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Now or Never: Environmental Protection under Hyperbolic Discounting

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Abstract

The author analyzes the optimal investment in environmental protection in a model of an infinite series of non-overlapping hyperbolically discounting agents. He shows that without a commitment mechanism society is eventually stuck in a situation where all agents prefer further investment in the long run, yet neither present nor future agents will actually ever invest. Such an outcome is not only unsatisfactory for each generation but may also be inefficient in a Pareto sense. The author's results are consistent with real world observations, and thus provide a new explanation for weak environmental policy performance.

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1 Introduction

A ubiquitous feature in environmental economics is that welfare costs and benefits of projects undertaken to mitigate environmental problems spread over decades or even centuries (e.g., global climate change, biodiversity loss, depletion of the ozone layer and disposal of radioactive waste). The problem with the standard exponential discounting approach, first introduced by Samuelson (1937) and put on an axiomatic basis by Debreu (1954) and Koopmans (1960), is that outcomes in the far distant future are worth close to nothing for any positive discount rate. In many people's view this is not the way we *do* think or *should* think about the far distant future. Therefore, discounting has been a controversial topic, with proposals ranging from ad-hoc adjustments to alternative axiomatic derivations (e.g., Lind 1982, Rabl 1996, Portney and Weyant 1999, Heal 1998).

One recent approach to deal with the shortcomings of exponential discounting is *hyperbolic discounting*, i.e., the discount rate is not constant but declining over time. It has been advocated for three reasons (for an overview see Pearce et al. 2003, Groom et al. 2005). First, empirical evidence suggests that decision makers use declining rather than constant discount rates (e.g., Gintis 2000, Frederick et al. 2002). Second, uncertainty over the future state of the world leads to declining certainty-equivalent discount rates (e.g., Weitzman 1998, Azfar 1999, Gollier 2002). Third, declining discount rates are consistent with a rule, which balances the welfare of current and future generations (e.g., Chichilnisky 1996, Li and Löfgren 2000).

In this paper, we analyze the optimal investment in environmental protection, given a hyperbolically discounting society, which consists of a series of non-overlapping generations, each represented by a unique agent. To capture the common pattern of many environmental problems, we assume that the present generation faces the costs of investment, while the benefits spread over all subsequent generations.

It is well known from the literature that hyperbolic discounting bears the problem of *time-inconsistency*. As Strotz (1956) has pointed out, this implies that an ex ante optimal decision is not carried out, because a later re-evaluation suggests that it is not optimal anymore. Although the time-inconsistency property of hyperbolic discounting has been used to model 'irrational' behavior, such as addiction and procrastination (e.g., Ackerlof 1991, O'Donoghue and Rabin 1999, Brocas and Carrillo 2001, Gruber and Koszegi 2001), there is a debate on how serious is the problem of time-inconsistency in long-term and intergenerational decision making (e.g., Henderson and Bateman 1995, Heal 1998, Pearce et al. 2003). In fact, if declining discount rates stem from uncertainty over future states of the world there is no issue of time-inconsistency if plans are updated as soon as better information is availbale. If, however, declining discount rates are due to declining pure rates of time preference, either because of imperfect altruism towards future generations or due to ethical considerations balancing the welfare of present or future generations, time-inconsistency has to be taken seriously.

Assuming that declining discount rates stem from declining pure rates of time preference, we distinguish three different behavioral patterns of agents. *Naive* agents do not recognize that their preferences are non-stationary. Thus, they do not anticipate that subsequent agents will not stick to their ex ante optimal plan. If agents are aware of the time-inconsistency problem, they are either *committed* if the first agent can commit all subsequent agents to her ex ante optimal plan, or are called *sophisticated* if no commitment mechanism is availbale. Then, time-consistent planning is equivalent to a non-cooperative sequential investment game all agents play against each other (Phelps and Pollak 1968). We show that, although it is ex ante optimal to do so, neither naive nor sophisticated agents invest in later periods but not in the first period. Although awareness of the time-inconsistency problem may pose a short run remedy, without a commitment mechanism society is eventually stuck in a situation where all agents prefer further investment in the long run, yet neither present nor future agents will actually ever invest. Such an outcome is unsatisfactory for each generation and may also be inefficient in a Pareto sense. Our results give rise to concern for the performance of long run environmental policy, as they are consistent with real world observations.

The paper is organized as follows. In section 2 the model is introduced. The ex ante optimal plan is analyzed in section 3, while section 4 is devoted to the ex post implemented plan. In section 5, we examine some of our model assumptions and discuss implications for environmental policy. Section 6 concludes.

2 A simple model of environmental protection

Consider the following situation: Society can invest in a project that is aimed to decrease the impact of the society's economic activity on the natural environment. We call this project *environmental protection*. Environmental protection in period t, k_t, is assumed to be a capital good, i.e., investments i_t in different periods t accumulate over time. As we focus on long-run environmental problems, we consider environmental protection to be long lasting, and thus abstract from depreciation. Then, the equation of motion for environmental protection is given by:

$$k_{t+1} = k_t + i_t \ . \tag{1}$$

For given k_0 this implies that $k_t = k_0 + \sum_{\tau=0}^{t} i_\tau$. In addition, we assume that environmental protection is non-negative and bounded, i.e., $k_t \in \left[0, \bar{k}\right]$, $\forall t \geq 0$. Investments i_t in environmental protection are assumed to be sunk, i.e., de-investment is not possible and, thus, $i_t \geq 0$ holds for all periods t.

Society's payoff of environmental protection in period t is given by $P(i_t, k_t)$, which is a strictly concave function in both arguments (partial derivatives are indicated by subscripts throughout the paper: $P_{ii} < 0$, $P_{kk} < 0$, $P_{ii}P_{kk} - P_{ik}^2 > 0$). A central assumption in this model is that utility costs and benefits of investments in environmental protection do not accrue at the same time. We assume that P is strictly decreasing in i_t ($P_i < 0$) and strictly increasing in k_t ($P_k > 0$). This implies that costs (investments in environmental protection i_t) occur before the benefits (stock of environmental protection k_t), as investments today accumulate the stock of the next period.[1] In addition, we assume that the marginal costs of investment are non-decreasing with the level of environmental protection, i.e., there are no economies of scale in environmental protection ($P_{ik} \leq 0$). This amounts to the assumption that cheap options to enhance environmental quality are chosen first and, thus, marginal costs increase with the level of environmental protection.

In each period t, there is a decision maker, in the following called *agent* t, who is in charge of the investment decision i_t in environmental protection in period t. Each agent t cares about current and future payoffs, but treats past decisions as bygone. All agents are supposed to exhibit Markov beliefs, i.e., their decisions depend only on the payoff-relevant state variable (environmental protection k_t) and not on the history of past decisions. Moreover, all agents are symmetric with respect to intertemporal preferences. Thus, agent t's present value of all future discounted payoffs W_t equals

$$W_t = \sum_{\tau=0}^{\infty} \delta_\tau P\left(i_{t+\tau}, k_{t+\tau}\right) \ , \tag{2}$$

[1] Examples include the disposal of nuclear waste and the abatement of CO_2 to slow down the anthropogenic greenhouse effect. While the costs occur today, the benefits spread over several decades or even centuries.

where $\delta_\tau > 0$ denotes the discount factor in period $t+\tau$, which is the product of all per-period discount factors σ_ν for $\nu \leq \tau$

$$\delta_\tau = \prod_{\nu=0}^{\tau} \sigma_\nu \ . \tag{3}$$

Without loss of generality we normalize $\delta_0 = \sigma_0$ to unity. Following Karp (2007) and Karp and Fujii (2008) we further assume that the per-period discount factors σ_ν become constant after a finite time T. That is, $\sigma_\nu = \beta,\ \forall \nu > T$, and thus $\delta_\tau = \delta_T \beta^{\tau - T},\ \forall \tau \geq T$. We further concentrate on the case of declining discount rates, which corresponds to (weakly) increasing per-period discount factors. Therefore, we impose $\beta > \sigma_{\nu+1} \geq \sigma_\nu,\ 0 < \nu < T$. We achieve the standard exponential discounting for $T = 0$, i.e., $\delta_\tau = \beta^\tau$. Quasi-hyperbolic discounting (e.g., Laibson 1997, Laibson 1998, Harris and Laibson 2001) corresponds to $T = 1$ and $\sigma_1 = \alpha\beta,\ 0 < \alpha < 1$.

In the following, we analyze the optimal investment in environmental protection given that agents are committed, naive or sophisticated. In particular, we derive conditions under which all agents never invest in environmental protection although all agents prefer investments in environmental protection in the long run.

3 Ex ante optimal investment plan

First, we derive the ex ante optimal investment plan. This is the plan agent t achieves by maximizing intertemporal utility, assuming that all future investment decisions will be carried out according to this plan. Note that the ex ante optimization problem is non-stationary if $T \geq 1$. This implies that an investment rule which only depends on the stock of environmental protection, $i_t = \phi(k_t)$, does not exist. As a consequence, we cannot derive the ex ante optimal plan via a dynamic programming approach.

The ex ante optimal control problem of agent t is given by:[2]

$$\max_{\{i_\tau\}_{\tau=t}^{\infty}} \sum_{\tau=0}^{\infty} \delta_\tau P(i_{t+\tau}, k_{t+\tau}) \tag{4}$$

subject to (1), $i_\tau \geq 0,\ \forall \tau \geq t$ and given k_t. As we do not assume Inada conditions to hold

[2]We assume that the optimization problem is well defined, i.e., the infinite sum does not diverge. Formally, this requires that in the long run P does not grow faster than the discount factors decline: $\lim_{t\to\infty} P(i_t, k_t) \leq A\beta^t$ with some constant $A > 0$.

for the payoff function P, we explicitly consider corner solutions, i.e., $i_\tau = 0,\ \forall\tau \in [t,\infty)$. Introducing the shadow price p^k_τ for the stock of environmental protection and a Kuhn-Tucker variable p^i_τ to control for the non-negativity of investment, we obtain the following Lagrangian $\mathcal{L}$:

$$\mathcal{L} = \sum_{\tau=0}^{\infty} \delta_\tau P(i_{t+\tau}, k_{t+\tau}) + \sum_{\tau=0}^{\infty} p^k_{\tau+1} \left[k_{t+\tau} + i_{t+\tau} - k_{t+1+\tau}\right] + \sum_{\tau=0}^{\infty} p^i_\tau i_{t+\tau} \ . \tag{5}$$

Hence, the first order conditions for an optimal intertemporal investment plan read:

$$p^k_{\tau+1} = -\delta_\tau P_i(i_{t+\tau}, k_{t+\tau}) - p^i_\tau \ , \tag{6a}$$

$$p^k_{\tau+1} = p^k_\tau - \delta_\tau P_k(i_{t+\tau}, k_{t+\tau}) \ , \tag{6b}$$

$$p^i_\tau \geq 0 \ , \quad p^i_\tau i_{t+\tau} = 0 \ . \tag{6c}$$

Because of the strict concavity of the Lagrangian (strictly concave objective function and linear restrictions), these necessary conditions are also sufficient if, in addition, the following transversality condition holds:

$$\lim_{\tau\to\infty} p^k_\tau k_{t+\tau} = 0 \ . \tag{7}$$

The strict concavity of the Lagrangian $\mathcal{L}$ also ensures the uniqueness of the optimal investment path $\{i_{t+\tau}\}^{\infty}_{\tau=0}$.

Equation (6b) is a difference equation, which can be solved unambiguously by taking into account the transversality condition (7):

$$p^k_\tau = \sum_{\nu=0}^{\infty} \delta_{\tau+\nu} P_k(i_{\tau+\nu}, k_{\tau+\nu}) \ . \tag{8}$$

Thus, along the optimal investment path the shadow price of environmental protection equals the present value of the accumulated future utility gains of an additional marginal unit of environmental protection. Inserting the shadow price of environmental protection (8) into equation (6a), we obtain the following necessary and sufficient condition for an ex ante optimal plan:

$$-\delta_\tau P_i(i_{t+\tau}, k_{t+\tau}) - p^i_\tau = \sum_{\nu=1}^{\infty} \delta_{\tau+\nu} P_k(i_{t+\tau+\nu}, k_{t+\tau+\nu}) \ , \qquad \forall\, \tau \geq 0 \ . \tag{9}$$

Equation (9) states that if positive investment in environmental protection is optimal in period $t+\tau$ (i.e., $p^i_\tau = 0$), agent t invests to such an extent that the marginal utility loss at time $t+\tau$ due to the investment in environmental protection (left hand side) equals the present value of the future marginal utility gains of this investment (right hand side). However, if this condition cannot be met for any positive investment $i_{t+\tau}$, then no investment in environmental protection is optimal (i.e., $p^i_\tau \geq 0$). The following proposition characterizes the ex ante optimal investment plan:

Proposition 1 (Ex ante optimal investment plan)
For the ex ante optimal investment plan of agent t the following statements hold:

1. *Optimal investment equals zero for all periods $\tau \geq 0$ if and only if*

$$-\frac{P_i(0,k_t)}{P_k(0,k_t)} \geq \frac{\beta}{1-\beta} \ . \tag{10}$$

2. *Optimal investment in period $\tau = 0$ equals zero if*

$$-\frac{P_i(0,k_t)}{P_k(0,k_t)} \geq \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\sum_{\nu=1}^{T} \delta_{\nu-1}\left(\sigma_\nu - \beta\right) \ . \tag{11}$$

3. *The unique steady state of environmental protection, $k^\star$, is given by*

   a) $k^\star = k_t$ if $i_{t+\tau} = 0, \ \forall \tau \geq 0$,

   b) the unique solution of the implicit equation

$$-\frac{P_i(0,k^\star)}{P_k(0,k^\star)} = \frac{\beta}{1-\beta} \ , \tag{12}$$

   if $i_{t+\tau} > 0$ for some $\tau \geq 0$.

The proof of Proposition 1 is given in the appendix.

The first part of Proposition 1 says that for investment to be optimal from an ex ante point of view only the long-run discount factor β matters. As $\beta > \sigma_\tau, \ \forall \tau \leq T$, the benefits of the first marginal investment into environmental protection in period $t+\tau$ increase with τ until $\tau = T$ and stay constant thereafter. Thus, if there is any investment along the ex ante optimal plan, that is condition (10) is violated, then investment in period $t+T$ is strictly positive but investment in former periods may be zero. In fact, the second part gives a sufficient condition for which investment in the first period $\tau = 0$ is not optimal. The condition is only sufficient but not necessary as investment in environmental protection in later periods decrease the marginal benefits of the investment in the first period. The third part of Proposition 1 establishes the existence of a unique steady state of environmental protection $k^\star$ of the ex ante optimal plan, which is equal to the initial stock k_t if no investment is optimal, and implicitly given by equation (12) otherwise.

Proposition 1 also implies that it may be optimal from an ex ante perspective to invest in environmental protection in the long run but not in the short run. In the following, we will say agent t *postpones* investment if $i_t = 0$ but $i_{t+\tau} > 0$ for some $\tau > 0$. The following corollary gives sufficient conditions for postponing investment to be ex ante optimal.

Corollary 1
It is ex ante optimal for agent t to postpone investment in environmental protection if

$$\frac{\beta}{1-\beta} > -\frac{P_i(0,k_t)}{P_k(0,k_t)} \geq \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\sum_{\nu=1}^{T} \delta_{\nu-1}\left(\sigma_\nu - \beta\right) \ . \qquad (13)$$

The proof of Corollary 1 follows directly from Proposition 1.

Corollary 1 says that postponing investment is ex ante optimal if condition (11) holds but condition (10) is violated. A main insight of Corollary 1 is that postponing investment can be ex ante optimal only for hyperbolically discounting agents. If $T = 0$ the last term of the RHS of condition (13) vanishes, and thus the condition cannot hold.[3]

4 Ex post implemented investment

For $T \geq 1$ the ex ante optimal plan of agent t is, in general, not ex ante optimal for future agents because of the non-stationarity of their preferences.[4] Therefore, future agents may not stick to the ex ante optimal plan of their predecessors. As a consequence, the investment that is actually implemented ex post may differ from the ex ante optimal

[3]Note, however, that postponing investment can also be optimal for exponentially discounting agents within other model frameworks. In particular, this is the case if future outcomes are risky (see, for example, the option value framework developed by Dixit 1992 and Dixit and Pindyck 1994).

[4]This is the case if investment is ex ante optimal, i.e., condition (10) is violated.

plan, depending on the behavioral pattern assumed. In the following, we analyze the ex post implemented investment plans under commitment, and for naive and sophisticated agents. We show that, if condition (13) holds, i.e., investment is ex ante optimal in the long run but not in the short run, only committed agents will actually invest in environmental protection.

4.1 Commitment

If the first agent, agent zero, has the power to enforce her ex ante optimal plan, she would certainly do so, as this is, by definition, the unique plan which maximizes her intertemporal utility. According to the analysis of the former section, investment in environmental protection is ex ante optimal, if condition (10) is violated. Obviously, in the case of such a commitment, the ex post implemented plan equals to the ex ante optimal plan. The following proposition summarizes this insight.

Proposition 2 (Commitment)
If all subsequent agents are committed to the ex ante optimal plan of agent zero, the following statements hold:

1. *There is no investment in environmental protection if and only if*

$$-\frac{P_i(0,k_0)}{P_k(0,k_0)} \geq \frac{\beta}{1-\beta} \ . \tag{14}$$

2. *The unique steady state of environmental protection, $k^\star$, is given by*

   a) $k^\star = k_0$ *if* $i_t = 0, \ \forall t \geq 0$,

   b) *the unique solution of the implicit equation*

$$-\frac{P_i(0,k^\star)}{P_k(0,k^\star)} = \frac{\beta}{1-\beta} \ , \tag{15}$$

   if $i_t > 0$ *for some* $t \geq 0$.

The proof of Proposition 2 follows directly from Proposition 1.

If no commitment mechanism is available, the ex post outcome depends on the agents' awareness of the time-inconsistency problem. Following the standard approach, we distinguish two different behavioral patterns.

4.2 Naive agents

Agents are naive, if they are not aware that the ex ante optimal plans of subsequent agents may differ from their own ex ante optimal plan. As a consequence, the naive agent invests in environmental protection if and only if it is ex ante optimal to invest in the first period. If, however, agent t does not invest because investment is not ex ante optimal in period t, then all subsequent agents do not invest either. This holds as their ex ante optimal plans are identical to the ex ante optimal plan of agent t because $k_{t+1} = k_t$ if $i_t = 0$. The following proposition elaborates on the ex post implemented investment in case of naive agents.

Proposition 3 (Naive agents)
If agents are naive and cannot be bound to the ex ante optimal plan of agent zero, the following statements hold:

1. *Agent t invests in environmental protection if and only if it is ex ante optimal to invest in period t.*
2. *A sufficient condition for optimal ex ante investment in period t to equal zero is given by condition (11).*
3. *For the unique steady state of environmental protection, k^n, the following conditions hold:*
   a) $k^n = k_0$ *if* $i_t = 0,\ \forall t \geq 0,$
   b) $k^n < \bar{k}^n$ *where* $\bar{k}^n$ *is given by the solution of the implicit equation*

$$-\frac{P_i\left(0, \bar{k}^n\right)}{P_k\left(0, \bar{k}^n\right)} = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\sum_{\nu=1}^{T} \delta_{\nu-1}\left(\sigma_\nu - \beta\right) \ , \tag{16}$$

if $i_t > 0$ *for some* $t \geq 0$.

The proof of Proposition 3 is given in the appendix.

Proposition 3 implies that naive agents never invest in environmental protection if agent zero does not invest in environmental protection. In particular, this holds if condition (13) holds. In this case it is ex ante optimal for agent zero not to invest herself but for future agents to invest. As naive agents do not forsee that future agents do not stick to their ex ante optimal plan, there is no investment at all in environmental protection over the infinite time horizon although all agents prefer investment in the long run.

4.3 Sophisticated agents

In contrast to naive agents, sophisticated agents anticipate future agents' deviations from their ex ante optimal plan. As agents can only influence future agents' decisions by influencing the stock of environmental protection, time consistent planning of all agents can be viewed as playing a non-cooperative sequential game. We seek symmetric Markov perfect equilibria, i.e., stationary investment rules only depending on the stock of environmental protection, $i_t = \phi(k_t)$, which are mutually best responses for all agents. Thus, an equilibrium investment rule satisfies

$$\phi(k_t) = \arg\max_{i_t} \left[P(i_t, k_t) + \sum_{\tau=1}^{\infty} P\left(\phi(k_{t+\tau}), k_{t+\tau}\right) \right] \ , \tag{17}$$

subject to equation (1) and $\phi(k_t) \geq 0, \ \forall t \geq 0$.

In order to apply a dynamic programming approach we rewrite the intertemporal utility (2) of agent t to yield (Karp and Fujii 2008):

$$\begin{aligned} W_t &= P\left(i_t, k_t\right) + \sum_{\tau=1}^{\infty} \delta_\tau P\left(i_{t+\tau}, k_{t+\tau}\right) \\ &= P\left(i_t, k_t\right) + \sum_{\tau=1}^{T} \delta_\tau P\left(i_{t+\tau}, k_{t+\tau}\right) + \sum_{\tau=T+1}^{\infty} \beta\delta_{\tau-1} P\left(i_{t+\tau}, k_{t+\tau}\right) \\ &= P\left(i_t, k_t\right) + \sum_{\tau=1}^{T} \left(\delta_\tau - \beta\delta_{\tau-1}\right) P\left(i_{t+\tau}, k_{t+\tau}\right) + \beta \sum_{\tau=0}^{\infty} \delta_\tau P\left(i_{t+1+\tau}, k_{t+1+\tau}\right) \ . \end{aligned} \tag{18}$$

Then, we can write agent t's optimization problem recursively by introducing the value function V:

$$V(k_t) = \max_{i_t} \left[P\left(i_t, k_t\right) + \sum_{\tau=1}^{T} \left(\delta_\tau - \beta\delta_{\tau-1}\right) P\left(\phi(k_{t+\tau}), k_{t+\tau}\right) + \beta V\left(k_{t+1}\right) \right] \ . \tag{19}$$

Assuming a differentiable equilibrium investment rule, we obtain the following Euler equation (as shown in the appendix):

$$\begin{aligned}-P_i\left(\phi(k_t),k_t\right) \;\geq\; & \sum_{\tau=1}^{T}\left(\delta_\tau-\beta\delta_{\tau-1}\right)\left[P_i\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\phi'(k_{t+\tau})+P_k\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\right] \\ & +\beta\left[P_k\left(\phi(k_{t+1}),k_{t+1}\right)-P_i\left(\phi(k_{t+1}),k_{t+1}\right)\right]\ , \qquad (20)\end{aligned}$$

where the inequality corresponds to the corner solution $\phi(k_0)=0$. In general, the equilibrium investment rule and the corresponding steady state are not unique (see Karp 2005, Karp 2007 and Karp and Fujii 2008). The reason is that the equation of motion for the stock of environmental protection (1) and the Euler equation (20) constitute an underdetermined system of equations for the unknowns k_t, $\phi(k_t)$ and $\phi'(k_t)$. However, the following proposition establishes that under certain conditions the unique equilibrium is that all agents do not invest.

Proposition 4 (Sophisticated agents)
If agents are sophisticated and cannot be bound to the ex ante optimal plan of agent zero, the following statements hold:

1. *No investment in all periods $t\geq 0$ is the unique equilibrium if and only if*

$$-\frac{P_i(0,k_0)}{P_k(0,k_0)}\geq\frac{\beta}{1-\beta}+\frac{1}{1-\beta}\sum_{\nu=1}^{T}\delta_{\nu-1}\left(\sigma_\nu-\beta\right)\ . \qquad (21)$$

2. *For the steady state(s) of environmental protection, k^s, the following condition holds:*

    a) $k^s=k_0$ *if* $i_t=0$ *for all* $t\geq 0$.

    b) $k^s\leq\bar{k}^s$ *where* $\bar{k}^s$ *is given by the solution of the implicit equation*

$$-\frac{P_i\left(0,\bar{k}^s\right)}{P_k\left(0,\bar{k}^s\right)}=\frac{\beta}{1-\beta}+\frac{1}{1-\beta}\sum_{\nu=1}^{T}\delta_{\nu-1}\left(\sigma_\nu-\beta\right)\ , \qquad (22)$$

if $i_t>0$ *for some* $t\geq 0$.

The proof of Proposition 4 is given in the appendix.

The first part of Proposition 4 says that no agent will invest in environmental protection if condition (21) holds. Note that if condition (21) holds the equilibrium is only unique with respect to the outcome $i_t = 0$, $k_t = k_0$, $\forall t \geq 0$. In fact, all investment rules $\phi(k_t)$ with $\phi(k_0) = 0$ and $\phi'(k_0) \leq 0$ are equilibrium investment rules. Moreover, no investment in all periods can still be an equilibrium if condition (21) is violated, but it is not the unique equilibrium.

For a stable steady state to exist, $\phi'(k^s) \leq 0$ has to hold. Thus, at least in a neighborhood around a stable steady state investment in environmental protection of subsequent agents are strategic substitutes. That is, the more agent t invests the less agent $t+1$ will invest. As $\phi'(k^s)$ is not determined, there exists an interval of stable steady states $k^s = \left[k_0, \bar{k}^s\right]$ if condition (21) is violated. The maximal steady state is given by equation (22) which corresponds to $\phi'(k^s) = 0$ (i.e., investment decisions of agents are independent of the stock at the steady state).

4.4 Comparison

Using Propositions 2–4, we now compare the ex post outcomes of committed, naive and sophisticated agents. First, we compare the investment decisions between the three behavioral regimes.

Corollary 2 (Investment comparison: now or never)
For both naive and sophisticated agents the following statement holds:

$$i_0 = 0 \qquad \Rightarrow \qquad i_t = 0 \;, \quad \forall\, t > 0 \;. \tag{23}$$

The proof of Corollary 2 follows directly from Propositions 3 and 4.

Corollary 2 says neither naive nor sophisticated agents postpone investment in environmental protection, i.e., no investment in period $t = 0$ but positive investment in some later period $t > 0$. The reason is straightforward. Naive agents always stick to their ex ante optimal plan. If investment is not optimal in period t the ex ante optimal plan of agent $t+1$ is identical to the ex ante optimal plan of agent t, as $k_{t+1} = k_t$. Sophisticated agents invest according to an investment rule $\phi(k_t)$. If $\phi(k_t) = 0$ then $\phi(k_{t+1}) = 0$, as $k_{t+1} = k_t$. Thus, postponing investment can only occur if agent zero can commit future agents to her ex ante optimal plan. In particular, if condition (13) of Corollary 1 holds, there will be no investment in environmental protection in all periods, no matter whether agents are sophisticated or naive, although all agents would prefer investment in the long run.

This does not imply, however, that investment decisions of naive and sophisticated agents are identical. On the contrary, if, for example, condition (21) is just violated[5] then there exist an equilibrium investment rule with $\phi(k_0) > 0$ while it is still not ex ante optimal to invest in $t = 0$ and, therefore, naive agents will never invest. The intuition is that the benefits of investments today decrease with investments in future periods. If condition (21) is just violated this also holds for condition (14). Accordingly, it is ex ante optimal for agent zero to invest at least in the long run, which implies $i_T > 0$. Thus, as naive agents wrongly believe that agents in the future will invest according to their ex ante optimal plan, investment seems less beneficial for them compared to sophisticated agents who correctly anticipate future agents' deviations from their ex ante optimal plan.

Second, the following corollary compares the steady state for the different behavioral regimes.

Corollary 3 (Steady state comparison)
For the steady states of committed, naive and sophisticates agents the following relationships hold:

1. $k^\star = \bar{k}^s = k^n = k_0$ *if condition (14) holds,*
2. $k^\star > \bar{k}^s = k^n = k_0$ *if condition (13) holds,*
3. *and* $k^\star > \bar{k}^s > k^n \geq k_0$ *if condition (21) is violated.*

The proof of Corollary 3 follows directly from Propositions 2–4.

The first part of Corollary 3 captures the case that no investment in environmental protection is ex ante optimal. In this case also naive and sophisticated agents will never invest in environmental protection. The second part says that if condition (13) holds, it is ex ante optimal to postpone investment, but there will be no investment in case of naive or sophisticated agents. The third part is the general case, for which investment is ex ante optimal and also sophisticated agents invest. As already outlined, this does not necessarily imply that naive agents also invest. Moreover, the maximal achievable steady state for sophisticated agents $\bar{k}^s$ exceeds the steady state in case of naive agents k^n. Thus, awareness of the time-inconsistency problem may overcome the no investment outcome of naive agents if sophisticated agents can coordinate on the equilibrium investment rule leading to $\bar{k}^s$.

[5] More formally: $-\frac{P_i(0,k_0)}{P_k(0,k_0)} = \epsilon + \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\sum_{\nu=1}^{T} \delta_{\nu-1}(\sigma_\nu - \beta)$ for some sufficiently small $\epsilon > 0$.

Nevertheless, even the maximal steady state sophisticated agents can reach falls short of the ex ante optimal steady state level of environmental protection. Thus, both naive and sophisticated agents are eventually stuck in a situation in which it is ex ante optimal for them and all subsequent agents to further invest in environmental protection in the long run, but neither they nor future generations will actually invest. This is not only unsatisfactory for all agents but may also be inefficient in a Pareto sense, as the following proposition states.

Proposition 5 (Efficiency)
For the intertemporal utility of committed, naive and sophisticated agents the following statements hold:

1. *Enforcement of the ex ante optimal plan is always Pareto optimal.*
2. *The ex post implemented investment plans of naive and sophisticated agents may be inefficient in the sense that there exist Pareto superior investment plans.*

The proof of Proposition 5 is given in the appendix.

Obviously the ex ante optimal plan of agent t is Pareto optimal as any deviation from it decreases intertemporal utility of agent t. It is not surprising that the ex post implemented plans of naive and sophisticated agents may be inefficient (although naive agents are not aware of this fact as they always assume their ex ante optimal plan will be carried out), it is rather surprising that they may *not* be inefficient. The reason is that utility cannot be directly transferred between different agents. The only way of utility transfers in this model is via investments in environmental protection. As environmental protection is assumed to be bounded, it is not possible to compensate the utility loss of one agent due to an additional investment by ever increasing investments of future agents (i.e., Ponzi games are ruled out). However, whether a decreasing series of additional investments can constitute a Pareto improvement among all agents depends on the series of per-period discount factors.

5 Discussion

In a simple model of environmental protection we have shown that neither naive nor sophisticated agents postpone investment even if this is ex ante optimal. Thus, if agent zero does not invest, so do all subsequent agents although all agents prefer investment in the long run. Of course, not all environmental problems exhibit the property that postponing investment is ex ante optimal. But even if naive or sophisticated agents invest, society will eventually reach a steady state for which further investment is ex ante optimal, at least in the long run, but neither naive nor sophisticated agents will actually invest. Such an outcome is not only unsatisfactory for each generation, it may also be inefficient in a Pareto sense. In the following we discuss some of our model assumptions and hint at immediate policy implications.

We assumed that the stock of environmental protection does not depreciate. First note that due to continuity all our results also hold for sufficiently small rates of depreciation. However, if depreciation is sufficiently large condition (21) is not sufficient anymore for the no investment equilibrium to be unique for sophisticated agents. The reason is that for positive depreciation investment decisions of subsequent agents are not necessarily strategic complements in a neighborhood of the steady state, as $\phi'(k^s) \leq \gamma$, where $\gamma > 0$ denotes the constant rate of depreciation of the stock of environmental protection. As a consequence, the RHS of a corresponding sufficient condition for no investment to be the unique equilibrium would increase. Then, depending on the depreciation rate γ we might not find a sufficient condition such as condition (13) for which it is ex ante optimal to postpone investment and neither naive nor sophisticated agents invest. However, the core results that neither sophisticated nor naive agents postpone investment, and no commitment to the ex ante optimal plan results in a steady state in which both naive and sophisticated agents would prefer further investment in the long run but no investment is actually carried out, remain untouched.

The payoff function P was assumed to be time-invariant. This neglects problems where doing nothing worsens the environmental problem such that marginal benefits of investments in environmental protection increase over time. Further, it does not capture technological progress which decreases marginal costs of investment over time. Both extensions may lead to investment of future agents although agent zero did not invest and agent zero cannot commit future agents to her ex ante optimal plan. However, even if naive or sophisticated agents do invest in later periods they invest later and less than would be ex ante optimal, again leading to a steady state where further investment would be ex ante optimal for all subsequent agents but is not actually carried out.

As our results are qualitatively robust, this gives rise to severe concern for the performance of long run environmental policy if decision makers exhibit time-inconsistent preferences and cannot easily commit themselves and future decision makers to an ex ante optimal plan. First, we want to emphasize that the unsatisfactory policy performance with respect to some long run environmental problems is consistent with the assumption of declining discount rates of decision makers. As an example think of the problem of nuclear waste disposal.[6] It was obvious from the very beginning of the civilian utilization of nuclear fission for energy generation in the 1950s that there will be non-recycable wastes which are highly radioactive for up to ten thousands of years. Yet, the solution to the disposal problem has been continually postponed and still no long run storage site for radioactive waste exists. Moreover, it was not until the 1970s that nuclear waste disposal became a source of concern and governments commissioned research in this area. Another example is slow progress in stabilizing the emissions of greenhouse gases to prevent, or at least reduce, anthropogenic climate change.[7] The Framework Convention on Climate Change, which was open to signature in Rio de Janeiro in June 1992 and at the UN headquarters thereafter, received the signatures of 186 states. The signatory developed countries agreed as a first step to stabilize their greenhouse gas emissions at their 1990 levels by 2000. Most countries have failed to do so. Similar outcomes can be observed with the subsequent Kyoto protocol which was signed in December 1997. In this treaty the developed countries agreed to reduce their greenhouse gas emissions to 95 % of their 1990 levels by 2008–2012. Some countries which signed the protocol refused to ratify it (e.g., USA and Australia). Also many countries which ratified Kyoto are still far from their promised emission targets. Moreover, the countries which already met (or are likely to meet) their targets have done so more by accident than by deliberate action (Pearce 2003). Both examples fit well with the behavior we would expect if governments make decisions on the basis of hyperbolic discounting (although they might be not aware of it) and the ex ante optimal plan suggests to postpone investment to later periods. Although governments might have intended to act in the future they fail to do so because of the time-inconsistency of their preferences.[8]

Our analysis suggests that awareness of the time-inconsistency problem may be a short run remedy for the no investment outcome. As argued, sophisticated agents may invest

[6]For a more detailed exposition of the nuclear fuel cycle see, for example, Proops (2001) and Wilson (1996).

[7]Although anthropogenic climate change is a *stock* pollutant problem which is likely to exhibit increasing marginal damage, the case is applicable to the results of the model presented as the Framework Convention on Climate Change and the Kyoto protocol only limit *emissions*.

[8]We do not deny, however, that also the global public good problem of mitigating climate change, that is all countries benefit from the abatement efforts of all other countries, plays a crucial role for the weak performance of climate change mitigation.

in environmental protection although it is ex ante optimal to postpone investment and, thus, naive agents never invest. However, sophisticated agents would have to coordinate on the "right" equilibrium and even the maximal reachable steady state falls short of the ex ante optimal levels of environmental protection. As a consequence, also sophisticated agents are eventually stuck in a situation where further investment in the long run is ex ante optimal for all subsequent agents but no one is ever investing.

6 Conclusion

In this paper we have analyzed optimal intertemporal investment in environmental protection for a society consisting of hyperbolically discounting agents. Because of the non-stationarity of hyperbolic preferences, the ex post observed outcome crucially depends on additional behavioral constraints. As prime examples we have discussed the committed, the naive and the sophisticated agents. In the model framework analyzed we have shown that neither naive nor sophisticated agents postpone investment even if this is ex ante optimal. Thus, if agent zero does not invest, so do all subsequent agents although all agents prefer investment in the long run. Such an outcome is not only unsatisfactory for each generation, it may also be inefficient in a Pareto sense.

Our result gives rise to concern as they are consistent with real world observations of unsatisfactory policy performance with respect to long run environmental problem. Awareness of the time-inconsistency problem may act as short-run remedy, yet inevitably results in long run steady states of environmental protection which are below the ex ante optimal level. Obviously, a commitment mechanism would help. Cropper and Laibson (1999), for example, suggest to Pareto improve the outcome by subsidizing the interest rate. Their crucial assumption is that the effect of implemented policies occur with a time-lag, which is in fact a commitment for the next period. However, they only consider quasi-hyperbolic discounting implying that only a commitment mechanism for one period is necessary. In the general setting of our model commitment for T periods would be needed. However, especially in a long-term intergenerational setting, the enforcement power of the present generation is very limited (and also questionable on ethical grounds as this implies a dictatorship of the present over the future generations). Hence, the solution of this problem is open to future research.

Finally, it is worth noting that, although our model was primarily designed to address long-run environmental problems, the results extend to other investment decisions of a long-run and intergenerational nature, such as education, health insurance and pension schemes.

Appendix

Proof of Proposition 1

Ad 1. Assume that it is optimal not to invest in all periods, i.e., $i_{t+\tau} = 0,\ \forall \tau \geq 0$. Then, it follows from (1) that $k_{t+\tau} = k_t,\ \forall \tau \geq 0$. Inserting into the necessary and sufficient condition (9) and recalling that the Kuhn-Tucker parameter $p^i_\tau \geq 0$ if $i_{t+\tau} = 0$ yields

$$-\frac{P_i(0,k_t)}{P_k(0,k_t)} \geq \sum_{\nu=1}^{\infty} \frac{\delta_{\tau+\nu}}{\delta_\tau} = \left\{ \begin{array}{ll} \displaystyle\sum_{\nu=\tau+1}^{T} \frac{\delta_\nu}{\delta_\tau} + \frac{\delta_T}{\delta_\tau} \sum_{\nu=T+1}^{\infty} \beta^{\nu-T} \ , & \tau < T \\ \displaystyle\sum_{\nu=1}^{\infty} \beta^\nu \ = \frac{\beta}{1-\beta} \ , & \tau \geq T \end{array} \right\} . \tag{A.1}$$

This condition has to hold for all τ for the no-investment path $i_{t+\tau} = 0, \forall \tau \geq 0$ to be optimal from an ex ante point of view. Note that the expression for $\tau \geq T$ is larger than the expression for $\tau < T$ as $\sigma_\tau < \beta,\ \forall \tau \leq T$. Thus, the inequality holds for all $\tau \in [0, \infty)$ if it holds for $\tau \geq T$.

Ad 2. By assumption $P_{ik} < 0$ and $P_{kk} < 0$ hold, thus

$$P_k(0,k_t) \geq P_k(i_{t+\tau}, k_{t+\tau}) \ , \qquad \forall\, \tau \geq 0 \ . \tag{A.2}$$

Inserting into the necessary and sufficient condition (9) for $\tau = 0$, we obtain the following condition for $i_t = 0$ to be optimal

$$\begin{aligned} P_i(0,k_t) \ &\geq \ \sum_{\nu=1}^{\infty} \delta_\nu P_k(i_{t+\nu}, k_{t+\nu}) \ \geq \ P_k(0,k_t) \sum_{\nu=1}^{\infty} \delta_\nu \\ &= \ P_k(0,k_t) \left[\sum_{\nu=1}^{T} \delta_\nu + \sum_{\nu=T+1}^{\infty} \delta_\nu \right] \ = \ P_k(0,k_t) \left[\sum_{\nu=1}^{T} \delta_\nu + \delta_T \frac{\beta}{1-\beta} \right] \\ &= \ P_k(0,k_t) \left[\frac{\beta}{1-\beta} + \frac{1}{1-\beta} \sum_{\nu=1}^{T} \delta_\nu (1-\beta) + \frac{\delta_T \beta}{1-\beta} - \frac{\beta}{1-\beta} \right] \\ &= \ P_k(0,k_t) \left[\frac{\beta}{1-\beta} + \frac{1}{1-\beta} \sum_{\nu=1}^{T} \delta_{\nu-1} (\sigma_\nu - \beta) \right] \ . \end{aligned} \tag{A.3}$$

Note that the term in brackets is smaller than $\beta/(1-\beta)$ because the sum is negative as $\beta > \sigma_\tau, \ \forall \tau \geq T$.

Ad 3. The case of no investment is trivial. If investment is ex ante optimal, there exists a $\tau \geq T$ with $i_{t+\tau} > 0$. Thus, the steady state cannot be reached before period $t+\tau$. As there is no depreciation, investment has to equal zero in the steady state. Thus, the steady state stock of environmental protection is given by the solution of equation (12). This solution is unique because due to the assumed curvature properties of P we obtain:

$$\frac{\partial}{\partial k}\left(-\frac{P_i(0,k)}{P_k(0,k)}\right) = \frac{P_{kk}(0,k)P_i(0,k) - P_{ik}(0,k)P_k(0,k)}{P_k(0,k)^2} > 0 \ . \tag{A.4}$$

Thus, the steady state is higher the larger is the long-run per-period discount factor β.

□

Proof of Proposition 3

Ad 1. By definition naive agents believe that all succeeding agents stick to their ex ante optimal plan. Given this believe it is optimal for agent t to follow her ex ante optimal plan. As a consequence, agent t only invests in period t if this is ex ante optimal. Ad 2. A sufficient condition for investment in period t to be ex ante optimal is given by condition (11). Ad 3. The case of no investment is trivial. Suppose investment is positive for some $t \geq 0$. Any level of environemental protection k can only be a steady state if all subsequent agents do not invest, for which a sufficient condition is condition (11).

□

Derivation of the Euler equation

Using the value function (19) the optimization problem of agent t reads

$$\max_{i_t}\left[P(i_t,k_t) + \sum_{\tau=1}^{T}\left(\delta_\tau - \beta\delta_{\tau-1}\right)P(i_{t+\tau},k_{t+\tau}) + \beta V(k_{t+1})\right] \tag{A.5}$$

subject to equation (1) and $0 \geq i_\tau = \phi(k_\tau), \ \forall \tau \geq t$.

First, note that the following conditions hold:

$$\frac{\partial k_{t+\tau}}{\partial k_t} = \frac{\partial k_{t+\tau}}{\partial k_{t+\tau-1}} \cdot \ldots \cdot \frac{\partial k_{t+1}}{\partial k_t} = 1 \ , \quad \frac{\partial k_{t+\tau}}{\partial i_t} = \frac{\partial k_{t+\tau}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial i_t} = 1 \ , \quad \forall \tau > t \ . \tag{A.6}$$

Then, we obtain for the first-order condition

$$\begin{aligned}-P_i\left(\phi(k_t),k_t\right) &= \sum_{\tau=1}^{T}\left(\delta_\tau-\beta\delta_{\tau-1}\right)\left[P_i\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\phi'(k_{t+\tau})\right.\\ &\quad \left.+P_k\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\right]+\beta V'(k_{t+1})\ ,\end{aligned} \tag{A.7}$$

where the inequality sign corresponds to $\phi(k_t)=0$. By the envelope theorem,

$$\begin{aligned}V'(k_t) &= P_k(\phi(k_t),k_t)+\sum_{\tau=1}^{T}\left(\delta_\tau-\beta\delta_{\tau-1}\right)\left[P_i\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\phi'(k_{t+\tau})\right.\\ &\quad \left.+P_k\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\right]+\beta V'(k_{t+1})\ ,\end{aligned} \tag{A.8}$$

Inserting the first-order condition into $V'(k_t)$ yields:

$$V'(k_t)=P_k(\phi(k_t),k_t)-P_i(\phi(k_t),k_t)\ . \tag{A.9}$$

Inserting back into the first-order condition, we obtain the Euler equation

$$\begin{aligned}-P_i\left(\phi(k_t),k_t\right) &= \sum_{\tau=1}^{T}\left(\delta_\tau-\beta\delta_{\tau-1}\right)\left[P_i\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\phi'(k_{t+\tau})+P_k\left(\phi(k_{t+\tau}),k_{t+\tau}\right)\right]\\ &\quad +\ \beta\left[P_k\left(\phi(k_{t+1}),k_{t+1}\right)-P_i\left(\phi(k_{t+1}),k_{t+1}\right)\right]\ .\end{aligned} \tag{A.10}$$

However, it may be that the Euler equation does not hold for non-negative investments $i_t=\phi(k_t)$. In this case, the optimal investment is $i_t=\phi(k_0)=0,\ \forall t\geq 0$ and the following inequality holds

$$\begin{aligned}-P_i(0,k_0t) &\geq \sum_{\tau=1}^{T}(\delta_\tau-\beta\delta_{\tau-1})\left[P_i(0,k_0)\phi'(k_0)+P_k(0,k_0)\right]\\ &\quad +\ \beta\left[P_k(0,k_0)-P_i(0,k_0)\right]\ .\end{aligned} \tag{A.11}$$

Proof of Proposition 4

Ad 1. First, note that a stable steady state requires $\phi'(k^s)\leq 0$. Second, exploding equilibria with an ever increasing stock of environmrntal protection are ruled out by the boundedness of $k\in\left[0,\bar{k}\right]$. Now, we show that if no investment is the unique equilibrium then condition (21) holds and vice versa.

"⇒": Suppose, no investment in all periods is the unique equlibrium. Inserting into the Euler equation and re-arranging terms yields

$$\begin{aligned} -\frac{P_i(0,k_0)}{P_k(0,k_0)} &\geq \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\left(1 + \frac{P_i(0,k_0)\phi'(k_0)}{P_k(0,k_0)}\right)\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta) \qquad \text{(A.12)} \\ &\geq \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta)\ . \end{aligned}$$

"⇐": Suppose that condition (21) holds. Suppose further that there exists a stable steady state with $k^s > k_0$. Inserting into the Euler equation and re-arranging terms yields:

$$\begin{aligned} -\frac{P_i(0,k^s)}{P_k(0,k^s)} &= \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\left(1 + \frac{P_i(0,k^s)\phi'(k^s)}{P_k(0,k^s)}\right)\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta) \qquad \text{(A.13a)} \\ &\leq -\frac{P_i(0,k_0)}{P_k(0,k_0)} + \frac{1}{1-\beta}\frac{P_i(0,k^s)\phi'(k^s)}{P_k(0,k^s)}\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta) \qquad \text{(A.13b)} \\ &< -\frac{P_i(0,k_0)}{P_k(0,k_0)} + \frac{1}{1-\beta}\frac{P_i(0,k^s)\phi'(k^s)}{P_k(0,k^s)}\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta)\ . \qquad \text{(A.13c)} \end{aligned}$$

The '$\leq$' sign in the second line holds due to condition (21), the '$<$' sign in the third line holds due to equation (A.4). This implies that

$$0 < \frac{1}{1-\beta}\frac{P_i(0,k^s)\phi'(k^s)}{P_k(0,k^s)}\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta)\ , \qquad \text{(A.14)}$$

which can only hold for $\phi'(k^s) > 0$ as $P_i < 0$, $P_k > 0$ and the sum is negative because $\sigma_\nu < \beta, \forall \nu \geq T$. However, $\phi'(k^s) > 0$ contradicts the assumption of a stable steady state. As a consequence, the unique equilibrium is given by no investment of all agents.

Ad 2. The case of no investment is trivial. For $i_i > 0$ for some $t \geq 0$ the following condition has to hold in the steady state:

$$\begin{aligned} -\frac{P_i(0,k^s)}{P_k(0,k^s)} &= \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\left(1 + \frac{P_i(0,k^s)\phi'(k^s)}{P_k(0,k^s)}\right)\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta) \qquad \text{(A.15a)} \\ &\leq \frac{\beta}{1-\beta} + \frac{1}{1-\beta}\sum_{\nu=1}^{T}\delta_{\nu-1}(\sigma_\nu - \beta)\ . \qquad \text{(A.15b)} \end{aligned}$$

The inequality holds as $\phi'(k^s) \leq 0$. Thus, the maximal steady state $\bar{K}^s$ corresponds to an equilibrium rule with $\phi'\left(\bar{k}^s\right) = 0$. □

Proof of Proposition 5

Ad 1. A commitment to the ex ante optimal plan is always Pareto optimal, as the ex ante optimal plan is unique, due to the assumed curvature properties. Thus, any deviation from it would decrease the utility of agent zero (otherwise it would not have been optimal in the first place).

Ad 2. To show that the outcome of naive and sophisticated agents may be inefficient, we construct an example, for which we assume that agents discount *quasi-hyperbolically*, that is $T = 1$ with $\sigma_1 = \alpha\beta,\ 0 < \alpha < 1$. We further assume that condition (13) holds, which implies that neither naive nor sophisticated agents will invest although investment is ex ante optimal. Thus, the following condition holds:

$$\frac{\beta}{1-\beta} > -\frac{P_i(0,k_0)}{P_k(0,k_0)} = -\frac{P_i^0}{P_k^0} \geq \frac{\alpha\beta}{1-\beta} \qquad \text{(A.16)}$$

To show that a Pareto improvement for naive and sophisticated agents may be possible, consider the utility effect of marginal investments Δi_0 and Δi_1 of agents zero and 1. To keep the analysis simple we assume that all other agents do not invest. Note that the utility of all other agents increases if agents zero and 1 increase their investments. Then, the net utility effects of the investments Δi_0 and Δi_1 for agents zero and 1 are given by:

$$\Delta W_0 = \Delta i_0 \left(P_i^0 + P_k^0 \frac{\alpha\beta}{1-\beta}\right) + \Delta i_1 \alpha\beta \left(P_i^0 + P_k^0 \frac{\beta}{1-\beta}\right) , \qquad \text{(A.17a)}$$

$$\Delta W_1 = \Delta i_1 \left(P_i^0 + P_k^0 \frac{\alpha\beta}{1-\beta}\right) + \Delta i_0 \left(P_k^0 + P_k^0 \frac{\alpha\beta}{1-\beta}\right) . \qquad \text{(A.17b)}$$

According to condition (A.16), for both equations the first term is negative and the second term is positive. If agent 1 invests to such an amount that her net utility gain is zero, we derive for Δi_1:

$$\Delta i_1 = -\Delta i_0 \frac{P_k^0 + P_k^0 \frac{\alpha\beta}{1-\beta}}{P_i^0 + P_k^0 \frac{\alpha\beta}{1-\beta}} > 0 . \qquad \text{(A.18)}$$

Inserting into ΔW_0 and dividing by Δi_0 yields:

$$\frac{\Delta W_0}{\Delta i_0} = P_i^0 + P_k^0 \frac{\alpha\beta}{1-\beta} - \frac{\alpha\beta \left(P_i^0 + P_k^0 \frac{\beta}{1-\beta}\right)\left(P_k^0 + P_k^0 \frac{\alpha\beta}{1-\beta}\right)}{P_i^0 + P_k^0 \frac{\alpha\beta}{1-\beta}} \,. \tag{A.19}$$

If, for example, $P_i^0 = -12$, $P_k^0 = 1$, $\alpha = 0.5$ and $\beta = 0.95$ then $\Delta W_1 / \Delta i_1 > 0$ and, therefore, a Pareto improvement can be achieved if both agents depart from the no investment equilibrium. □

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