

Report on "Reserve Price When Bidders Are Asymmetric", by H. Gunay, X. Meng, and M. Nagelberg

The paper studies optimal reserve prices in a second price auction with asymmetric bidders when the reserve price cannot be type-dependent. The optimal auction for asymmetric bidders is an asymmetric auction with type-dependent, implicit "reserve prices". That is, bidder  $i$  would face a minimum valuation  $r_i$  under which the seller never trades with her. That cut-off depends on the distribution of her valuation,  $F_i$ , but not on the distribution of other bidders' valuations or on the number of those bidders. Even if we restrict attention to second price auctions, the optimal reserve price  $r_i$  for each bidder  $i$  is still a function of  $F_i$  that does not depend on the number or types of other bidders: conditional on bidder  $i$  having the largest valuation, in which case she will be the winner, and the rest of bidders having a valuation below  $r_i$ , so that the price will be  $r_i$ , the expected price for the seller is  $(1 - F(r_i))r_i$ , and the (opportunity) cost of a sale is the seller's valuation. Thus, the optimal  $r_i$  is indeed the value that equates the "marginal revenue" from the bidder to the seller's cost. That is, the virtual valuation of bidder  $i$  to the seller's valuation.

However, if for whatever reason, the seller cannot set different reserve prices for different bidders, then what the authors show is that the optimal reserve price equates the "average marginal revenue" with the seller's valuation. That is, the bidders' average virtual valuation to the seller's valuation.

Consider the following "reason" for not using different reserve prices: bidders privately draw their type from some type space  $\Theta$  according to some common distribution,  $G$ . Given type  $\theta \in \Theta$ , then a bidder's valuation is a draw from a distribution  $F(v_i; \theta)$ . At a time after each bidder has learnt her type but before their valuation is realized, we would have an asymmetric model. However, from the point of view of the seller, the situation is still the same as before types are realized: with no new information, all bidders are symmetric from the seller's perspective, and their valuations are draws from  $F \cdot G$ . Thus, her best choice of a reserve price is the one that equates the marginal revenue computed for  $F \cdot G$  with her valuation. Since the expectation is a linear operator, the marginal revenue computed for  $F \cdot G$  is the expected value of the marginal revenue computed for  $F$ , where the expectation is taken with respect to  $G$ . Thus, indeed, the optimal reserve price equates the "average marginal revenue" to the seller's valuation. Or more properly, the expected virtual valuation of the winner conditional on the winner having that valuation.

In the paper, the authors assume (what amounts to) that the seller "observes" the realization of  $\theta$ . This does not change matters, as long as the seller cannot use this information to discriminate among bidders, and as long as all realizations of valuations are still possible for all bidders (common support of  $F(v_i; \theta)$ ).

The authors claim that, contrary to the model where reserve prices can be bidder-specific, with a common reserve price, its optimal value depends on the number of bidders. This is only as long as a change in the number of bidders changes the average virtual valuation. This is not an interesting comparison. If we "replicate" the demand by, for instance, bringing a new bidder of each type, the expected virtual valuation of the winner conditional on any valuation  $r$  does not change, and then the optimal reserve price does not change either.

That is the meaningful exercise: In the two stage framework that I mention above, not holding the "average virtual valuation" constant for every winning value is equivalent to changing  $G$  as the number of bidders changes. In this regards, the wording in the paper is misleading.

Theorem 3 is rather a corollary of Lemma 1. I am not sure that this should be considered among the most important results of the paper. On the contrary, it would be more important to know what are the conditions under which (3) in Lemma 1 is sufficient for an optimum. These conditions appear as a proposition in itself, Proposition 5, which makes for a surprising structure.

Perhaps the most interesting result in the paper appears now as an example, Example 6. What it basically says is that an increase in the virtual valuation of a bidder does not imply an increase in the expected virtual valuation. The change in virtual valuation also implies changes in the density function for each player's probability of winning, and these changes do not have to be monotone. Exploring the conditions under which this may be so deserves more than just a simple example.