RESPONSE TO REFEREE

The referee has raised several important questions. They concern a justification for the criterion function and the stochastic processes that I used. My response draws upon: Wendell H. Fleming and Jerome L. Stein (F/S), Stochastic Optimal Control, international finance and debt, Journal of Banking and Finance, 28 (5) May 2004, 979-96.

Criterion or Value Function

Greenspan appropriately stresses that the function of financial markets is to allocate saving to investment to maximize the growth of the economy. However, risk should also be considered. I approached this by using as the criterion, select the debt or leverage to:

(i) \( \text{Max } E \ln X(T)/X(0) \)

Maximize “growth” the logarithm of net worth at some later date. The logarithm builds in risk aversion.

Alternatively, in F/S we used as a criterion function equation (ii), the maximization of the expected discounted value of a concave function of consumption \( C(t) \) over an infinite horizon. The risk aversion is \( (1-\gamma) > 0 \) and \( \gamma \) can take on very negative values, a lot of risk aversion. The discount rate is \( \delta > 0 \).

(ii) \( \text{Max } E \int (1/\gamma)C(t)^{\gamma}e^{-\delta t} dt \quad 0 < t < \infty, \gamma \neq 0. \)

One difficulty with (ii) is that risk aversion will differ between borrowers and lenders. The borrower will probably have low risk aversion and the lender will probably have a higher one. Alternatively, any debt can be considered optimal if a sufficiently low risk \( \gamma \) aversion is selected. Therefore specifying \( \gamma \neq 0 \) is of dubious value.

If \( \gamma = 0 \), then equation (ii) becomes (iii) the logarithmic function. If consumption is proportional to net worth, then (iii) is like (i).

(iii) \( \text{Max } E \int \ln C(t)e^{-\delta t} dt \quad 0 < t < \infty, \)

The social interest is to maximize expected growth where risk is built in through concavity. Greenspan certainly could accept (i). That is why (i) is much more acceptable as a criterion that (ii) with an arbitrary risk aversion.

It turns out, that the optimal debt ratio in F/S BOX 2/equation (12) is (iv). My point is that risk aversion only affects \( (1-\gamma)\sigma^2 \) a multiple of the appropriate variance in the equation for the optimal debt ratio \( f^* \). Therefore one gains a questionable amount by using a criterion function (ii) compared to the logarithmic.

(iv) \( f^* = (b-r)/(1-\gamma)\sigma^2 + f(0) \) F/S BOX 2/equation (12)
Response to referee

**Stochastic processes**

The next question raised by the referee is the choice of stochastic process. This is very important. The variable of interest was primarily the drift of the asset price. I considered a few models. In Model I, the asset price has a drift and the deviation is ergodic mean reversion. In Model Ia, there was not ergodic mean reversion. In the housing price bubble, the market assumed that the appreciation of the asset was greater than the interest rate. That led to the “free lunch” syndrome: borrow at the mortgage rate of interest of about 5%, spend the money on consumption and refinance/repay the loan from the higher value of the house. This is precisely what happened and it was unsustainable and disastrous.

I therefore constrained the drift in Model I/Ia to preclude the free lunch.

The market model II was indeed naïve and extrapolated the unsustainable drift. That is precisely what produced the bubble and its subsequent collapse. I contrast sensible Model I/Ia with the market behavior “socially irrational” Model II.

The next question is whether there are alternative sensible stochastic processes and what are their implications for the optimal debt? The referee is most justified in raising the question. To a large extent, I discussed this issue in my previous paper A Tale of Two Debt Crises, Economics/Open Access, vol. 4 January 3, 2010, BOX 1. There I consider three stochastic processes on (a) the capital gain plus productivity of capital (b) the interest rate. In then show in BOX 2 how the optimal debt ratio varies in each case.

The answer is that the qualitative structure is similar, but the precise form is different. Hence when I do the empirical work, I tried to do it in a way that is consistent with the alternative formulations. That is, the normalized “excess debt” that I derived is consistent with the alternative formulations.

In the revision of the current paper, I shall be more explicit concerning the issues raise by the referee.