

A Case Where Barro Expectations Are Not Rational

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Abstract The paper generalizes Feldstein's criticism (*Perceived Wealth in Bonds and Social Security*, 1976) of Barro's analysis (*Are Government Bonds Real Net Wealth?*, 1974) for the case that the interest rate exceeds the growth rate. This is done by considering an economy in steady state where all agents hold 'Barro expectations': they believe that government debt must necessarily be repaid and therefore leave the present value of their income streams unchanged. In this scenario, a change in the mode of taxation affects the present value of disposable income in the private sector. This violates their Barro expectations.

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1 Introduction

Feldstein (1976) has shown that a permanent government deficit may increase the present value of the households' income if the rate of growth exceeds the rate of interest. The present note generalizes Feldstein's remarks to the case that the rate of interest exceeds the rate of growth. In this sense, it invalidates Barro's (1974) analysis also for that case. The generalization is proved by presenting an example of an economy in steady state where government decides to reduce taxation and generate a permanent deficit. All members of the private sector hold "Barro expectations": they believe that government debt must necessarily be repaid and the present value of their income streams remain unchanged. Hence they do not change their expenditure. It is shown that the policy switch from a pay-as-you-go regime to a debt regime increases the present value of disposable income in the private sector, contrary to Barro's assertion.

In this sense the example disproves the logic of Barro's interpretation of Ricardian equivalence.

2 An Example

Consider a closed economy that grows with the nominal rate g . Gross income at time t is Y_t , private expenditure (consumption plus investment) is E_t , taxes are T_t and government expenditure is G_t . Let D_t denote government debt. Initially there is no government debt:

$$D_0 = 0. \tag{1}$$

Income and government expenditure grow both with rate g . So we have

$$Y_t = (1 + g)^t Y_0 \tag{2}$$

$$G_t = (1 + g)^t G_0 \tag{3}$$

The economy is initially in full equilibrium with an interest rate $i > g$ and private expenditure E_0 . Private expenditure E and government expenditure G add

up to total production, and the expectation held by all parties is that this will continue in the future:

$$E_t + G_t = Y_t. \quad (4)$$

Up to $t = -1$, the government budget was balanced, and the taxes levied in any period $t < 0$ were equal to government spending G_t in that period. Call this the “pay-as-you-go regime.” All parties have expected and expect that this policy would continue throughout the future, but government changes its policy and decides to run a permanent deficit of a fraction $\alpha \in (0, 1)$ of its expenditure G_t in each period, beginning at $t = 0$ while leaving government expenditure G_t unchanged. So government expenditure remains as described in equation (13). Call this the “debt regime.”

In line with Barro’s (1974; 1989) analysis, the households and firms expect that the change in policy does not affect the present value of their lifetime income stream. Hence they believe that “rearrangements of the timing of taxes – as implied by budget deficits – have no first-order effect on the economy” (Barro, 1989, 51). They conclude that, sooner or later, the government has to increase taxes, leaving the present value of their incomes unaltered. So they change neither consumption nor investment. In short, everybody in the private sector holds “Barro expectations” and behaves accordingly. (It will turn out, however, that these expectations will not be fulfilled.)

In each period $t = 0, 1, 2, \dots$ the deficit is αG_t , and outstanding government debt D increases in each period by this amount. Therefore we have

$$D_0 = 0 \quad (5)$$

$$D_{t+1} = D_t + \alpha G_t. \quad (6)$$

This implies together with (3).

$$D_t = \frac{((1+g)^t - 1)}{g} \alpha G_0. \quad (7)$$

So debt grows asymptotically in proportion with production. The ratio of government debt to government expenditure approaches α/g and the ratio of government

debt to production approaches α/g times the share of government expenditure in total production.

$$\lim_{t \rightarrow \infty} \frac{D_t}{G_t} = \frac{\alpha}{g}, \quad \lim_{t \rightarrow \infty} \frac{D_t}{Y_t} = \frac{\alpha}{g} \cdot \frac{G_0}{Y_0}. \quad (8)$$

As the government keeps its expenditure on goods, services and manpower G unaltered and runs a deficit, its budget, denoted by B , will exceed expenditure G by interest payments iD on public debt:

$$B_t = G_t + iD_t. \quad (9)$$

The share of interest payments in the government budget is

$$\frac{iD_t}{B_t} = \alpha \frac{i}{g} \frac{(1+g)^t - 1}{(1+g)^t} \quad (10)$$

and approaches $\alpha \frac{i}{g}$:

$$\lim_{t \rightarrow \infty} \frac{iD_t}{B_t} = i \frac{\alpha}{g}. \quad (11)$$

So for growth a rate of 2 percent ($g = 0.02$), a rate of interest of 4 percent ($i = 0.04$) and a deficit rate of 10 per cent ($\alpha = 0.1$) this ratio would approach 17 percent.

The present value of government debt D_t is

$$\left(\frac{1}{1+i} \right)^t D_t = \frac{\alpha}{g} \left(\frac{(1+g)^t - 1}{(1+i)^t} \right) G_0. \quad (12)$$

As $i > g$ is assumed, the present value of the debt is a positive number and goes to zero for $t \rightarrow \infty$ although debt is never retired.

The deficit in period 0 is αG_0 . It is entailed by the tax reduction of the same size. So we have tax receipts of $T_0 = (1 - \alpha)G_0$ in period 0. In period 1 government debt is $D_1 = \alpha G_0$. This requires interest payments iD_1 . The deficit in period 1 is the sum of government expenditure G_1 plus interest payments iD_1 minus tax receipts T_1 . The deficit is to be αG_1 . Hence we have

$$G_1 + iD_1 - T_1 = \alpha G_1.$$

A similar consideration applies to all periods:

$$G_t + iD_t - T_t = \alpha G_t. \quad (13)$$

Solving for T_t gives the amount of taxes to be collected in period t :

$$T_t = (1 - \alpha)G_t + iD_t. \quad (14)$$

Furthermore, the ratio of taxes to production approaches

$$\lim_{t \rightarrow \infty} \frac{T_t}{Y_t} = \left(1 + \frac{(i - g)\alpha}{g}\right) \cdot \frac{G_o}{Y_o} > \frac{G_o}{Y_o}. \quad (15)$$

For $i > g$ the government collects higher taxes under the debt regime than under the pay-as-you-go regime. Comparing the tax increase entailed by switching from the pay-as-you-go regime to the debt regime (which is $T - G$) with interest payments iD necessary under a debt regime yields

$$\lim_{t \rightarrow \infty} \left(\frac{T_t - G_t}{Y_t} - \frac{iD_t}{Y_t} \right) = \frac{(i - g)\alpha G_o}{g Y_o}. \quad (16)$$

In the long term, the tax increases entailed by switching to the debt regime exceed the interest payments necessary to serve the debt.¹

Now consider the households. If the government would run a balanced budget all the time, their discounted disposable income would have been

$$\begin{aligned} W &= \sum_{t=0}^{\infty} \left(\frac{1}{1+i} \right)^t (Y_t - G_t) \\ &= (Y_0 - G_0) \sum_{t=0}^{\infty} \left(\frac{1+g}{1+i} \right)^t \\ &= \frac{1+i}{i-g} (Y_0 - G_0). \end{aligned} \quad (17)$$

¹ This holds true for the case considered here that the rate of interest exceeds the growth rate ($i > g$). As the analysis up to this point is valid for $i \leq g$ as well, these conclusions would be reversed for $i < g$: Switching to a debt regime would lower taxes in this case while interest payments on debt would remain positive.

The debt policy, however, results in disposable income

$$Z_t = Y_t - T_t + iD_t \quad (18)$$

which is

$$\begin{aligned} Z_t &= (1+g)^t (Y_0 - (1-\alpha)G_0). \\ &= Y_t - (1-\alpha)G_t > Y_t - G_t. \end{aligned} \quad (19)$$

Under the pay-as-you-go regime, disposable income in each period would have been $Y_t - G_t$. Hence *the switch from the pay-as-you-go regime to the debt regime has increased disposable income for all periods by the fraction α of government expenditure G_t .*

The present value of disposable income is

$$Q = \sum_{t=0}^{\infty} \left(\frac{1}{1+i} \right)^t Z_t.$$

This is calculated as

$$Q = \frac{1+i}{i-g} \cdot (Y_0 - (1-\alpha)G_0) \quad (20)$$

The difference between this present value of disposable income under the debt regime and the corresponding present value under the pay-as-you-go regime (17) is

$$Q - W = \frac{1+i}{i-g} \alpha G_0 > 0. \quad (21)$$

Hence the present value of the households' lifetime income has increased by switching from a pay-as-you-go regime to the debt regime. The Barro expectations held by the subjects are not fulfilled.

As the value of their lifetime income stream has increased, they could have afforded higher expenditure, with more consumption and more investment, but this would have presumably affected the rate of interest and the value of production and income in turn. Hence the assumption (2) – which is a cornerstone of Barro's (1974) analysis – can not usefully be upheld.

3 Conclusion

It has been urged elsewhere that Ricardian equivalence in Barro's (1989) interpretation is quite irrelevant regarding fiscal policy (Schlicht, 2006, Sect. 9). The above example points out that Barro's view is *internally* contradictory. In conclusion, arguments alluding to "Ricardian equivalence" in Barro's interpretation ought not to be applied to real economies. Despite their apparent logical appeal such arguments are misleading, not only empirically, but also theoretically (Romer, 1995, 72).

References

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Mathematica notebook with the calculations for
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by Ekkehart Schlicht

Note : all calculations are elementary and can be done by hand. I have added the calculations done in Mathematica for making it easier to check the results.

■ **Income Y and government expenditure G (Eqs. 2 and 3)**

In[1]:= $Y[t_]:= (1 + g)^t Y_0;$
 $G[t_]:= (1 + g)^t G_0;$

■ **Government debt (Eqs. 5 to 7)**

In[3]:= $d[t_]:= \frac{(-1 + (1 + g)^t) \alpha G_0}{g}$

Note: The command `expression1 == expression2` returns True if `expression1` and `expression2` are mathematically the same.

In[4]:= $d[0] == 0$

Out[4]= True

Note: The command `Simplify[expression1 == expression2]` gives True if `expression1` and `expression2` are mathematically equivalent.

In[5]:= $\text{Simplify}[d[t + 1] == d[t] + \alpha G[t]]$

Out[5]= True

■ **Asymptotic ratio of debt to government expenditure (Eq.8)**

In[6]:= $\text{Limit}\left[\frac{d[t]}{G[t]}, t \rightarrow \infty, \text{Assumptions} \rightarrow (i > 0) \ \&\& \ (g > 0)\right]$

Out[6]= $\frac{\alpha}{g}$

■ **Asymptotic ratio of debt to production (Eq. 8)**

In[7]:= $\text{Limit}\left[\frac{d[t]}{Y[t]}, t \rightarrow \infty, \text{Assumptions} \rightarrow (i > 0) \ \&\& \ (g > 0)\right]$

Out[7]= $\frac{\alpha G_0}{g Y_0}$

■ **Share of interest on government debt in the government budget (Eqs. 9 and 10)**

In[8]:= $\text{Simplify}\left[\frac{i d[t]}{G[t] + i d[t]} == \frac{((1 + g)^t - 1) i \alpha}{(1 + g)^t g + ((1 + g)^t - 1) i \alpha}\right]$

Out[8]= True

■ **Asymptotic share of interest on debt in the government budget (Eq. 11)**

In[9]:= $\text{Limit}\left[\frac{i d[t]}{G[t] + i d[t]}, t \rightarrow \infty, \text{Assumptions} \rightarrow (i > 0) \ \&\& \ (g > 0)\right]$

Out[9]= $\frac{i \alpha}{g + i \alpha}$

■ Numerical example after Eq. 11

$$\text{In[10]:= } \frac{i \alpha}{g + i \alpha} / . g \rightarrow 0.02 / . i \rightarrow 0.04 / . \alpha \rightarrow 0.10$$

Out[10]= 0.166667

■ Present value of government debt (Eq. 12)

$$\text{In[11]:= } \text{Simplify}\left[\left(\frac{1}{1+i}\right)^t d[t] == \frac{\alpha (1+g)^t - 1}{g (1+i)^t} G_0, \text{Assumptions} \rightarrow (i > g) \ \&\& \ (g > 0)\right]$$

Out[11]= True

$$\text{In[12]:= } \text{Limit}\left[\left(\frac{1}{1+i}\right)^t d[t], t \rightarrow \infty, \text{Assumptions} \rightarrow (i > g) \ \&\& \ (g > 0)\right]$$

Out[12]= 0

■ Taxes (Eq. 14)

$$\text{In[13]:= } T[t_] := (1 - \alpha) G[t] + i d[t]$$

■ Asymptotic ratio of taxes to income (Eq. 15)

$$\text{In[14]:= } \text{Limit}\left[\frac{T[t]}{Y[t]}, t \rightarrow \infty, \text{Assumptions} \rightarrow (i > 0) \ \&\& \ (g > 0)\right]$$

$$\text{Out[14]= } -\frac{(g(-1+\alpha) - i\alpha)G_0}{gY_0}$$

$$\text{In[15]:= } \text{Simplify}\left[-\frac{(g(-1+\alpha) - i\alpha)G_0}{gY_0} == \left(1 + \frac{(i-g)\alpha}{g}\right)\frac{G_0}{Y_0}\right]$$

Out[15]= True

■ Tax increase from pay-as-you-go to debt regime minus interest payments necessary under the debt regime (Eq. 16)

$$\text{In[16]:= } \text{Limit}\left[\frac{T[t] - G[t]}{Y[t]} - \frac{iD[t]}{Y[t]}, t \rightarrow \infty, \text{Assumptions} \rightarrow (i > 0) \ \&\& \ (g > 0)\right]$$

$$\text{Out[16]= } -\frac{(g-i)\alpha G_0}{gY_0}$$

$$\text{In[17]:= } \text{Simplify}\left[-\frac{(g-i)\alpha G_0}{gY_0} == \frac{(i-g)\alpha G_0}{gY_0}\right]$$

Out[17]= True

■ Household wealth with balanced budget (Eq. 17)

$$\text{In[18]:= } W = \text{Simplify}\left[\sum_{t=0}^{\infty} \left(\frac{1}{1+i}\right)^t (Y[t] - G[t])\right]$$

$$\text{Out[18]= } \frac{(1+i)(G_0 - Y_0)}{g-i}$$

■ Disposable income (Eqs. 18 and 19)

$$\text{In[19]:= } Z[t_] := Y[t] - T[t] + i d[t]$$

$$\text{In[20]:= } Z[t]$$

$$\text{Out[20]= } -(1+g)^t (1-\alpha)G_0 + (1+g)^t Y_0$$

■ Present value of disposable income (Eq. 20)

$$\text{In[21]:= } Q = \text{Simplify}\left[\sum_{t=0}^{\infty} \left(\frac{1}{1+i}\right)^t (z[t])\right]$$

$$\text{Out[21]= } -\frac{(1+i) \left((-1+\alpha) G_0 + Y_0\right)}{g-i}$$

■ Difference of present values (Eq. 21)

`Simplify[Q - W]`

$$-\frac{(1+i) \alpha G_0}{g-i}$$

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