Comment on “Reserve Price When Bidders are Asymmetric,” by Hikmet Gunay, Xin Meng, and Mark Nagelberg.

Summary: The paper derives the optimal reserve price in second-price auctions when bidders are asymmetric. The number and composition of bidders both influence the optimal reserve price. Sufficient conditions are established under which the reserve price is unique.

Main comments: The results in the paper are for the most part unsurprising. The surprising part – that the reserve price may increase when a bidder becomes stronger – is based on an example (Example 6) in which some of the distribution functions have an unusual property. In particular, the virtual valuations are non-monotonic. An example with more well-behaved distributions would be more compelling.

On the other hand, providing sufficient conditions for uniqueness is useful. While I am not overly surprised that the hazard rate and the reverse hazard rate play a role in the proof, the restriction to convex distribution functions (increasing densities) is less intuitive and should be relaxed. As far as I can tell, the assumption enters only in the first part of the proof to ensure that $(-2f_i(r) - f_i'(r)(r - x_0))$ is negative. Note, however, that this term is the second derivative of $F_i(r)x_0 + (1 - F_i(r))r$ or $x_0 + (1 - F_i(r))(r - x_0)$, this being the seller’s problem if he only had bidder $i$ to sell to. Thus, it is sufficient to assume this problem is concave; $f_i'(r) > 0$ is overkill. This has a more immediate interpretation and is more closely related to the virtual valuations we are used to thinking about in design problems (the monotonicity of virtual valuations does not ensure concavity, only that the problem is single-peaked).

Minor comments:

1. The use of the word “type” to refer to bidders with different identities or distributions functions is confusing, since the term usually refers to bidders’ valuations. Footnote 5 at least acknowledges this issue, but could another term not be used? How about different “kinds” of bidders, or instead of saying that the type is unknown, how about just saying that the distribution is unknown?

2. Bottom of page 4: The formula at the bottom of the page reportedly gives expected revenue. This is incorrect, since it also incorporates the seller’s own-use value. Naturally, the seller is not attempting to maximize revenue. It would be preferable to use a term such as payoff (or even profit) instead of revenue.
3. Theorem 3 (page 7): what does $\sum_{i}^{N} H_i(x) f_i(x)$ mean? There seems to be too many $i$'s. In the proof, the same paragraph seems to be repeated twice in the proof of part a.

4. Proof of Proposition 5 (page 10): The proof mentions likelihood ratio dominance, which is not one of the assumptions in the proof (although it is mentioned in footnote 14). This should be deleted.