

Reply to Referee n. 2

Let us thank the second referee for carefully reading out paper and providing useful advice.

After reading the two reports and further discussing this paper during presentations at key conferences and institutes (a further presentation of this paper is scheduled for June 19th, 2012), we do believe that major revisions are needed. In particular, we only presented an exact analytical result without providing analytical or numerical evidence of its usefulness. This is the main reason for several objections also raised by the second referee. Therefore, the revised version will include at least an appendix or supplementary material with calculation and comparison of option prices with different methods. If interesting enough, this material will be included in one or two appropriate sections in the main body of the paper.

Having said that, let us now reply to the specific points of the second report.

1. *The distribution of sums of two independent random variables (convolutions) is a topic treated in every introductory course in probability and statistics, see e.g. Section 3.6.1 in Rice (1995) or Section 2.5.3 in Ross (2009). Therefore the computations on p.4 can be removed completely.*

These results make our paper self-contained and there is no space problem in an on-line journal. However, we agree that this is standard material and we can move it to an appendix.

2. *The case when the time point t does not coincide with a renewal epoch time is in our view irrelevant (case 2 on p.7). The reason for this is that during a typical trading day there are several thousands of tick-by-ticks (renewal epoch times) and one can therefore without any further loss of accuracy approximate the time point t with the closest renewal epoch time during that day (as in case 1 on p.7). Then all of the computations on pp.9-10 becomes rather irrelevant.*

The referee considers our main result irrelevant, but he/she does only provide a qualitative argument. Indeed, our result is exact and we originally believed it deserved publication for this reason. The referee might be right, but one can prove this quantitatively and convincingly only by comparing our exact option prices with the renewal approximation.

3. A subsequent remark of the referee concerns the difficulties in numerically computing Mellin convolutions and ordinary convolutions for durations. For what concerns the calculation of Erlang type distributions, in [1], we used standard Matlab integration commands. However, in the literature, there are several papers on the efficient calculation of convolution integrals using Fourier analysis techniques, etc.. In the revised version, we will include suitable pointers to this literature and to further papers on convolution integrals. As for Mellin convolution, the logarithm of the product of independent and identically distributed (i.i.d.) random variables is a sum of i.i.d random variables and, using this trick, the calculation of Mellin convolutions can be reduced to the calculation of ordinary convolution integrals. Springer & Thompson (1966) and Lomnicki (1967) were mainly included for historical purposes, but we can easily include more material on this point if these two references are considered misleading.

4. *The authors points out that the durations J_i in Equation (6) should not follow exponential distributions, according to previous studies. To this end we would like the authors to concretize the distribution providing at least one (hopefully more than one) explicit model for J_i . Furthermore, we would like the authors to demonstrate how to realistically calibrate or estimate the parameters in*

these explicit models. Finally, some numerical studies should be provided showing the model in practice (as already mentioned above).

As mentioned above, we agree that a numerical part is missing making the current version of our paper incomplete.

In the past, we extensively studied the empirical distribution of the durations J_i (see [2] and references therein, for instance, where we also estimated parameters). A good model is the Weibull distribution. Based on the analysis of a tick-by-tick data set used in the previous work by one of us (DJIA stocks traded at NYSE in October 1999), in that paper, we rejected the hypothesis that tails of the empirical inter-trade distribution are described by a power law. We further argued that the Tsallis q -exponentials are a viable tool for fitting and describing the unconditional distribution of empirical inter-trade durations and they compare well to the Weibull distribution.

5. Does options with a one-day maturity really exists on the market? If so, please provide a reference to which exchange such options are traded at etc.

So far, we are not aware of any such option being traded.

We have detected several typos and we will correct them. We thank the referee for pointing us to the typo in eq. (14).

References

[1] Politi, M., Kaizoji T. and Scalas, E. (2011). Full Characterization of the Fractional Poisson Process. *Europhysics Letters*, 96(2): 20004–20009.

[2] Politi M. and Scalas E. (2008). Fitting the empirical distribution of intertrade durations. *Physica A* Volume 387,(Issues 8–9, 15 March 2008), Pages 2025–2034.