

Yes, We should Discount the Far-Distant Future at Its Lowest Possible Rate:  
A Resolution of the Weitzman-Gollier Puzzle  
E-Economics MS 346

Suppose that the constant interest rate  $r$  that will prevail in the future is constant but uncertain. Weitzman (1998) proposes that one takes in that case the term-decreasing discount rate  $r_d(t)$  that is defined as

$$e^{-r_d(t)t} = E [e^{-rt}] .$$

He suggested that this solution is based on the assumption that stakeholders are risk neutral, therefore they implement any investment with a positive expected net present value. However, Gollier (2004) explained that this proposition is not theoretically founded. To illustrate, he claimed that it is equally justified in that case to implement any project with a positive net future value, which would be equivalent to discount future cash flows at the term-increasing discount rate  $r_c(t)$  defined as

$$e^{r_c(t)t} = E [e^{rt}] .$$

This paper provides three very different analyses to determine which of these two opposite proposals is right.

The more promising analysis is the one proposed in section 3.1. In this section, the author assumes that the decision maker is risk neutral, but is averse to consumption fluctuations over time. Then, as in Gollier (2009), he takes into account of the fact that consumption is optimized over time, so that there is a relationship between the expected growth of consumption  $u'(c_t)/u'(c_0)$  and the interest rate  $r$ . This yields Theorem 1, in which the superiority of the Weitzman solution is claimed. I have several problems with this section:

- It is not clear at all how the uncertainty is resolved over time in this model, in spite of the fact that it is a crucial question.
- The author assume that the exogeneous source of uncertainty comes from the growth rate of consumption in a Lucas economy, and that the uncertainty about the equilibrium interest rate is derived endogenously from it. This is an obscure alternative presentation to the existing models in the literature on this question. Of course, this does not change anything. But it can be a source of difficulty for the readership. In particular, it forces the author to express complex conditions in footnote 2 to guarantee that the short term rate is constant between dates 0 and  $T$ .
- The author disentangle risk aversion and time preferences by considering a recursive model à la Kreps-Porteus (or Epstein-Zin). It is not clear why the author uses this alternative approach, and how it affects the results. I believe that going back to the standard discounted expected utility model would yield the same results.

- This model and the one in Gollier (2009) are completely equivalent in one special case, i.e., when the information at date 0 completely eliminates the uncertainty. Gollier (2009) shows that introducing a concave utility function (which means solely aversion to consumption fluctuations over time in that context, as in this paper) fully reconciles the two approaches. *Gollier's conclusion is absolutely incompatible with the result of Theorem 1 in this paper.* The author should explain the source of the difference between the two papers. Again, contrary to the author's claim, this is not because he assumes that the decision maker is risk neutral.
- I believe that the crucial difference comes from the assumption that  $c_0$  is safe, which is not the case in Gollier (2009). I would like the author to think about the alternative assumption in which it would be  $\bar{c}_T$  that would be safe. I guess that it will imply that Gollier's  $r_c$  solution would be efficient in that case. Anyway, I don't believe that any of these two assumptions is compatible with the assumption that the interest rate is constant ex post. As is well-known in macroeconomics and consumption theory, a change in interest rate does affect optimal saving and consumption, which means that  $c_0$  is a function of  $r$ .

I don't like the other two models presented respectively in sections 3.2 and 4. These models are completely disconnected from each other. The one in section 3.2 requires reading the underlying paper by Ang and Liu, because the assumptions of Ang and Liu's paper are not made explicit in this paper. Only the result is presented. This is frustrating. The model presented in section 3.3 discusses a third model in which people have different beliefs about future interest rates. This assumption is completely disconnected from the standard assumption made in the context of the Weitzman-Gollier puzzle.

The bottom line is that the model in section 3.1 has some potential, but many details remain to be made more explicit. The other two models should be removed.