Response to the comment 3 "The BIP Trilogy (Bipolarization, Inequality and Polarization):
One Saga but Three Different Stories"

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First of all, we would like to thank the third anonymous reader for his/her useful comments and for helping us clarifying various points. We will take his/her suggestions into account when preparing a revised version of this paper. Below we indicate our answers to his/her various remarks/suggestions.

MAJOR REMARKS

First comment

"I found interesting the distinction between polarization and bipolarization. In fact, I believe that some misconceptions may arise between these two concepts and I thus appreciated the effort to clarify the distinctions between the two concepts. The origin of misconceptions may also be due to the fact that, in physics (and physical) terminology, polarization means “an agglomeration towards two extremes” something that can lead to think economic polarization as the agglomeration on TWO poles. The latter concept may be inaccurate in economic terms and there is thus a difference in terminology between the economic term and the physics (and possibly widespread) term, generating a confusion that is worth to be clarified and that the paper could stress a bit more. From this point of view I have to admit that several past economic analyses highlight the existence of polarization whereas they are probably highlighting the existence of bi-polarization: the contribution in the present paper is thus even more useful within the economic discipline.”
**Answer:** We agree with the reader's comment and will stress in section 2 the respective desirable properties of measures of inequality, bi-polarization and polarization and hope that the distinction between these three concepts will be clearer. Moreover we will extend somehow our introduction by stressing the respective policy relevance of each of these three notions.

**Second comment**

"Partly related to what I said in the previous comment, I believe that there may be some widespread misconceptions in the three concepts (Inequality, Bipolarization and Polarization). Therefore I suggest that at the very beginning of the paper a clear and direct definition to the three terms should be immediately given. This would avoid possible confusion later on. From this point of view, I found a bit confusing the discussion on page 3 taken from the field of international relations: without a clear definition of the three terms, and possibly having in mind the “physics” definition of polarization, the discussion sounded misleading to me. I would suggest moving this discussion after a clear definition for the three terms is given, so that it appears as a possible application of the three concepts."

**Answer:** We will follow the reader’s advice and put the paragraphs where the three concepts are defined before that mentioning that a similar distinction exists in the field of international relations.

**Third comment**

"Given the concept of polarization that you are describing in the paper, is it correct to say that bipolarization is simply a special case of polarization? If so, the authors could explicitly say so; if not, the authors should stress more what makes them apart."

**Answer:** It is indeed correct to say that bipolarization is a special case of polarization but it should be kept in mind that the two strands of literature on bipolarization and
polarization reported in the paper have followed distinct approaches: the former defines ex-ante the two groups while the latter let generally the data determine how many poles exists (one possibility being that two poles are identified). As mentioned previously we will list in section 2 the respective desirable properties of an inequality, a bi-polarization and a polarization index to clarify the differences between these three notions.

Fourth comment

"At the beginning of section 2.1, the authors claim that “the concept of bi-polarization is clearly related to that of middle class”. I am not sure what the authors are hinting to. Actually I think that bi-polarization may be related to the LACK of middle-class as several studies argue how, with the emergence of bi-polarization of income, the middle-class is disappearing. If the authors were also referring to the absence of middle class, they should be clearer on it. If on the contrary they were referring to something else, I was not able to follow their reasoning."

Answer: We will clarify this point and after writing in the introduction that “The analysis of bi-polarization is hence linked to that of the importance of the middle class” we will add “More precisely an increased degree of bi-polarization over time implies a declining middle class.”

Fifth comment

"Section 2.2 deals with the measurement of polarization. In the earlier parts of the paper, polarization had been described as a concept that could be applied to several aspects but, in section 2.2, measurement is strictly and directly related to income. The authors should signal clearly and motivate this change of approach and also discuss if and how other dimensions (for example education, skills, health, family background etc…) of polarization can be measured."

Answer: It is important to stress that, in the economic literature, in addition to the distinction between the degrees of inequality, bi-polarization and polarization of an
income distribution, there exists also an important difference between the notions of income polarization and social polarization. More precisely pure income polarization refers to the case where the division of the population into subpopulations and the distances between them are derived on the basis of income while in the case of pure social polarization the definition of groups and the measurement of the distances between groups are not based on income but on categorical variables such as race, religion or ethnicity. Finally the economic literature on polarization includes also "hybrid" measures, where the group partition is based on a social characteristic while the measurement of distances between these groups is derived from the distribution of income (or some other variable representing the standard of living) in the whole population. The focus of this paper is however on the concepts of income inequality, income bi-polarization and income polarization. We will stress these points in the new version of the paper.

It should also be clear that a hybrid approach could be similarly adopted when analyzing the distribution of variables such as education or health, provided they may be considered as continuous variables.

**Sixth comment**

"The results on the Shapley decomposition in section 4 are interesting and in my opinion they stir some further questions that could be discussed. In particular, some policy implications could be given: in fact, if the three different concepts are originated by different economic sources, this information can used to tailor specific policies that aim at mitigating each of these inequality dimensions. For example, the authors show that the main source of (bi-)polarization is income from labour, and thus policies that aim to mitigate this aspect should focus probably on the redistribution of labour income; on the contrary, the main component of the Gini inequality index appears to be income from capital, so that policies aiming at reducing this dimension of inequality, should focus probably on wealth redistribution."

**Answer:** We will try to better emphasize the potential policy implications of the Shapley decomposition. For example, some transfers may lead to a decrease in bi-polarization
while at the same time they may lead to an increase in polarization if their introduction implies the appearance of an additional local pole. This is, for example, the case of unemployment benefits which at the margin decrease bi-polarization but increase polarization (compare Tables 3 and 4).

**Seventh comment**

"The results of Shapley decomposition of the polarization index (Polarization DER) appear to be quite dependent on the value of \( \alpha \). However, very little is said about what represents \( \alpha \) (I could not even find an exact definition for it) and in particular, more effort should be put forth to explain why, in economic terms, when \( \alpha \) changes, the results also change. This would probably also explains while, overall, the authors suggest that the main contribution to polarization comes from income from wealth."

**Answer:** We had indeed forgotten to define \( \alpha \) and thank the reader for drawing our attention to this omission. The parameter \( \alpha \) measures the sensitivity of the index DER to the local identification. A detailed analysis of the role played by the parameter \( \alpha \) is beyond the scope of this paper. It may be found in the original paper of Duclos et al. (2004) where these authors wrote that

“Our measure bears an interesting resemblance to the Gini coefficient. Indeed, if \( \alpha = 0 \), the measure *is* the Gini coefficient. However, our arguments ensure that not only is \( \alpha \) \( >0 \), it cannot go below some uniformly positive lower bound, which happens to be 0.25. Where, in the axioms and in the IA (identification – alienation) structure, does such a bound lurk? To appreciate this, consider Axiom 2, which refers to a double-squeeze of two “side” basic densities. Such squeezes bring down internal alienations in each component density. Yet the axiom demands that overall polarization not fall. It follows, therefore, that the increased identifications created by the squeeze must outweigh the decreased within-component alienation. This restricts \( \alpha \). It cannot be too low.

By a similar token, \( \alpha \) cannot be too high either. The bite here comes from Axiom 1, which decrees that a single squeeze (in an environment where there is just one basic component)
cannot increase polarization. Once again, alienation comes down and some identifications go up (as the single squeeze occurs), but this time we want the decline in alienation to dominate the proceedings. This is tantamount to an upper bound on $\alpha$.”

**Eighth comment**

"Section 2.2 argues how the measure of polarization should determine the number of poles endogenously and propose some procedures that actually do so. Given this premise, I could not understand whether, in section 4, the authors also determine endogenously the number of poles. This would be particularly interesting and if they actually identify endogenously several poles, they could also display some characteristics of these poles, like the average education, age, family background of the individuals within each single pole."

**Answer:** We have adopted the empirical procedure suggested by Duclos et al. (2004). In their approach the number of poles is indeed determined by the data, that is, we did not fix exogenously the number of population subgroups. It is true that it would be nice to “display some characteristics of these poles, like the average education, age, family background of the individuals within each single pole”, as suggested by the referee. However to get such results, we would need to implement an alternative procedure, based on what is called a mixture model. Such an approach has been implemented by Pittau et al. (2010). We are not yet familiar with such a procedure but plan to use it, in future work.

**MINOR REMARKS**

a) "Throughout the paper I had the strong feeling that the authors were really interested in measuring the three dimensions of inequalities rather than explaining the economic rationale that generate them. This was also true when, in section 4, they actually obtain some insights on the economic mechanisms but the authors do not exploit them to describe those mechanisms. I would probably thus suggest to state from the beginning
that the main focus of the paper is on the “definition”/”measurement” of these concepts (or as an alternative, to expand the discussion on the economic mechanisms).

**Answer:** We will add the following sentence in the introduction: “Although the main focus of this paper is on issues related to the measurement of inequality, bi-polarization and polarization, it should be clear that the type of exercise we implement is likely to be of great importance to policy makers.”

b) "At the end of page 3 the authors argue how bi-polarization refers to the case where there is a large share of individuals that are poor and a relevant share of individuals that are rich. I was wondering whether bi-polarization can also refer to a case where most individuals belong to the rich and middle class (so that poor are negligible) or to a case where most individuals belong to the middle and poor class (so that rich are negligible). Would these cases still be considered a bi-polarization?"

**Answer:** By definition (see, Foster and Wolfson (1992; 2010)) bi-polarization assumes that there are two groups in the population: those having an income smaller than the median income (the "poor") and those having an income higher than the median income (the "rich").

c) The claim of the authors at the beginning on page 4 “The analysis of bi-polarization is hence linked to that of the importance of the middle class” sounded to me as taken for granted rather than explained and motivated.

**Answer:** The clear (negative) link between the concepts of bi-polarization and that of middle class is well explained in Foster and Wolfson (1992; 2010). A clear reference to this paper appears now in the paper so that we decided not to repeat their arguments.

d) "Partly related to point 5) in the main remarks: should the title of section five be changed to “On the measurement of polarization of income” or can it be generalized to other dimensions?"
Answer: We believe that the question concerned section 4. We will add the word income and give as title to this section “4. On the differential impact of various income sources on income inequality, bi-polarization and polarization: the case of Luxembourg”

e) "I was a bit surprised in finding on page 10 a reference to an item listed as (1999, forthcoming): is this a misprint? If not, I may still have hope for some of my old working papers."

Answer: This paper was indeed written in 1999 but was never published. It will appear very soon in a special section of the *Journal of Economic Inequality* entitled “Classics Rediscovered”.

**References**