

Report on MS 734: “The power and the weakness of rational expectations” by Eduard Gracia

General comments

This paper states the importance of carefully assessing the predictability of asset prices (and associated macro variables), as well as the role of rational expectations, by the use of relevant stochastic models. I think that the largest potential of the paper lies in the stochastic modelling of asset prices and in the analysis of the role of asset prices in the ‘business cycle’. I’d like to see that a revised version reflects this clearer, in the title and in the organization of the paper. I give some detailed suggestions below.

The model of periodic stock market cycles in section 3 is well explained. It is driven by the interaction between rent-seeking producers and capital owners who can only imperfectly control the effects of rent seeking. Section 4 contains an empirical assessment of this model. The relevance of the predator-prey dynamics for the stock market is easy to accept, since rent-seeking affects the distribution of the value (created). In section 5, the author goes a step further and applies (also with reference to already published work) the same modelling tools to the aggregate production function. The relevance of this is not so immediately compelling, since it is not so intuitive why (rational) agents should find it worth while to work to “reduce the (growth of the) cake”, not only the distribution. The author makes reference to research on emerging and poorly institutionalized economies, that have indicated that the productivity growth may actually have suffered severely from selfish agents’ rent seeking behaviour, but in a revised version of this paper a strengthening of the argument for empirical relevance would be welcome. That said, when the link between the rent seeking behaviour and productivity is accepted, one sees that this is a logical coherent model of a low frequency macroeconomic cycle with interesting properties (genuinely endogenous cycles, asymmetric up- and downturns, etc) and predictions (literally promising to forecast the cycle).

Like other theories, this model concentrates on the one single business cycle mechanism. Realistically, real world recessions and booms are the results of more than one mechanism. Most likely, the model of this paper will therefore have to be combined with other cycle generating mechanisms in order to become a model with large enough explanatory/predictive power to be of practical use. It would be interesting to hear what the author thinks would be a sensible research programme that aims at setting up an eclectic econometric models of the real world aggregate dynamics.

I found this paper an interesting read, with several analytically based results that should interest many readers, in particular in a time when events have forced renewed attention to the interaction between financial markets and the macroeconomy. Overall I found the exposition good and the paper is well organized. I have included some suggestions for improvements in the detailed comments that the author might want to consider.

Detailed comments

Abstract: Consider re-phrasing “...the mean (i.e. ‘expected’) path must be a random walk” since the mean is either a number/parameter or a deterministic function.

Introduction. I think that the “logical fallacy” bit is quite apart from the rest of the paper, which makes a distinct contribution in terms of the dynamic model of the cycle. Consider shortening this part, perhaps leaving the logical fallacy argument for another (methodological?) paper.

Page 3, line 3. Typo: “mush”.

The example on page 3 and 4. It is good with an intuitive example, so it should be kept. On the other hand, as noted below, it is possible that the even more relevant argument (i.e., for the actual model used) in Appendix 1, can be integrated moved to the main text after some simplification.

Paragraph starting in the last line of p 8. The reference to TFP (and the discussion) is not needed. It becomes clear later in the paper that your model does not depend on an exogenous TFP cycle, and your theory of technical progress is not a theory of TFP either (not a criticism).

Page 16. Item d. Decreasing amplitude of the autocorrelation function is a property of stationarity. I would truncate the function at lag 48 say, since that will be enough to show/check the qualitative predictions, and to keep a reasonable sample size.

Page 16-17. The empirical test strategy of estimating the autocorrelation function seems to be very basic and simple, but statistical inference nevertheless rests on stationarity. Moreover, it is “1-1” with an estimated AMA model. Hence, for consistency, and without much more work you can estimate a data coherent AR model (approximating the ARMA) to allow valid statistical inference about the features you are interested in. The associated characteristic roots are then immediately available, and at the different business cycle frequencies. Persistent cycles should have associated characteristic roots of magnitude close to one. I have experimented a little with modelling the CAPE series and found a real root very close to one at the long run frequency (of course this is typical of economic time series), and complex roots of magnitude less than one at the higher frequencies. However, the diagnostics of an AR model show that the model is statistically inadequate for valid inference. This suggests that there is “more” non stationarity in the series than captured by the AR (and its cycles). Location shifts, structural breaks in the mean for example, represent one relevant candidate for further modelling. In the light of this, statistical inference about the significance of the lags of the autocorrelation function (after direct estimation as in the paper) is also dubious. More modelling is required to substantiate approximately valid inference.

Page 18, Figure 4. Much shorter lags (20?)

Page 19, Figure 5 can be dropped, since as you say it is just an illustration, The length of the cycles can instead be lifted from the characteristic roots of the AR model.

Page 23, equation (2). Why is there no productive role of capital in this production function? This also reminds me of other theories of the cycle where investments and savings behaviour give the mechanism. As noted above, I would

think that this new theory is complementary, not alternative, to the older ones. The author might comment on this (preferably in the Introduction).

Page 23, equation (3). How fast can productivity drift to zero and still be realistic for real world aggregate productivity?

Appendix 1

I think the gist of your results follows more or less directly from the definition of the process, and that the appendix can be shortened and maybe be incorporated in the main text. Below are some notes.

If P_t is a geometric Brownian motion starting from $P_0 = p$ with drift parameter μ , then $Y_t = \ln P_t$ is a Brownian motion with drift parameter $\gamma = \mu - \frac{1}{2}\sigma^2$ and variance σ^2 . Equivalently

$$(1) \quad P_t = pe^{X_t} = pe^{(\mu - \frac{1}{2}\sigma^2)t + \sigma Z_t}$$

where Z_t is a standard Brownian motion starting from $Z_0 = 0$. The mean is

$$\begin{aligned} E(P_t | P_0 = p) &= pE(e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma Z_t}) \\ &= pe^{(\mu - \frac{1}{2}\sigma^2)t} E(e^{\sigma\sqrt{t}\xi_t}) \quad (\xi_t = Z_t/\sqrt{t}) \end{aligned}$$

Using

$$E(e^{\sigma\sqrt{t}\xi_t}) = e^{\frac{1}{2}\sigma^2 t}$$

from the properties of the normal distribution, we have

$$(2) \quad E(P_t | P_0 = p) = pe^{(\mu - \frac{1}{2}\sigma^2)t} e^{\frac{1}{2}\sigma^2 t} = pe^{\mu t}$$

showing that μ is the mean growth rate of P_t .

These equations have interesting implications when $\mu > 0$ but small in magnitude to σ^2 . On the one hand:

$$E(P_t | P_0) \longrightarrow \infty \text{ as } t \longrightarrow \infty$$

but as seen from (1), $\ln P_t = (\mu - \frac{1}{2}\sigma^2)t + \sigma Z_t$ is drifting in the negative direction when $\mu < \frac{1}{2}\sigma^2$. As a consequence of the law of large numbers we have that $\ln P_t \longrightarrow -\infty$ as $t \longrightarrow \infty$ under this assumption meaning that $P_t \longrightarrow 0$. The market value P_t is drifting towards zero even though its mean or expected value is continually increasing.

This of course is a reflection of the property that the true drift parameter of the series is $\mu - \frac{1}{2}\sigma^2$, what you refer to as the median, while the mathematical expectation is μ .

Of course, may well have, $\mu > \frac{1}{2}\sigma^2$ and then the positive drift is still smaller than indicated by the mean, which is your result.

Page 39. The reference to Goodwin is important and it deserves to appear in the main text.