The Focus Axiom and Poverty: On the Co-existence of Precise Language and Ambiguous Meaning in Economic Measurement

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Abstract Despite the formal rigour that attends social and economic measurement, the substantive meaning of particular measures could be compromised in the absence of a clear and coherent conceptualization of the phenomenon being measured. A case in point is afforded by the status of a 'focus axiom' in the measurement of poverty. 'Focus' requires that a measure of poverty ought to be sensitive only to changes in the income-distribution of the poor population of any society. In practice, most poverty indices advanced in the literature satisfy an 'income-focus' but not a 'population-focus' axiom. This, it is argued in the present paper, makes for an incoherent underlying conception of poverty. The paper provides examples of poverty measures which either satisfy both income and population focus or violate both, or which effectively do not recognize a clear dichotomization of a population into its poor and non-poor components, and suggests that such measures possess a virtue of consistency, and coherent meaning, lacking in most extant measures of poverty available in the literature.

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…"That is not what I meant at all.
That is not it, at all."
-T. S. Eliot: *The Lovesong of J. Alfred Prufrock*

'The question is,' said Alice, 'whether you *can* make words mean so many different things.'
'The question is,' said Humpty Dumpty, 'which is to be master – that's all.'
-Lewis Carroll: *Through the Looking Glass*

1. Introduction

What do we mean when we say we are 'measuring poverty'? It would be easy enough to respond by suggesting that what we mean is reflected precisely in what we say – and that would be the end of that, if the response were an accurate one. The difficulty is that the literature on the measurement of poverty affords reason to doubt the accuracy of the response. In particular, it is not at all obvious that the measurement of poverty, as it is largely practiced today, is informed by a clear and coherent notion of *what* it is that is really being measured. This want of coherence, it will be argued in this note, can be traced to an inconsistent – because it is partial - deference to a widely employed axiom in the measurement of poverty. The axiom in question is the so-called 'Focus Axiom' (see Sen 1981).

The focus axiom (potentially) comes into play whenever poverty measurement involves the so-called 'identification' exercise, namely the stipulation of a 'poverty line' level of income whose job it is to separate the poor section of a population from what is perceived to be that population's definitely non-poor component. The axiom is a particular application, in the context of poverty, of a more general principle in population ethics which Broome (1996) calls the 'Constituency Principle'. This latter principle can be stated, in loose terms, as the requirement that in comparing the goodness of alternative states of the world, one should confine attention to how good the states are from the point of view of some identified constituency of individuals who alone are judged to be the relevant and interested parties to the outcome of the comparison exercise. For instance, in comparing the goodness of alternative histories of the world, it could be claimed that which history is better ought to be determined exclusively by which history is better for the constituency yielded by the intersection of the populations that exist in both histories. As applied to poverty comparisons, the Constituency Principle/Focus Axiom would demand that in determining the relative poverty status of two societies, we ought to confine our concern to, or ‘focus’ our attention on, the condition of the *poor constituency* in the two societies.

In practice, the constituency principle in poverty measurement has resolved itself into what one may call an ‘*Income Focus Axiom*’, which is the requirement that, other things equal, any increase in the income of a non-poor person ought not to affect one’s assessment of measured poverty. A properly thoroughgoing appreciation of the constituency principle ought to extend the scope of the principle also to an appropriately formulated ‘*Population Focus Axiom*’, which is the requirement that, other things equal, any increase in the size of the non-poor population ought not to affect one’s assessment of measured poverty. With few exceptions (see, among others, Subramanian 2002, Paxton 2003, Chakravarty, Kanbur and Mukherjee 2006, Hassoun 2010, Hassoun and Subramanian 2011), the Population Focus Axiom has received little attention in the poverty measurement literature. Effectively, extant measurement protocols, as reflected in a number of real-valued measures of poverty in current use, seem to suggest that Income Focus must be respected though Population Focus may (indeed, must) be violated. This partial and inconsistent deployment of the constituency principle in measurement exercises which nevertheless are governed by rigorous axiom systems is ultimately a reflection of some confusion on precisely what one means when one claims to be engaged in measuring poverty.

Here is an attempt, through the employment of an analogy, to uncover the nature of the confusion. Suppose we are interested in measuring the ‘blue-ness’ of a purple mixture of given quantities of blue and red paint. It appears to me that there are at least two different notions of the ‘measurement of “blue-ness”’ that one could have in mind:

(a) The first notion would relate to the question of ‘how blue’ is the purple.

(b) The second notion would relate to the question of ‘what is the quantity of blue’ in the purple.

We can tell these two notions apart with the aid of the following test. Suppose, first, that we employed a darker shade (without altering the amount) of red paint in the purple. Suppose, next, that we employed a greater quantity of (the same shade of) red. In either case, the described changes to the purple mixture would certainly lead to the judgment that the purple has become ‘less blue’, though neither change is compatible with the notion that ‘the quantity of blue’ in the purple has changed. We can think of the blue as the ‘poor’ and the red as the ‘non-poor’. Employing a darker shade of red is analogous to increasing the income(s) of the non-poor, while employing a greater quantity of red is analogous to increasing the size of the non-poor population. If measured poverty is to be invariant with respect to increases in non-poor incomes or populations – as a properly exhaustive concession to a focus axiom would demand - then we are effectively subscribing to a measure of poverty that seeks to assess the 'quantity of poverty in a society'. If, however, we do not find the focus axiom to be persuasive, then we are effectively subscribing to a measure of poverty that seeks to assess 'how poor' a society is.

The distinction, in terms of Population Focus, can be clarified with the help of yet another analogy. Imagine a very small cup of coffee with two spoons of sugar in it, and a very large cup with three spoons of sugar in it. It would be natural to judge that the first cup of coffee is more sugary than the second cup, even though it is also true that there is a smaller quantity of sugar in the first cup than in the second.

The distinction made above between 'how poor a society is' and 'the quantity of poverty there is in a society' echoes an invitation, in Hassoun (2010; p.8), to recognize just such a difference: '…it may be important to clearly distinguish between a population's poverty and how much poverty there is in a population.' This strikes me as being a key insight, but while Hassoun seems to regard 'how much poverty there is in a population' to be the only proper object of poverty measurement, my own inclination would be to allow both notions of poverty alluded to above to be quantified by a measure of poverty, provided there is a clear declaration of which conception of poverty it is that is being measured.

It is perhaps understandable that one's intuitions on how compelling a focus axiom is could be shifting and uncertain. An unstructured view of the matter, which is informed largely by immediate apprehension, is compatible with positions both for and against a focus axiom, an issue that is discussed with the help of some illustrations in the Appendix. The examples reviewed in the Appendix suggest that there is reason to believe that both views of the intended meaning of a measure of poverty ('How poor is a society?' [which flows from denying any version of a focus axiom], and 'What is the quantity of poverty in a society?' [which flows from deferring to both income focus and population focus]) are valid ones, although, of course, it would be a great help for the practitioner to explicitly specify which view she espouses. It is important to add here that the 'how poor is a society?' view of poverty does not derive only from denying the appeal of focus (while accepting a clear demarcation between the poor and the non-poor segments of a population). It can also arise in a situation where 'focus' is an *irrelevant* concept. This would happen if the 'identification exercise' is not seen to be an integral or essential part of the poverty measurement exercise. In such a situation, no recourse is had to the specification of a 'realistic' poverty line intended to serve the purpose of certifying individuals with incomes in excess of the line as being wholly and unambiguously non-poor. As we shall see later, 'fuzzy' views of poverty, in which (effectively) every individual in a society is seen as being more or less poor, are compatible with this perspective on poverty.

There are two implications to the issue of whether or not one subscribes completely and consistently to a constituency principle in the measurement of poverty. The first implication is that partial acknowledgement of a constituency principle (as reflected, for instance, in acceptance of the income focus axiom and denial of the population focus axiom) is unreasonable and inconsistent – and also a feature of most poverty indices in use: if the inconsistency is sought to be rectified by requiring respect for the population focus axiom as well, then this, in conjunction with other commonly accepted properties of poverty measures, can result in logically incoherent aggregation. The second implication is that in either of the events of a comprehensive acceptance or comprehensive rejection of a constituency principle, or in the event of a by-passing of the identification exercise, poverty indices would have to be specified differently from the way in which they have thus far been specified in the bulk of the poverty measurement literature. These issues, which are both simple and basic, but arguably also of importance for the cause of meaningful measurement, are reviewed in the rest of this essay.

2. Focus, and the Possibility of Coherent Aggregation

An income distribution is an *n*-vector $\mathbf{x} = (x_1,...,x_i,...,x_n)$, where $x_i (\geq 0)$ is the (finite) income of individual i in an *n*-person society, and *n* is any positive integer. The set of all *n*-vectors of income is $\mathbf{X}_n$, and the set of all possible income vectors is the set $\mathbf{X} \equiv \cup_n \mathbf{X}_n$. We shall let $\mathbf{D} \subseteq \mathbf{X}$ stand for the comparison set of income distributions, viz. the set of conceivable income vectors whose poverty ranking we seek. The poverty line is a positive level of income z such that persons with incomes less than z are designated poor, and the rest non-poor. For all $\mathbf{x} \in \mathbf{D}$ and $z \in \mathcal{R}_{++}$ (where $\mathcal{R}_{++}$ is the set of positive real numbers), $n(\mathbf{x})$ is the dimensionality of $\mathbf{x}$, $N(\mathbf{x})$ is the set of all people represented in **x**, $Q(\mathbf{x}; z)$ is the set of poor people represented in **x**, and $\mathbf{x}_z^P$ is the sub-vector of poor incomes in $\mathbf{x}$. It will be convenient to see $\mathbf{D}$ as a collection of *ordered* income vectors, that is, for every $\mathbf{x} \in \mathbf{D}$, the individuals in $N(\mathbf{x})$ will be indexed so as to ensure that $x_i \leq x_{i+1}$ for all $i \in N(\mathbf{x}) \setminus n(\mathbf{x})$. For every $\mathbf{x} \in \mathbf{D}$ and $i \in N(\mathbf{x})$, the rank-order of the *i*th poorest person in the vector $\mathbf{x}$ is defined as $r_i(\mathbf{x}) \equiv n(\mathbf{x}) + 1 - i$ (with income-ties taken to be broken arbitrarily). If $\mathcal{R}$ is the set of reals, then a poverty measure is a function $P:\mathbf{D}\times\mathcal{R}_{++}\rightarrow\mathcal{R}$ such that, for all $\mathbf{x}\in\mathbf{D}$ and $z\in\mathcal{R}_{++}$, P assigns a real number which is supposed to be a measure of the poverty associated with the regime $(\mathbf{x};z)$. We define $\mathbf{D}^*$ to be the set of all vectors in which each person's income is zero: $\mathbf{D}^*\equiv\{\mathbf{x}\in\mathbf{D}\,|\,x_i=0\forall i=1,...,n(\mathbf{x})\}$. In everything that follows, we shall assume the poverty index P to be *anonymous*, that is to say, invariant to interpersonal permutations of income.

The *Income Focus (IF) Axiom* requires that for all $\mathbf{x},\mathbf{y}\in\mathbf{D}$ and $z\in\mathcal{R}_{++}$, if $n(\mathbf{x})=n(\mathbf{y})$ and $\mathbf{x}_z^P=\mathbf{y}_z^P$, then $P(\mathbf{x};z)=P(\mathbf{y};z)$.

The Population Focus (PF) Axiom requires that for all $\mathbf{x},\mathbf{y}\in\mathbf{D}$ and $z\in\mathcal{R}_{++}$, if $\mathbf{y}=(\mathbf{x},x)$ for any $x\geq z$, then $P(\mathbf{x};z)=P(\mathbf{y};z)$.

The *Maximality (M) Axiom* requires that for all $\mathbf{x},\mathbf{y}\in\mathbf{D}$ and $z\in\mathcal{R}_{++}$, if $\mathbf{x}\in\mathbf{D}^*$ and $\mathbf{y}\notin\mathbf{D}^*$, then $P(\mathbf{x};z)\geq P(\mathbf{y};z)$.

The Poverty Growth (PG) Axiom requires that for all $\mathbf{x},\mathbf{y}\in\mathbf{D}$ and $z\in\mathcal{R}_{++}$, if $\mathbf{x}_z^P=(x,...,x)$ for any x satisfying $0\leq x<z$, $N(\mathbf{x})\setminus Q(\mathbf{x})\neq\Phi$, and $\mathbf{y}=(\mathbf{x},x)$, then $P(\mathbf{y};z)>P(\mathbf{x};z)$.

Axiom IF requires measured poverty to be invariant with respect to increases in non-poor incomes, while Axiom PF – see also Paxton's (2003) Poverty Non-Invariance Axiom - requires measured poverty to be unchanging with respect to increases in the non-poor population. It may be mentioned here that Axiom PF is diametrically opposed in spirit to what Kundu and Smith (1983) call the 'Nonpoverty Growth Axiom', which is the requirement that poverty should decline with an increase in the non-poor population. Population Focus, clearly, takes a 'quantity of poverty in a society' view of a poverty measure, while the Nonpoverty Growth property takes a 'how poor is a society' view of a poverty measure. As stated earlier, it may well be hard to display dogmatism, one way or the other, with respect to the appeal of a focus axiom. In any event, it is not a concern of this paper to argue the substantive merits of one position or the other. What, however, *can* be said is that it does seem to be inconsistent to see merit in one of the two focus axioms but not in the other. Inasmuch as virtually all known poverty indices satisfy Axiom IF, there appears to be a strong case for votaries of IF to require a poverty measure to also fulfill Axiom PF. The difficulty is that Axiom PF, in conjunction with Axioms M and PG, both of which are standard features of most known poverty indices, results in incoherence. Before examining this problem, let us take quick stock of the Maximality and Poverty Growth axioms.

The Maximality axiom simply requires that poverty is never worse than when every person in the population has zero income. This property is satisfied by most known poverty indices, and is compatible with a normalization axiom due to Pattanaik and Sengupta (1995) which requires, in part, that the poverty measure should simply be the headcount ratio when the entire population has zero income: as the authors point out, this accords with a standardization procedure in which the upper bound on the measure is defined by unity – the case of 'extreme' poverty where every person is maximally poor (that is, has zero income). (It has been pointed out to the author in personal communication by Manimay Sengupta that a weaker version of the Maximality axiom can be obtained by simply normalizing the poverty index to lie in the interval [0,1] and requiring that, for all $\mathbf{x} \in \mathbf{D}^*$ and $z \in \mathcal{R}_{++}$, $P(\mathbf{x}; z) = 1$.) Finally, the Poverty Growth axiom requires that in a situation where there is at least one non-poor person and where all the poor have the same income, the addition of another person to the population with this income should cause measured poverty to rise. This is a weakened version (see Subramanian 2002 and Hassoun and Subramanian 2011) of a similar axiom, with the same name, proposed by Kundu and Smith (1983).

Notice that the Poverty Growth Axiom would make sense whether we understood a poverty measure to signify 'how poor' a society is or 'what the quantity of poverty' in the society is. Population Focus, clearly, takes a 'quantity view' of poverty. Maximality, on the other hand, clearly adopts a 'how poor' view of poverty. The combination of these clashing views on what constitutes a 'measure of poverty' must inevitably result in contradiction, as suggested by the following simple Proposition.

Proposition: There exists no poverty measure $P : \mathbf{D} \times \mathcal{R}_{++} \to \mathcal{R}$ satisfying the Maximality (M), Population Focus (PF) and Poverty Growth (PG) axioms.

Proof: Let z be the poverty line and let x be a level of income such that $x \geq z$. Consider the income vectors $\mathbf{a} = (0,...0)$, $\mathbf{b} = (\mathbf{a}, x)$ and $\mathbf{c} = (\mathbf{b}, 0)$. By Axiom PG, $P(\mathbf{c}; z) > P(\mathbf{b}; z)$, and by Axiom PF, $P(\mathbf{b}; z) = P(\mathbf{a}; z)$, whence $P(\mathbf{c}; z) > P(\mathbf{a}; z)$ - which, however, contradicts $P(\mathbf{a}; z) \geq P(\mathbf{c}; z)$, as dictated by Axiom M. (*Q. E. D.*)

What the Proposition above suggests – and similar impossibility results can be found in Subramanian (2002) and Hassoun and Subramanian (2011) - is the following. If we wish to defer to a constituency principle, we should do so in its entirety, that is, we must accept the Population Focus, and not only the Income Focus, axiom. When, in the cause of consistency, this latter requirement is explicitly imposed on a poverty measure, then we find that its combination with other properties that are a standard feature of known poverty indices of the type that satisfy Income Focus, leads to impossibility. Virtually all available real-valued measures of poverty in the literature satisfy Income Focus, but they also incorporate the headcount ratio, which – as pointed out in Hassoun (2010) – violates Axiom PF (notice that the headcount ratio declines with a rise in the non-poor population). This inconsistent attitude toward focus is troublesome, and leads to the problem of 'incoherent aggregation' alluded to earlier.

3. Where Focus is Comprehensively Respected: Examples of Measures which Assess the 'Quantity of Poverty'

The preceding observations suggest that extant measures of poverty are essentially confused about what view of poverty is actually sought to be captured by its measurement. If a 'focus' view – one that upholds a 'quantity of poverty' interpretation – is favoured, then it will not suffice to defer to Income Focus alone: Population Focus must also be deferred to. An implication of such comprehensive deference to a focus principle is, as mentioned earlier, that one cannot continue to advance the sorts of poverty measures that abound in the literature. This problem is, however, susceptible of a ready solution. Specifically, one particularly simple means to the end of satisfying both Income and Population Focus would be to take any of the many widely employed poverty measures in current use – measures which incorporate the headcount ratio – and simply multiply them by the size of the total population (see Subramanian 2000, Hassoun 2010). For instance, if $P^S(\mathbf{x};z)$ is the Sen (1976a) measure of poverty for an income vector $\mathbf{x}$ when the poverty line is z, then the simplest way of deriving from this measure one which also satisfies Population Focus is to advance the cause of the measure

$$\hat{P}^S(\mathbf{x};z) \equiv n(\mathbf{x})P^S(\mathbf{x};z)\,. \tag{1}$$

The proposal, that is, is simply to replace – in the composition of any 'standard' poverty index which satisfies Income Focus – the headcount ratio by the aggregate headcount. It must be added here that one well-known poverty index that adopts a consistent stance toward focus by satisfying both the Income Focus and the Population Focus axioms is the so called

Income-Gap (or IG) Ratio which, for any income vector $\mathbf{x}$ and poverty line z, and given that $\mu_z^P(\mathbf{x})$ is the mean income of the poor population, is defined by

$$P^{IG}(\mathbf{x};z) \equiv [z - \mu_z^P(\mathbf{x})]/z\,. \qquad (2)$$

P^{IG} is not a generally favoured index of poverty because it takes no account of the prevalence – as measured by any headcount – of the poor. Nevertheless, it bears remarking that P^{IG} is one of the few known indices in the poverty measurement literature that manifests a consistent response to the notion of focus.

4. Where Focus is Comprehensively Violated: Examples of Measures which Assess the 'Poorness' of a Society

One is not, of course, obliged to see any merit in any version of a focus axiom: one is, after all, free to take a view of poverty which measures 'how poor' a society is, rather than 'how much poverty there is' in the society. A consistent response to the merits of a focus axiom, that is, resides as much in accepting both Income Focus and Population Focus as in rejecting both. This raises the question of the sort of poverty measure one might expect to derive when one comprehensively discards focus, even while accepting the view that a section of a population can be unambiguously non-poor. As it happens, there is at least one known index in the poverty measurement literature, due to Anand (1977), which rejects both Population and Income Focus. For any income vector $\mathbf{x} \in \mathbf{D}$ and poverty line z, and given that $\mu(\mathbf{x})$ is the mean income of the distribution, Anand's modification of the Sen index of poverty (see also Thon 1979) is yielded by

$$P^A(\mathbf{x};z) \equiv \left[\frac{z}{\mu(\mathbf{x})}\right] P^S(\mathbf{x};z)\,. \qquad (3)$$

Notice that like any other 'standard' poverty measure that incorporates the headcount ratio, the index P^A violates Population Focus; in addition, it violates Income Focus too: an increase in the income of a non-poor person would change the value of μ, and therefore of the poverty measure. Despite the consistent stance displayed by P^A toward the notion of focus, it is ironical that its appeal *as a measure of poverty* (as distinct from its appeal as a measure of '*the difficulty of alleviating poverty*') has been questioned (see, for example, Sen 1981). It could be argued that this criticism misses the point that P^A is a consistent measure of poverty which, by denying any merit to any version of a focus axiom, asserts the validity of a view of poverty that reflects the notion of 'how poor a society is'. Indeed, and by contrast, it is not quite clear precisely what view of a measure of poverty an index such as P^S upholds, since it endorses one focus axiom (Income) and violates the other (Population).

A second example of a poverty measure which violates both focus axioms is one due to the present author (Subramanian 2009a). This measure of poverty seeks to incorporate the possibility that an individual's deprivation status is determined not only by her own income but also by the average income of the social or ethnic or geographic (or other appropriately relevant) group that she is affiliated with. The idea is to capture some element of 'horizontal' (inter-group) inequality, in addition to the more familiar phenomenon of 'vertical' (inter-personal) inequality, in the measure of poverty, by postulating an externality arising from group-affiliation such that, other things equal, a poor person's poverty status is seen to be a declining function of the average level of prosperity of the group to which she belongs. Suppose the population to be partitioned into a set of mutually exclusive and exhaustive subgroups (such as on racial lines), that $\mathbf{x}$ is the given income distribution, and that $\mu^i(\mathbf{x})$ is the average income of the group to which person i belongs. Then, person i's deprivation status can be written as: $d_i = \max[\frac{z - x_i}{z + \mu^i}, 0]$ (which is just a specialization of the requirement that a poor person's poverty status be a declining function of both her own income and of her group's average income). One can now define a 'group-affiliation externality'-adjusted measure of poverty, P^G (where the superscript G is a reminder of the 'group'-mediated deprivation which the measure incorporates), as a simple average of the values of the individual deprivation functions d_i: for all $\mathbf{x} \in \mathbf{D}$ and $z \in \mathcal{R}_{++}$,

$$P^G(\mathbf{x}; z) = \left[\frac{1}{n(\mathbf{x})}\right] \Sigma_{i \in Q(\mathbf{x})} \left[\frac{z - x_i}{z + \mu^i(\mathbf{x})}\right]. \tag{4}$$

(Note that, properly speaking, the precise 'grouping', or partitioning of the population into groups, ought to be entered as an argument in the poverty function, but this has been omitted in expression (4) in order to lighten the notational burden.) Motivationally, the distinctive feature of the poverty measure P^G is that it takes account of the inter-group distribution of poverty: specifically, for any given $\mathbf{x}$ and associated distribution $\{\mu^i(\mathbf{x})\}$ of the mean incomes of the groups to which individuals belong, poverty is minimized when $\mu^i(\mathbf{x}) = \mu(\mathbf{x}) \forall i = 1, ..., n(\mathbf{x})$. Of interest in the context of the present paper's concerns is that P^G, like most conventional poverty indices, violates Population Focus; additionally, and like the index P^A, it also violates Income Focus: changes in non-poor incomes will typically change group means and, therefore – given (4) – the value of the poverty index.

5. Where Focus is Irrelevant: Further Examples of Measures which Assess the 'Poorness' of a Society

Yet another means of addressing the question of how to measure poverty when focus is not deferred to, is to adopt a thoroughgoingly fuzzy approach, in one specific sense, to reckoning poverty. In this approach, one is under no compulsion to truncate a distribution at some specified poverty line and focus one's attention only on the income distribution below the poverty line. The idea, rather, is to allow for the possibility that everybody in a society is more or less poor (which is compatible, as we shall see, with specifying a 'pseudo poverty line', that is, a line pitched high enough that no-one's income is likely to exceed it): in effect, one can, under this procedure, avoid engaging in the messy 'identification exercise' of specifying a 'realistic' and satisfactory poverty line. In particular, a vague approach to reckoning poverty is compatible with postulating a 'fuzzy membership function', which assigns a 'poverty status' in the interval [0,1] to each individual income in a distribution. On fuzzy poverty measurement, the reader is referred to, among others, Kundu and Smith (1983), Shorrocks and Subramanian (1994), Chiappero-Martinetti (2000), Qizilbash (2003), and Subramanian (2009b). (It should be noted that the fuzzy approach described in this paper corresponds to what Qizilbash 2003 calls the 'degree' approach, and which he distinguishes from 'epistemic' and 'supervaluationist' approaches to accounting for vague predicates. In particular, my concern is not to claim any particular and superior merit for the fuzzy approach considered here, but rather to present illustrative examples of how one variant of such an approach might effectively deny a focus axiom any role in the measurement of poverty.)

Before we deal with membership functions, some further investment in notation is required. With this in mind (see Subramanian 2009b), let the highest level of income in any of the vectors contained in $\mathbf{D}$ be designated by $\bar{x}$: that is, $\bar{x} \equiv \max_{\mathbf{x}\in\mathbf{D}} \{x_{n(\mathbf{x})}\}$. It is useful now to define a sort of 'pseudo-poverty line' as a very large, finite level of income Z which is 'sufficiently larger than $\bar{x}$' to ensure that every person in each of the distributions contained in $\mathbf{D}$ would have to be regarded as being (more or less) poor. The precise value assigned to Z is not a matter of any great significance: as we shall see, Z will simply be employed as a 'device', dictated by considerations of arithmetical convenience, for deriving a fuzzy poverty index.

Fuzzy poverty membership functions can be – to borrow the terminology even if not the exact context of Barrientos (2010) - 'relational' or 'non-relational' (corresponding respectively to 'egalitarian' and 'prioritarian' social valuations of income, in the sense of Parfit 1997, though it could be needlessly misleading to employ these latter terms in the present context). 'Relational' membership functions assign poverty status to an income after locating that income within the overall distribution of income, that is to say, in relation to other incomes, in a distinctly 'menu-dependent' way. 'Non-relational' membership functions, on the other hand, assign the same poverty status to any given level of income irrespective of what relation the income level in question bears to other incomes in the income distribution, that is to say, in a distinctly 'menu-independent' way. In either case, poverty status may be expected to be a non-increasing function of income (a simple and appealing monotonicity requirement), and to be bounded from below by zero (no poverty) and from above by unity (complete poverty), which is a simple zero-one normalization. Formally, 'relational' membership functions are drawn from the set $M_R \equiv \{m : \mathcal{R}_+ \times \mathbf{D} \rightarrow [0,1] | m(x;\mathbf{x})$ is continuous and non-increasing in x $\forall\ \mathbf{x} \in \mathbf{D}$, $m(0;\mathbf{x}) = 1$ $\forall\ \mathbf{x} \in \mathbf{D}$, and $Lim_{x \to Z} m(x;\mathbf{x}) = 0\ \forall\ \mathbf{x} \in \mathbf{D}\}$. 'Non-relational' membership functions are drawn from the set $M_{NR} \equiv \{m : \mathcal{R}_+ \rightarrow [0,1] | m(x)$ is continuous and non-increasing, $m(0) = 1$, and $Lim_{x \to Z} m(x) = 0\}$. ($\mathcal{R}_+$ is the non-negative real line.)

Here is an example of a 'relational' membership function (employed in Subramanian 2009b). Given an ordered income vector $\mathbf{x} = (x_1, ..., x_i, ..., x_{n(\mathbf{x})}) \in \mathbf{D}$ and some Z, consider the membership function $m^1(x_i;\mathbf{x}) \equiv \left[\frac{Z - x_i}{Z}\right]\left[\frac{r_i(\mathbf{x})}{n(\mathbf{x})}\right]$, where, to recall, $r_i(\mathbf{x}) \equiv n(\mathbf{x}) + 1 - i$ is the rank-order of the ith poorest person in the vector $\mathbf{x}$. The function $m^1(x_i;\mathbf{x})$ assigns to the ith poorest person in the vector $\mathbf{x}$ a poverty status that depends on i's income and also – in a 'relational' way – on her rank-order in the vector. Notice that m^1 declines as we move up the income ladder, that $m^1(x_i;\mathbf{x}) = 1$ when $x_i = 0$, and that $m^1(x_i;\mathbf{x})$ goes to zero as x_i goes to Z, so m^1 satisfies the 'monotonicity' and 'normalization' properties alluded to earlier. A fuzzy poverty index can be written as a simple average of the poverty statuses, as reflected in the values of their membership functions, of all the individuals in a society. Such a poverty index, corresponding to the fuzzy membership function m^1, will then be given, for any $\mathbf{x} \in \mathbf{D}$ and Z, by

$$P^1(\mathbf{x};Z) = [1/n(\mathbf{x})]\Sigma_{i\in N(\mathbf{x})} m^1(x_i;\mathbf{x}) = [1/n(\mathbf{x})]\Sigma_{i\in N(\mathbf{x})} \left[\frac{Z - x_i}{Z}\right]\left[\frac{r_i(\mathbf{x})}{n(\mathbf{x})}\right]. \quad (5)$$

Consider Sen's (1976b) welfare index, or 'distributionally adjusted measure of real national income', which is given, for any $\mathbf{x} \in \mathbf{D}$, by

$$W^S(\mathbf{x}) = \mu(\mathbf{x})[1 - G(\mathbf{x})], \quad (6)$$

where $G(\mathbf{x})$ is the Gini coefficient of inequality in the distribution of incomes in $\mathbf{x}$. It has been shown in Subramanian (2009b) that if $n(\mathbf{x})$ is large enough to allow $[n(\mathbf{x})+1]/[n(\mathbf{x})]$ to be approximated to 1, then the fuzzy poverty measure P^1 can be approximated to the expression $1/2 - (1/2Z)\mu(\mathbf{x})[1 - G(\mathbf{x})]$. That is to say, for any $\mathbf{x} \in \mathbf{D}$ and Z, the following is true for sufficiently 'large' values of $n(\mathbf{x})$:

$$P^1(\mathbf{x};Z) = 1/2 - (1/2Z)W^S(\mathbf{x}). \quad (7)$$

As pointed out in Subramanian (2009b), the fuzzy poverty index P^1 is simply a re-cardinalization of the 'crisp' welfare index W^S: the one is just a negative affine transform of the other, and this enables us to see that the particular value attached to Z (so long as Z satisfies $\overline{x} << Z < \infty$) is of no great significance.

An example of a 'non-relational' membership function is the following one. Given $\mathbf{x} \in \mathbf{D}$ and Z, consider the fuzzy membership function $m^2(x_i) \equiv \left[\frac{Z^\beta - x_i^\beta}{Z^\beta}\right]$, $0 < \beta < 1$. Notice that the poverty status of any person i now depends only on person i's income, reckoned on its own, and not in relation to any other person's income in the distribution: in this sense, m^2 is a 'non-relational' membership function. To register this more clearly, imagine two income vectors $\mathbf{x}$ and $\mathbf{y}$ and two individuals $j \in N(\mathbf{x})$ and $k \in N(\mathbf{y})$ such that $x_j = y_k$ and $r_j(\mathbf{x}) \neq r_k(\mathbf{y})$; then, it is easy to see that for the 'menu-dependent', 'relational' membership function m^1, $m^1(x_j;\mathbf{x}) \neq m^1(y_k;\mathbf{y})$ (because $r_j(\mathbf{x}) \neq r_k(\mathbf{y})$), while for the 'menu-independent', 'non-relational' membership function m^2, $m^2(x_j) = m^2(y_k)$ (because $x_j = y_k$). It is also easy to see, of course, that m^2 satisfies the 'monotonicity' and 'normalization' properties: m^2 declines in income, is unity when income is zero, and approaches zero as income approaches Z. As before, a fuzzy poverty index – call it P^2 - can be obtained as a simple average of the values of the membership functions of all individuals in the society: for any $\mathbf{x} \in \mathbf{D}$ and Z,

$$P^2(\mathbf{x};Z) = [1/n(\mathbf{x})]\Sigma_{i\in N(\mathbf{x})} m^2(x_i) = [1/n(\mathbf{x})]\Sigma_{i\in N(\mathbf{x})}\left[\frac{Z^\beta - x_i^\beta}{Z^\beta}\right]. \quad (8)$$

It may be noted that the fuzzy membership function m^2 is in all essential respects exactly like the individual deprivation function employed in his poverty measure by Chakravarty (1983); and it appears to be perfectly legitimate to propose a 'crisp' normalized aggregate welfare function, increasing in each income and strictly concave (whence also equity-preferring), which one could christen W^C after Chakravarty, and which is given, for all $\mathbf{x} \in \mathbf{D}$, by

$$W^C(\mathbf{x}) = \left[\frac{1}{n(\mathbf{x})}\right]\Sigma_{i\in N(\mathbf{x})} x_i^\beta. \quad (9)$$

Given (8) and (9), it is straightforward that

$$P^2(\mathbf{x};Z) = 1-(1/Z^\beta)W^C(\mathbf{x}). \quad (10)$$

Once more we see that P^2 is just a negative affine transform of the welfare index W^C, and that, in a substantive sense, Z has no real role to play in deriving the poverty index we are after.

Note, finally, that the 'fuzzy poverty indices' in Expressions (7) and (10) are actually what we are more accustomed to viewing as 'crisp "illfare" indices'.

6. Concluding Observations

Consider the following focus principle, here called the Comprehensive Focus (CF) Axiom, which combines both Income Focus and Population Focus into a single property (one that was earlier introduced in Subramanian 2002 as the Strong Focus Axiom):

The *Comprehensive Focus (CF) Axiom* requires that for all $\mathbf{x},\mathbf{y} \in \mathbf{D}$ and $z \in \mathcal{R}_{++}$, if $\mathbf{x}_z^P = \mathbf{y}_z^P$, then $P(\mathbf{x};z) = P(\mathbf{y};z)$.

We can now conceive four categories of practitioners engaged in the enterprise of measuring poverty: Category A is constituted by those that subscribe to Axiom CF; Category B is constituted by those that reject both Axioms IF and PF; Category C is constituted by those for whom the focus axiom is essentially irrelevant, from the consideration that their view of poverty does not entail any serious engagement with the identification exercise; and Category D (which seems to account for the majority of experts in the field) is constituted by those that subscribe to Axiom IF but not Axiom PF. Members of Category A can be seen to uphold a view of a measure of poverty that is concerned with the question of 'the quantity of poverty there is in a society'; members of Categories B and C can be seen to uphold a view of a measure of poverty that is concerned with the question of 'how poor a society is'; and members of Category D uphold a view of a measure of poverty which – at least to this author's mind – does not add up to a consistent or coherent conceptualization. The phrase 'a measure of poverty', it seems, *could* mean a measure such as is reflected in Equation (1) or Equation (2) – these measures are potential candidates for endorsement by Category A members – just as the phrase *could* mean a measure such as is reflected in one of Equations (3), (4), (7) and (10) – the first two of these measures are potential candidates for endorsement by Category B members, and the latter two are potential candidates for endorsement by category C members. It is the view of poverty upheld by Category D members that appears to be troublesome: whether a 'measure of poverty' could mean a measure that supports Axiom IF and violates Axiom PF is doubtful, because – presumably – words can only 'mean' something 'meaningful'.

This suggests that Alice's query ('The question is whether you *can* make words mean so many different things') is amenable to a qualifiedly affirmative response: 'Yes, within limits, a word *can* mean different things, the limits being supplied by the requirements of inherent plausibility and logical coherence.' Further, Humpty Dumpty's apparently peremptory and *non sequitur* response to Alice's query ('The question is which is to be master – that's all') appears, on reflection, not to be so unreasonable after all. Specifically, the conceptions of poverty espoused by members of Categories A, B and C suggest that these members are in command of their respective notions of poverty in a way that is denied to Category D members: it is as if the practitioner is the master of the poverty measure in the Category A, Category B and Category C cases, while the unruly poverty measure is the master of the practitioner in the Category D case.

Finally, it is important to underline that for any measure to reflect a meaning that can be comprehended, a necessary condition would be that the premises underlying the measure should be made clear and explicit. In the instant case, if one's particular stance on a focus axiom is left unspecified, then one must deem it natural that a poverty measure advanced by a member of any one of the four categories A, B, C and D just defined would be met with rather complete incomprehension – given their own respective and distinctive perspectives on focus – from the members of each of the other three categories. Each, with justice, might be expected to elicit the Eliot denial from the other three: '…That is not what I meant at all./That is not it, at all.' The importance of a presumed shared set of underlying premises of meaning and context for communication to be successful is well brought out in a piquant passage in one of G. K. Chesterton's Father Brown stories ('The Invisible Man'), where we hear the priest saying:

> Have you noticed this – that people never answer what you say? They answer what you mean – or what they think you mean. Suppose one lady says to another in a country house, 'Is anybody staying with you?' the lady doesn't answer 'Yes; the butler, the three footmen, the parlour-maid, and so on', though the parlour-maid may be in the room, or the butler behind the chair. She says: 'There is *nobody* staying with us', meaning nobody of the sort you mean. But suppose a doctor inquiring into an epidemic asks, 'Who is staying in the house?' then the lady will remember the butler, the parlour-maid, and the rest. All language is used like that; you never get a question answered literally, even when you get it answered truly…

As with house-guests, so with a measure of poverty. This essay is an invitation to recognize that, while there are invalid senses in which the term 'a measure of poverty' can be used, there is also more than one valid sense in which it can be invoked, and that there is always a case for explicating the sense(s) in which the term is employed as clearly and cogently as possible.

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APPENDIX

FOCUS: PRO OR CON?

As mentioned in the text, one's intuition, unsupported by any seriously rigorous formal reasoning, can sometimes be sympathetic and at other times antagonistic to the demands of a focus axiom. To see what is involved, consider the following examples, which involve the use of alternative non-decreasingly ordered income vectors ($\mathbf{x}, \mathbf{y}$, etc.) and poverty lines (the poverty line z being a threshold level of income such that those with incomes less than the line are regarded as poor, and the rest as non-poor).

Example 1. $z = 10$, $\mathbf{x} = (0,0,100,100)$, $\mathbf{y} = (9,9,15,15)$. In **x** the poor are utterly destitute while in **y** the poor just fall short of the poverty line, so that though the redistributive capacity for relieving poverty in **x** is greater than in **y** because the non-poor are so much better off in **x** than in **y**, it is compatible with a reasonable assessment - it can be maintained - to suggest that **x** is poverty-wise worse than **y**: in this instance, the income focus axiom is indeed appealing.

Example 2. $z = 10$, $\mathbf{x} = (9,9,10,10)$, $\mathbf{y} = (9,9,100,100)$. In both **x** and **y** the poor are equally badly off, but the burden of poverty is much higher in **x** than in **y** because the non-poor population is so much better off in **y** than in **x**: the income focus axiom is less appealing in this example than in the previous one. It would be excusable to hold the view that **x**, in the present instance, is poverty-wise worse than **y**.

Example 3. $\mathbf{x} = (10,10,10.1,10.2,10.3)$, $\mathbf{y} = (10,10,10.2,10.3,10.4)$, $\mathbf{u} = (10,10,10.3,10.4,10.5)$. In this example, the income focus axiom could be unattractive for a practical reason revolving around the possibility of measurement errors. If the poverty line z is taken to be 10.1 (and we accept income focus), then it would be reasonable to pronounce all three vectors **x**, **y**, and **u** as displaying the same measure of poverty; if, however, the true value of z were marginally different, at 10.2, then it would be reasonable to suggest that **x** is poverty-wise worse than **y**, and **y** indifferent to **u**. If the true value of z were slightly different, at 10.3, then it would be reasonable to rank **x**, **y** and **u** is descending order of poverty. Minor variations in the specification of the poverty line could thus precipitate major variations in the poverty ranking of distributions, and this could again be a reason for not perceiving unqualified merit in the income focus axiom.

Example 4. $\mathbf{x} = (0,0,10)$, $\mathbf{y} = (0,0,10,10)$, $\mathbf{u} = (0,0,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10)$. In moving from **x** to **y**, it is hard to see how the addition of one non-poor person to the population makes a difference to one's perception of poverty; however, in moving from **x** to **u**, the potential for relieving poverty through redistribution increases so considerably through an expansion in the size of the non-poor population that it is not immediately unreasonable to pronounce **x** to be poverty-wise worse than **u**. The population focus axiom is less persuasive when we consider the transition from **x** to **u** than when we consider the transition from **x** to **y**.

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