Referee report for Pascal Stiefenhofer's
"The natural projection approach to production and uncertainty."

This paper aims to extends the natural projection approach to the case of a two-period private ownership production model with uncertainty. In doing so, the author studies the structure of the equilibrium set showing that it is closed, a smooth manifold and diffeomorphic to Euclidean space. He also studies the projection map from the equilibrium set to the set of economies, showing that it is a proper and smooth map of degree one. This completes the proof that an equilibrium always exist in this extension. Furthermore, the author extends the study to analyze the sets of critical equilibrium prices and the set of critical economies. He shows that both sets are closed, and the latter also has Lebesgue measure zero. The author continues to establish that around regular economies, locally equilibria vary continuously as functions of the parameters that define the economy.

Overall, a paper that extends the natural projection approach to production should be interesting to a general equilibrium theorist, although difficult to justify its appropriateness for a general interest journal unless it gives new insight or solves a well-known open problem. I don't feel that this work does either.

My main worry is that the paper seems to overlap considerably with an unpublished paper of Balasko (2010) that is not cited. Unless it is clear which is the additional contribution of this paper, which to me seems thin, I would tend to reject it the way it is. Balasko's paper is interesting since it reformulates the production model, by suitably adjusting demand functions, to restore the exchange economy framework. He then applies this machinery to show that, among other results, the equilibrium set for production economies is a manifold diffeomorphic to Euclidean space and that the natural projection is proper of degree one. The introduction of uncertainty in this paper seems like a rather thin contribution.

However, given that the author was not aware of Balasko's paper, given that the contribution is mainly technical, it would have seemed important anyway to compare these results with those of Kehoe (1983, 1985), Fuchs (1974), Jouini (1993) and Smale (1974). For example, Fuchs and Smale es-

tablish generic finiteness, Kehoe establishes parity of equilibria, and Jouini shows topological properties of the equilibrium set. All of these are results that this paper aims to establish.

Finally, given that the introduction is so important to motivate the paper, it seems appropriate to fix some of its problems and inaccuracies, some of which are detailed below.

Other Suggestions.

1. The introduction is disorganized and the author should probably aim to make it self contained. For example, he mentions the maps $(p, \omega) \mapsto \omega$ and $z(p, \omega) = 0$ without saying what any of these terms are. They seem standard through Balasko's papers and books, but perhaps not outside of them.

2. Also, within the introduction, there are several inaccuracies:

   (a) The author mentions that existence is a consequence of the projection mapping being smooth and proper. However, it is a consequence of the projection map having degree one (mod 2).

   (b) "The number of equilibria is not only finite but always odd and constant for some sections of the parameter space" is not correct. It should be corrected to reflect that this is true generically and locally. Also, the word "sections" is too vague.

3. There are several historical inaccuracies:

   (a) The author mentions in the abstract that the natural projection approach was introduced in (Balasko, 1988). A more accurate paper would be (Balasko, 1978) as the author himself mentions in the introduction.

   (b) It is mentioned that "Originally [...] the natural projection approach is studied in the context of static exchange models." There are many papers in the literature studying time of the form $t \in \{0, 1, \ldots, T\}$, $t \in [0, T]$ or $t \in \mathbb{N}$ with the natural projection approach or through excess demand functions. It can be shown that for time of the form $t \in \{0, 1, \ldots, T\}$, it reduces to the static model

(Debreu himself knew this) while a good starting point might be Mas-Colell and Zame (1991) for the other two time sets.

References

[1] Balasko, Y. "The natural projection approach to smooth production economies." (2010) mimeo.

[2] Kehoe, T.J. "Regularity and index theory for economies with smooth production technologies." Econometrica 51 (1983) 895-918.

[3] Kehoe, T.J. "Multiplicity of equilibria and comparative statics." Econometrica 100 (1985) 119-147.

[4] Mas-Colell, A. and Zame, W.R. "Equilibrium theory in infinite dimensional spaces." Handbook of Mathematical Economics (1991) volume IV, chapter 34, p. 1835- 1898.

[5] Smale, S. "Global analysis and economics IV: finiteness and stability with general consumption sets and production." Journal of Mathematical Economics 1 (1974) 119-128.