Reply to Referee Report No. 2

I would like to thank the second referee for his/her very helpful comments. I also agree with some of them. For example, he/she recommends the introduction be shortened and some of the mathematical sections committed to appendices (which was already suggested by the first referee), and also to explain better why the money market interest rate is corrected by the growth rate of M2 in the empirical model[1]. I agree with all this.

Nevertheless, there are in particular three recommendations in this second review which appear to come from a misreading of the paper's content. Specifically:

A. **Recommendation 3:** The referee claims that "the author develops his alternative production function for a closed economy." This can only be a misunderstanding. The model is based on nine well defined assumptions, and none of them requires the economy to be closed. On page 30, when defining the empirical strategy, there is a side discussion about the fact that, in a closed economy, one would expect the value of $\bar{\sigma}^2 - \bar{\delta}$ to be negative (whereas in an open economy it could perfectly be positive or zero), but this has absolutely nothing to do with the general applicability of the model as developed throughout pages 13 to 26.

B. **Recommendation 4:** The referee states that "it should be made much clearer why the author uses the production function to replicate short-run output fluctuations, although originally the neoclassical production function was developed for long-term growth analyses." This must also be a misunderstanding, as it should be clear

[1] The reason, incidentally, is that credit (i.e. money) creation entails a flow of purchasing power from creditors to debtors that runs opposite to the flow from debtors to creditors represented by interest rates, so the creditors' net rents at any point in time should be regarded as the balance between the two.

from the mere fact that the empirical analysis is based on a series of 60 *annual* observations that we are not testing against "short term" data. Instead, as explained in the paper itself (pg. 5), it "puts forward the hypothesis that observed GDP fluctuations are better modeled by regarding capital […] as a 'sunk cost' […] than as a variable input whose reward is its marginal product, as the basic Cobb-Douglas function assumes."

What the results prove is therefore that long-term GDP data (60 years is definitely not a short run) are better explained through a model that could otherwise be regarded as "short-term focused" than through another that incorporates value-weighted capital as an explanatory variable (just as the Cambridge school would have predicted, by the way). The claim that the Cobb-Douglas' theoretical rationale is "long term" simply cannot overrule the fact that it fails the test.

C. **<u>Recommendation 6</u>:** The referee requests that "the author should try to estimate the production functions in levels […]. In such an exercise, the Cobb Douglas production function will certainly deliver reasonable results." This, however, is questionable on two counts:

   a. First, estimating in levels makes no difference: testing a Cobb-Douglas function with a constant growth trend under these conditions (even in intensive form) yields not just as bad but even worse results than in the paper (I included these tests in the Appendix after this note to prove the point). The reason why many authors fail to highlight this is that, as the paper explains (pgs. 4 and 36-37), because they perform the regression either without a time trend (as in Cobb & Douglas 1928) or with calibrated instead of estimated factor coefficients (as in Solow 1957). This has nothing to do with regressing against levels instead of increments.

b. Second, from a methodological perspective, estimates in levels should be avoided when, as in this case, a data series may have a unit root, for otherwise they can lead to spurious results. This was not known in the days of Cobb & Douglas or Solow, as the concept came up the early 1970s (e.g. Granger & Newbold 1974), but has been regarded as a basic principle ever since. Moreover, its relevance to macroeconomic time series has been well established at least since Nelson & Plosser (1982). This in fact explains why the regressions in the Appendix display an (unbelievably high) R^2 around 99% while the Durbin-Watson test reveals a strong autocorrelation bias. When a unit root may exist, the standard procedure is to estimate against increments instead of levels, as was done in the paper.

In sum, as much as I would like to follow the referee's recommendations, in these three cases they appear to be based either on misinterpretations of the paper or on misconceptions of various natures, and therefore, as explained above, they cannot be applied.

On the other hand, as mentioned at the start of this reply note, I fully agree with recommendations 1, 2 and 5, and would be happy to incorporate them in a revised version.

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Appendix

<u>Cobb-Douglas Estimation in Levels</u>:

Given the usual definition of aggregate GDP ('Y_t') and the usual Tornquist aggregates for labor ('L_t') and capital ('K_t'), the Cobb-Douglas function with a time trend is defined as $Y_t = Ae^{at}K_t^{\alpha_K}L_t^{\alpha_L}$ where t represents time, 'A', 'a', 'α_K' and 'α_L' are constants and where, *ex hypothesi*, $\alpha_K > 0$, $\alpha_L > 0$ and $\alpha_L = 1-\alpha_K$.

To estimate this function in levels, we simply apply a logarithmic transformation:

$$\ln Y_t = \ln A + at + \alpha_K \ln K_t + \alpha_L \ln L_t + u_t$$

Or, applying the standard naming convention for the regression constants:

$$\ln Y_t = \alpha + \beta_1 t + \beta_2 \ln K_t + \beta_3 \ln L_t + u_t$$

We can then proceed to perform the regression on the BLS U.S. data series from 1948 to 2008 (i.e. the same as in the paper, only now taking the levels instead of the differences as the basis for the regression). The results appear on Table A:

Table A

Results of an OLS regression on a standard Cobb-Douglas function (based on levels)

Analytical Expression: $\ln Y_t = \alpha + \beta_1 t + \beta_2 \ln K_t + \beta_3 \ln L_t + u_t$

Independent Variables:

Constant	*Coefficient ('$\hat{\alpha}$')*	7.36661***
	Standard Deviation	(1.09093)
Time (t)	*Coefficient ('$\hat{\beta}_1$')*	0.0383010***
	Standard Deviation	(0.00607211)

K_t	*Coefficient ('$\hat{\beta}_2$')*	-0.132756
	Standard Deviation	(0.157891)
L_t	*Coefficient ('$\hat{\beta}_3$')*	0.173938
	Standard Deviation	(0.151266)

Statistical Fit Metrics:

Degrees of Freedom	57
R^2	99.6201%
F-statistic	4982.095
Akaike Information Criterion	-215.2865
Bayesian (a.k.a. Schwarz) Information Criterion	-206.8430
Hannan-Quinn Information Criterion	-211.9774
Durbin-Watson Test (critical values 1.48468 and 1.69035)	0.440417

From the point of view of testing the Cobb-Douglas functional form, therefore, this yields an even worse result than in the paper, as neither of the regression parameters associated to capital nor the one associated to labor can be distinguished from zero at any level of statistical significance (at least in the paper labor appeared to be significant!).

Cobb-Douglas Estimation in Levels and Intensive Form:

If, following again the referee's indications, we now estimate the function in intensive form (i.e. with the variables weighted by labor hours), then it becomes:

$\ln y_t = \ln A + at + \alpha_K \ln k_t + u_t$ (where evidently $y_t \equiv \frac{Y_t}{L_t}$ and $k_t \equiv \frac{K_t}{L_t}$)

Or, applying the standard naming convention for the regression constants:

$$\ln y_t = \alpha + \beta_1 t + \beta_2 \ln k_t + u_t$$

We can then proceed to perform the regression also on BLS U.S. data series from 1948 to 2008. The results appear on Table B:

Table B

Results of an OLS regression on an intensive-form Cobb-Douglas function (based on levels)

Analytical Expression: $\ln y_t = \alpha + \beta_1 t + \beta_2 \ln k_t + u_t$

Independent Variables:

Constant	*Coefficient ('$\hat{\alpha}$')*	2.29981***
	Standard Deviation	(0.536072)
Time (t)	*Coefficient ('$\hat{\beta}_1$')*	0.0178698***
	Standard Deviation	(0.00472512)
k_t	*Coefficient ('$\hat{\beta}_2$')*	0.117851
	Standard Deviation	(0.168333)

Statistical Fit Metrics:

Degrees of Freedom	58
R^2	98.5618%
F-statistic	1987.384
Akaike Information Criterion	-199.1487
Bayesian (a.k.a. Schwarz) Information Criterion	-192.8161
Hannan-Quinn Information Criterion	-196.6669
Durbin-Watson Test (critical values 1.51886 and 1.65396)	0.120161

Once again, the result seems clear: only the time variable is statistically significant, whereas the hypothesis of capital intensity being irrelevant cannot be statistically rejected.

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