t-1-j} + \epsilon_{11,t-1} < FSI^0 | FSI_{t-2} < FSI^0) \\
&= \Pr(\epsilon_{11,t-1} < FSI^0 - c_{1,1} - \sum_{j=1}^{p_1} A_{1,1..j} y_{t-1-j}) \\
&\quad \times \Pr(\epsilon_{11,t-2} < FSI^0 - c_{1,1} - \sum_{j=1}^{p_1} A_{1,1..j} y_{t-2-j})
\end{aligned}$$

Assuming normal distribution for the residuals $\epsilon_{11,t} \sim N(0, \sigma_1^2)$, we have

$$\begin{aligned}
\Pr(\mathcal{S}_t = 1 | \mathcal{S}_{t-1} = 1) &= \Phi\left(\frac{1}{\sigma_1}(FSI^0 - c_{1,1} - \sum_{j=1}^{p_1} A_{1,1..j} y_{t-1-j})\right) \\
&\quad \times \Phi\left(\frac{1}{\sigma_1}(FSI^0 - c_{1,1} - \sum_{j=1}^{p_1} A_{1,1..j} y_{t-2-j})\right). \quad (23)
\end{aligned}$$

In a similar way we can calculate $\Pr(\mathcal{S}_t = i | \mathcal{S}_{t-1} = j)$ for $i = 1, 2$ and $j = 1, 2$. Equation (23) and equation (21) make the difference between the two models clear. While in the Markov switching model the transition probability is invariant over time, in the MRVAR model the transition probability is a function of realized past values of y_t and hence path dependent.

The interpretation of the state variable in both Markov switching VAR model and MRVAR model is that it reflect the state of the economy. It is therefore of interest to quantify the degree to which the economy is in a particular state, such as boom or recession. The MRVAR model allows this quantification through variable transition probabilities. Equation (23) shows that the smaller the expected value of the financial stress index FSI_{t-1} the large is the probability of staying in the low financial stress state. Intuitively, it is appealing to image that an economy far away from a high financial stress state is unlikely to fall into the state of high financial stress, while an economy close to a high financial stress state is more likely to fall into the state of high financial stress and this likelihood should be reflected in the transition probability. In the Markov-switching model the transition probability from one state to the other is assumed to be constant without reflecting the difference in the likelihood of switching VAR from one state into the other under known different economic condition.

It is to note that both the Markov-switching VAR model and the MRVAR model cannot nest each other. The transition probability from one state to the other in the MRVAR model have to be time-varying, depending on the values of y_{t-1-j} it cannot be a time-invariant constant. Thus the MRVAV model cannot nest the Markov-switching VAR model, and the Markov-switching VAR model

does not nest the MRVAR model either. Here we would like to point out that the state described in the two models are two different entities. While in the Markov switching VAR model the underlying state is reflected in the whole set of variables in y_t and hence often interpreted as the state of the economy under consideration, in the MRVAR model the state is reflected only in the threshold variable which is often only one component in y_t and hence we call it financial stress state. These two state variables can be related to each other but they are by no means the same.

Difference in Empirical Modeling

In this subsection we want to compare these two competing but not encompassing models in modeling of the links between the financial stress and the output. We summarize the results (see Fig. 19). It shows that the high financial stress regime defined in our MRVAR (see the fourth graph in Fig. 19) corresponds, to a large extent, to the state predicted by the estimated Markov-switching model (see the third graph in Fig. 19). The MRVAR model estimates 6 high financial stress regime episodes, four of the 6 episodes (1 3 5 6) correspond quite well to the NBER dating of economic recession during the periods from 1980:1 to 2012:2. (see the last graph in Fig. 19). The other two high financial stress episodes (2 4) reflected mainly the stock market stress respectively in 1987, including the event of Black Monday, and in 1998 including a decrease of Dow Jones by 11.5%, within 3 days at the end of August, amid fears about the impact of Asia and Russia financial crisis.

The Markov switching VAR model predicts 5 episodes of high probability for the "stressed" state (see the third graph in Fig. 19). Three of the five episodes (1 2 5) correspond quite well to the NBER dating of economic recession and the episodes of the high financial stress states as well. Four of the 5 episodes (1 2 3 5) correspond to four high financial stress episodes identified in MRVAR.

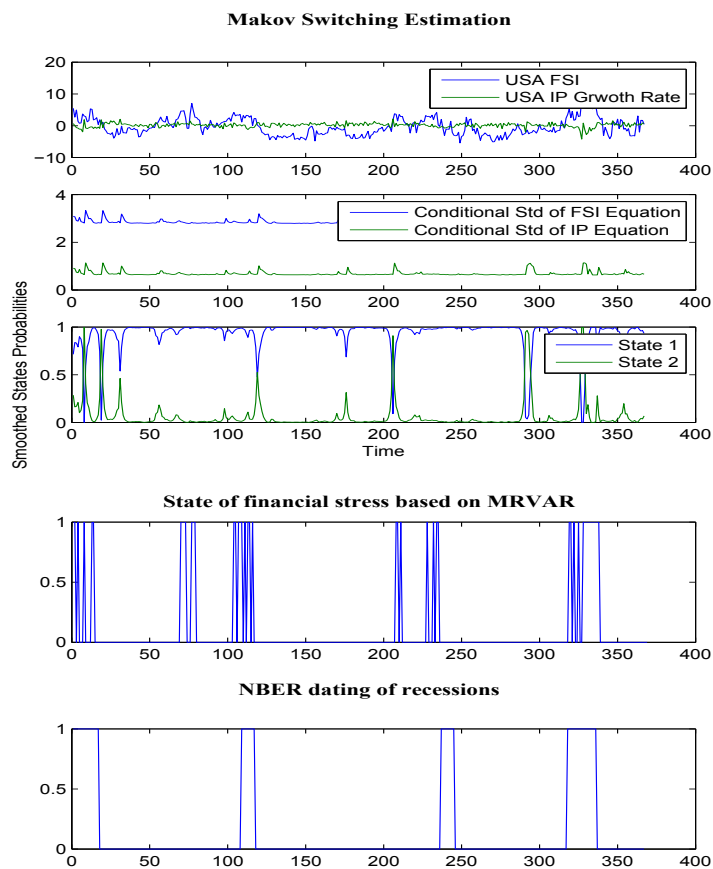


Figure 19: Comparison between MRVAR and Markov Switching for USA

The MRVAR model and the Markov switching VAR model capture to a large extent same features in the data, but these two model results still differ from each other significantly.

The Markov-switching model is able to capture the effect of changes of an unobservable state on the observed variables, it is however not always easy to interpret the state, because the estimated state does not always fit the economic intuition. The nature of a presumed constant transition probability leaves not room for an appealing economic idea that under certain circumstances a transition one state to the other may be more much probable than under some other circumstances.

MRVAR model ankles the regime switching on an observable variable and thus models the switching mechanism explicitly, which allows a clear interpretation of the switching between regimes. However, we should avoid over interpreting this explicitly formulated financial stress state as a state of the economy,

and it is barely the state of the financial stress. High financial stress situations do not necessarily always lead to recessions in the business cycle. The event of the black Monday in 1987 and the event of the Russian financial crisis with its impact on the US financial market were two examples (see the second and the fourth episodes of high financial stress states in the third graph in Fig. 19 and compare them with the NBER dating of economic recession in the fourth graph in the same Figure.)

Statements on which model is a better one is not a proper way to address the issue on comparison between these two types of models. Two models serve different purpose. If one wants to model links between financial market and the real activities, like what we are doing in this paper, MRVAR provides a good framework. If one wants to uncover the state of the economy and its impact on the financial market and the real activity as well, the Markov-switching VAR can be a choice.