

**REFEREE'S REPORT FOR ECONOMICS ON
"A GENERALIZED UNIFICATION THEOREM FOR CHOICE
THEORETIC FOUNDATIONS: AVOIDING THE NECESSITY OF PAIRS
AND TRIPLETS"
FOR ONLINE PUBLICATION**

It is well-known that if the domain of an individual choice correspondence includes all budget sets containing up to three elements, then this choice correspondence can be rationalized by a unique preference ordering if and only if it satisfies the Weak Axiom of Revealed Preference (WARP). An alternative way of stating this "all budget sets containing up to three elements" condition is to say that the union of any three (not necessarily distinct) singleton budget sets belongs to the budget domain. This paper shows that one can replace each "singleton budget set" in this condition by "a budget set from which the singleton element is chosen" without affecting the equivalence. The paper also demonstrates the equivalence between a number of rationality axioms under this new assumption.

The novel assumption in this paper (Assumption 1) takes the following form: the budget domain contains a collection of budget sets with the following properties: First, each consumption bundle is chosen from at least one of the budgets in the collection; Second, the union of any three (not necessarily distinct) budgets in this collection belongs to the budget domain. Under this new assumption, the WARP is equivalent to the Strong Axiom of Revealed Preference (SARP). Hence every choice correspondence satisfying this new assumption and the WARP can be rationalized by a unique preference ordering. If every budget set in the budget domain is finite, Assumption 1 implies equivalence between the WARP and a modified version of Sequential Path Independence (Bandyopadhyay, 1988). If the budget domain contains only finite sets and is closed under finite union, any choice correspondence satisfying Assumption 1 and WARP if and only if it satisfies Arrow's Condition (Arrow, 1959).

While the author motivates this paper by claiming that Assumption 1 is superior to the "all budget sets containing up to three elements" condition, I am afraid that I have to disagree, from both a theoretical and a practical point of view.

First, Assumption 1 is an assumption on the choice correspondence, not just the budget domain. (It requires each consumption bundle to be chosen from at least one of the budget sets in the collection constructed.) An experimentalist wishing to see whether her subjects' choices in a laboratory can implement the "up to three element condition" and check WARP,

but she wouldn't be able to tell whether Assumption 1 is satisfied prior to observing her subjects' responses.

Second, Assumption 1 is not easy to verify. It is a "there exists" condition. Given an arbitrary choice correspondence, it may not be obvious how one would construct the desired collection of budget sets. It is even harder to show that Assumption 1 is not satisfied as this amounts to demonstrating a "for all" condition. Thus it is hard to *know* whether the theorems in this paper can be applied. Moreover, since Assumption 1 is an assumption on the choice correspondence, two choice correspondences can be defined on the same domain but one satisfies Assumption 1 while the other does not. The "up to three elements" assumption, on the other hand, would not have run into such difficulties.

Third, it is also unclear whether Assumption 1 is easy to satisfy in a practical sense. The requirement that the union of any three budget sets in a particular collection is included in the budget domain is not trivial. The author suggested in his/her motivation section (Section 2) that one can apply the theorem to a setting in which the collection required is the set of all finite discretized budget triangles, claiming that unions of budget triangles "can be observed in price cut or wholesale situations" (p. 5). True I may observe *some* unions of budget triangles with block pricing, but is it easy to observe *all* three-set unions of budget triangles? Indeed, since there are uncountably many price vectors allowed by the author (this is not obvious in Section 2 but is explicit in Example 2 in Section 3), I need to observe uncountably many unions! Even if I discretize the price space and bound it from above, we are still talking about a huge domain. At least I fail to see how this is "more plausible" (p. 5) than requiring the budget domain to include all budget sets containing up to three elements.

Last but not least, if the budget domain contains all singleton budget sets and is closed under finite union, it contains all finite budget sets. The equivalence between WARP and Arrow's Condition when the budget domain contains all finite budget sets has long been established (Arrow, 1959). The new assumption adds nothing. If instead the budget domain does not contain all singleton budget sets, the requirement that every consumption bundle is chosen from at least one budget in the collection required is less innocuous than the author may wish it to be.

The strength of this paper, instead, is the recognition that it is the *choices* from budget sets that is crucial for rationalizability. Thus instead of considering unions of three singleton budget sets, we can take larger sets from which the singleton element is chosen.

This idea is not uncommon in the social choice literature. In particular, Grether and Plott (1982) show that the choice-theoretic version of Arrow's Impossibility Theorem (c.f.: Le Breton and Weymark, 2011, Theorem 19), which assumes Arrow's Condition, does not rely on the agenda domain (i.e., the budget domain) containing all pairs of social alternatives.

A key step in their proof is that one can extend a social choice correspondence to smaller agendas by “pretending” that unavailable alternatives are available but will not be chosen. This is similar to the idea in the current paper.

Granted, there is a crucial difference between one-individual social choice¹ and consumer choice theory. A social choice correspondence is defined on a preference domain — it takes preferences as (part of the) input. On the other hand, consumer choice theory asks which, if any, preference ordering can rationalize a choice correspondence. Thus social choice theorems typically involve two more axioms: some version of a Pareto optimality axiom and an “independence of irrelevant alternatives” axiom. Indeed, under these extra axioms the dictatorship result in Arrow’s Impossibility Theorem remains even when Arrow’s Condition is weakened² (see Man and Takayama, 2012). On the other hand, we do need the full strength of Arrow’s Condition in Theorem 2 of this paper.

But this difference makes the observation in this paper interesting. It prompts me to ponder on our interpretation of bundles outside a budget set — under what conditions can we treat unavailable alternatives as undesirable alternatives? While this paper does not address this question directly, it sheds light on our treatment of budget sets in consumer choice theory.

REFERENCES

- ARROW, K. J. (1959): “Rational Choice Functions and Orderings,” *Economica*, 26(102), 121–127.
- BANDYOPADHYAY, T. (1988): “Revealed Preference Theory, Ordering and the Axiom of Sequential Path Independence,” *Review of Economic Studies*, 55(2), 343–351.
- GRETHER, D. M., AND C. R. PLOTT (1982): “Nonbinary Social Choice: An Impossibility Theorem,” *Review of Economic Studies*, 49(1), 143–149.
- LE BRETON, M., AND J. A. WEYMARK (2011): “Arrowian Social Choice Theory on Economic Domains,” in *Handbook of Social Choice and Welfare*, ed. by K. J. Arrow, A. K. Sen, and K. Suzumura, vol. 2, chap. 17, pp. 191–299. North-Holland, Amsterdam.
- MAN, P. T. Y., AND S. TAKAYAMA (2012): “A Unifying Impossibility Theorem,” Working Paper, University of Queensland.

¹Grether and Plott (1982) state their theorem for at least two individuals, but it holds also for one individual.

²The existence of a dictator in a one-individual social choice implies the rationalizability of the choices at any fixed preference.