Response to Referee reports of our paper:
“Financial Stress, Regime Switching and Macrodynamics: Theory and Empirics for the US, EU and Non-EU Countries”

October 2, 2013

Response to the Anonymous - Referee Report 1
The referee raises three main issues in the second paragraph of the report. To address the first issue of "the difference between the methodology that they are suggesting and the regular Markov switching models"(the second invited reader and the third invited reader also raised the same issue) we elaborate a comparison between these two approaches in their formal formulations and in empirical results as well in the appendix of the paper. The second issue concerns the way presenting the NMPC numerical method. We can add a paragraph elaborating on the intuition of NMPC procedure. The third issue is about the presentation of the theoretical model. We can modify the formulation to emphasize the difference among the three scenarios considered in the theoretical model, in particular the difference between Equ (3) and Equ (6).

To the minor points pointed out in the referee report the following are our responses: Answers:
1. Function \( \psi \) is a quadratic adjustment cost of investment.
2. BEL is the abbreviation of Belgium. We add a list of abbreviations in the appendix.
3. Change of Title?
Answer: We think the title could stay the same, unless the editor has some strong preference

Response to the anonymous invited reader comments
1. p.5: Answer: The interest rate \( r \) is assumed to be \( r = 0.04 \).
2. p.12: Answer: We add the following table of descriptive statistics of the variables used in the model to provide an intuitive interpretation for the data.
3. p.10/13: Answer: The countries and the time horizon analyzed should be stated explicitly at the beginning of section 3 or 4. They are mentioned in table 1 for the first time. And we give the time horizon of our empirical estimation at the beginning of section 4.
Table 1: Abbreviation of Country Names

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>The United States</td>
</tr>
<tr>
<td>JPN</td>
<td>Japan</td>
</tr>
<tr>
<td>DEU</td>
<td>Germany</td>
</tr>
<tr>
<td>FRA</td>
<td>France</td>
</tr>
<tr>
<td>GBR</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>ITA</td>
<td>Italy</td>
</tr>
<tr>
<td>ESP</td>
<td>Spain</td>
</tr>
<tr>
<td>DNK</td>
<td>Denmark</td>
</tr>
<tr>
<td>SWE</td>
<td>Sweden</td>
</tr>
<tr>
<td>FIN</td>
<td>Finland</td>
</tr>
<tr>
<td>AUS</td>
<td>Austria</td>
</tr>
<tr>
<td>BEL</td>
<td>Belgium</td>
</tr>
<tr>
<td>NLD</td>
<td>Netherlands</td>
</tr>
</tbody>
</table>

Notes: Table shows the correlation coefficients between Financial Stress Index and Output of a country and their correlations with the variables in USA.

4. p.13: For me, it is not completely clear, why the support of the threshold parameter is restricted to be positive. A high stress regime with negative output consequences might not be necessarily associated with a stress variable exceeding zero. For some countries a FSI above 0.5, for instance, could already imply a high stress regime. If the argument in the paper holds, the statistical procedure should also find a value above zero.

Answer: FSI is a constructed variable (see IMF(2011)), such that the index value of zero indicates a usual situation and large positive index values indicate high financial stress situation. Therefore, it is reasonable to search the threshold value which defines high financial stress on in the domain of positive FSI values. Indeed from statistical point of view, without restricting the support of the threshold value on the positive domain would have made the statistical model more general. This would however be inconsistent with the construction of the financial stress index.

5. p.14: It is not clear what is meant by following sentence: The AIC (M = 2, pho = 3, phi = 2) = 202965.4 The number seems to be way too low.

Answer: We correct the typos. It is \( AIC(M = 2, p_1 = 3, p_2 = 3) = -202.965 \)

\( M, p_1 \) and \( p_2 \) the number of regimes, the VAR lags in regime 1 and the VAR lags in regime 2 respectively.

6. p.17: Our Answer: The issue raised by the reviewer is totally justified. The issue of asymmetric IRF in this nonlinear MRVAR model has been thoroughly studied by one author of this paper in (Mittnik, S. and W. Semmler (2012b)). What we present here is the within – regimeIRF under the assumption of no inter regime migration, which is used to assess the local dy-
Table 2: Descriptive Statistics of the Data

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Country_p FSI</th>
<th>Country_p OUTPUT</th>
<th>USA_p FSI</th>
<th>USA_p ADV</th>
<th>OUTPUT USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTPUT USA</td>
<td>0.340</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.340</td>
</tr>
<tr>
<td>JPN_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT JPN</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>DED_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT DED</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>PTA_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT PTA</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>GBR_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT GBR</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>ITA_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT ITA</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>ESP_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT ESP</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>DEN_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT DEN</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>FIN_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT FIN</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>NOE_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT NOE</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>SWF_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT SWF</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>MLG_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT MLG</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>HIL_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT HIL</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>AUT_p FSI ADV</td>
<td>1</td>
<td>0.240</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
<tr>
<td>OUTPUT AUT</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
<td>1</td>
<td>0.240</td>
</tr>
</tbody>
</table>

Notes: Table 2 shows the correlation coefficients between the Financial Stress Index and the Output of a country and their correlations with their counter parts in USA.

7. p. 21 Conclusion: "On the other hand, large reductions in financial stress tend to induce stronger expansionary effects in low rather than in high growth regimes." I am not sure whether this relates perfectly to the analyses conducted in the paper.

Answer: In many of similar studies some exception of the general case frequently comes out: Some countries experience spillover effects from other countries so that the typical result, namely lower stress, less effects of policies does not always come out.

8. p. 37: I really like the comparison of the results of the Markov Switching model with the multi regime VAR, particularly the dating of stress periods. It delivers valuable insights in the dynamics and supports the derived results. I would recommend putting these comparisons/analyses in the main part. Moreover, I could not find a cross reference in the main text to the Markov Switching analyses in the appendix.

Answer: In the revised version, we elaborate on a comparison between these two approaches in their formal formulations and in empirical results as well. Still we think it is better to put this in the appendix than in the main text, because this would otherwise distract the focus of the paper.

9. The authors focus on short run dynamics, not allowing for regime switching. The shortcomings are mentioned in the paper. Two questions came to my mind: a. Plotting (G)IRFs with a 24 month horizon seems too long for a short
run analysis. It is likely that another shock might have occurred within this
time frame (see for instance Euro area crisis).

Answer: This comment is in that 24 months is a too long horizon for a short run. However, plotting a IRF with a longer horizon contains no less information than plotting it on short shorter one. One aspect in using a 24 month horizon is to show the level to which the IRG will converge.

b. On p.15 the sentence "So any further financial stress shock [...][]" is used to explain the outcomes of the multi regime VAR. This explanation does not fit if one focuses on a setting/model set-up in which no other shock takes place.

Answer: Sentence can be corrected.

t. In all figures showing IRFs with respect to different regimes it should be labeled which are low and high stress regime responses.

Answer: We modified the labels.

Response to the second invited reviewer Christian R. Proano

Comments and Suggestions 1 In order to keep the argument of the paper as straight-forward as possible, I would described directly the model given by eqs. (4) - (6), instead of first introducing the model given by eqs. (1) - (3).

Answer: Can be revised to make the model clear

2. Since the application of the NMPC approach is still not widely used in macroeconomics, it would be helpful for the understanding of the dynamics of the model if the authors could briefly discuss the functioning of this algorithm.

Answer: This can be done or reference made to better descriptions of the algorithm

3 There are some works mentioned in the text which are not however in the list of references such as Mendoza and Semmler (2012).

Answer: correction will be made

4. The lables in the axes of Fig. 3 are too small.

Answer: can be changed

5. The comparison between the MRVAR and a Markov-Switching model is an interesting exercise which indeed seems to further empirical support for the MRVAR results discussed in the main text. The discussion of the MS-VAR models for the U.S. and Germany is not sufficiently detailed, and seems to be unnecessary for the argument of the paper.

Answer: will be done, see below a sketch.

6. Is there a specific reason for presenting the accumulated impulse-response functions, and not the standard ones?

Answer: IRF and accumulated IRF contain essentially the same set of information. while in a stationary VAR, which is the case of our with – inregime IRF, the IRF will generally converge to zero and thus the main message of a IRF is how quickly the system will restore itself, the accumulated IRF gives the message in which direction the system is going to evolve if the impulses continue. In our paper the later is what we want to look at.
MRVAR v.s. Markov Switching Models—Could be put into an appendix

In this paper we choose the MRVAR approach to model the regime changes. An often used approach in modeling regime changes is the Markov-switching VAR model. It is of interest to compare these two modeling approaches from both theoretical and empirical perspectives. We restrict our comparison to two regime cases, which is most relevant in this paper.

Formal Difference

A Markov-switching model with two regimes is defined as follows.

\[
y_t = \begin{cases} 
  c_1 + \sum_{j=1}^{p_1} A_{1,j} y_{t-j} + \epsilon_{1t}, & \text{for } s_t = 1 \\
  c_2 + \sum_{j=1}^{p_2} A_{2,j} y_{t-j} + \epsilon_{2t}, & \text{for } s_t = 2 
\end{cases}
\]

where the variable \( s_t \) that controls the selection of the regime, follows a Markov process:

\[
\Pr(s_t | \{s_{t-j}\}_{j=1}^{\infty}, \{y_{t-j}\}_{j=1}^{\infty}) = \Pr(s_t | s_{t-1}).
\]

The variable \( s_t \) is usually assumed to be an unobservable. In our context \( s_t \) describes the state of the economy. \( y_t = (FSI_t, 100\Delta \log(\text{IP}_t))^\prime \) contains the two variables in our model.

A MRVAR model is a threshold VAR model, formally defined as follows.

\[
y_t = \begin{cases} 
  c_1 + \sum_{j=1}^{p_1} A_{1,j} y_{t-j} + \epsilon_{1t}, & \text{for } f_{t-1} < \tau^0 \\
  c_2 + \sum_{j=1}^{p_2} A_{2,j} y_{t-j} + \epsilon_{2t}, & \text{for } f_{t-1} \geq \tau^0 
\end{cases}
\]

where \( f_{t-1} = FSI_{t-1} \) is the threshold variable observed at time \( t - 1 \); and the regimes are defined by the pre-specified threshold value \( -\infty = \tau_0 \leq \infty \).

These two models are common in that given a choice of the regime \( i = 1, 2 \), the time series \( y_t \) is generated by a \( VAR(p_t) \) process. They differ in the mechanism how a regime is chosen. While in the Markov switching VAR model the switching from a state \( s_{t-1} \) to the next state \( s_t \) is a random process with a constant transition probability

\[
\Pr(s_t = i | s_{t-1} = j) = h_{ji} \text{ with } i, j = 1, 2,
\]

the regime switching in the MRVAR model is governed by the value of the switching variable \( f_{t-1} \). To facilitate we can define a state variable for the MRVAR model in the following way:

\[
s_t = \begin{cases} 
  1 \text{ for } FSI_{t-1} < FSI^0 \\
  2 \text{ for } FSI_{t-1} \geq FSI^0 
\end{cases}
\]

With the help of the above defined state variable the MRVAR (3) takes the same form of the Markov switching model (1). Then we can calculate the transition
probability in the MRVAR model:

\[
\Pr(s_t = 1|s_{t-1} = 1) = \Pr(FSI_{t-1} < FSI^0|FSI_{t-2} < FSI^0)
\]

\[
= \Pr(c_{1,1} + \sum_{j=1}^{p_1} A_{1,1,\ldots,j} y_{t-1-j} + \epsilon_{11,t-1} < FSI^0|FSI_{t-2} < FSI^0)
\]

\[
= \Pr(\epsilon_{11,t-1} < FSI^0 - c_{1,1} - \sum_{j=1}^{p_1} A_{1,1,\ldots,j} y_{t-1-j})
\]

Assuming normal distribution for the residuals \(\epsilon_{11,t} \sim N(0, \sigma_1^2)\), we have

\[
\Pr(s_t = 1|s_{t-1} = 1) = \Phi\left(\frac{1}{\sigma_1} (FSI^0 - c_{1,1}) - \frac{1}{\sigma_1} \sum_{j=1}^{p_1} A_{1,1,\ldots,j} y_{t-1-j}\right). \tag{5}
\]

In a similar way we can calculate \(\Pr(s_t = i|s_{t-1} = j)\) for \(i = 1, 2\) and \(j = 1, 2\). Equation (5) and equation (4) make the difference between these two models clear. While in the Markov switching model the transition probability is invariant over time, in the MRVAR model the transition probability is a function of realized past value of \(y_t\) and hence path dependent.

Assuming that the state variable \(s_t\) describes the state of the economy, it is of interest to quantify the degree to which the economy is in a state, such as a boom or recession. The MRVAR model allows this quantification through variable transition probabilities. Equation (5) shows that the smaller the expected value of the financial stress index \(FSI_{t-1}\) the larger is the probability of staying in the low financial stress state. Intuitively, it is appealing to image that an economy far away from a high financial stress state is unlikely to fall into the state of high financial stress, while an economy close to a high financial stress state is more likely to fall into the state of high financial stress and this likelihood should be reflected in the transition probability. In the Markov-switching VAR model the transition probability from one state to the other is assumed to be constant, without reflecting the difference in the likelihood of switching from one state into the other under different economic condition.

It is to note that both the Markov-switching VAR model and the MRVAR model cannot nest each other. The transition probability from one state to the other in the MRVAR model have to be time-varying, depending on the value of \(y_{t-1-j}\) it cannot be a time-invariant constant. Thus the MRAV model cannot nest the Markov-switching VAR model, and the Markov-switching VAR model does not nest the MRVAR model either.

**Difference in Empirical Modeling**

In this subsection we want to compare these two competing but not encompassing models in modeling of the links between the financial stress and the output. We summarize the results (see Fig. ). It shows that the high financial stress regime defined in our MRVAR (see the fourth graph in Fig. ) corresponds, to a large extend, to the state predicted by the estimated Markov-switching model.
(see the third graph in Fig. ). The MRVAR model estimates 6 high financial stress regime episodes, four of the 6 episodes (1 3 5 6) correspond quit well to the NBER dating of economic recession during the periods from 1980:1 to 2012:2, (see the last graph in Fig. ). The other two high financial stress episodes (2 4) reflected mainly the stock market stress respectively in 1987, including the event of Black Monday, and in 1998 including a decrease of Dow John by 11.5%, within 3 days at the end of August, amid fears about the impact of Asia and Russia financial crisis.

The Markov switching VAR model predicts 5 episodes of high probability for the "stressed" state (see the third graph in Fig. ). Three of the five episodes (1 2 5) correspond quit well to the NBER dating of economic recession and the episodes of the high financial stress states as well. Four of the 5 episodes (1 2 3 5) correspond to four high financial stress episodes identified in MRVAR.

**Figure 1:** Comparison between MRVAR and Markov Switching for USA
The MRVAR model and the Markov switching VAR model capture to a large extent same features in the data, but these two model results still differ from each other significantly.

The Markov-switching model is able to capture the effect of changes of an unobservable state on the observed variables, it is however not always easy to interpret the state, because the estimated state does not always fit the economic intuition. The nature of a presumed constant transition probability leaves not room for an economic consideration of the mechanism leading to changes from one state into the other.

MRVAR modelanko the regime switching on an observable variable and thus models the switching mechanism explicitly, which allows a clear interpretation of the switching between regimes. However, we should avoid ever interpreting this explicitly formulated financial stress state as a state of the economy, and it is barely the state of the financial stress.

Statements on which model is a better one is not a proper approach. Two model serve different purpose. If one wants to model links between financial market and the real activities, MRVAR provides a good framework. If one wants to uncover the state of the economy and its impact on the financial market and the real activity, the Markov-switching VAR can be a choice.