

Yes, we should discount the far-distant future at its lowest possible rate: A resolution of the Weitzman-Gollier puzzle

Response to Referees 1 & 2

I thank Referees 1 and 2 very much for their thoughtful and constructive comments. I welcome this opportunity to briefly explain further the contribution of my paper and then reply in more detail to three specific points that the referees raise.

My motivation for writing this paper was that I was unsure why Criterion 1 should hold in Gollier (2004): “Different investment projects should be ranked according to their expected net future value.” Most standard corporate finance textbooks strongly advocate NPV as the preferred capital budgeting technique and therefore it was not clear to me why social planners should choose something different. The primary purpose of this paper is to show that, within a Weitzman-Gollier setting, the ENFV criterion, $D_T - pe^{r_c(T)T} > 0$, is potentially inconsistent with expected utility maximisation and therefore should not be used by the social planner. This is important as recent work in the area has concluded that Weitzman and Gollier are both right (or in one case, that Gollier is to be preferred), while I conclude that Gollier (2004) is “wrong” and Weitzman (1998) is “right” within the pure exchange economy of my paper.

1 Risk aversion vs. risk neutrality

The central critique of Referee 1 is:

“This model and the one in Gollier (2009) are completely equivalent in one special case, i.e., when the information at date 0 completely eliminates the uncertainty. Gollier (2009) shows that introducing a concave utility function (which means solely aversion to consumption fluctuations over time in that context, as in this paper) fully reconciles the two approaches. *Gollier’s conclusion is absolutely incompatible with the result of Theorem 1 in this paper.* The author should explain the source of the difference between the two papers. Again, contrary to the author’s claim, this is not because he assumes that the decision maker is risk neutral.”

My reading of Gollier’s paper is that his concave utility function does not solely mean aversion to consumption fluctuations over time. As he says on page 4 “In this paper, we reconcile the two approaches, and we link them to the Ramsey rule. To do this, we introduce risk aversion ... In their (Hepburn & Groom) conclusion they recognize that introducing risk aversion into the picture would provide a road to solve the puzzle. This is exactly what is done in this paper”. As a consequence, although all uncertainty is resolved in my model at time 0, (“As interest rates are non-stochastic after time 0...” last paragraph, page 6 of my paper. See also footnote 2) the two papers are not completely equivalent. The use of Epstein-Zin utility functions in my paper

to generate curvature in the utility function without introducing risk aversion is the central methodological contribution that I make to this debate. This is important as the initial Weitzman paper is set in a risk-neutral economy. As the third anonymous referee to the discussion paper version of Gollier (2009) asks “Would these results hold in a more general model where the representative agent smooths consumption over time, but is not necessarily risk averse at any given point in time? The Weitzman/Gollier paradox was raised in a risk-free setting, and answering this question would give us more traction on whether it is optimality or risk aversion or both that is required to resolve it.” My paper provides the solution to this question.

Where I agree with the referee is that my central conclusions would still be valid within a standard discounted expected utility model where the curvature of the utility function captures both risk aversion and the intertemporal marginal rate of substitution of consumption. That is, my results hold *despite* the fact that the agent is risk neutral rather than *because* of this risk neutrality. Because of this, there are a number of similarities between my own paper and Gollier (2009) and it is not clear to me where any incompatibility lies. I expand on this below.

2 The assumption that c_0 is non-stochastic

Referee 1 makes a very interesting observation about the reliance of my results on the assumption that c_0 is non-stochastic at time 0. If c_0 is stochastic at time $-\delta$ then the generalized form of equation (13) in the paper is:

$$E_{-\delta}[\Delta u] = -pE_{-\delta}[u'(c_0)] + D_T \left\{ E_{-\delta}[u'(c_0)]E_{-\delta}[e^{-rT}] + Cov_{-\delta} [u'(c_0), e^{-\tilde{r}T}] \right\}$$

Therefore this project increases expected utility if and only if:

$$D_T \left\{ e^{-r_a T} + \frac{Cov_{-\delta} [u'(c_0), e^{-\tilde{r}T}]}{E_{-\delta}[u'(c_0)]} \right\} - p > 0 \tag{1}$$

Gollier (2009) uses a risk-neutral expectation operator and standard expected utility in his equations (9)–(10) to derive a similar expression. In both his paper and mine, c_0 must be independent of \tilde{r} to establish that the ENPV criterion is consistent with expected utility maximisation. Gollier and I achieve this in different ways. He takes a highly stylised production economy where (i) that there exists a risk-free production asset, (ii) there is constant marginal productivity of capital and (iii) there are no transactions costs from investing in production. Then, in lemma 1, he proves that if the coefficient of risk aversion is equal to 1, c_0 is independent of r and therefore, in proposition 2, that the ENPV criterion is correct even if consumption is allowed to adjust to the interest rate.

I have preferred to work in the standard consumption-based asset pricing framework, where the interest rate adjusts to consumption, rather than vice versa. If aggregate consumption is constrained to equal aggregate income, and forthcoming aggregate income

is known while future income is not, then this endogenises the interest rate uncertainty while ensuring that c_0 is non-stochastic at time $-\delta$. In my opinion, this strengthens this stream of literature because it has never been clear to my *why* the interest rate is unknown in the Weitzman/Gollier economy nor what will determine its eventual resolution.

What is clear from this is that Proposition 2 of Gollier (2009) and my Theorem 1 are similar. We both derive situations under which the ENPV criterion is correct, but use different stylised, but well established, theoretical frameworks to get there.

The main focus of my paper, though, is on the ENFV criterion. Gollier (2009) does not consider situations under which this is the appropriate evaluation criterion. I argue that it should never be used, which is why Gollier (2004) is “wrong”, and this is my main result. To see this, from equation (11) of my paper:

$$e^{\rho T} E_{-\delta}[\Delta u] = D_T E_{-\delta}[u'(c_T)] - p \left\{ E_{-\delta}[u'(\bar{c}_T)] E_{-\delta}[e^{rT}] + Cov_{-\delta} [u'(\bar{c}_T), e^{\tilde{r}T}] \right\}$$

and now the expected utility is positive if and only if:

$$D_T - p \left\{ e^{rcT} + \frac{Cov_{-\delta} [u'(\bar{c}_T), e^{\tilde{r}T}]}{E_{-\delta}[u'(\bar{c}_T)]} \right\} > 0 \quad (2)$$

This is now similar to equations (11)–(12) in Gollier (2009). In my pure exchange economy, due to the standard Euler equation, $e^{-rT} = e^{-\rho T} u'(\bar{c}_T)/u'(c_0)$, if c_0 is fixed then \bar{c}_T is positively correlated with r . This, again, is a standard finance result that higher interest rates result from high expected consumption growth, which underlies the risk-free rate puzzle of Mehra and Prescott (1985) and Weil (1989). In this case, it is clear from equation (2) that the ENFV criterion is always “too harsh” and therefore should not be used.

Referee 1, though, asks an important question. Could we construct an economic framework where \bar{c}_T is non-stochastic at time $-\delta$, which would then lead to the ENFV criterion being correct. I believe that this is not possible for a number of reasons. First, if \bar{c}_T is fixed then c_0 must be negatively correlated with r . This means that, the higher r , the more the investor is saving (c_0 decreases) and the higher the interest that is being received on that investment. Therefore, this will lead to higher future financial wealth and, *ceteris paribus*, higher future consumption. This contradicts the assumption that \bar{c}_T is fixed. I can conceive that, by modelling current wealth in a way that is suitably negatively correlated with the interest rate, this decreased wealth effect might be constructed to exactly offset the increase in saving and interest rates with higher r . Even if this were possible mathematically and credible economically, other issues would still arise. For example, if we wish to apply the ENFV criterion generically then \bar{c}_T must be known at time $-\delta$ for all horizons, $T > 0$. This would fully reveal at time $-\delta$ the forward rate (the term structure of which is flat) that would emerge at time 0. But as interest rates are non-stochastic after time 0, this would also reveal the risk-free rate at time $-\delta$ as this must equal the forward rate. This removes

all uncertainty at time $-\delta$. Finally, it does seem economically realistic to believe that we can perfectly identify aggregate consumption in 200 years' time, but not tomorrow.

Given this discussion, in the crudest terms, Gollier (2004) is “wrong”, while Weitzman (1998) is right only when c_0 and \tilde{r} are independent, which is guaranteed to hold in a pure exchange economy (as I show) or given certain conditions on the utility function and assumptions about production (Gollier). Ultimately, the correlation between c_0 and \tilde{r} is an empirical question that neither paper addresses.

3 Contribution of Jacquier et al. (2005)

I am disappointed that I have not effectively communicated to either referee the importance of Jacquier et al. (2003, 2005 — henceforth JKM) for this debate. I believe this is crucial for understanding why social planners cannot effectively use the ENFV criterion. Gollier (2004) provides a simple numerical example to illustrate his theoretical result.¹ Here the social planner is choosing between two projects. In the first, the rate of return is unknown. A group of “optimistic” advisors tell her that the return will be 5%, while a group of “pessimistic” advisors say it will be 0%. After 1000 years, the ENFV is 7.7×10^{20} if a social planner puts equal weight on each expert’s advice. This is much greater than can be achieved from a project with a sure rate of return of 2.5% as $1.025^{1000} = 5.3 \times 10^{10}$. Therefore, “We conclude that, if the representative agent is risk neutral, it is better not to invest in the project with a sure 2.5% rate of return.” (Gollier 2004, p.86).

JKM, following an established stream of literature in the theory of investment, would argue that the ENFV measure is not the most appropriate way to compare future expected investment values. They would say, let r denote the true rate of return from the first project, which is currently unknown. Then the true future value of each \$1 invested will be e^{rT} . The social planners’ objective should be to try to construct a forecast of future value that is as close to e^{rT} as possible given the limited information available at the time.²

In this case, one way of proceeding would be to ask N experts to give estimates, r_i , $i \in \{1, \dots, N\}$ of r . We assume that each expert is unbiased, so $r_i = r + e_i$ where $E[e_i] = 0$ and e_i has bounded variance. We will also assume that e_i has the same probability density function for all i and that the experts are independent. Following the numerical example, Gollier’s approach is to estimate the ENFV as $N^{-1} \sum_{i=1}^N e^{r_i T}$. However, JKM argue that, instead, the social planner should average the estimates first, $\bar{r} = N^{-1} \sum_{i=1}^N r_i$. Assuming that N is sufficiently large, we can invoke the Central Limit Theorem (given the other assumptions) and approximate $\bar{r} \sim N(r, \sigma^2)$, for some variance σ^2 that reflects N and the dispersion of e_i . Now, JKM contend that

¹Gollier actually compares the [0%,5%] case against 1% with certainty.

²They use unbiasedness and minimum expected least squared errors as alternative definitions of “as close as possible”. I concentrate here on unbiasedness.

the best estimate of the future value of the uncertain project is:

$$\text{Adjusted } ENFV = e^{\bar{r}T - 0.5T^2\sigma^2}$$

This is because the Adjusted ENFV is an unbiased measure of the true realized future value.

$$E \left[e^{\bar{r}T - 0.5T^2\sigma^2} \right] = e^{rT}$$

By the usual Jensen's inequality effect it is clear that:

$$\text{Adjusted } ENFV = e^{\bar{r}T - 0.5T^2\sigma^2} < e^{\bar{r}T} < \frac{1}{N} \sum_{i=1}^N e^{r_i T} = ENFV$$

and thus it is clear that the ENFV is an upward biased measure of the true future payoff available from an asset with an uncertain rate of return.

Referee 1 correctly observes that the JKM argument is not placed in the economic framework of the Weitzman-Gollier puzzle — it focuses on equity investment. JKM assume that stock returns are drawn from an i.i.d normal distribution $N(\mu, s^2)$ and where estimates of $\mu, \hat{\mu}$ are based on the arithmetic average of observed historic returns rather than as an average of experts' forecasts. Further, as the underlying distribution is normal, there is no need to invoke the Central Limit Theorem and s^2 is easily identified. This detail, though, is not important for the intuitive story. One reason why social planners cannot use ENFV is because it is an upward biased measure of future value.

I personally believe the Ang and Liu (2004) framework also adds understanding in two areas (i) by comparing it with the JKM framework above, it shows the asymmetric effect that uncertainty has on discounting and compounding. The ENPV measure correctly captures the effects of uncertainty on discounting, while the ENFV measure does not for compounding and (ii) it shows that even a risk neutral social planner has to be careful about discounting stochastic cash flows at the risk-free rate, which is a somewhat counter-intuitive result. This is relevant as the payoff from the ENFV strategy is unknown at time $-\delta$.