

Review of *Economics* manuscript 580, “The social cost of carbon on an optimal balanced growth path”

Summary

This paper examines the conditions for balanced optimal growth in the presence of climate change damages and the growth rate of the social cost of carbon (SCC) on an optimal balanced growth path. The author addresses these questions by transferring previous findings from the economic growth literature, in the vein of Solow and others, to a Ramsey-style neoclassical growth model with emissions replacing labor in the aggregate production function. The main results include: a balanced growth path requires technological change to be “emissions-augmenting;” for a balanced growth path to be optimal the elasticity of the marginal utility of consumption must be constant; and the SCC decreases over time on a balanced growth path.

I have two major comments on this paper. The first pertains to the general research question that the paper aims to address, and the second pertains to one of the main conclusions of the paper.

Is this the right question?

The growth rate of the SCC is a worthy topic of investigation, as it is not immediately obvious how this marginal value will change over time based on intuition alone. In fact, I have heard policy makers and advisors ask this very question: How fast will the SCC grow over time, and why? I have not heard a pat answer and explanation. However, the most pressing question here is how fast the SCC will grow *in the near term*—that is, in the coming years and decades—since this is the time frame over which current decision-makers must design a carbon tax or cap and trade system or other forms of regulations. It could take several centuries for the climate to finally stabilize, considering the very slow removal rate of carbon from the atmosphere and the slow rate of progressive heat uptake in the deep ocean. If one were to answer those policy advisors who ask about the growth rate of the SCC by saying, “Well, after the climate system reaches a steady-state several centuries from now the SCC will then grow at such and such a rate, but until that time I’m not sure...,” they would not be impressed. (An overused quote from Keynes would be highly appropriate here.) So, unless it can be shown how the near term SCC growth rate relates to this very long run rate, it is of academic interest only. I would strongly encourage the author to expand the formal analysis to examine the growth rate of the SCC in the transition phase from current conditions to a balanced growth path. Or at least give the reader some qualitative indication of how the growth rate in the near term relates to the rate on the balanced growth path.

The SCC on a balanced growth path

My second major comment is about the final main conclusion of the paper: the author finds that on a balanced growth path the SCC will decrease over time. I am not confident that I can follow the author's derivations closely enough to confirm or deny them on their own terms, so I will address this question from a different angle. My approach here is much more elementary than that used by the author, and in some ways is less general. However, this simplistic example should be sufficient to double-check the author's result.

Using a standard representative agent model and a CRRA utility function, a general expression for the SCC is:

$$SCC_t = - \sum_{\tau=t}^{\infty} \frac{\partial C_{\tau}}{\partial x_t} e^{-[\eta \bar{g}_{t\tau} + \rho](\tau-t)},$$

where C_{τ} is aggregate (market and non-market) consumption in year τ , x_t is emissions in year t , η is the elasticity of the marginal utility of consumption, $\bar{g}_{t\tau}$ is the time-averaged growth rate of per capita consumption between years t and τ , and ρ is the pure rate of time preference or utility discount rate.

To simplify matters, I will assume that $C_{\tau} = (1 - aT_{\tau}^b)(1 - s)Y_{\tau}$, where T_{τ} is the temperature anomaly in year τ , s is a constant savings rate, and Y_{τ} is aggregate income in year τ ; $T_{\tau} = \lambda \ln(X_{\tau}/X_{pl})/\ln 2$, where λ is the climate sensitivity parameter (i.e., the steady-state temperature anomaly due to a sustained doubling of the atmospheric carbon concentration), X_{τ} is the atmospheric carbon concentration in year τ , and X_{pl} is the preindustrial atmospheric carbon concentration; and $X_{\tau} = X_t e^{-\beta(\tau-t)} + \sum_{z=t}^{\tau} x_z e^{-\beta(\tau-z)}$, where β is a constant atmospheric carbon decay rate. These simplifications ignore several important processes, including: the cumulative effect of climate damages through lost investment, the time lag as the atmospheric temperature slowly converges toward its long run equilibrium level due to the progressive uptake of heat in the deep ocean, and the complexities of the carbon cycle that cause carbon to "decay" from the atmosphere at a non-constant rate. Nevertheless, this model still contains the bare-bones features of most climate change IAMs and is sufficiently realistic for my present purposes. With these simplifying assumptions we get:

$$\frac{\partial C_{\tau}}{\partial x_t} = -abT_{\tau}^{b-1}(1-s)Y_{\tau} \frac{\lambda}{X_{\tau} \ln 2} e^{-\beta(\tau-t)}.$$

Now assume that the economy is on a "steady-state," or "balanced," growth path, as described by the author—i.e., the atmospheric carbon concentration, temperature, and population are

constant, and aggregate (and per-capita) income is growing at the constant rate g . In this case we get the following expression for the social cost of carbon:

$$SCC_t = \sum_{\tau=t}^{\infty} abT^{b-1} \frac{\lambda}{X \ln 2} (1-s) Y_t e^{-[(\eta-1)g + \rho + \beta](\tau-t)} = \frac{abT^{b-1} \lambda / (X \ln 2) (1-s)}{(\eta-1)g + \rho + \beta} Y_t.$$

Because Y_t is the only state variable in this expression that is changing over time, the SCC must grow at the same constant rate, g .¹

If I understand the model in the paper correctly, then the simple example given above is a restricted version of the general model analyzed by the author. So it would seem that the author's main conclusion—that the SCC must *decrease* over time on a balanced growth path—should hold in this case. But this is in direct contradiction to the result derived above, so either I have erred in my derivation, or I have misunderstood the author's model, or there is an error in the author's analysis. If my derivation is in error, then I trust that the author will be able to quickly identify the flaw in my algebra. If our results are in fact consistent and I have merely misunderstood the author's model, then further clarification in the text would be helpful to save other readers from unnecessary confusion. If there is an error in the author's analysis, then this should be corrected and the results rewritten accordingly.

(Here's one possibility: The author's derivation begins with a present value Hamiltonian in equation 4, so perhaps the author's result pertains to the present value of the SCC rather than the current value as in my version above. Footnote 24 on page 11 seems consistent with this conjecture, since that footnote draws an analogy between the SCC and discounted rather than current instantaneous utility.)

Minor comments

This section contains a collection of minor comments, mostly but not exclusively editorial in nature. Nothing crucial here, but might as well clean these up while you're at it.

Page 1: "The literature usually employs a constant social discount rate that is inferred from historical data of the market rate of return." For climate change problems, I would say that the

¹ It turns out that this highly stripped-down model can give a reasonable first-order numerical approximation of the SCC, even ignoring the differences between the current levels of the atmospheric carbon concentration and temperature and their long run steady-state values. For example, following Nordhaus (2008) assume: $a = 0.0028388$, $b = 2$, $\lambda = 3$ degrees C, $\eta = 2$, $\rho = 0.015$ per year, $s = 0.22$, $X_{pj} = 594$ GtC, and $X_t = 804$ GtC. Also assume $g = 0.015$ per year, $\beta = 0.01$ per year, and $Y_t = 6.5 \times 10^9 \times 7000$ \$ per year. Using the observed current value of $T_t \approx 0.8$ degrees C gives $SCC = 5.9$ \$/tCO2. Using the equilibrium value of $T_t = \lambda \ln(X_t / X_{pj}) / \ln 2 \approx 1.3$ degrees C gives $SCC = 9.6$ \$/tCO2. Both of these crude estimates of the SCC are in the ballpark of the value calculated by Nordhaus (2008) using DICE2007, which was 7.7 \$/tCO2.

literature usually employs a Ramsey-style model where the discount rate moves over time with the change in the growth rate of per capita consumption.

Page 1: Change “have not been witness in the past” to “have not been witnessed in the past.”

Page 2: Consider changing “The latter requirement seems according to literature’s consensus view to be fulfilled in reality” to “The latter requirement is consistent with what seems to be the predominant view in the climate economics literature.”

Page 2: “...a deterministic Ramsey growth model with a standard carbon stock accumulation equation commonly employed in environmental economics.” The model of accumulation and decay used in this paper is the simplest possible version and is far simpler than most carbon cycle models used in standard IAMs. However, I think it is true that more realistic versions of this model would imply a steady state level of carbon and temperature in the (very) long run, so this is probably an acceptable simplification for the purposes this paper.

Page 6: “Uzawa (1961) has shown in a theorem that in such a model balanced growth requires technological progress to be constant and labor-augmenting, i.e., that we need $\Omega_{hat}=g=constant$ and the production function to have the form $Y=F(K,\Omega_{hat}*L)$. Applying the Uzawa theorem to our model implies that on the balanced growth path we need as well $\Omega_{hat}=g=constant$ and we need technical progress to be emission-augmenting.” The analysis here is aided by replacing labor with emissions in the aggregate production function. This makes me wonder if anything of substance would change if both labor and emissions were included in the production function and both were subject to technological change.

Page 7: Insert “for” before “this balanced growth path to be optimal requires...”

Page 8: Replace “...henceforth the index (*) denotes the steady state-level or the balanced growth path of the variable” with “...henceforth the index (*) denotes the steady-state level of the variable on the balanced growth path.”

Page 7+: Section 5 gives restrictions on preferences for balanced growth to be optimal. I agree that this is interesting to know, but it still makes me wonder why (or if) we should give such pride of place to balanced growth. Wouldn’t it make more sense to reverse the logic and derive the characteristics of the optimal growth path from specified restrictions on technology and preferences specified ex ante?

Page 11: Replace “...which is all what is required in the Proposition” with “...which is all that is required in the Proposition.”