Indirect Taxation of Monopolists: A Tax on Price

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Abstract   A digressive tax like a variable rate sales tax or a tax on price gives firms an incentive for expanding output. Thus, unlike unit and ad valorem taxes which amplify the harm from monopoly, a digressive tax lessens the harm. We analyse a tax on price with respect to efficiency and practical policy appeal. Using a tax on price in combination with ad valorem taxation it is possible to achieve the Ramsey solution. That is, the combination of the two taxes secures tax revenue in the least distortive way. We also show how tax reforms based only on observation of price and quantity can make use of a tax on price in order to improve welfare. That is, it is practical to use a tax on price.

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Keywords   Tax on price; ad valorem tax; tax incidence

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1. Introduction.

According to their widespread use in public finance unit and ad valorem taxes are popular taxes. For this reason there is a clear interest in analysing the workings of precisely these two taxes in spite of the fact that there are, as noted by Hamilton (2009), many other instruments. On the other hand, it is well known that both taxes are shifted into the price consumers pay. That is, when used in a monopolised market, or other imperfectly competitive markets, these taxes drive the price even further above the marginal cost. On this basis it is of interest to look at the efficiency of digressive taxes, and, moreover, if the application of such taxes are equally practical to unit and ad valorem taxes.

Digressive taxes provide the opposite incentive to unit and ad valorem taxes; hence, under such taxes firms expand output. When applied in imperfectly competitive markets the margin between price and marginal cost narrows under such taxes (Dalton, 1929 and Robinson, 1933). A tax on price is an example of a digressive tax. In this paper we study the workings of such a tax in monopoly. As a practical matter it is possible to implement a tax on price as variable rate sales tax (Hamilton, 1999). There are other ways to introduce digressive tax schemes. Assuming that the marginal cost is non-decreasing a tax based on the Lerner index will do. When the tax relates positively to the index the marginal tax is decreasing and, in turn, gives the monopolists incentive to expand output. A tax scheme based on the difference between price and average cost has similar effects.

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The idea that a tax on price counteracts monopolistic behaviour and, at the same time, secures revenue is discussed in Shilling (1969). Subsequently Tam (1991) shows some results on the workings of a tax on price but, as argued by Sumner (1993) it is unclear how a tax on price relates to welfare.\(^3\) It is also unclear whether it is practical to use a tax on price. The purposes of this paper are twofold. First, we want to see how efficient a tax on price actually is. Especially, we analyse the relationship between the allocation under Ramsey pricing and under a combination of a tax on price and ad valorem taxation, respectively.\(^4\) The Ramsey price is the price set by a regulator so that the monopolistic firm is ensured some fixed revenue whilst social welfare is maximised. The monopoly profit is of course the maximum that it is possible to extract by taxation. Hence, in the absence of lump sum taxes Ramsey pricing gives the most efficient allocation that can be reached subject to some restriction on tax revenue.\(^5\)

Second, an objection against digressive taxes is that they are impractical because of the information needed in order to use them. If information difficulties make it impractical to apply the theoretically ideal tax structure it is relevant to ask when a practical reform of existing taxes results in a gain. To demonstrate this way of reasoning, consider excise versus ad valorem taxes. A practical reform is a matched-pair tax reform; that is, an increase in the ad valorem tax rate that matches the decrease in the excise tax measured at before-reform price. This kind of reform does not require knowledge about demand and cost conditions. Based on first-round effects ad valorem taxes are better than unit taxes. What Suits and Musgrave (1953) show, is that inclusion of second-round effects will not

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\(^3\) Tam (1991) analyses a tax on price. However, as pointed out by Sumner (1993) the neglect of the restriction that profit is non-negative results in shortcomings and some results are (according to Sumner, 1993) plainly wrong. Neither of these papers related the tax on price to Ramsey pricing, and neither of the papers considers how to practically conduct tax reforms.

\(^4\) It is well-known that ad valorem taxes welfare dominates unit taxes. For monopoly Suits and Musgrave (1953) show that the optimum unit tax is zero. Hence we leave out the unit tax and compare ad valorem taxes and a tax on price.

\(^5\) Sumner (1993) and Tam (1993) use some numerical calculations to discuss this. Neither uses the Ramsey pricing approach.
change the dominance of ad valorem over excise taxes. By parallel reasoning, we ask about a practical welfare improving tax reform based on a tax on price to see if such reform in fact is stopped by information constraints.\textsuperscript{6}

In Section 2 we analyse an example to demonstrate the idea.\textsuperscript{7} In Section 3 we make the point in general and discuss practically feasible tax reform. Section 4 concludes.

2. Example

We consider a monopolistic firm where \( p(x) \) is demand for the monopolist’s output, called \( x \), and \( c(x) \) is production cost. It is assumed that cost and demand are linear functions of output, that is \( c(x) = cx \) and \( p(x) = a - bx \). There are (at least) three ways to generate tax revenue; a tax on the firm’s turnover, a unit tax, and a tax on the price set by the monopolist. Revenue under an ad valorem tax is \( R^\tau = \tau p^\tau x^\tau \), where the ad valorem tax rate is \( \tau \). Output and price under ad valorem taxation is \( x^\tau = \frac{1}{2}b^{-1}(a - (1 - \tau)^{-1}c) \) and \( p^\tau = \frac{1}{2}(a + (1 - \tau)^{-1}c) \), respectively. Ad valorem taxation harms consumers through changing the final price.

Although we do not focus on unit taxes notice that the monopolist’s output is \( x^t = \frac{1}{2}b^{-1}(a - c - t) \) where \( t \) is the tax rate. The price is \( p^t = \frac{1}{2}(a + c + t) \) showing that consumers are harmed by the excise tax. Comparing tax rates with the same final price, \( p^\tau = p^t \), gives \( \tau (1 - \tau)^{-1}c = t \) and

\textsuperscript{6} Sumner (1993) discusses product quality as a source to information problems although he does not address the issue formally.

\textsuperscript{7} In fact, the example used in Section 2 is the same as that used by Sumner (1993). However, we explicitly compare a tax on price to an ad valorem tax which allows us to demonstrate that there will not always exist a tax on price that can match the revenue produced by some ad valorem tax rates.
Revenue generated by the ad valorem tax exceeds that generated by the excise tax when $\tau p^T x^T > tx^T$ or $\tau p^T > t$. Using $\tau (1 - \tau)^{-1} c = t$, the condition $\tau p^T > t$ comes down to $a > (1 - \tau)^{-1} c$ after some rewriting. This result is what is shown more generally in Suits and Musgrave (1953).

As a substitute for the ad valorem tax, consider a linear tax paid on the basis of the price charged by the monopolist. When the tax rate is $s$, the monopolist’s profit is $(a - bx)x - cx - s(a - bx)$. Output and price are $x^s = 1/2 b^{-1} (a - c + sb)$ and $p^s = 1/2 (a + c - sb)$, in that order. Plainly, a tax on price drives the price below the price in an unregulated monopoly and, contrary to ad valorem taxes, the price change is a benefit to consumers. Obviously, it is impossible to compare ad valorem taxes to a tax on price on an equal price basis. In the example in this section, we therefore take a more direct approach and ask whether the revenue attained by use of ad valorem taxation can be obtained with a tax on price set by the monopolist. That is, it is a non-marginal tax change where one tax fully replaces the other tax.

As noted above, under a tax on price the monopolist’s profit maximising output is $x^s = 1/2 b^{-1} (a - c + sb)$ and the price is $p^s = 1/2 (a + c - sb)$. For positive tax rates output is positive and the price is well-defined when $a + c > bs$. The tax revenue is $R^s = s 1/2 (a + c - sb)$ and we want to ask whether it is possible that $R^s = R^\tau$ for some given $R^\tau$, say $\hat{R}^\tau$, whilst $p^s < p^\tau$. Clearly, tax revenue is the same under the two taxes when $s = 1/2 b^{-1} \left( a + c \pm \sqrt{(a + c)^2 - 8b \hat{R}^\tau} \right)$.

Suppose that $(a + c)^2 > 8b\hat{R}^\tau$ and use the positive radical so that $s$ is positive. It is easy to see that the solution satisfies $a + c > bs$. Hence, when the solution to $R^s = R^\tau$ for some $R^\tau$, say $\hat{R}^\tau$, is well-

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8 The condition $a > (1 - \tau)^{-1} c$ holds since otherwise there would be no production under the ad valorem tax.

defined, we can find a tax on price that is of equal yield to the ad valorem tax but with lower consumer price.

The restriction \((a + c)^2 > 8b \hat{R}^r\) gives a restriction on the possibility of replacing the ad valorem tax with a tax on price when one uses either one or the other of the two taxes. Under ad valorem taxation revenue is \(4b \hat{R}^r = \tau(a^2 - (c/(1 - \tau))^2)\). Hence, the restriction \((a + c)^2 > 8b \hat{R}^r\) comes down to \((a + c)^2 > 2\tau(a^2 - (c/(1 - \tau))^2)\). For ad valorem tax rates less than half, the inequality is easily seen to be satisfied. For ad valorem tax rates higher than a half, the inequality is satisfied dependent on parameters. An example where the inequality fails to be satisfied is in the case of vanishing marginal costs and ad valorem tax rates higher than one half. This is unsurprising. The monopolist maximises revenue when marginal costs are imperceptible and can be ignored. Thus, tax revenue is equal or close to \(\tau a^2/4b\). A tax on price drives the price down, thus limiting the revenue that can be generated.

3. Generalisations

The example in the preceding section is based on the assumption that a tax on price fully replaces ad valorem taxation. The example makes clear that it is not in general possible to find a tax on price which can match the revenue from ad valorem taxation. This does not suggest that a tax on price is irrelevant. Rather, the result only shows the limitations of the specific functional forms used. That is, the example illustrates why it is more proper to ask about the effect of a tax reform where ad valorem and taxes on price are both in use.
Under a combination of an ad valorem tax and a tax on price the monopolist’s profit is \( \pi = (1 - \tau)p(x)x - c(x) - sp(x) \). In obvious notation first- and second-order conditions are \((1 - \tau)(p_xx + p) - c_x - sp_x = 0\), and \(\theta = (1 - \tau)(2p_x + 2p_xxx) - c_xx - sp_xx < 0\), respectively. It is easy to see that \(x_\tau = \theta^{-1}(p_xx + p)\) and \(x_s = \theta^{-1}p_x\), showing that the monopolist’s output decreases with an increase of the ad valorem tax rate whilst it increases as the rate of tax on price goes up.\(^{10}\) Plainly, an increase of the ad valorem tax rate reduces marginal revenue which explains why output goes down. Although an increase of the rate of tax on price shifts net demand inwards, marginal revenue increases, explaining why this tax expands output. It is easy to see how the positive effect on output is derived. Selling one more unit of output the monopolist must lower the price not just on the last unit sold but also on all other units sold. Under a tax on price there is a lessening of the tax burden since setting a lower price applies to all units sold. Hence marginal revenue under a tax on price is \(p + p_x(x - s)\), which is why this tax has opposite effects from the ad valorem tax.

Following Suits and Musgrave (1953) there are two ways to compare taxes: either to find a set of taxes which will result in the same final price and output and inspect the resulting revenues, or to find a set of taxes that produce equal revenue, and then to compare prices and outputs. For obvious reasons we apply the latter comparison. Consider combinations of tax rates with a fixed yield of \(\check{R} = \tau p x + sp\), where output and price are functions of the tax rates, \(x = x(\tau, s)\) and \(p = p(\tau, s)\) (but for brevity written without the arguments). Unsurprisingly, we restrict attention to ad valorem tax rates for which \(0 \leq \tau \leq 1\). The tax on price must satisfy \((1 - \tau)x - s > 0\) since the monopolist closes down otherwise. Throughout we maintain the next assumption to characterise feasible tax plans.

\(^{10}\) From now on we drop the arguments from the functions. Subscripts are used to denote a partial derivative.
Assumption 1. There is a pair of tax rates, \( \{\tau_0, s_0\} \) so that \((1 - \tau)x - s > 0\) for \(0 \leq \tau \leq \tau_0\) and \(s \leq s_0\).

Unless Assumption 1 is satisfied it clearly does not make sense to compare the two taxes. From
\[
\hat{R} = \tau px + sx \text{ the change in tax rates that keeps intact revenue is:}
\]
\[
\{px + (\tau p_x x + sp_x + \tau p)x_t\}d\tau + (\tau p_x x + sp_x + \tau p)x_s ds = 0 \tag{1}
\]
For \(\tau = s = 0\) the marginal revenue of the ad valorem tax—\(R_t = px + (\tau p_x x + sp_x + \tau p)x_t\)—is positive and the marginal revenue of the tax on price—\(R_s = (\tau p_x x + sp_x + \tau p)x_s\)—is zero. We maintain the following assumption about marginal tax revenues.

Assumption 2. There is a pair of tax rates, \(\{\tau_1, s_1\} \) so that \(R_t\) is positive and decreasing with \(\tau\) for \(\tau \leq \tau_1\) and \(s \leq s_1\).

Together the assumptions make clear that \(0 \leq \tau \leq \bar{\tau}\) where \(\bar{\tau} = min(\tau_0, \tau_1)\) and \(s \leq \bar{s}\) where \(\bar{s} = min(s_0, s_1)\) is a feasible tax plan. That is, the monopolist will not close down under the tax scheme, and, moreover, the ad valorem tax’s marginal revenue is positive. Clearly, a tax scheme characterised by tax rates \(0 < \tau \leq \bar{\tau}\) and \(s = 0\) is feasible. The output effect of a revenue-neutral tax reform is \(x_t d\tau + x_s ds\), and using equation (1):
\[
x_t d\tau + x_s ds = -\frac{px}{\tau p_x x + sp_x + \tau p} d\tau \tag{2}
\]
Clearly, for $0 < \tau \leq \bar{\tau}$ and $s = 0$ output is below unregulated monopoly output. Thus, the right hand side of equation (2) is positive if the tax reform involves less ad valorem taxation. In turn, because $x_{\tau} > 0$ and $x_s > 0$, a revenue-neutral reform implies more unit taxation when ad valorem taxation is downsized at least for a combination of positive ad valorem taxation and no tax on price taxation. Clearly, output increases under such a reform. The tax change is feasible only if the monopolist continues production after the tax change, that is, the profit restriction $((1 - \tau)x - s)p - c \geq 0$ is satisfied under the new tax rates. We can show Proposition 1.

Proposition 1. When a pair of tax rates satisfies Assumptions 1 and 2, and if output under the pair of tax rates is less than the unregulated monopoly output, a tax reform can increase output to unregulated monopoly output without revenue loss.

Proof.

When $0 < \tau \leq \bar{\tau}$ and $s = 0$, a tax reform will not make the new tax plan infeasible. To see this write the monopolists profit as $\pi = xp - c - R$ where $R = (\tau x + s)p$. Next, let us define $x_m = \text{Argmax}_x xp - c$. Clearly, when $\tau > 0$ and $s = 0$ the monopolist’s output is less than $x_m$. The tax reform defined by equation (2) increases output and hence $xp - c$ goes up. Since revenue by construction is unchanged the profit net of tax cannot go down.

End of proof.
Proposition 1 shows that if a pair of taxes, $0 < \tau \leq \bar{\tau}$ and $s = 0$, is feasible, so is $\{\tau + d\tau, s + ds\}$ when tax changes satisfy equation (2). The argument can be repeated starting out at $\{\tau + d\tau, s + ds\}$. It is easy to see why this tax change is feasible. When there is ad valorem taxation on no tax on price output is below monopoly output. Hence, the tax change means that the monopolist’s profit net of tax increases. Thus the profit restriction $((1 - \tau)x - s)p - c \geq 0$ is satisfied. After a series of tax reforms, output is brought to that level the monopolist chooses in the absence of taxation ($x_m$). At this point the effect of further tax changes is uncertain because the monopolist’s net of tax profits goes down. Trivially, shifting taxation further away from ad valorem taxation and towards unit taxation comes to a stop when $(x - s)p - c = R$. We summarize it as Proposition 2.

Proposition 2. When output under a pair of taxes equals output in unregulated monopoly, further tax reform with increased ad valorem and increased price taxation reduces price without harming revenue. The possibility for tax reform stops when $((1 - \tau)x - s)p - c = 0$.

When output under taxation exceeds that of unregulated monopoly output, the rate of tax on price is driven into a region where marginal revenue is negative. This is seen immediately since the term $\tau p_x x + s p_x + \tau p$ is negative from the first order condition. In turn, $R_s$ is negative and $R_{\tau}$ is positive in this situation. The reason that such reforms make sense is of course that increasing the tax on price benefits price more than the adverse price effect of the higher ad valorem taxation that is needed to keep revenue unchanged. What Propositions 1 and 2 show is that maximisation of consumers’ surplus, subject to a revenue constraint, will never involve output falling below unregulated monopoly output. As a matter of fact, subject to satisfaction of the profit restriction, the tax policy sustains output exceeding unregulated monopoly output.
To explain the result further and relate it to Ramsey pricing, suppose a regulator picks a point on the demand curve under the restriction that the price is no less than the sum of average cost and average revenue, that is, \( p \geq c/x + R/x \). Now, if the sum of average cost and average revenue exceeds the price for all prices, there is clearly no solution: there is never enough revenue to cover production costs and simultaneously meet the revenue restriction. Figuratively, the demand curve lies below the average cost curve adjusted with average revenue. If the monopoly price precisely satisfies \( p = c/x + R/x \), then the restriction on the mix of ad valorem and price tax is that output under the taxes must be equal to unregulated monopoly output, that is, the tax rates must satisfy \( \tau(p_{x}x + p) + sp_{x} = 0 \), evaluated at the price in unregulated monopoly. Finally, suppose the unregulated monopoly price satisfies \( p > c/x + R/x \). In terms of tax reforms, suppose that a pair of tax rates secure the needed revenue at an output that equals unregulated monopoly output. Increasing output by increasing the tax on price (and lowering the ad valorem tax rate) reduces the price. If, the price after the reform is characterised by \( p \geq c/x + R/x \), there is room for another round of tax reform. This goes on until the price satisfies \( p = c/x + R/x \) which is the Ramsey price. We summarise this in Proposition 3.

Proposition 3. When ad valorem taxes and a tax on price are combined to minimise price subject to a revenue restriction, the resulting price is the Ramsey price.

Proposition 3 shows that supplementing ad valorem taxation with a tax on price—or some output-related profit tax—is efficient in the sense that the deadweight loss that follows from taxing a
monopolist is minimised. Combinations of ad valorem and unit taxes can do the same only if tax rates are allowed to be negative (Myles, 1996).

Digressive taxes are only rarely discussed in the literature. They are dismissed by Dalton (1929) and Robinson (1933). Also, Glaister (1987) suggests that it is impractical to use such taxes because of the information required to design them. Hence, it is relevant to ask whether there are circumstances when an ad valorem tax can be partly replaced by a tax on price without the need for a lot of information. It is clear from equation (1) that specification of a revenue-neutral tax reform requires some detailed knowledge of demand and cost conditions, including information about the monopolist’s second-order condition. This is not practical. To the contrary, the tax reform \( xd\tau = -ds, d\tau < 0 \) and \( ds > 0 \), is obviously practical. Tax reforms defined this way are what Suits and Musgrave (1953) call matched-pair reforms. Such reforms do not call for knowledge about demand and cost relations.

Suppose that tax revenue derives from an ad valorem tax alone and consider the effects of a series of matched-pair reforms. First, consumers benefit from a series of reforms because the price goes down \( d\tau + xsd = dts > 0 \). Second, the reforms are feasible because the monopolist’s profit is not harmed. To see this, observe that the profit change is \( \Delta \pi = -pxd\tau - pds + \pi_x(xd\tau + xsd) \). By specification of the tax reform \( -pxd\tau - pds = 0 \). The term \( \pi_x(xd\tau + xsd) \) also cancels because of the first-order condition. Thus, the monopolist’s profit is unchanged. This implies that the matched-pair tax reforms are not stopped by feasibility. Third, consider the effect on revenue. The change in revenue is:
\[ \Delta R = p [x d\tau + d\tau + \tau (x d\tau + x d\tau)] + [\tau x + s] p x (x d\tau + x d\tau) \]

This reduces to \( \Delta R = (\tau p + [\tau x + s] p_x)(p/\theta) d\tau \). Using the first-order condition \( \tau p + [\tau x + s] p_x = p_x x + p - c_x \) is positive whenever output is less than unregulated monopoly output.

The term \((p/\theta)d\tau\) is positive the reform involves less ad valorem taxation \((d\tau\) is negative) since the second-order condition implies that \(\theta\) is negative. That is, starting out with a combination of positive ad valorem taxation and no tax on price, the first round of reform is positive. After a series of reforms, revenue starts to fall. This occurs for tax rates giving the monopolist incentives to produce as he would do without taxation. We summarise this in Proposition 4.

Proposition 4. Starting with pure ad valorem taxation, a series of matched-pair reforms continuing until tax revenue begins to fall brings output to that chosen by the monopolist in the absence of taxes.

Proposition 4 is interesting because it demonstrates that there are practical reforms which are welfare-improving without harmful revenue effects. In particular, there is no need for extraordinary information to avoid the negative output changes that go along with ad valorem (and unit) taxes. Of course, Proposition 4 lists sufficient conditions for the existence of a simple tax reform. As is clear from Proposition 3, knowledge about cost and demand conditions allows the combination of taxes that equals the efficiency of Ramsey pricing.

In this paper we have re-examined a tax on price. The appeal of such a tax is that it simultaneously provides revenue and incentives for firms to reduce price (Tam 1991 and 1993, and Sumner 1993). Taking the profit constraint explicitly into consideration we have worked out how far the efficiency of a tax on price goes (Propositions 1 and 2). Moreover, we have shown that a combination of ad valorem tax and a tax on price produces the allocation corresponding to Ramsey pricing (Proposition 3). In this way, the combination of the two taxes is an efficient tax policy in the sense that the unavoidable deadweight loss that goes with taxing a monopolist (given non-availability of lump-sum taxes) is minimised.

Unsurprisingly, identifying fixed revenue combinations of a tax on price and an ad valorem tax calls for knowledge about demand and cost conditions. On this account Glaister (1987) suggests that the tax is of limited practical value. Similarly, Dalton (1929) and Robinson (1933) discuss output-based subsidies but dismisses them as a practical possibility. This is surely the case when the tax scheme is to be constructed so as to induce the Ramsey optimum. Nevertheless, it is possible to design a practical and beneficial tax reform that combines ad valorem taxation with a tax on price. First, the tax reform is practical since it is based on matched-pair tax reforms, i.e., it is based on observation of price and output. Second, it is beneficial because it goes some way in minimising the deadweight loss that is unavoidable when taxing a monopolist.
References


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