Solving the Paradox of Monetary Profits

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Abstract
Bruun and Heyn-Johnsen (2009) state the paradox that economics has failed to provide a satisfactory explanation of how monetary profits are generated, even though the generation of a physical surplus is an established aspect of non-neoclassical economics. They emphasise that our ability to explain phenomena like the Global Financial Crisis (GFC) will be limited while ever we are still unable to explain this fundamental aspect of capitalism. In fact this paradox can be solved very simply, using insights from what is known as “Circuit Theory”. In this paper the author shows how monetary profits are generated, and introduces a multisectoral dynamic disequilibrium monetary model of production.

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Solving the Paradox of Monetary Profits

Bruun and Heyn-Johnsen (2009) state the paradox that economics has failed to provide a satisfactory explanation of how monetary profits are generated, even though the generation of a physical surplus is an established aspect of non-neoclassical economics. They emphasise that our ability to explain phenomena like the Global Financial Crisis (GFC) will be limited while ever we are still unable to explain this fundamental aspect of capitalism.

In fact this paradox can be solved very simply, using insights from what is known as “Circuit Theory” (Graziani (1990) and Graziani (1995)). Graziani’s brilliant insight was that a credit economy must be using a non-commodity as money, since the alternative of “an economy using as money a commodity coming out of a regular process of production, cannot be distinguished from a barter economy” Graziani (1995, p. 518). From the insight that in a credit economy, an intrinsically valueless token is nonetheless accepted as full payment in the exchange of goods, Graziani derived the conclusion that:

any monetary payment must therefore be a triangular transaction, involving at least three agents, the payer, the payee, and the bank… Since in a monetary economy money payments go necessarily through a third agent, the third agent being one that specialises in the activity of producing means of payment (in modern times a bank), banks and firms must be considered as two distinct kinds of agents. Graziani (1995, pp. 518-519)

Unfortunately, attempts by Graziani and subsequent Circuitist authors to turn this insight into a viable model of the creation of monetary profits in a pure credit economy have to date been a failure—a situation that was well expressed in the Rochon’s lament in 2005: "How does M become M+?" Rochon (2005, p. 125).

However, this failure was not due to any weakness in the underlying vision of a pure credit economy, but to the inappropriate tools that these authors employed to consider the issue. A simple model that is explicitly both monetary and dynamic, and uses bank accounts as its fundamental concept, explains how capitalists can and do make a profit. In brief, "M becomes M+" via the price mechanism converting the physical surplus generated in production into money.

The basic Circuit model

Though Graziani verbally specifies that his model includes a Central Bank, this was not actually implemented in his or any other Circuit model. I will stick with this tradition of modelling a Wicksellian pure credit economy (though the model can be extended to include a government sector, a Central Bank and fiat money), leaving the topic of the creation of fiat money (and the relationship between credit and fiat money) to a later paper.

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1 At a simplistic level, Neoclassical theory has its marginal productivity theory of income distribution, but that is subject to all the flaws pointed out in the Cambridge Controversies. The most succinct refutation of this model was given by Bhaduri, A. 1969, ‘On the Significance of Recent Controversies on Capital Theory: A Marxian View’, The Economic Journal, vol. 79, no. 315, pp 532-539. See Debunking Economics pp. 134-137 for a simple elucidation of Bhaduri’s argument.
The starting point of Graziani’s model is the issuance of loans by the banking sector to the firm sector to finance production (Graziani (2003, pp. 26-27)). I will start at that point, and express this entirely in terms of the flows this starting point sets in motion. Initially I use the absolutely minimal set of sectoral bank accounts needed: a Loan account and a Deposit account for the firm sector \((F_t \& F_D\) respectively), a Deposit account for workers \((W_D)\) and a transactions account for the Banking Sector \((B_t)—\) using \(I\) rather than \(D\) as a subscript here to indicate that transactions through this account are the source of bank income, and also to distinguish it from accounts on which the Banking sector is obliged to pay a deposit rate of interest).

The model begins with the banking sector extending a loan of \$.A to the firm sector; this initialises the system by creating \$.A of credit money stored in the \(F_D\) account, for which there is a matching record of debt in \(F_t\).

The minimum set of flows that this creation of credit money sets in train is:

1. Accrual of interest \((A)\) compounds the outstanding debt in \(F_t\) at the rate \((r_t)\) specified in the loan contract;
2. Assuming that the firm sector meets its debt-servicing obligations in full, a flow of money \((\text{also } A)\) from \(F_D\) to \(B_t\) offsets the compounding of debt in the first operation;
3. A flow of money \((B)\) from \(B_t\) to \(F_D\) pays the firm sector interest on its deposits at the rate \(r_D\)—a lower rate than that charged on debt;
4. A flow of money \((C)\) from \(F_D\) to \(W_D\) pays wages to workers (who are then employed in factories to produce output for sale);
5. A flow of money \((D)\) from \(B_t\) to \(W_D\) pays workers interest on their bank balances; and
6. A flow of money \((E \& F)\) from both \(B_t\) and \(W_D\) goes to \(F_D\) to pay for the output from the factories owned by the firm sector.

These flows are set out in Table 1 using simply the alphabetic markers \(A \text{ to } F\).

Table 1: Minimal Circuit Model

<table>
<thead>
<tr>
<th>Account/Operations</th>
<th>Firm Loan (F_t)</th>
<th>Firm Deposit (F_D)</th>
<th>Worker Deposit (W_D)</th>
<th>Bank Income (B_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&amp;2</td>
<td>(A)</td>
<td>(-A)</td>
<td>+A</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(B)</td>
<td>(-B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(-C)</td>
<td>(+C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(+D)</td>
<td>(-D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(E)</td>
<td>(-E)</td>
<td>(-F)</td>
<td></td>
</tr>
</tbody>
</table>

We can now consider the conditions under which this financial system will reach equilibrium. These conditions are summarised in Table 2.

Table 2: Stability conditions for the financial system in Table 1

<table>
<thead>
<tr>
<th>Account</th>
<th>Alphabetic condition</th>
<th>Verbal Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_D)</td>
<td>(B+D+E+F=A+C)</td>
<td>Deposit interest plus workers’ and bankers' consumption equals debt interest plus wages</td>
</tr>
<tr>
<td>(W_D)</td>
<td>(C+D=E)</td>
<td>Wages plus deposit interest equals workers’ consumption</td>
</tr>
<tr>
<td>(B_t)</td>
<td>(A=B+D+F)</td>
<td>Loan interest equals deposit interest plus bankers’ consumption</td>
</tr>
</tbody>
</table>
These conditions can be specified more precisely by making simple constant parameter substitutions:

1. \( A \) is the loan rate of interest \( r_L \) times the amount outstanding in the loan account \( F_L \). Since this model considers a single loan of \( \Lambda \) and no compounding or repayment of the loan, we can write \( A = r_L \cdot \Lambda \);
2. \( B \) is the deposit rate of interest \( r_D \) times the balance in \( F_D \);
3. \( C \) is some factor (say \( w \)) of the current balance in \( F_D \); thus we can substitute that \( C = w \cdot F_D \);
4. \( D \) is the deposit rate of interest times the balance in \( W_D \); \( D = r_D \cdot W_D \); and
5. \( E \) and \( F \) will be some factor (say \( \omega \) and \( \beta \) respectively) of the balances in the accounts \( W_D \) and \( B_i \); thus \( E = \omega \cdot W_D \) and \( F = \beta \cdot B_i \).

Now we can solve for the equilibrium levels of these accounts—which will give us the conditions that generate a stable financial system, and by implication a stable production system driven by these financial flows:

\[
F_L = \Lambda \\
F_D = \frac{(\beta - r_L) \cdot (\omega - r_D)}{(\beta - r_D) \cdot (\omega - r_D + W)} \cdot \Lambda \\
W_D = \frac{\omega \cdot (\beta - r_L)}{(\beta - r_D) \cdot (\omega - r_D + W)} \cdot \Lambda \\
B_i = \frac{r_L - r_D}{(\beta - r_D)} \cdot \Lambda \\
\]

The conditions under which these four equilibrium conditions result in positive bank balances for all sectors are that:

1. The loan \( \Lambda \) is positive;
2. \( \beta > r_L \) and \( \beta > r_D \);
3. \( \omega > r_D \);
4. \( w > 0 \) and \( w + \omega > r_D \); and
5. \( r_L > r_D \).

Since 1 and 5 are obvious, these conditions collapse to:

1. \( \beta > r_L \);
2. \( \omega > r_D \); and
3. \( w > 0 \)

This raises the question of what the parameters \( \beta \), \( \omega \) and \( w \) signify. The first two describe the rate of flow of money out of the \( B_i \) and \( W_D \) accounts respectively to pay for consumption. The values of \( \beta \) and \( \omega \) are therefore numerical estimates of how often bankers and workers respectively turn over the balances in their accounts in a year (the time period can be specified precisely here because interest rates are also in terms of interest per year). These parameters are thus the ratios between...
consumption over a year, and the average balance in the respective accounts at any point in the year.

A reasonable value for $\omega$ would be 26, since workers tend to spend their wages on a fortnightly basis. For $\omega$ to be less than $r_D$—which would in turn imply a negative balance in $F_D$ and hence bankrupt firms—workers would need to be able to survive on the balance in their accounts for something like 50 years (if $r_D$ was 2%). That option is fairly easily dismissed.

Bankers, who are much wealthier, can afford to spend their account balances much more slowly than workers—a figure of 1 could be in the ballpark. But only if bankers made Ebenezer Scrooge look like a spendthrift—by taking more than 20 years to spend the balance outstanding in their accounts at any one point in time—would $\beta$ be less than $r_L$ (if the real rate of interest was 5%), and hence workers bank balances would be negative.

The third condition simply states that firms must pay workers wages; so long as they do, and the other two conditions are satisfied, all account balances will be positive in equilibrium. Figure 1 to Figure 3 show the impact of varying these three parameters on the equilibrium balances in the three accounts (given $A=$$100, $r_L=5\%$ p.a. and $r_D=1\%$ p.a.).

![Account Balances as function of $\beta$](image)

**Figure 1**
Solving the Paradox of Monetary Profits

Account Balances as function of $\omega$

$\omega$ from rd to 100 (given $\beta=1, w=4$)

Figure 2

Account Balances as function of $w$

$w$ from 0 to 100 (given $\beta=1, \omega=26$)

Figure 3

**From Account Balances to Incomes**

The above calculations establish that a single act of credit money creation can lead to a financial system in which all sectoral account balances are positive (prior to the consideration of debt repayment). From this information we can also derive the conditions under which all income flows will be positive—including profits.
The annual equilibrium income levels from wages and interest income are easily derived: wage income per annum equals \( w F_0 \) while gross interest income is \( r_c A \). Given the assumption of a constant rate of loan interest \( r_c = 5\% \) and a constant level of debt \( A = \$100 \), the latter is simply \$5 per annum (where this would be \$5 billion if the loan were \$100 billion). The former depends on the value of \( w \), as indicated in Figure 4.

Annual Wage Income as a function of \( w \)

\[
\begin{array}{c|c}
w & \text{annual wage income (} \$\text{)} \\
\hline
0 & 0 \\
1 & 100 \\
2 & 200 \\
3 & 300 \\
4 & 400 \\
5 & 500 \\
\end{array}
\]

![Annual Wage Income as a function of \( w \)](image)

w from 0 to 10 (given \( \beta = 1, \quad \omega = 26 \))

**Figure 4**

What should be apparent from Figure 4 is that for any value of \( w \) slightly above 1, the annual wages bill exceeds the value of the initial loan \( A \): *the flows initiated by the loan over a year exceed the size of the loan itself*. This highlights one of the key reasons why Circuit theorists have thus far failed to show how capitalists can borrow money, pay interest and make a profit: they have confused the stock of money \( (A) \) with the flows that stock can generate over time.\(^2\)

In fact, as the firms’ deposit accounts are depleted by wage and interest payments, they are replenished by consumption expenditure by the other classes in the economy (and interest earned on deposit). The stock money can thus circulate several times in one year as a result—something that Marx clearly enunciated over a century ago in Volume II of *Capital*:

“*Let the period of turnover be 5 weeks, the working period 4 weeks... In a year of 50 weeks ... Capital I of £2,000, constantly employed in the working period, is therefore turned over 12½ times. 12½ times 2,000 makes £25,000.*” Marx and Engels (1885, Chapter 16: The Turnover of Variable Capital)

Aggregate wages and aggregate profits therefore depend in part upon the turnover period between the outlay of money to finance production and the sale of that production. This turnover period can be substantially shorter than a year, in which case \( w \) will be substantially larger than 1.

---

\(^2\) This statement from Graziani 1989 is indicative of the error of confusing the initial loan with the volume of transactions that can be generated by such a loan over a year: “If on the other hand, wage-earners decide to keep part of their savings in the form of liquid balances (that is, banking deposits), firms will get back from the market less money than they have initially injected in it” (Graziani 1989, p. 520).
Solving the Paradox of Monetary Profits

A second fundamental insight from Marx lets us explain what \( w \) is, and simultaneously derive an expression for profits: the annual wages bill reflects both the turnover period, and the way in which the surplus value generated in production is apportioned between capitalists and workers. The value of \( w \) therefore reflects two factors: the share of surplus (in Sraffa’s sense) that accrues to workers (I depart from Marx and follow Sraffa here by specifying the division of surplus between capitalists and workers in such a way that the sum is 1, so that if capitalists get \( s\% \) of the surplus, workers get \( (1-s)\% \)); and the turnover period measured in years—the time between \( M \) and \( M^+ \). Labelling the latter \( \tau_S \) (p.a.) and expressing it as a fraction of a year, I can perform the substitution shown in Equation (2):

\[
\frac{1 - s}{\tau_S} \quad (2)
\]

Aggregate wages are therefore \((1-s).F_o/\tau_S\). Since national income resolves itself into wages and profits (interest income is a deduction from other income sources), we have also identified that gross profit equals \( s.F_o/\tau_S \). The next two figures show the dependence of both wages and profits on \( s \) and \( \tau_S \).

Wages and Profits as functions of \( \tau_S \)

![Graph showing the dependence of wages and profits on \( \tau_S \)](image)

\( \tau_S \) (given \( \beta=1, \omega=26, s=0.3 \))

Figure 5

---

\(^3\) If this seems like a Milton Friedman magic trick (“Putting a rabbit into a hat in full view of the audience, and then expecting applause when he later pulls it out again”, to quote Joan Robinson from a talk she gave to Sydney University students in 1974), bear with me—later I show that profits can also be derived from the production system.
Solving the Paradox of Monetary Profits

Wages and Profits as functions of \( s \)

\[ \text{Income in $/year} \]

\[ s \text{ (given } \beta=1, \ \omega=26, \ \tau_S=0.25) \]

Figure 6

We can now specify the conditions under which profits net of interest payments will be positive. Given that wages typically represent 70% of national income, a value 0.3 for \( s \) is acceptable, and I use that in Figure 7. As this indicates, the condition for net profit to be positive is hardly onerous. Only if \( \tau_S \) is greater than 7—so that it takes more than seven years to go from \( M \) to \( M^+ \)—will net profit be negative.

Class incomes as functions of \( \tau_S \)

\[ \tau_S \text{ from 0 to 10 (given } \beta=1, \ \omega=26, \ s=0.3) \]

Figure 7

Given that the annual wages bill was between 2 and 3 times the level of corporate debt in America before it became the Ponzi-dominated charade it is today, a value of \( \tau_S=0.25 \)—which asserts that the time between \( M \) and \( M^+ \) is about 3 months—seems apt. For our hypothetical pure credit economy, this implies the following table of equilibrium account balances, gross and net incomes:

Table 3

<table>
<thead>
<tr>
<th>( \tau_S ) from 0 to 10</th>
<th>Account Balances</th>
<th>Gross Income p.a.</th>
<th>Net Income p.a.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solving the Paradox of Monetary Profits

<table>
<thead>
<tr>
<th>Firms/Capitalists</th>
<th>86.627</th>
<th>103.952</th>
<th>99.819</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>9.333</td>
<td>242.555</td>
<td>242.649</td>
</tr>
<tr>
<td>Bankers</td>
<td>4.04</td>
<td>5</td>
<td>4.04</td>
</tr>
</tbody>
</table>

Even though capitalists are net debtors in this model (something that Schumpeter was both conscious of and quite comfortable about), it should be obvious that with an equilibrium net profit of $99.819 p.a., it would not be difficult to repay this debt.4

**Production and Prices**

This simple model does not yet have the structure needed to explicitly model fixed capital, so I rely upon a simple production system in which output is produced by labour:

\[
Q = a \cdot L
\]  

Labour employed in turn equals the monetary flow of wages divided by the wage rate:

\[
L = \frac{1 - s}{\tau_s} \cdot F_D \div W
\]

Finally prices are needed to link this physical output subsystem to the financial model above. In equilibrium, it must be the case that the physical flow of goods produced equals the monetary demand for them divided by the price level. We can therefore derive that in equilibrium, the price level will be a markup on the monetary wage, where the markup reflects the rate of surplus as defined in this paper. To answer Louis-Philippe Rochon’s vital question, M becomes M+ via a price-system markup on the physical surplus produced in the factory system.

This markup can be derived simply by considering demand and supply factors in equilibrium. The flow of demand in equilibrium will be the sum of wages and profits (since interest payments are a transfer and do not contribute to the value of output—despite Wall Street’s bleatings to the contrary). The monetary value of demand is thus:

\[
D_M = \frac{1 - s}{\tau_s} \cdot F_D + \frac{s}{\tau_s} \cdot F_D = \frac{F_D}{\tau_s}
\]

The physical units demanded equals this monetary demand divided by the price level:

\[
D = \frac{D_M}{\bar{P}} = \frac{1}{\tau_s} \cdot \frac{F_D}{\bar{P}}
\]

In equilibrium this physical demand will equal the physical output of the economy:

---

4 The issue of debt repayment raises yet another series of conundrums that have to date hobbled Circuit Theory. I therefore delay this topic until after a production component has been added to the model.
Solving the Paradox of Monetary Profits

\[ Q = a \cdot \frac{1 - s}{\tau_S} \cdot \frac{F_D}{W} \]
\[ D = \frac{1}{\tau_S} \cdot \frac{F_D}{P} \]

(7)

Solving for \( P \) yields:

\[ \frac{1}{\tau_S} \cdot \frac{F_D}{P} = a \cdot \frac{1 - s}{\tau_S} \cdot \frac{F_D}{W} \]
\[ P = \frac{1}{\tau_S} \cdot \frac{F_D}{a \cdot \frac{1 - s}{\tau_S} \cdot \frac{F_D}{W}} \]
\[ P = \frac{1}{(1 - s)} \cdot \frac{W}{a} \]

(8)

The markup is thus the inverse of workers' share of the surplus generated in production. Circuit theory therefore provides a monetary expression of Marx's theory of surplus value, as it was always intended to do.\(^5\)

With these physical and price variables added to the system, we are now able to confirm that profit as derived from the financial flows table corresponds to profit as the difference between the monetary value of output and the wage bill (in this simple single-sectoral model).

As a preliminary step here I recast all parameters in this system into time constants\(^6\)—thus \( \beta \) is replaced by \( \tau_B \) and \( \alpha \) by \( \tau_W \) (with the dimension of years). The reason for this is that time constants provide a consistent way of describing the components of a dynamic model in terms of its fundamental frequency. A value of \( \omega = 26 \) may look arbitrarily large; the corresponding value of \( \tau_W = 1/26 \) asserts that workers spend their accounts at a frequency of 1/26th of a year, and is rather more easily accepted as feasible. Time constants also show the dimensional relationships in the model more clearly.

The form of the equilibrium is much more complex—given the use of time constants and the addition of three more terms (\( Q, L \) and \( P \))—but the numerical values confirm that this is the same result as earlier:

---


\(^6\) See [http://en.wikipedia.org/wiki/Time_constant](http://en.wikipedia.org/wiki/Time_constant) for an explanation of this concept.
A simple calculation shows that the two measures of equilibrium gross profit—(s.\(F_{De}/\tau_s\)) and (\(P_eQ_e−W.L_e\))—both equal 103.952.

The price relation given above applies only in equilibrium. Out of equilibrium, it is reasonable to postulate a first-order convergence to this level, where the time constant \(\tau_p\) reflects the time it takes firms to revise prices. This implies the following dynamic pricing equation:

\[
\frac{d}{dt} P = -\frac{1}{\tau_p} \left( P - \frac{1}{1-s} \cdot \frac{W}{a} \right)
\]

Empirical research by Blinder and others implies that \(\tau_p\) is roughly one year (Blinder (1998)). We are now able to assemble a simple dynamic model of a pure credit economy. To do so, I employ a novel method of generating models of financial dynamics, which makes use of the power of many computer algebra programs to perform symbolic as well as numeric calculations.

**Dynamics and Symbolic Mathematics**

A table like Table 1 is input as a matrix (see Figure 8):
Solving the Paradox of Monetary Profits

The alphabetic placeholders are then replaced with appropriate functions:

\[
\begin{align*}
A &:= r_L F_L(t) \\
C &:= r_D F_D(t) \\
E &:= r_D W_D(t) \\
G &:= \frac{B_I(t)}{\tau_B} \\
B := A & \quad D := \frac{1 - s}{\tau_S} F_D(t) \\
F := \frac{W_D(t)}{\tau_W} \\
\end{align*}
\]

The program returns a set of differential equations that can be analysed (as is done above), or simulated (as will be done below):

\[
\text{System } \left( S_2 \right) \rightarrow \begin{cases}
\frac{d}{dt} F_L(t) = 0 \\
\frac{d}{dt} F_D(t) = r_D F_D(t) - r_L F_L(t) + \frac{B_I(t)}{\tau_B} + \frac{W_D(t)}{\tau_W} + \frac{F_D(t)(s - 1)}{\tau_S} \\
\frac{d}{dt} W_D(t) = r_D W_D(t) - \frac{W_D(t)}{\tau_W} - \frac{F_D(t)(s - 1)}{\tau_S} \\
\frac{d}{dt} B_I(t) = r_L F_L(t) - r_D F_D(t) - r_D W_D(t) - \frac{B_I(t)}{\tau_B} \\
\end{cases}
\]

Not only is the table in Figure 8 much easier to read than the equations in Figure 10, the tabular approach enables models of arbitrary complexity to be assembled in confidence, as is demonstrated later.

Having shown that the model is coherent, explains where profit comes from, and shows that a constant stock of credit money can sustain a constant level of economic activity, I now turn to another conundrum in Circuit Theory—the impact of the repayment of debt.

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7 In this table I separate the operation of debt compounding (A) from the payment of interest on the debt (B). Since I assume here that \( B=A \), this table is equivalent to Table 1, but this formulation allows for the real world phenomenon that some firms do not pay some of the interest due on their debt, in which case \( B\lt A \) and the bank then adds the unpaid interest to the outstanding debt.
Debt repayment and anti-Keynes Post Keynesianism

There is a long tradition in Post Keynesian economics of arguing that the repayment of debt destroys the credit money that was created by that debt. Comments to that effect can be found in Minsky and Graziani as well as any number of other Circuit theorists:

“To the extend that bank debts are repaid, an equal amount of money is destroyed” (Graziani (2003, pp. 29-30))

“Money is created as banks lend-mainly to business-and money is destroyed as borrowers fulfill their payment commitments to banks. Money is created in response to businessmen’s and bankers’ views about prospective profits, and money is destroyed as profits are realized.” (Minsky (1982, p. xxii))

This conventional Post Keynesian/Circuit position directly contradicts Keynes on the same topic. Keynes did not see the repayment of debt as destroying money, but instead argued that the repayment of debt by one entrepreneur made that same credit available to be relent to another one:

If investment is proceeding at a steady rate, the finance (or the commitments to finance) required can be supplied from a revolving fund of a more or less constant amount, one entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment. Keynes (1937, p. 247)

So we have a clash of authorities over what is really a dispute over banking practice. In this section I show that Keynes’ vision of a “revolving fund” is consistent with a constant-output economy as envisaged in Keynes (1937), whereas the “money destruction” scenario results in an economy that tapers to zero output—unless there are matching acts of new money creation to balance the destruction of money by loan repayment. In both scenarios, repayment of debt is easily accomplished.

The revolving fund

Modelling Keynes’s concept of a revolving fund of finance requires the introduction of an additional "bank reserve account" ($B_0$) into which money affecting debt repayment is transferred, and where it is stored prior to be relent to new borrowers as outlined in Keynes’s statement above. Two further operations are also needed—one to record the repayment of debt, and another to record the relending of current idle reserves:

1. A flow of money ($H$) from $F_0$ to $B_r$ repays the debt, and the bank records this payment by reducing the recorded amount of the debt in $F_1$ by $H$, and
2. A flow of money ($I$) from $B_r$ to $F_0$ relends the currently out of circulation money to the firm sector, and the bank records this as an increase in the indebtedness of the firm sector in $F_1$. 
Solving the Paradox of Monetary Profits

The model is implemented by making the substitutions that $H=FL/\tau_L$ and $I=BR/\tau_R$; in the simulations below $\tau_L=7$ years and $\tau_R=2$ years. The outcome confirms Keynes’s expectations: a constant level of economic activity can be sustained as money repaid by one entrepreneur is lent to another (See Figure 12 and Figure 13).

Bank Accounts with a Revolving Fund

Figure 11

Figure 12
Money destruction
Implementing the conventional Circuitist money destruction perspective also requires the introduction of an additional account in which money is stored prior to its destruction, which I label $M_B$ (for "Money Bin") rather than $B_B$—since its purpose is similar to a waste disposal bin, in that whatever is thrown in there is scheduled for later destruction—where $J = M_B / \tau_{MD}$ and $\tau_{MD} = 1/12$th of a year (one month). The debt repayment operation is otherwise equivalent to the repayment of debt in the revolving fund model above. The new operation in this model is therefore the destruction of money:

1. A flow of money in $M_B (J)$ is destroyed.

$$M_D := \begin{pmatrix}
    "Type" & "Firm Loan" & "Money Bin" & "Firm Deposit" & "Worker Deposit" & "Bank Income" \\
    "Account" & F_L(t) & M_B(t) & F_D(t) & W_D(t) & B_I(t) \\
    "Compound Interest" & A & 0 & 0 & 0 & 0 \\
    "Pay Interest on Loan" & -B & 0 & -B & 0 & B \\
    "Interest on Deposit" & 0 & 0 & C & 0 & -C \\
    "Wages" & 0 & 0 & -D & D & 0 \\
    "Interest on Deposit" & 0 & 0 & 0 & E & -E \\
    "Consumption" & 0 & 0 & F + G & -F & -G \\
    "Debt Repayment" & -H & H & -H & 0 & 0 \\
    "Money Destruction" & 0 & -J & 0 & 0 & 0 \\
\end{pmatrix}$$

Figure 14 describes a doomed economy: as debt is repaid and money destroyed, all accounts head monotonically towards zero—except the money destruction bin, which rises initially as the inflow of money to be destroyed exceeds the rate of destruction. Output and employment likewise descend to zero; only price remains constant to the bitter end. Contrary to conventional Circuitist belief, firms
have no difficulty in repaying the debt, but it is in everybody's interests—workers, bankers and capitalists alike—that they do not do so.

The situation can be remedied by introducing new money creation at a rate that exactly balances money destruction—in which case the difference between the two approaches becomes semantic. In the next section I introduce money creation while continuing with Keynes’s concept of a revolving fund for the existing stock of money.
Endogenous money creation, physical output and economic growth

The expansion of the monetary system to accommodate growth necessitates just one more row to the transactions table in which new money and debt are created simultaneously by matching entries in the $F_L$ and $F_D$ columns:

1. A flow of new money ($J$) is added to $F_D$, which is also recorded as an increase in the level of debt in $F_L$.

Figure 17 shows the implementation of this step in the symbolic matrix:

\[
E_M := \begin{pmatrix}
"Type" & 1 & 0 & -1 & -1 & 0 \\
"Account" & "Firm Loan" & "Bank Reserves" & "Firm Deposit" & "Worker Deposit" & "Bank Income" \\
"Account" & $F_L(t)$ & $B_R(t)$ & $F_D(t)$ & $W_D(t)$ & $B_I(t)$ \\
"Compound Interest" & A & 0 & 0 & 0 & 0 \\
"Pay Interest on Loan" & -B & 0 & -B & 0 & B \\
"Interest on Deposit" & 0 & 0 & C & 0 & -C \\
"Wages" & 0 & 0 & -D & D & 0 \\
"Interest on Deposit" & 0 & 0 & 0 & E & -E \\
"Consumption" & 0 & 0 & F + G & -F & -G \\
"Debt Repayment" & -H & H & -H & 0 & 0 \\
"Relending Reserves" & I & -I & I & 0 & 0 \\
"Money Creation" & J & 0 & J & 0 & 0 
\end{pmatrix}
\]

Several extensions to the physical side of the model are required. In the absence of Ponzi speculation (which will be introduced in a subsequent paper), growth in the money supply is only warranted if economic growth is occurring, which in turn requires a growing population and/or labour productivity. These variables introduce the issue of the employment rate, and this in turn raises the possibility of variable money wages in response to the rate of unemployment—a Phillips curve.  

These additional variables are specified in Equation (11):

\[
\begin{align*}
\frac{da}{dt} &= \alpha \cdot a \\
\frac{dN}{dt} &= \beta \cdot N \\
\frac{dW}{dt} &= p_h \left( \frac{L}{N} \right) \cdot W
\end{align*}
\]

---

8 The functional form used here is a generalised exponential function $g(x) = (x_0 - m) \cdot e^{s \cdot (x - x_0)} + m$, where $x$ is the argument (in this case, the unemployment rate), $(x_0, y_0)$ is a coordinate on the curve, $s$ the slope of the curve at that point and $m$ the minimum value of the function. In this simulation $(x_0, y_0) = (0.92, 0)$, $s=1$ and $m=0.04$; this means that at an unemployment rate of 8%, money wages do not change, they rise by 25% p.a. at full employment (0% unemployment), and they fall at a maximum rate of 4% p.a. at high levels of unemployment.
In the simulations shown in Figure 18 and Figure 19, \( \alpha = 1.5\% \), \( \beta = 1\% \), initial labour productivity was set at 1 unit of output per worker per year and the initial money wage at $1 per worker per year (these are obviously index numbers, and more realistic figures derived from national accounts could also be used).

Even though this model is still very simple, in Keen (2009a) I show that it reaches substantive policy conclusions that differed from neoclassical and "Keynesian" equilibrium doctrine, by considering the effectiveness of a government stimulus injection in response to a "credit crunch." The "money
multiplier" perspective on money creation asserts that a government stimulus would be more effective if given to the banks than the debtors, since this would create additional money equivalent to the injection divided by the reserve ratio—so that there would be a "multiplier effect" if the stimulus were given to the banks, but no such effect if the stimulus were given directly to the public (and indeed Obama used precisely this argument to explain his stimulus package in April 2009; Obama (2009)).

However capitalism itself is not as stable as this model. To approach the complex cyclical—and sometimes chaotic—nature of capitalism itself, we need to go beyond the simple abstraction of single-sectoral production and consider the multi-sectoral nature of production.

What follows is a brief outline of a very recently completed preliminary multisectoral monetary model of production. This research is still tentative, and substantial work is still needed to validate its structure.9

**Combining Goodwin, Minsky and Graziani**

Richard Goodwin's growth cycle model, which gave mathematical form to Marx's income-distribution-driven verbal model of growth cycles in Marx (1867, Chapter 25, Section 1), was one of the outstanding achievements of non-neoclassical economic thought Goodwin (1967). In Keen (1995) I showed that this model could be based to model Minsky's theory of financial instability by incorporating a nonlinear desire to invest and a banking sector that lent whenever desired investment exceeded profits. However money was implicit in the model: there was debt—incurred whenever the desire to invest exceeded profits—but no explicit consideration of the monetary dynamics.

Armed with a mathematical model of monetary dynamics courtesy of Graziani's vision, it is now possible to build a strictly monetary version of Goodwin's model. This also necessitates considering the multi-commodity nature of production, because there has to be a capital goods sector—distinct from consumer and other goods—from which capital goods are purchased by monetary transfers.10

---


10 A genuine dynamic multi-sectoral model of production has eluded economics to date. Almost all attempts to build such a model have worked with abstractions such as ignoring money, working in discrete rather than continuous time, and assuming equilibrium prices and levels of output applied at all time steps to sidestep the complexities of disequilibrium dynamics.

This line of research has largely been abandoned by input-output theorists because dynamic input-output models with some or all of the above characteristics proved to have critical instabilities: magnitudes that by necessity had to be positive—such as prices and sectoral outputs—necessarily returned negative values in dynamic simulations.

The roots of this problem lie in the nature of matrix algebra. Whatever “production function” is used necessarily has a linear input-output matrix as its dominant component near equilibrium—this is simply a result of a Taylor expansion. That matrix necessarily has non-negative entries since production requires non-zero levels of inputs, and it was proven decades ago—as an exercise in pure mathematics—that the scalar
Solving the Paradox of Monetary Profits

Goodwin’s model is easily specified in six causal steps:

1. Capital ($K$) determines Output ($Y$) via an accelerator relation ($\nu$): $Y = K/\nu$;
2. Output determines employment ($L$) via labour productivity ($a$): $L = Y/a$;
3. Employment determines the employment rate ($\lambda$) given population ($N$): $\lambda = L/N$;
4. The employment rate determines the rate of change of wages ($w$) via a Phillips Curve; 
   \[ \frac{1}{w} \frac{d}{dt} w = p_n(\lambda) ; \]
5. Output minus Wages times Employment determines Profit ($\Pi$); $\Pi = Y - w \cdot L$; and
6. All profits are invested ($I$), the integral of which (minus depreciation $\gamma \cdot K$) determines the 
capital stock, completing the model: 
   \[ \frac{d}{dt} K = I - \gamma \cdot K = \Pi - \gamma \cdot K \]

My 1995 Goodwin-Minsky model (Keen (1995)) replaced the unrealistic assumption that all profits are 
invested with a nonlinear investment function ($I/I/K$), and introduced the rate of change of debt ($D$), which was driven by the difference between this investment function and net profits ($Y-W\cdot r\cdot D$): 
   \[ \frac{d}{dt} D = I - \Pi \]. In Keen (2009b) I added a "Ponzi Investment" component to the model where 
Ponzi Investment— speculation on asset prices rather than the construction of new assets—was a 
nonlinear function of the rate of economic growth: 
   \[ \frac{d}{dt} \bar{p}_k = \kappa(g) \cdot Y . \] This model, specified in 
absolute values rather than ratios as in Goodwin (1967), is shown in Equation (12); see Keen (2009b) 
for full details.

representation of the stability of such a matrix, known as its "dominant eigenvalue", is 
necessarily greater than zero (See http://en.wikipedia.org/wiki/Perron%E2%80%93Frobenius_theorem).
An inconvenient truth for economics derived from this mathematical fact is that any operation that depends 
for its stability on both a matrix $A$ and its inverse $A^{-1}$ will necessarily be unstable—because $A^{-1}$ will have a 
dominant eigenvalue that is the scalar inverse of $A$’s. Since $A$’s dominant eigenvalue is greater than zero, either 
A or its inverse will therefore have a dominant eigenvalue that is greater than 1. The dominant eigenvalue 
determines whether a dynamic process driven by a matrix is stable or unstable, and a value greater than 1 
guarantees instability whether the process described is carried out in either continuous or discrete time. 
This is a dilemma for dynamic equilibrium input-output models, because if the matrix $A$ describes the quantity 
dynamics of the system, then its inverse describes the price dynamics. Consequently either prices or quantities 
must be unstable. A detailed heuristic explanation of this problem is given in Blatt, J. M. 1983, Dynamic 
This is not a problem for a dynamic disequilibrium input-output model however, in which all processes 
necessarily take time—including price setting. From a systems dynamics perspective, neoclassical "general 
equilibrium" input-output models presume that the constant in price setting is zero. This does not mean that 
prices adjust in an instant, but that they adjust with "infinite" speed—and necessarily overshoot their target of 
"equilibrium".

Page 20
\[ \frac{d}{dt} y = g \cdot y \]
\[ \frac{d}{dt} w = P_c(\lambda) \cdot w \]
\[ \frac{d}{dt} D = I(\pi_r) \cdot Y - \Pi + P_k \]
\[ \frac{d}{dt} P_k = \kappa(g) \cdot Y \]
\[ \frac{d}{dt} a = \alpha \cdot a \]
\[ \frac{d}{dt} N = \beta \cdot N \]

(12)

This monetary multi-sectoral implementation of this extended Goodwin model essentially reproduces the Circuit model of money creation and Goodwin’s production model within each sector, while also adding in the intersectoral purchases that are now necessary to enable production, and allowing for multi-sectoral consumption as well. The monetary component of the model is constructed by a dramatically expanded version of the financial table in Figure 17, where each column reproduces the structure of Figure 17, and rows for investment and inter-sectoral input-output purchases occur that are proportional to labour input (Figure 20 shows a 2 sectoral version, while the graphics show the results of a simulation with a 4 sectoral model). The physical components include a lagged price setting equation for each sector identical to Equation (10), and the following three equations for investment, output and employment:

\[ \frac{d}{dt} K_s = \frac{F_{DS}}{\tau_s(\pi_s)} \cdot P_k - nK_s \]
\[ \frac{d}{dt} Q_s = -\frac{1}{\tau_{QS}} \left( Q_s - \frac{K_s}{V_s} \right) \]
\[ \frac{d}{dt} L_s = -\frac{1}{\tau_{LS}} \left( L_s - \frac{Q_s}{a_s} \right) \]

(13)

The first equation specifies that the rate of change of capital stock reflects the flow of funds for investment divided by the current price for capital minus depreciation; the second states that output is a lagged function of the capital stock; and the third portrays employment as a lagged function of output. Each sector is also divided into two identical halves, so that intra-sectoral purchases can also be shown as being made via monetary exchanges between bank accounts.

Net profit in this multi-sectoral model is now a more complicated (and more realistic) entity: it is the revenue from sales for each sector minus wages, intermediate goods purchases, and interest payments on debt:

\[ \Pi_s = P_s \cdot Q_s - W \cdot L_s - \sum_{i \in S} \left( \sigma_{si} \cdot W \cdot L_s \right) - \left( r_l \cdot F_{LS} - r_d \cdot F_{DS} \right) \]

(14)
Here $\sigma_2$ represents the purchases of the output of sector $i$ by sector $s$, which are proportional to labour employed in sector $s$.

Being necessarily monetary, and having nonlinear investment functions based on the rate of profit, as well as nonlinear money circulation functions in place of the constant parameters of the previous model, this model incorporates the essential of Minsky’s Financial Instability Hypothesis as well as Goodwin’s growth cycle model. True to Minsky’s guidance that, since capitalism itself suffers from financially-driven booms and slumps that include Depressions, “it is necessary to have an economic theory which makes great depressions one of the possible states in which our type of capitalist economy can find itself”, this model can generate both sustained cycles and, with suitable initial conditions, financially-driven Depressions. The simulation shown below however has sustained cycles only, because the degree of Ponzi behaviour in this simulation is mild, given the parameters used for the behavioural relations.

![Matrix of Equations]

\[ \begin{array}{c|cccccccc} \text{Type} & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 \\ \hline \text{Name} & \text{K1} & \text{K2} & \text{K3} & \text{K4} & \text{K5} & \text{K6} & \text{K7} & \text{K8} & \text{K9} \\ \text{Symbol} & \text{FLK1} & \text{FLK2} & \text{FLK3} & \text{FLK4} & \text{FLK5} & \text{FLK6} & \text{FLK7} & \text{FLK8} & \text{FLK9} \\ \text{Composing} & 0 & A_1 & A_2 & A_3 & A_4 & 0 & 0 & 0 & 0 \\ \text{Day Int} & 0 & 0 & 0 & 0 & 0 & B_1 & B_2 & B_3 & B_4 \\ \text{Wages} & 0 & 0 & 0 & 0 & 0 & C_1 & C_2 & C_3 & C_4 \\ \text{Consumption} & 0 & 0 & 0 & 0 & 0 & D_1 & D_2 & D_3 & D_4 \\ \text{Int W} & 0 & 0 & 0 & 0 & 0 & E_1 & E_2 & E_3 & E_4 \\ \text{Pay Int} & 0 & 0 & 0 & 0 & 0 & F_1 & F_2 & F_3 & F_4 \\ \text{Recycle Reserves} & 0 & 0 & 0 & 0 & 0 & G_1 & G_2 & G_3 & G_4 \\ \text{Repay Loans} & 0 & 0 & 0 & 0 & 0 & H_1 & H_2 & H_3 & H_4 \\ \text{New Money} & 0 & 0 & 0 & 0 & 0 & I_1 & I_2 & I_3 & I_4 \\ \end{array} \]

**Figure 20**

The model generates both growth and cycles, as illustrated in the next few Figures:
Solving the Paradox of Monetary Profits

The Rate of Profit in a Monetary Multisectoral Model of Production

Real Rate of Economic Growth

Figure 21

Figure 22
Solving the Paradox of Monetary Profits

Debt and Growth Dynamics

Rate of Growth
Change in Debt Ratio (RHS)

Distribution of National Income

Wages Share
Profit & Interest Share and Rate of Growth

Wages
Profit (RHS)
Interest (RHS)
Rate of Growth (RHS)
Solving the Paradox of Monetary Profits

Conclusion

The multi-sectoral model above is preliminary only, and much checking of its structure is needed to validate that it is logically consistent, let alone calibrate it to empirical data. However it appears that this model renders in a multi-sectoral framework the basic insights of the single-sectoral model outlined earlier in this paper (that industrial capitalists can and do make a monetary profit in a pure credit economy) while adding the inherently cyclical nature of an economy driven by struggles over the distribution of income, and the prospect for financial instability caused by a monetary system that endogenously finances the degree of speculation as well as investment that capitalists wish to undertake.

The insights from the preceding single sectoral model are, on the other hand, unequivocal. The results of that model are derived simply from avoiding the problem so wittily expressed by Kalecki, that economics is "the science of confusing stocks with flows". With that confusion removed by working in a framework that is explicitly based on recording the flows between bank accounts and the production and consumption they drive, it is obvious that Circuit Theory achieves what it set out to do, to provide a strictly monetary rendition of the Marx-Schumpeter-Keynes-Minsky tradition in economics. As an explicitly monetary model, it provides an excellent foundation for explaining the processes that led to the Global Financial Crisis.
References


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